Sovereign Default in a Monetary Union

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Abstract

In the aftermath of the global financial crisis, sovereign default risk and the zero lower bound have limited the ability of policy-makers in the European monetary union to achieve their stabilization objective. This paper investigates the interaction between sovereign default risk and the conduct of monetary policy, when borrowers can act strategically and they share with their lenders a single currency in a monetary union. We address this question in an endogenous sovereign default model of heterogeneous countries in a monetary union, where the monetary authority may be constrained by the zero lower bound. We uncover three main results. First, in normal times, debtors have a stronger incentive to default to induce more expansionary monetary policy. Second, the zero lower bound, or constraints on monetary policy may act as a disciplining device to enforce repayment of sovereign debt. Third, sovereign default risk induces countries with a preference for tight monetary policy to accept a laxer policy stance. These results help to shed light on the recent European experience of high default risk, expansionary monetary policy and low nominal interest rates.

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1 Introduction

In the aftermath of the global financial crisis, two major developments have constrained policy-makers in the European monetary union in reaching their stabilization objective. First, high interest rates on government debt have limited the conduct of fiscal policy in several member countries of the monetary union. In turn, high interest rates reflected the perception of heightened risk of sovereign default. Second, the zero lower bound on nominal interest rates constrained the ability of monetary policy to achieve its objectives via its conventional policy tool. Figure 1 presents data on the fall in nominal, risk-free interest rates and on the rise of interest rates on government debt in the Euro Area.

This paper investigates the interaction between sovereign default risk and the conduct of monetary policy, when borrowers can act strategically and they share a single currency with their lenders in a monetary union. We address this issue in a model of a monetary union with endogenous sovereign default, where the monetary authority may be constrained by the zero lower bound. We ask three related questions. First, how does the stance of monetary policy affect strategic debtors' incentive to default on external liabilities? Second, how do default incentives change in the presence of the zero lower bound? Third, how does default risk affect countries' preferences over the stance of monetary policy? We uncover three main results. First, in normal times, debtors have a stronger incentive to default to induce more expansionary monetary policy. Second, constraints on monetary policy may act a disciplining device to enforce repayment of sovereign debt. Third, sovereign default risk induces countries with a preference for tight monetary policy to accept a laxer policy stance.

Our first result shows that debtors are more likely to default when they understand that they can induce more expansionary monetary policy. This occurs if the monetary authority is not constrained in pursuing his mandate. When countries act strategically, they consider the implications of their default and repayment decisions on the conduct of monetary policy. The monetary authority of the union is bound by its mandate to achieve a price-stability objective. Debtors are aware that default has deflationary effects, to which the monetary authority must react with expansionary measures. In turn, this expansionary monetary policy is beneficial for borrowers, whose default incentive thus strengthens.

Second, we show that constraints on monetary policy can induce repayment of sovereign debt. When the monetary authority is constrained by the zero lower bound, debtor countries cannot exploit their default decision to induce expansionary monetary policy. The presence of the zero lower bound constrains the ability to expand monetary policy, since interest rates cannot fall below zero. Due to this constraint, the monetary authority cannot achieve its price-stability objective. Deflationary pressure ensues, is detrimental to welfare. The deflationary pressure induced by the zero lower bound may be stronger when countries default than when countries repay debt. Hence, constraints on monetary policy may reduce welfare associated with default more than they reduce welfare associated with debt repayment. In particular, the zero lower bound reduces welfare upon default more than upon repayment when the natural nominal interest rate associated with default is lower than the one associated with repayment. Intuitively, the natural nominal interest rate may be lower if default creates a shortage of assets and thus impedes saving by

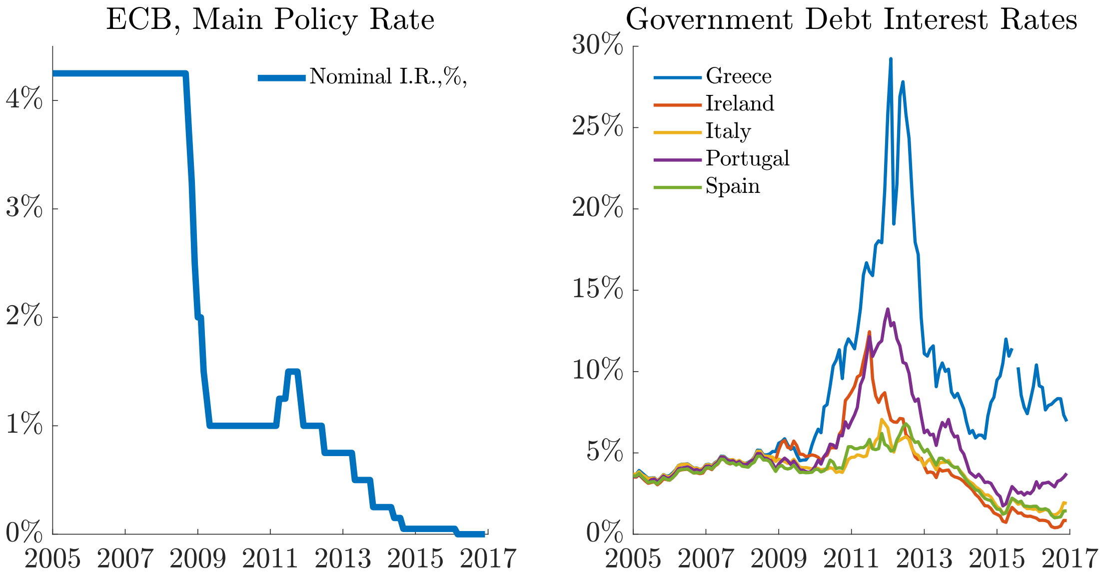

Figure 1: Nominal interest rate in the Euro Area, and interest rates on government debt issued by Greece, Ireland, Italy, Portugal and Spain. The nominal interest rate set by the European Central Bank fell after the global financial crisis, reaching values close to the zero lower bound at the end of 2013. At the same time, interest rates on government debt rose since the beginning of 2010, reflecting a high perceived risk of sovereign default by these countries.

countries with a desire to do so. When this is the case, the presence of constraints on monetary policy strengthens countries' incentive to repay debt. Due to this fact, countries with positive external assets have no incentive to overcome the constraint posed by the zero lower bound, as this constraint acts as an enforcement device of sovereign debt repayment.

Finally, the presence of sovereign default risk may change saver countries' preferences over the conduct of monetary policy. A looser stance of monetary policy increases debtors' incentive to repay debt. Countries who hold external assets benefit from a loosening of monetary policy if this incentivizes repayment by debtors. Hence, countries who might otherwise prefer a tight stance of monetary policy favour a lax monetary policy, if this reduces the losses on their external assets due to default.

To study this issue we develop a model where heterogeneous countries with limited commitment to repay debt form part of a monetary union. The economy lasts two periods, and we plan to extend this framework to an infinite-horizon setting in upcoming work.

The countries in the monetary union differ in terms of their intertemporal income path and, consequently, in terms of preferences over the conduct of monetary policy. In the initial period, one set of countries has low income relative to the future, and therefore it has a desire to borrow against future resources. In addition, these countries inherit a stock of debt from the past. We describe this group of debtor countries as the "Periphery". The complementary group of countries represents the "Core" of the monetary union. These countries have high income relative to the future, and therefore a desire to save. In addition, these countries enter the initial period with a positive level of external assets. These assets represent claims against countries in the "Periphery" of the monetary union.

Monetary policy has real effects due to the presence of a form of nominal rigidity.

Nominal wages are downwardly rigid. If wages are against their lower bound, domestic output is demand-determined and countries experience unemployment. Thus, expansionary monetary policy stimulates domestic demand and output by increasing aggregate prices. However, the monetary policy authority is constrained in its conduct of expansionary policy by the presence of the zero lower bound on nominal interest rates.

We study in this framework how the stance of monetary policy affects default incentives. We consider three different kinds of default decisions, depending on how decision makers internalize the effects of default on nominal variables, domestically and in the union. First, if countries ignore the effect of their default on the severity of nominal rigidities, the default decision is entirely analogous to that of a real model in the tradition of Arellano (2008). Second, countries may understand that default stimulates domestic demand. When this is the case, default reduces unemployment in debtor countries. Hence, the incentive to default strengthens, and default occurs for lower debt levels.

Third, we consider how countries may act strategically in their default decision. The potential of strategic behavior arises when countries understand how their actions condition the monetary authority's behavior in pursue of its price stability objective. In this setting, the presence or absence of constraints on monetary policy has crucial implications for default decisions. Default has deflationary effects, as it reduces demand in lending countries. In normal times, the monetary authority reacts to the deflationary pressure by loosening monetary policy. Debtor countries benefit from the expansionary response of the monetary authority, as this relaxes nominal rigidities. Hence, their incentive to default strengthens further. When constraints on monetary policy bind, this channel is muted, as the monetary authority cannot loosen policy. Hence, the incentive to default weakens, if constraints on monetary policy are more severe under default than they are under repayment.

2 Literature Review

This paper contributes to two strands of the literature on international macroeconomics that studied the sovereign debt crisis in the euro area.

First, we contribute to the literature on endogenous sovereign default, by analyzing the interaction between borrowing and lending countries when they both share the same currency in a monetary union. To this purpose, we develop a model of an economy subject to sovereign default risk in the spirit of Eaton and Gersovitz (1981) and Arellano (2008). We depart from the baseline model by analyzing the impact of debtors' default and borrowing decisions on lenders and on the conduct of monetary policy, in a setting where nominal variables have real effects. Na et al. (2018) also study how nominal rigidities affect a country's optimal default decision. We turn attention instead to how default interacts with the conduct of monetary policy when debtors and creditors form part of a monetary union. In addition, we study the optimal default decision when nominal rigidities co-exist with constraints on the action of monetary policy, such as the zero lower bound. Cole and Kehoe (2000) is a seminal paper dealing with the interaction between debtors and lenders in a default crisis. Their emphasis is on how such interaction can give rise to debt crises due to a lack of confidence in the government's ability to repay. We study instead its implications for the conduct of policy in a monetary union and, in turn, for optimal default incentives.

Second, this paper falls in the strand of research that analyzes monetary unions. We contribute by analyzing how the objectives and constraints that the monetary authority of a union faces determine the optimal default incentives of member countries. In addition, we analyze how default incentives shape the preferences that union residents have over how monetary policy should be conducted. Many papers, among which Eggertsson and Woodford (2003) and Eggertsson and Krugman (2012), have studied how the presence of the zero lower bound imposes a constraint on monetary policy that affects real quantities and welfare. More recently, Benigno and Fornaro (2018), Eggertsson et al. (2016) and Eggertsson and Mehrotra (2014) studied how the presence of the zero lower bound can have a permanent effect on aggregate output. We focus on how the default decision of members of a monetary union differs in the presence of this constraint. Farhi and Werning (2017) analyze optimal risk-sharing across countries in a monetary union, is an incomplete-markets setting without endogenous default risk. Fornaro (2018) analyzes how debt-deleveraging episodes may lead to recessions and to the emergence of a liquidity trap in a monetary union, while Benigno and Romei (2014) address the implications of debt deleveraging and the zero lower bound with flexible exchange rates. Here, we show that the presence of the zero lower bound affects default incentives, and it may act as a force that enforces debt repayment. Corsetti and Dedola (2016) study whether unconventional monetary policy may rule out self-fulfilling debt crisis. Our emphasis is on default episodes that are driven by fundamentals, and on their implications for a monetary authority that aims to achieve a nominal stabilization objective. In the framework in Aguiar et al. (2015) the distribution of debt across member countries of a monetary union determines the probability of rollover debt crises, which are not the driver of default in our setting. In addition, we focus on a setting where debt is denominated in real terms and where nominal rigidities imply that monetary policy may affect output. Their work analyzes a setting where welfare-detrimental inflation may reduce the real value of debt. Previous work in de Ferra (2018) studies how subsidies on asset holdings in a monetary union affect countries' decisions on external saving and borrowing in the absence of nominal rigidities. This paper builds, in part, on the framework developed there. Our results on how countries' incentives to coordinate policies are crucially affected by the presence of the zero lower bound are close related to the findings of Fornaro and Romei (2018).

3 Model

3.1 Environment

The world economy is composed of a unitary-mass continuum of countries. Each country is belongs to one of two groups, Home and Foreign. The two groups are denoted by H and F, respectively. The two groups of countries have equal measure. Within each group, all individual countries are identical and have zero measure. Time is discrete, and the world economy lasts for two periods. Each country is inhabited by a continuum of household of unitary mass, by a continuum of identical firms, and by a government. The government is composed of a national fiscal authority and by a unitary-mass continuum of identical subnational fiscal authorities, or regions. All regions are identical within each country. In addition, the world economy is inhabited by a supranational monetary authority.

All households have identical preferences defined over two goods, tradable and non-tradable. We refer to the two goods as T and N, respectively. Preferences of the representative household in each country are as follows:

$$U_{i,j} = \log\left(c_{T,i,j,1}^{a} c_{N,i,j,1}^{1-a}\right) + \beta \mathbb{E}\left[\log\left(c_{T,i,j,2}^{a} c_{N,i,j,2}^{1-a}\right)\right] \tag{1}$$

where $c_{T,i,j,t}$ and $c_{N,i,j,t}$ denote consumption by the representative household in region j of country i in period $t = 1, 2$ of goods T and N, respectively.[1] $\mathbb{E}$ denotes the mathematical expectation operator conditional on information available in the initial period.[2] Households in all countries supply inelastically their endowment of labor l to firms in the same country.

The two goods differ in terms of their tradability across countries. Good T can be traded internationally at no cost. Conversely, good N cannot be shipped across countries but it can be traded at no cost across regions within a given country. The countries receive endowments of good T in both periods. Countries in H and F differ in terms of the inter-temporal profile of their good-T endowment.[3] H enjoys positive growth, and its endowment is relatively scarce in the initial period, while the reverse is true in F:

$$y_{T,H,1} < y_{T,H,2},$$
$$y_{T,F,1} > y_{T,F,2}.$$

The total endowment of good T in the world economy is constant: $y_{T,H,t} + y_{T,F,t} = y_T$. We denote by $y_{\mathcal{L}}$ and $y_{\mathcal{H}}$ the low and high value of the endowment, respectively, and we assume that the endowment profiles of the two countries are the mirror image of each other:

$$y_{T,H,1} = y_{T,F,2} = y_{\mathcal{L}}$$
$$y_{T,F,1} = y_{T,H,2} = y_{\mathcal{H}}$$

Money is the numéraire of the world economy. The two groups of countries are in a monetary union, hence they share the same currency, or numéraire. The law of one price holds and the price of good T in units of currency is the same in all countries:

$$p_{T,H,t} = p_{T,F,t} = p_{T,t}.$$

Firms in each country have access to a linear technology to produce good N by using

[1]Both i and j lie in the unit interval $I = [0, 1]$.

[2] Aggregate consumption in either country is definded by the Cobb-Douglas aggregator of consumption of the two goods: $c_{i,j,t} = \frac{1}{a^a(1-a)^{1-a}} c_{T,i,j,t}^{a} c_{N,i,j,t}^{1-a}$. The assumption that the inter-temporal elasticity of substitution is equal to the inverse of the intra-temporal elasticity of substitution is convenient to derive several of the analytical results presented below.

[3]This endowment is identical across regions in a given country. Thus, we omit the region subscript for brevity.

labor as an input:[4]

$$y_{N,i} = l_i. \tag{2}$$

3.2 Households

Households in all countries purchase consumption of both types of goods in each period. Their resources are given by the wages they receive in return for the labor $l_{i,t}$ that firms employ. In addition to labor income, they receive an endowment of good T. Households have limited access to international financial markets and, in the initial period, they can only purchase positive amounts of risk-free assets $b_{M,i}$, denominated in units of money. These bonds mature in the terminal period and they pay a nominal interest rate, $\mathfrak{i}$ which is the policy rate of the monetary authority in the union. Finally, households receive in each period a lump-sum transfer in units of good T from the subnational fiscal authority, $s_{i,j,t}$, whose terminal-period realization is stochastic. The budget constraint of the representative household in region j of a generic country i in the initial period is as follows:

$$p_{T,1}c_{T,i,j,1} + p_{N,i,1}c_{N,i,j,1} + \frac{b_{M,i,j}}{1+\mathfrak{i}} = p_{T,1}\left(y_{T,i,1} + s_{i,j,1}\right) + w_{i,1}l_{i,1}, \tag{3}$$

where $p_{N,i,1}$ is the price of good N that prevails in country in i in the initial period. In the terminal period, the budget constraint is given by:

$$p_{T,2}c_{T,i,j,2} + p_{N,i,2}c_{N,i,j,2} = p_{T,2}\left(y_{T,i,2} + s_{i,j,2}\right) + b_{M,i,j} + w_{i,2}l_{i,2}, \tag{4}$$

where the key difference is given by the presence of nominal wealth among the resources available to households. In addition, the terminal-period equilibrium values of some variables are stochastic, and not known to the households in the initial period.

The maximization problem of households in all countries is to maximize their expected lifetime utility subject to the two budget constraints:

$$\begin{aligned} V_{HH,i,j}\left(x, \{y_{T,i,t}\}_{t=1}^{2}, \{s_{i,j,t}\}_{t=1}^{2}\right) = & \max_{b_{M,i,j}, \left\{c_{T,i,j,t}, c_{N,i,j,t}\right\}_{t=1,2}} U_{i,j} \\ & \text{s.t. } b_{M,i,j} \geq 0, \\ & (3), (4). \end{aligned} \tag{5}$$

where we define x_i as the vector of aggregate state variables that are relevant for the problem of the household. This is given by the vector $x = \left[\{y_{T,i,t}\}_{i\in I, t=1,2}, \{s_{i,j,t}\}_{\{i,j\}\in I^2, t=1,2}\right]$ of endowment realizations and transfers in all countries and regions which, given monetary policy, determines in equilibrium the vector of all prices $[\{p_{T,t}\}_{t=1,2}, \{p_{N,i,t}\}_{i\in I, t=1,2}, \{w_{i,t}\}_{i\in I, t=1,2}, \mathfrak{i}\}]$.

The intra-temporal optimality condition associated with the choice between consumption of goods T and N is given in each country by:

$$c_{N,i,j,t} = \frac{1-a}{a}\frac{p_{T,t}}{p_{N,i,t}}c_{T,i,j,t}, \tag{6}$$

[4] The location of firms across regions within a country is irrelevant. Hence, we omit again the region subscript. We assume that firms hire an identical amount of labor from each household in the country.

which implies that the relative amount of good N demanded by households is increasing in the amount of good T they consume, and it is decreasing in the relative price of the good itself.

The inter-temporal optimality condition associated with the purchase of nominal assets is as follows:

$$\frac{1}{c_{T,i,j,1}} = \beta \left(1 + \mathfrak{i}\right) \mathbb{E} \left[\frac{p_{T,1}}{p_{T,2}} \frac{1}{c_{T,i,j,2}} \right] + \mu_{i,j}, \tag{7}$$

where $\mu_{i,j}$ denotes the Lagrange multiplier on the non-negativity constraint for nominal assets.

From the maximization problem of the households we derive the vector-valued policy function $f_{HH,i}$ associated with their problem for the consumption of T and N good and for the purchase of nominal assets:

$$\begin{bmatrix} c_{T,i,j,t} \\ c_{N,i,j,t} \\ b_{M,i,j,t} \end{bmatrix} = f_{HH,i} \left(x, \{s_{i,j,t}\}_{t=1}^{2}, \{y_{T,i,t}\}_{t=1}^{2} \right). \tag{8}$$

3.3 Firms and Nominal Rigidities

Each country is populated by a continuum of atomistic firms that behave competitively and take prices and wages as given. The problem of the representative firm in each country is to maximize profits given by the revenue from the sale of output of good N, net of labor costs:

$$\begin{aligned} &\max_{y_{N,i,t}, l_{i,t}} \; p_{N,i,t} y_{N,i,t} - w_{i,t} l_{i,t} \\ &\text{subject to: } y_{N,i,t} = l_{i,t} \end{aligned} \tag{9}$$

The optimality conditions associated with the problem of the firm imply that the price of good N equals the wage in each country:

$$p_{N,i,t} = w_{i,t}. \tag{10}$$

The presence of a nominal rigidity affects the determination of wages in all countries. In the initial period, the nominal rigidity implies that the wage cannot fall below a given threshold $\kappa_{i,1}$, which may be country-specific:

$$w_{i,1} \geq \kappa_{i,1}. \tag{11}$$

In the terminal period, the wage cannot fall below a given fraction of its initial-period value. Again, we assume that the fraction $\kappa_{i,2}$ may be country-specific:

$$w_{i,2} \geq \kappa_{i,2} w_{i,1}. \tag{12}$$

We assume that, in each country, the nominal rigidity is only a concern when the endowment of good T is relatively scarce. In particular we assume that $\kappa_{i,t} = 0$ if $y_{T,i,t} = y_{\mathfrak{H}}$ and $\kappa_{i,t} = \kappa > 0$ otherwise. Hence, nominal rigidities may only bind in H in the initial period and in F in the terminal one.[5]

3.4 Government

3.4.1 Supranational Monetary Authority

A supranational monetary authority sets policy in order to achieve its objective for nominal variables. In the initial period, the objective of the monetary authority is to achieve a given target for the (geometric) average price of consumption good in all countries:

$$p_1^* = \exp\left(\int_0^1 \psi_i \times \log(p_{i,1})di\right), \tag{13}$$

where the parameter ψ_i denotes the weight assigned to countries i in the monetary authority objective. The price index $p_{i,t}$ of the consumption basket in each country is defined as:

$$p_{i,t} = p_{T,i,t}^{a} \times p_{N,i,t}^{1-a}. \tag{14}$$

In the terminal period, the objective of monetary policy is stated in terms of inflation in the average price of the consumption good:

$$\pi^* = \exp\left(\int_0^1 \psi_i \times \log(\pi_{i,2})di\right), \tag{15}$$

where inflation is intuitively defined as:

$$\pi_{i,2} = \frac{p_{i,2}}{p_{i,1}}. \tag{16}$$

We consider a monetary authority that places identical weights to all countries within each group, H, or F. These weight are denoted by $\psi_F = \psi$ and $\psi_H = 1 - \psi$.

To achieve its target, the monetary authority sets the interest rate $\mathfrak{i}$ on one-period nominal assets that it issues in the initial period. The net supply of nominal assets issued by the monetary authority is equal to zero. Hence, the nominal interest rate has to be consistent with households' optimization, given the target and the zero net supply of nominal assets. The monetary authority faces the zero lower bound as a constraint on its conduct of policy:

$$\mathfrak{i} \geq 0. \tag{17}$$

If the nominal interest rate that the monetary authority needs to set to achieve its target is lower than zero, the monetary authority sets the lowest possible interest rate and it fails to achieve its objective. In the terminal period, the policy of the monetary authority is simply conducted by directly setting the price of good T in the monetary union, $p_{T,2}$.[6]

[5] This assumption helps in deriving analytical results and it may be relaxed.

[6] Alternatively, but equivalently, the monetary authority determines the amount of monetary aggregates in circulation in the terminal period which, given real quantities, determines nominal prices.

3.4.2 Subnational Fiscal Authorities

Each country is inhabited by a unit-mass continuum of identical subnational fiscal authorities, each with jurisdiction over one region. Their main role is to choose how many assets to issue or to buy on international financial markets, and to default or repay external debt. Subnational authorities understand that their choice for debt issuance affects their future default risk, and therefore their borrowing costs. However, subnational authorities do not internalize the effects of their actions on relative goods' prices and wages, as these are determined at the national level.

We analyze the problem of subnational fiscal authorities backwards. First, we analyze their terminal-period default decision. Second, we describe their choice for debt issuance or asset purchases in the initial period. Finally, we study the default decision that the subnational fiscal authorities take in the initial period.

Terminal-period default. The subnational fiscal authorities in a country i enter the terminal period with a pre-determined amount of assets $b_{i,j,2}$. Negative values of assets indicate that the fiscal authority is in debt. Fiscal authorities cannot commit to always repay debt, and they decide whether to repay or to default by comparing the costs and benefits of the two decisions. Default entails an output cost, which takes the form of a reduction in the amount of T-good endowment available for consumption.[7] The cost of default is stochastic and it is denoted by ζ_2. The stochastic process driving the default cost is defined as follows:

$$\zeta_2 = \begin{cases} \hat{\zeta} > 0 & \text{with probability } \omega, \\ 0 & \text{with probability } 1-\omega. \end{cases} \tag{18}$$

With probability ω, the fiscal authorities in all countries face a high default cost, given by the value $\hat{\zeta}$. With complementary probability $1-\omega$ the value of the default cost is instead zero. The realization of the default cost is the same in all countries.

In the terminal period, the fiscal authorities decide whether to default or to repay in order to maximize consumption of households and, therefore, their welfare.[8] This form of default costs implies a simple default policy in the terminal period. Default is optimal if the cost associated with it is smaller than the amount of debt to be repaid. Formally, default is optimal if $-b_{i,j,2} > \zeta_2$. In particular, if the realization of ζ_2 is zero, countries will find it optimal to default for any positive amount of debt. If the default-cost realization is given by $\hat{\zeta}$, countries will never default as long as debt is below $\hat{\zeta}$.[9] The subnational fiscal authority sets the lump-sum transfer $s_{i,j,2}$ according to the default decision it takes. Upon default, the transfer is a tax whose amount equals the default cost, otherwise the transfer is given by the amount of assets held against foreigners:

$$s_{i,j,2} = \max\left\{-\zeta_2, b_{i,j,2}\right\}. \tag{19}$$

[7] In the terminal period, we can ignore the possibility of exclusion from international financial markets, as no borrowing, nor lending would take place in any case.

[8] This problem is presented formally in Appendix B.1.

[9] In equilibrium, it is easy to verify that debt is always below $\hat{\zeta}$.

Initial-Period Asset Trading. Fiscal authorities in all regions of all countries in H enter the initial period with a predetermined and negative amount of assets, $b_{H,1} < 0$, which is denominated in units of good T. Symmetrically, fiscal authorities in F enter the initial period with positive external wealth, $b_{F,1} = -b_{H,1}$. We will also refer to negative assets as debt.

Fiscal authorities in countries in H may or may not have access to international financial markets. Access to markets depends on whether decision-makers in the country repay or default on external debt, as discussed below. If they have access to international financial markets, fiscal authorities can trade bonds, $b_{i,j,2}$ that mature in the terminal period. Countries in F can always trade in international financial markets and, in particular, they can trade bonds with countries in H.[10] These bonds are denominated in units of good T. $r_{i,j}$ denotes the interest rate in units of T-good associated with bonds issued by the fiscal authority j in country i. Each fiscal authority issues debt taking into account how the amount of debt that it issues affects the real interest rate associated with it, according to the function $\mathfrak{r}(b_{i,j,2})$.[11] The fiscal authorities choose the amount of assets to trade and the transfer they rebate to households the resources they obtain from financial markets. The initial-period budget constraint of a fiscal authority is as follows:

$$s_{i,j,1} = b_{i,j,1} - \frac{1}{1+r_{i,j}} b_{i,j,2}. \tag{20}$$

The fiscal authorities choose the amounts of assets to trade and transfers to rebate to households, in order to maximize their welfare, taking into account their own terminal-period default policy, as well as households' decisions according to (8). Formally, they solve the following problem:

$$\begin{aligned} V^R_{SN,i,j}\left(x, \{y_{T,i,t}\}_{t=1}^2\right) = & \max_{\{s_{i,j,t}\}_{t=1}^2, b_{i,j,2}, r_{i,j}} V_{HH,i,j}\left(x, \{y_{T,i,t}\}_{t=1}^2, \{s_{i,j,t}\}_{t=1}^2\right) \\ & \text{s.t. } s_{i,j,1} = b_{i,j,1} - \frac{1}{1+r_{i,j}} b_{i,j,2}, \\ & s_{i,j,2} = \max\{-\zeta_2, b_{i,j,2}\}, \\ & r_{i,j} = \mathfrak{r}(b_{i,j,2}). \end{aligned} \tag{21}$$

The intertemporal optimality condition associated with the choice of $b_{i,j,2}$ can be expressed as:

$$\frac{1}{1+r_{i,j}} \frac{1}{c_{T,i,j,1}} = \beta\omega \frac{1}{c_{T,i,j,2}} \tag{22}$$

where the fiscal authority understands that debt will only be repaid in the terminal period if default costs are high, with probability ω.[12]

[10] Since countries in F enter the initial period with positive external assets, they do not have an incentive to default.

[11] This function is determined in equilibrium from the optimality conditions of risk-averse lenders, as shown in Appendix B.1.

[12] In addition, we impose that the derivative of the real interest rate with respect to assets is equal to zero, as it is the case in equilibrium given the binary distribution of default costs.

Initial-Period Default In the initial period, the subnational fiscal authorities may default on the debt that they inherit from the past. If they decide to do so, they suffer from a default cost, ζ_1 and they are excluded from international financial markets. The default cost ζ_1 is public knowledge at the beginning of the initial period. If they default, fiscal authorities do not repay debt in the initial period but they lose the possibility to trade assets internationally. In addition, in the terminal period, the fiscal authority will suffer from the default cost ζ_2 in all states of the world. We assume that initial-period default costs are low, relative to those in the terminal period:

$$\hat{\zeta} > \zeta_1 + y_{\mathfrak{H}} - y_{\mathfrak{L}}. \tag{23}$$

The subnational fiscal authorities know that, given default in the initial period, welfare of the households is given by the following value function:

$$\begin{aligned} V^{D}_{SN,i,j}\left(x, \{y_{T,i,t}\}_{t=1}^{2}\right) =& V_{HH,i,j}\left(x, \{y_{T,i,t}\}_{t=1}^{2}, \{s_{i,j,t}\}_{t=1}^{2}\right) \\ \text{s.t. } & s_{i,j,1} = -\zeta_1, \\ & s_{i,j,2} = \{\underbrace{-\hat{\zeta}}_{\text{w.p. } \omega}, \underbrace{0}_{\text{w.p. } 1-\omega}\}. \end{aligned} \tag{24}$$

The subnational fiscal authorities take the default decision in the initial period by comparing the value of repaying with the value of defaulting, defined by (21) and (24). Formally, the default choice of subnational fiscal authority is given by:

$$V_{SN,i,j}\left(x, \{y_{T,i,t}\}_{t=1}^{2}\right) = \max_{D_{SN}\{0,1\}} = (1 - D_{SN})V^{R}_{SN,i,j}(\cdot) + D_{SN}V^{D}_{SN,i,j}(\cdot), \tag{25}$$

where D_{SN} is an indicator policy that takes the value of unity in the event of default, and $V_{SN,i,j}$ denotes the value to households when the subnational fiscal authority has the option to default or repay external debt.

3.4.3 National Fiscal Authority

Each country is inhabited by a national fiscal authority, in addition to the subnational fiscal authorities. The role of the national fiscal authority is twofold. First, it can alter the default decision of the subnational fiscal authorities. Second, it may coordinate with fiscal authorities in other countries in doing so.

Initial-Period Default by the National Authority The national fiscal authority can alter the default decision of the subnational fiscal authorities. The key reason why it may decide to do so is to internalize the effects of default on domestic demand and, therefore, on the amount of N good produced by firms subject to nominal rigidities. The national fiscal authority can simultaneously choose default or repayment for all regions in the country. When the national fiscal authority imposes default to all regions, the value to the representative household in country i is given by:

$$\begin{aligned} V^D_{NF,i}\left(x, \{y_{T,i,t}\}^2_{t=1}\right) =& V_{HH,i,j}\left(x, \{y_{T,i,t}\}^2_{t=1}, \{s_{i,j,t}\}_{j,t=1,2}\right) \\ &\text{s.t. } s_{i,j,1} = -\zeta_1 \ \forall j, \\ &\qquad s_{i,j,2} = \{\underbrace{-\hat{\zeta}}_{\text{w.p. } \omega}, \underbrace{0}_{\text{w.p. } 1-\omega}\} \ \forall j. \end{aligned} \tag{26}$$

If the national fiscal authority does not impose default, the debt issuance decision conditional upon repayment is left to the subnational fiscal authorities. Hence, the value for the representative household in country i under repayment is given by:

$$V^R_{NF,i}\left(x, \{y_{T,i,t}\}^2_{t=1}\right) = V^R_{SN,i,j}\left(x, \{y_{T,i,t}\}^2_{t=1}\right) \ \forall j, \tag{27}$$

given that all subnational authorities in i are identical and thus make identical asset-trading decisions.

The national fiscal authority takes its default decision by comparing the value to the representative household in the country of letting every subnational fiscal authority default agains letting them all repay debt:[13]

$$V_{NF,i}\left(x, \{y_{T,i,t}\}^2_{t=1}\right) = \max_{D_{NF}\{0,1\}} (1 - D_{NF}) V^R_{NF,i}(\cdot) + D_{NF} V^D_{NF,i}(\cdot). \tag{28}$$

where, again, D_{NF} denotes the indicator default policy function, and $V_{NF,i}$ denotes the value to households when the national fiscal authority has the option to default or repay external debt.

Initial-Period Default with Coordination across Borrowers The national fiscal authorities of all countries in H can form a coalition and coordinate their default decision.[14] When doing so, they internalize their impact on the instruments that the supranational monetary authority must set in order to achieve its nominal objectives, as described in Section 3.4.1.

When the national fiscal authorities of all countries in H coordinate their repayment

[13] The national fiscal authority cannot impose different default or repayment decisions across the subnational fiscal authorities. This would be discriminatory as different regions would have different welfare. A Rawlsian or max-min social welfare function would prevent such discriminatory behavior.

[14] The analysis of this setting can also be interpreted as the one that would be relevant for a large debtor country which can affect aggregate variables in the union.

decision, their value function is given by:[15]

$$
\begin{aligned}
V^{R}_{\widehat{NF},H}\left(\hat{x}\right) = & V^{R}_{NF,i}\left(x,\{y_{T,i,t}\}_{t=1}^{2}\right)\ \forall i\in H \\
& \text{s.t. } s_{i,1} = b_{i,1} - \frac{1}{1+r_i} b_{i,2}\ \forall i \\
& s_{i,2} = \max\left\{b_{i,2}, -\zeta_2\right\}\ \forall i\in H \\
& s_{i,2} = \{\underbrace{b_{i,2}}_{\text{w.p. }\omega}\quad \underbrace{0}_{\text{w.p. }1-\omega}\}\ \forall i\in F
\end{aligned}
\tag{29}
$$

where all countries understand how their decision to repay impacts the optimal transfers $s_{i,t}$ that are set in the monetary union, and, therefore, aggregate consumption and prices. The coordinated fiscal authorities take as given the aggregate state variable $\hat{x}$ which includes the distribution of external assets in the initial period $\left\{b_{i,1}\right\}_i$ as well as the distribution of endowment profiles across countries. Formally, the state variable is given by: $\hat{x} = [\{y_{T,i,t}\}_{i\in I,t=1,2}, \{b_{i,1}\}_{i\in I}]$. Differently than the individual national fiscal authorities, they understand the impact of their actions on the distribution of transfers in the monetary union.

On the other hand, if all countries in H coordinate and decide to default, their value function is as follows:

$$
\begin{aligned}
V^{D}_{\widehat{NF},H}\left(\hat{x}\right) = & V^{D}_{NF,i}\left(x,\{y_{T,i,t}\}_{t=1}^{2}\right)\ \forall i\in H \\
& \text{s.t. } s_{i,1} = -\zeta_1\ \forall i\in H \\
& s_{i,2} = \{\underbrace{-\hat{\zeta}}_{\text{w.p. }\omega}\quad \underbrace{0}_{\text{w.p. }1-\omega}\}\ \forall i\in H \\
& s_{i,1} = s_{i,2} = 0\ \forall i\in F,
\end{aligned}
\tag{30}
$$

where, again, all countries understand the implications of their default decision on aggregate quantities and prices in the monetary union.

The coordinated national fiscal authorities default decision is consequently defined as follows:

$$
V_{\widehat{NF},H}\left(\hat{x}\right) = \max_{D_{\widehat{NF}}\{0,1\}} (1-D_{\widehat{NF}})V^{R}_{\widehat{NF},H}\left(\hat{x}\right) + D_{\widehat{NF}}V^{D}_{\widehat{NF},H}\left(\hat{x}\right). \tag{31}
$$

$D_{\widehat{NF}}$ denotes the indicator default policy function for the coalition, and $V_{\widehat{NF},i}$ denotes the value to households when the coalition of national fiscal authorities has the option to default or repay external debt.

[15]When coordinating their repayment or default decision, the identical national fiscal authorities either all repay or all default. We abstract from cases where a fraction of countries defaults and the other repays. As when considering choices of regions, a rawlsian social welfare function over countries' welfare would make these choices sub-optimal, given that they would imply heterogeneous welfare across countries in H.

3.5 Market-clearing Conditions

In equilibrium, all markets must clear. In particular, the market for good T must clear within the monetary union in each period, and aggregate endowment of this good net of default costs, where relevant, must equal its aggregate consumption:[16]

$$\int_i c_{T,i,t} di = \int_i y_{T,i,t} - \zeta_t D_{i,t} \; di. \tag{32}$$

The market for good N must clear and consumption of this good equals output within each country:[17]

$$\int_j c_{N,i,j,t} dj = y_{N,i,t}. \tag{33}$$

The market for nominal assets clears, and the amount of assets demanded by all countries equals the zero quantity supplied by the supranational monetary authority:

$$\int_i b_{M,i} di = 0. \tag{34}$$

Note that since private agents cannot issue nominal assets, the condition above implies that in equilibrium, no agent can hold a positive amount of nominal assets: $b_{M,i} = 0 \; \forall i$.

The market for labor clears in each country, subject to the downward wage rigidity. In the initial period:

$$\left(l_{i,1} - l\right)\left(w_{i,1} - \kappa_{i,1}\right) = 0, \tag{35}$$

and, in the terminal period:

$$\left(l_{i,2} - l\right)\left(w_{i,2} - \kappa_{i,2} w_{i,1}\right) = 0. \tag{36}$$

Finally, the market for risky assets must clear and bonds issued by borrowing countries equal bonds purchased by savers:[18]

$$\int_i b_{i,2} d_i = 0. \tag{37}$$

3.6 Equilibrium

Given the distribution of initial assets, $\{b_{i,1}\}_{i \in I}$ and the distribution of endowment of good T $\{y_{i,t}\}_{t=1,2,i\in I}$, an equilibrium is a sequence of quantities, $\{c_{T,i,t}, c_{N,i,t}, b_{M,i}, l_{i,t}, y_{N,i,t}, b_{i,2}, s_{i,t}\}_{t=1,2,\in I}$, prices $\{p_{T,t}, p_{N,i,t}, \mathfrak{i}, w_{i,t}, r_i, p_{i,t}\}_{t=1,2,\in I}$ and default decision, $D_{i,t}$ such that:

- Household optimality conditions (3), (4),(6) and (7) as well as their inelastic labor supply, $l_{i,t} \leq l$ are satisfied.

[16]The variable $\zeta_{i,t}$ takes the value ζ_1 in the initial period and $\hat{\zeta}$ or zero in the terminal period.

[17]Note that, in equilibrium, all $c_{N,i,j,t} = c_{N,i,t}$ since all subnational decision makers are identical. However, (33) clarifies that the market for good N must clear at the country-level.

[18]Note that we can integrate across the bonds traded by the heterogeneous countries since, given the terminal-period default cost process assumed, these are all homogeneous bonds that repay in the terminal period, high-default cost state which realizes with probability ω.

- Firms' labor demand (10) is satisfied.
- The budget constraint (20) and the intertemporal optimality condition (22) of the subnational fiscal authority are satisfied.
- The default decision in the initial period $D_{i,1}$ is given by either $D_{SN,i}$, $D_{NF,i}$, $D_{\widehat{NF},i}$ and it is consistent with either, (25), (28) or (31).
- The default decision in the terminal period is consistent with (99).
- The markets for good T, (32), good N, (33), nominal assets, (34), labor in the two periods, (35) and (36) clear.
- The objectives of the monetary policy, (13) and (15), are satisfied, or the zero lower bound binds, $\mathfrak{i} = 0$.
- The price index is defined by (14).
- The market for risky assets clears, (37), as implied by Walras' law.

Despite its complexity, the model has a simple and intuitive solution. The presence of subnational fiscal authorities and the assumption of unit elasiticity of intratemporal and intertemporal substitution allow us to derive analytical solutions for the equilibrium real interest rate and for the amount of bonds that are traded across countries. The subnational fiscal authorities consider the effect of their actions on the interest rate on debt that they issue, but they have no impact on the relative price of goods N and T. Thus, despite the presence of nominal rigidities, the borrowing and saving decisions in our model are the same as those that would arise in a flexible-prices, one-good model. When initial-period debt is repaid, the subnational fiscal authorities smooth household consumption of good T across the two periods. Specifically, they equalize consumption of good T between the initial period and the terminal-period state of the world where debt is again repaid, meaning $c_{T,i,j,1} = c_{T,i,j,2}$ with probability ω.[19] If all countries in H repay initial-period debt, the equilibrium real interest rate on risky bonds is the same for all countries and it is equal to:

$$r_{i,j} = r = \frac{1}{\beta\omega} - 1. \tag{38}$$

The higher is the terminal-period repayment probability ω, the lower is the interest rate on risky bonds. In the limit where ω tends to one, bonds issued by countries in H are equivalent to real, risk-free bonds denominated in units of good T. The interest rate on these bonds would thus be the risk-free one, $\frac{1-\beta}{\beta}$.

4 Results

We present here the key results of our analysis. First, we characterize debtor countries' optimal default policy, and how this is influenced by the stance of monetary policy. Then,

[19] Note that neither savers nor borrowers can transfer resources to and from the terminal-period state of the world default costs are zero.

we discuss countries' preferences over the stance of monetary policy, and how sovereign default risk affects them.

4.1 Sovereign Default

The stance of monetary policy affects countries' default decisions. Monetary policy crucially determines the severity of nominal rigidities and thus it plays a key role in shaping default incentives. We document in this section the optimal default policy of debtor countries in H. The next subsections describe how optimal default policies differ according to the identity of the key decision-maker who chooses between repayment or default on external debt. First, we study optimal default when subnational authorities take the decision to default or repay. These decision-makers do not internalize how their decisions affect demand at the national level and the severity of nominal rigidities. Second, we discuss how the incentive to stimulate domestic demand makes default more attractive, when the national fiscal authority takes the default decision. Third, we study the possibility for countries to coordinate their default decision, to influence in their favor the conduct of monetary policy. We discuss this last result with special consideration to the effects of the zero lower bound. While coalesced countries are more likely to default than countries acting individually in the absence of constraints on monetary policy, the zero lower bound may lead such coalitions to repay debt instead. This occurs as default would tighten constraints on monetary policy, largely to the expense of debtor countries themselves.

4.1.1 Default by Subnational Fiscal Authorities

The key driver of subnational fiscal authorities' decision to repay or default on external debt is the impact of such decision on the amount of good T available for consumption in the jurisdiction of the authority itself, over the two periods. The stance of monetary policy has no impact on their default decision instead. This is because subnational fiscal authorities take the severity of nominal rigidities as given, and they understand that their choices have no effect on the amount of good N that is produced and available for consumption at the national level.

We define $\bar{b}_{SN}$ as the level of initial-period assets for which the subnational authority is indifferent between default and repayment: $V^R_{SN,H}\left(\bar{b}_{SN}\right) = V^D_{SN,H}$. The following Proposition establishes one key result on this decision-maker's optimal default threshold.[20]

Proposition 4.1 (Default threshold of the subnational fiscal authority.) *At the highest level of debt for which a subnational fiscal authority prefers repayment to default, T-good consumption under repayment equals the geometric average of initial- and terminal-period consumption of good T under default. The default threshold for the subnational fiscal authority $\bar{b}_{SN}$ is given by:*

$$\bar{b}_{SN} = (1+\beta\omega)\left[\left(y_{\mathfrak{L}} - \zeta_1\right)^{\frac{1}{1+\beta\omega}}\left(y_{\mathfrak{H}} - \hat{\zeta}\right)^{\frac{\beta\omega}{1+\beta\omega}}\right] - \left(y_{\mathfrak{L}} + \beta\omega y_{\mathfrak{H}}\right). \tag{39}$$

[20]Appendix B.2 provides further details on this result.

Proposition 4.1 implies that the debt default threshold for the subnational fiscal authority is increasing in the default costs in the two periods, in absolute value. We clarify both analytically and graphically how this threshold is determined.

For a generic subnational authority in a generic country in H, the value associated with default defined in (24) can be expressed as a function of quantities and prices, after imposing the intra-temporal choice of the household across goods T and N, as follows:[21]

$$\begin{aligned} V^D_{SN,H} =& a\left[\log\left(y_{\mathfrak{L}}-\zeta_1\right)+\beta\omega\log\left(y_{\mathfrak{H}}-\hat{\zeta}\right)+\beta\left(1-\omega\right)\log\left(y_{\mathfrak{H}}\right)\right]+ \\ & \left(1-a\right)\left[\log\left(l_{H,1}\right)+\beta\mathbb{E}\log\left(l_{H,2}\right)\right]. \end{aligned} \tag{40}$$

The value associated with repayment defined in (21), after imposing the optimal choice for assets traded and the equilibrium interest rate on risky debt (38), is given by a function of endowments, initial assets and prices and quantities in the N-good sector:

$$\begin{aligned} V^R_{SN,H} =& a\left[\left(1+\beta\omega\right)\log\left(\frac{y_{\mathfrak{L}}+\beta\omega y_{\mathfrak{H}}+b_{H,1}}{1+\beta\omega}\right)+\beta\left(1-\omega\right)\log\left(y_{\mathfrak{H}}\right)\right]+ \\ & \left(1-a\right)\left[\log\left(l_{H,1}\right)+\beta\mathbb{E}\log\left(l_{H,2}\right)\right]. \end{aligned} \tag{41}$$

The subnational authority takes its default decision by comparing the two values of repayment and default above described. The threshold in (39) is the level of assets for which these values are equal to each other. The subnational authority understands that its default decision has no impact on the equilibrium amount of labor employed in the N-good sector. Hence, the subnational authority ignores the contribution of good N to welfare when deciding whether to default or to repay, as it perceives this contribution to be identical across the two decisions.

We now turn to analyze the implications of the subnational authority's default decision for households' consumption profile.

Corollary 4.1.1 (Consumption dynamics under default and repayment.) *When a subnational fiscal authority of a country in H is indifferent between defaulting and repaying debt in the initial period, household consumption of good T in the initial period would be higher upon default:*

$$y_{\mathfrak{L}}-\zeta_1>\frac{y_{\mathfrak{L}}+\beta\omega y_{\mathfrak{H}}+\bar{b}_{SN}}{1+\beta\omega}. \tag{42}$$

Conversely, in the terminal period, high-default cost state, T-good household consumption is higher conditional on initial-period repayment.

$$y_{\mathfrak{H}}-\hat{\zeta}<\frac{y_{\mathfrak{L}}+\beta\omega y_{\mathfrak{H}}+\bar{b}_{SN}}{1+\beta\omega}. \tag{43}$$

In the terminal-period, low-default cost state, T-good household consumption is given by the endowment independently of initial-period choices.

[21] For ease of exposition, we also impose symmetry across countries in H and across subnational authorities, so that $c_{N,H,t}=\frac{w_{H,t}l_{H,t}}{p_{N,H,t}}$.

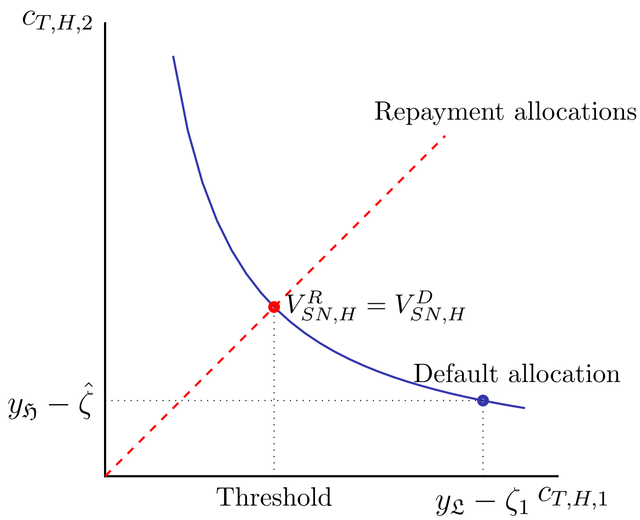

Figure 2: Default threshold for the subnational fiscal authority. The two axes represent consumption of T-good in the initial period and in the terminal period, in the high-default cost state. The blue, downward-sloping line is an indifference curve, representing the set of allocations that yield the same welfare as the default allocation to the representative household in H. From the point of view of the subnational fiscal authority, the contribution of good N to welfare is identical across all these allocations. The red, dashed, 45-degree, upward sloping line represents the set of allocations that are consistent with debt repayment and intertemporal optimality— i.e. with constant consumption of good T, across periods. Each point of the line corresponds to a level of initial assets, $b_{H,1}$, the higher the further out from the origin. The intersection of the two curves determines the repayment allocation that yields the same welfare as default. The default threshold $\bar{b}_{SN}$ is the level of debt to which this allocation corresponds. For higher levels of debt— i.e. closer to the origin than the intersection, default is preferred to repayment, and vice-versa for higher levels of debt.

Corollary 4.1.1 states that when the subnational fiscal authority defaults in the initial period, it causes its households' current consumption to be higher than it would be upon repayment. Given the default threshold (39), this result follows from the assumption in (23). The household consumption profile crucially determines the impact of nominal rigidities on default, when decision-makers take them into consideration. We study this important force in the following subsections.

Figure 2 introduces a graphical framework through which to analyze the optimal default decision, which will aid our analysis throughout the rest of this section. The figure displays the set of allocations that yield the same welfare as default, as well as those that are consistent with repayment, depending on initial-period assets. The default threshold is graphically determined as the intersection between these two sets of allocations— i.e. as the allocation where the subnational fiscal authority is indifferent between repayment and default. The location of the intersection can be use to gauge the relative attractiveness of default. The further is the intersection from the origin, the lower is the level of debt for which default is optimal.

4.1.2 Default by National Fiscal Authorities

When national fiscal authorities decide to repay or to default on external debt, they understand how their decision impacts on the severity of nominal rigidities in their country. National fiscal authorities internalize how the level of aggregate demand in the country depends on the default decision, and this consideration affects their relative incentive to default or repay. This is not the case for subnational authorities, who cannot affect demand at the national level. First, we discuss here the aggregate implications of the nominal rigidity that firms face, and how these affect the incentives of the fiscal authority. Second, we discuss the default decision of the national fiscal authority as a function of the severity of nominal rigidities.

Nominal Rigidities. Firms producing good N face downward wage rigidities. The implication of these nominal rigidities is that the production of good N in each country may be bounded from above by domestic demand for the good itself. When nominal rigidities bind, firms only hire the amount of labor that is necessary to satisfy domestic demand for good N, given prices and wages that the rigidities imply. In equilibrium, if the nominal rigidity binds in H in the initial period, the price of good N and the wage are given by:

$$p_{N,H,1} = w_{H,1} = \kappa, \tag{44}$$

and the amount of labor demanded by firms is given by

$$l_{H,1} = \min\left\{\frac{1-a}{a}\frac{p_{T,1}c_{T,H,1}}{\kappa}, l\right\}. \tag{45}$$

Nominal rigidities cause slackness in the economy, as firms only employ a fraction of the amount of labor that households supply inelastically. As a consequence, the amount of good N that is produced and consumed in equilibrium is lower than the one that would be technologically feasible:

$$c_{N,H,1} = l_{H,1} \leq l. \tag{46}$$

We graphically represent the equilibrium in the market for good N in Figure 3. The equilibrium allocation in this market lies at the intersection of the demand and supply for good N. Demand is the outcome of domestic households' intra-temporal optimization, (6). Supply is given by an inverse L-shaped line. The horizontal segment of supply is given by the price that nominal rigidites imply when they bind. The vertical segment is in correspondence of the level of output that firms could produce if they employed the entire endowment of labor supplied by households.

The national fiscal authority understands that the default decision has effects on households consumption of good T and, in turn, on demand for good N. Hence, the national fiscal authority has an incentive to alleviate the slackness present in the economy by increasing initial-period consumption of good T, thereby expanding demand and output of good N.

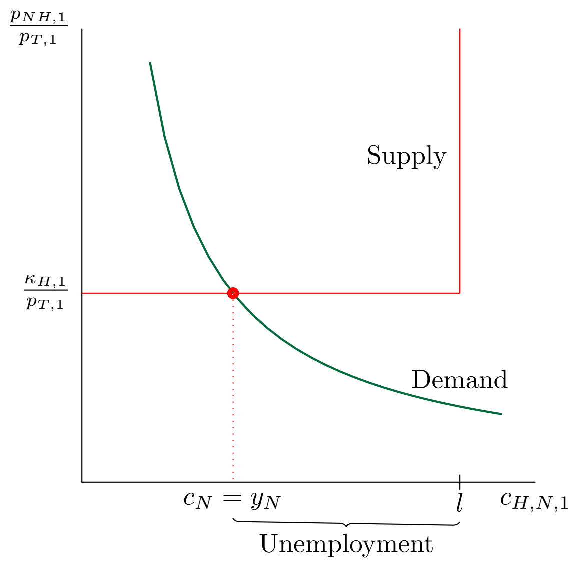

Figure 3: Equilibrium in the market for good N. The two axes represent consumption of N-good and the its price relative to good T, in the initial period, in a country in H. The green downward sloping curve represents demand for good N, (6). The red, inverse L-shaped line represents the supply of good N, the combination of (2) and (35). The supply is vertical in correspondence of output produced when the entire endowment of labor is employed, and horizontal in correspondence of the price implied by downward wage rigidities. The intersection of demand and supply determines the equilibrium amount of good N that is produced, and its relative price. The difference between the amount of labor supplied by households, l, and labor employed given output produced, y_N stands for involuntary unemployment.

Values of Default and Repayment. The values associated with default and repayment by the national fiscal authorities are analogous to the ones for subnational authorities (40) and (41), with the key difference that the former explicitly take into consideration domestic demand for good N. These values can be expressed as the sum of two terms. The first term equals the value to the subnational authority when nominal rigidities do not bind. The second term accounts for the severity of nominal rigidities.[22]

Formally, the value of default for the national fiscal authority can be written as:

$$V^D_{NF,H} = \begin{cases} V^D_{SN,H,FE} & \text{if } p_{T,1} \geq \tilde{p}_{T,D} \\ V^D_{SN,H,FE} + (1-a)\log\left(\frac{p_{T,1}}{\tilde{p}_{T,D}}\right) & \text{otherwise,} \end{cases} \tag{47}$$

where $\tilde{p}_{T,D} \equiv \kappa \frac{a}{1-a} \frac{l}{y_{\mathfrak{L}} - \zeta_1}$ is the minimum initial-period price of good T that ensures that nominal rigidities do not bind in a defaulting country in H. $V^D_{SN,H,FE}$ denotes the value to the subnational fiscal authority under default and full employment—i.e. when $c_{N,H,1} = l$. When nominal rigidities do not bind— i.e. when the price of good T is sufficiently high, the value for the national fiscal authority is identical to the full-employment one defined for the subnational authority. When the price of good T is sufficiently low, however, consumption of good N is low as well, and the national fiscal authority takes this fact into account when evaluating the effects of default on household welfare.

[22] Appendix B.3.1 provides further details on the derivation of these value functions.

The value under repayment for the national fiscal authority can be expressed in an analogous way:

$$V^R_{NF,H} = \begin{cases} V^R_{SN,H,FE} & \text{if } p_{T,1} \geq \tilde{p}^*_{T,R}(b_{H,1}) \\ V^R_{SN,H,FE} + (1-a)\log\left(\frac{p_{T,1}}{\tilde{p}_{T,R}\left(b_{H,1}\right)}\right) & \text{otherwise,} \end{cases} \tag{48}$$

where now $\tilde{p}_{T,R}\left(b_{H,1}\right) \equiv \frac{\kappa\, a}{1-a}\frac{l(1+\beta\omega)}{y_{\mathfrak{L}}+\beta\omega y_{\mathfrak{H}}+b_{H,1}}$ is defined similarly to $\tilde{p}_{T,D}$ and it represents the the minimum initial-period price of good T that ensures that nominal rigidities do not bind in a country in H that repays debt $|b_{H,1}|$. $V^R_{SN,H,FE}$ denotes the value to the subnational fiscal authority under repayment and full employment. Again, the national fiscal authority understands that its choice to repay would imply a certain level of good-T consumption and therefore a certain level of good-N output and consumption, given the effects of domestic demand on production in the presence of nominal rigidities.

Default Threshold. The severity of nominal rigidities crucially determines the level of debt for which it is optimal for the national fiscal authority to default. We will consider the three relevant cases, depending on whether nominal rigidities never bind, they bind only upon repayment, or upon both default and repayment.

First, suppose that nominal rigidities do not bind, neither if the national fiscal authority repays debt, nor if it defaults. This is the case when

$$p_{T,1} \geq \max\{\tilde{p}_{T,D}, \tilde{p}_{T,R}(\bar{b}_{NF,FE})\} \tag{49}$$

where $\bar{b}_{NF,FE}$ denotes the optimal default threshold, yet to be defined, for the national fiscal authority when nominal rigidities never bind, neither under default nor under repayment. In this circumstance, from comparing $V^D_{NF,H}$ and $V^R_{NF,H}$, the optimal default threshold for the national fiscal authority is the same as the one for the subnational fiscal authority, as domestic demand for good N plays no role in determining the optimal default decision:

$$\bar{b}_{NF,FE} = \bar{b}_{SN}. \tag{50}$$

The national and subnational fiscal authority both correctly understand that, in this instance, the level of consumption of good N does not depend on whether they default or repay. At this default threshold, following Corollary 4.1.1, consumption of good T is higher upon default than upon repayment. Hence, the inequality in (49) can be expressed as $p_{T,1} \geq \tilde{p}_{T,R}(\bar{b}_{NF,FE})$.

Second, consider the case where nominal rigidities always bind, both upon default and upon repayment— i.e. when

$$p_{T,1} < \min\{\tilde{p}_{T,D}, \tilde{p}_{T,R}\left(\bar{b}_{NF,NR}\right)\}. \tag{51}$$

The optimal default threshold in this setting is given by $\bar{b}_{NF,NR}$. Again, it follows from comparing the values under default and repayment (47) and (48). The default threshold can be expressed in a very similar form to (39):

$$\bar{b}_{NF,NR} = (1+\beta\omega)\left[(y_{\mathfrak{L}} - \zeta_1)\left(y_{\mathfrak{H}} - \hat{\zeta}\right)^{\beta\omega}\left(\frac{\tilde{p}_{T,R}\left(\bar{b}_{NF,NR}\right)}{\tilde{p}_{T,D}}\right)^{\frac{1-a}{a}}\right]^{\frac{1}{1+\beta\omega}} - (y_{\mathfrak{L}} + \beta\omega y_{\mathfrak{H}}). \quad (52)$$

The key difference between the two default thresholds (39) and (52) lies in the term that depends on the ratio of T-good prices that ensure full employment. This term captures the relative severity of nominal rigidities upon repayment and default, respectively. At the threshold, this ratio is greater than unity, since nominal rigidities are more detrimental to output upon repayment than upon default.[23] Hence, this threshold is higher than the one for the subnational fiscal authority, $\bar{b}_{SN}$ and default occurs for a lower level of debt (a higher level of assets) than when nominal rigidities are not taken into account. Finally, condition (53) can be expressed as $p_{T,1} < \tilde{p}_{T,D}$.

The left-hand side panel of Figure 4 graphically illustrates the impact of nominal rigidities on the default threshold for the national fiscal authority. The presence of nominal rigidities in the initial period increases the benefits of frontloading consumption in H. Graphically, this mechanism implies a north-eastward shift in the indifference curve that summarizes the allocations that yield the same welfare as default. Hence, the intersection with the set of allocations compatible with repayment lies to the right of the intersection in the absence of nominal rigidities. Thus, the level of debt for which default is optimal is lower when the national fiscal authority internalizes the effects of default and repayment on the level of domestic demand.

Finally, consider the case where nominal rigidities bind upon repayment but not under default. This is the case when

$$p_{T,1} \in \left[\tilde{p}_{T,D}, \tilde{p}_{T,R}\left(\bar{b}_{NF,FE}\right)\right). \quad (53)$$

In this setting, monetary policy crucially affects the default decision, as the economy would be at full employment conditional on default, but not upon repayment. By comparing the two value functions, (47) and (48), the default threshold is given by:[24]

$$\bar{b}_{NF,FE-D} = (1+\beta\omega)\left[(y_{\mathfrak{L}} - \zeta_1)\left(y_{\mathfrak{H}} - \hat{\zeta}\right)^{\beta\omega}\left(\frac{\tilde{p}_{T,R}\left(\bar{b}_{NF,FE-D}\right)}{p_{T,1}}\right)^{\frac{1-a}{a}}\right]^{\frac{1}{1+\beta\omega}} - (y_{\mathfrak{L}} + \beta\omega y_{\mathfrak{H}}). \quad (54)$$

Here, the term in the ratio of the prices $\tilde{p}_{T,R}$ and $p_{T,1}$ implies that the default threshold is increasing in the distance from full employment of the allocation under repayment. The more severe is unemployment upon repayment, the higher is that term and, therefore, the lower is the level of debt for which the national fiscal authority prefers to default.

We can now define the default threshold of the national fiscal authority as a function

[23] This follows from the result in Corollary 4.1.1. Appendix B.3.1 provides additional details, as well as an explicit solution for the threshold.

[24] Again, Appendix B.3.1 provides additional details and an explicit expression for the threshold $\bar{b}_{NF,FE-D}$.

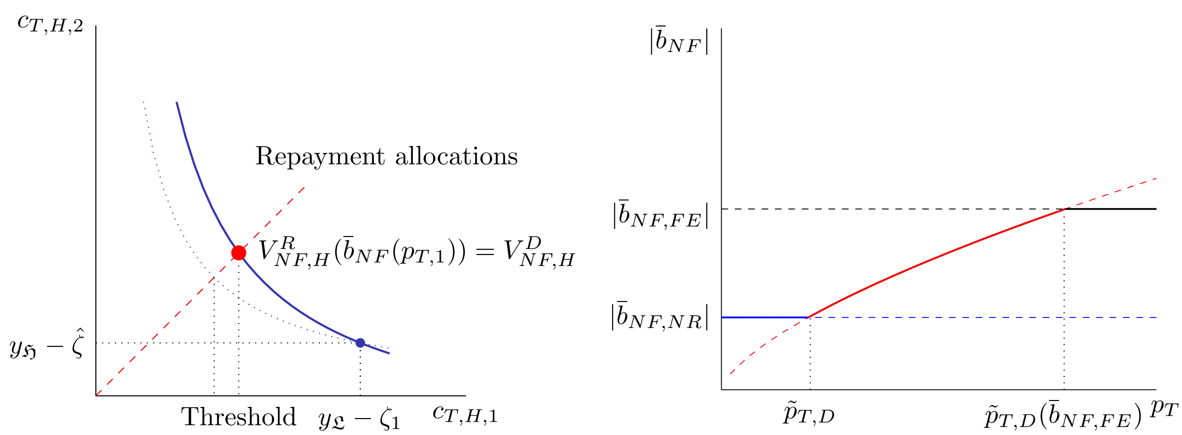

Figure 4: Optimal default threshold under nominal rigidities. The left-hand side panel describes how nominal rigidities affect the default threshold of the national fiscal authority. In the presence of nominal rigidities, the benefit of frontloading consumption is higher. Hence, in comparison to Figure 2, the indifference curve shifts north-east. For the national fiscal authrority it is thus optimal to default at a lower level of debt, as the intersection with the set of repayment allocation implies a higher level of initial-period consumption. The right-hand side panel displays graphically the default threshold (55) of the national fiscal authority. The threshold is increasing in absolute value in the initial-period price of good T: a higher price implies that nominal rigidities are less severe, and therefore default emerges for a higher level of debt (a lower level of assets).

of the price of good T in the initial period.

Proposition 4.2 *The default threshold of the national fiscal authority as a function of the price of good T is determined as the combination of the three thresholds that depend on the relative severity of nominal rigidities upon default and repayment:*

$$\bar{b}_{NF}(p_{T,1}) = \begin{cases} \bar{b}_{NF,NR} & \text{if } p_{T,1} < \tilde{p}_{T,D} \\ \bar{b}_{NF,FE-D}(p_{T,1}) & \text{if } p_{T,1} \in \left[\tilde{p}_{T,D}, \tilde{p}_{T,R}\left(\bar{b}_{NF,FE}\right)\right) \\ \bar{b}_{NF,FE} & \text{if } p_{T,1} \geq \tilde{p}_{T,R}\left(\bar{b}_{NF,FE}\right) \end{cases} \tag{55}$$

The right-hand side panel of Figure 4 presents graphically the default thresholds described above.

Having characterized its default threshold, we can show that the additional benefit of default given by the relaxation of nominal rigidities makes it optimal for the national fiscal authority to default at lower levels of debt than for the subnational authority.

Corollary 4.2.1 (Default threshold of national fiscal authority.) *The national fiscal authority finds it optimal to default for a lower level of debt than the subnational fiscal authority:*

$$\left|\bar{b}_{NF}(p_{T,1})\right| \leq \left|\bar{b}_{SN}\right|. \tag{56}$$

The result follows from comparing $\bar{b}_{SN}$ with $\bar{b}_{NF}(p_{T,1})$. Appendix B.3.1 provides explicit expressions for the thresholds.

Finally, if national fiscal authorities in all countries take the default-repayment decision in the initial period, an issue of absence or multiplicity of equilibria arises. This issue is due to the binary and non-convex nature of the default decision, and to the two-way interaction between default and monetary policy. We discuss this issue in greater detail in Appendix B.3.2.

4.1.3 Default by a Coalition of National Fiscal Authorities

The national fiscal authorities of all countries in H can form a coalition and take jointly their default or debt repayment decision. The coalesced countries are aware that their decisions affect aggregate variables in equilibrium. In particular, the coalition understands that its decisions determine the conduct of policy by the supranational monetary authority, in pursue of its price-stability objectives. This result occurs due to the effects of default by all countries in H on the level and the distribution of consumption within the monetary union.

Countries in H may benefit from the ability to influence monetary policy, given the presence of nominal rigidities. The ability to engineer higher prices through default and, thus, to relax the severity of unemployment, reduces the relative benefit of debt repayment. First, we discuss in detail the equilibrium implications on nominal variables of default and repayment by the coalition of countries in H. Second, we present the values of repayment and default for a representative household in H when the coalition of national fiscal authorities takes jointly the default or repayment decision. Third, we compare the optimal debt repayment threshold that emerges from this problem with the ones previously obtained for the individual national and subnational fiscal authorities. The absence of a limit on expansionary monetary policy plays a crucial role in this analysis. We analyze the presence of the zero lower bound in the next subsection, and we focus here instead on the setting where the action of the monetary authority is not constrained by this limit.

Aggregate Effects of Default and Repayment. A key force makes the default decision by a coalition of countries in H different from that of an individual national fiscal authority. Crucially, the coalition considers how its decisions influence the equilibrium determination of nominal variables in the monetary union.

The main nominal variable of interest is the equilibrium price of good T in the initial period, $p_{T,1}$. A higher price of good T implies that households reallocate demand towards good N, in turn reducing the severity of unemployment in countries in H, where nominal rigidities bind. In equilibrium, this price is determined by the intersection of the aggregate supply and demand for this good. Figure 5 provides a graphical representation of the equilibrium in this market.[25]

[25] For the purpose of Figure 5 and 6 below, we set $a = 0.25$, in line with the literature. The discount factor β is 0.99. We assume that $y_{\mathfrak{H}} = 0.6$ and $y_{\mathfrak{L}} = 0.4$, so that we normalize aggregate endowment to unity. The parameter that governs the downward wage rigidities, κ, equals unity. The monetary authority targets zero inflation in every period and it gives equal weights to the countries in the monetary union, meaning $p^* = \pi^* = 1$ and $\psi = 0.5$. The fiscal authorities in countries in H inherit debt equal to $y_{\mathfrak{L}}/4$. Default cost in the initial period is $\zeta_1 = 0.03$ while the highest default cost in the second period is $\hat{\zeta} = 0.3$. Finally, we set the probability to repay in the terminal period equal to $\omega = 0.75$.

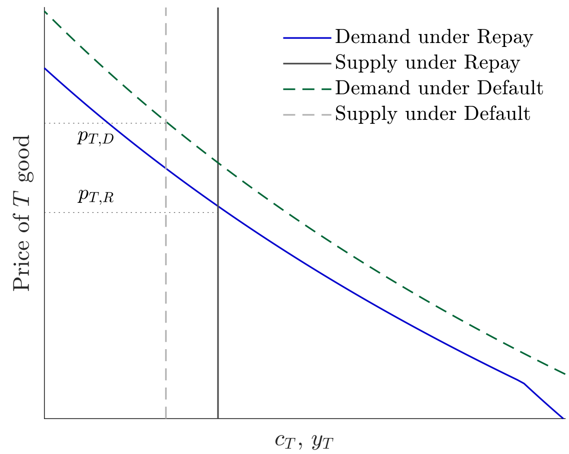

Figure 5: Equilibrium in the market for good T in the initial period, under default and repayment, in the absence of the zero-lower bound. Supply is given by the aggregate amount of good-T endowment, net of default costs if countries in H do not repay debt. Demand is given by the combination of the equilibrium conditions described in Section 3.6, with the exception of good-T market clearing. In particular, households' intratemporal consumption allocation, the price stability objective of the monetary authority, and the output of good N play a key role in determining demnad for good T.

First, the aggregate supply of good T is given by the aggregate endowment of this good in all countries of the monetary union, net of default costs if relevant.

Second, the aggregate demand for good T is the combination of all equilibrium conditions described in Section 3.6, except for the T-good market clearing condition (32). Three main forces determine the level of aggregate demand for good T: households' intratemporal demand for goods, the objective of the monetary authority, and the aggregate supply of good N. The combination of the three gives rise to a downward sloping schedule in the c_T-p_T space. For a given price of good N intratemporal optimality implies that households' desired T-good consumption is decreasing in the price p_T. In addition, for the objective of the monetary authority to be satisfied, a high price of good T must be associated with a low price of good N. Hence, when the absolute price of good T is high, its relative price is high, as well, implying a low desired consumption of good T, and contributing to the downward slope of the aggregate demand schedule.

Default and repayment by countries in H shift the intercept of the aggregate demand. When countries in H default, they consume in the initial period a higher amount of good T, for any given price. In addition, the aggregate supply of good T contracts, as resources are lost to the default cost ζ_1. Hence, the upward shift of aggregate demand, in conjunction with the lower aggregate supply of this good upon default, imply that the initial-period equilibrium price of good T is higher when the countries in H default on debt.

Values of Default and Repayment. The values associated by the coalition with default and repayment resemble the ones of the subnational fiscal authority (40) and (41), with two key differences. First, the members of the coalition consider the spillover effect of T-good consumption on domestic demand for good N, exactly as the individual national fiscal authorities do. Second, the coalition internalizes the effect of its action on aggregate equilibrium prices in the monetary union. Thus, when nominal rigidities bind for countries in H, the coalition understands how it would benefit from inducing an expansionary monetary policy. This is the crucial difference between the values of default and repayment for the individual national fiscal authorities and for the coalition of all countries in H.

Formally, the value of default for the coalition of national fiscal authorities can be expressed as follows:[26]

$$V^D_{\widehat{NF},H} = \begin{cases} V^D_{SN,H,FE} & \text{if } p_1^* \geq p_{1,D}^* \\ V^D_{SN,H,FE} + \frac{1-a}{a+\psi(1-a)} \log\left(\frac{p_1^*}{p_{1,D}^*}\right) & \text{otherwise,} \end{cases} \tag{57}$$

where $p_{1,D}^* \equiv \kappa\left(l\frac{a}{1-a}\right)^a (y_{\mathfrak{H}})^{\psi(1-a)} (y_{\mathfrak{L}} - \zeta_1)^{-(a+\psi(1-a))}$ is a threshold for the initial-period, price-level target of the monetary authority above which nominal rigidities would not bind in countries in H, conditional on their default. As previously, $V^D_{SN,H,FE}$ denotes the value to the subnational fiscal authority under default and full employment—i.e. when $c_{N,H,1} = l$. If the price-level target of the monetary authority is high enough, nominal rigidities do not bind in H and the value of default for the coalition is the same as for the subnational fiscal authorities. If, instead, the price target is low enough, countries in H face nominal rigidities and, due to low demand, firms in H produce a low amount of good N. In this instance, the coalition internalizes how households would benefit from the higher aggregate prices and less severe nominal rigidities that default implies.

Similarly, the value of repayment for the coalition depends on the initial-period price target for the monetary authority, as follows:

$$V^R_{\widehat{NF},H}(b_{H,1}) = \begin{cases} V^R_{SN,H,FE} & \text{if } p_1^* \geq p_{1,R}^*(b_{H,1}) \\ V^R_{SN,H,FE} + \frac{1-a}{a+\psi(1-a)} \log\left(\frac{p_1^*}{p_{1,R}^*(b_{H,1})}\right) & \text{otherwise,} \end{cases} \tag{58}$$

where $p_{1,R}^*(b_{H,1}) \equiv \kappa\left(l(1+\beta\omega)\frac{a}{1-a}\right)^a \frac{(y_{\mathfrak{H}}+\beta\omega y_{\mathfrak{L}}-b_{H,1})^{\psi(1-a)}}{(y_{\mathfrak{L}}+\beta\omega y_{\mathfrak{H}}+b_{H,1})^{a+\psi(1-a)}}$ is again a threshold for the initial-period, price-level target of the monetary authority. For a price-level target above the threshold, nominal rigidities would not bind in H upon repayment. The threshold is decreasing in the level of initial-period in assets in H. When debt is lower (assets are higher), repayment of debt is consistent with a relatively high level of demand in H, hence a relatively low target of the monetary authority is sufficient to ensure full employment. $V^R_{SN,H,FE}$ is the previously defined value for the subnational fiscal authority under repayment and full employment. The coalition of national fiscal authorities understand that its actions have effects on aggregate prices in the monetary union, as it does under default. In particular, the coalition understands that the severity of nominal rigidities in H depends on the level of initial assets $b_{H,1}$, given the implications of this quantity on the level of domestic aggregate demand, as well as on the equilibrium price $p_{T,1}$ that is determined in the monetary union.

The consideration of the equilibrium determination affects the threshold on initial

[26] Appendix B.4 provides additional details on the derivation of the expressions in (57) and (58).

assets for which the coalition finds it optimal to default. We now turn to discuss the determination of this threshold.

Default Threshold. The severity of nominal rigidities plays a key role in determining the optimal default threshold of the coalition of countries in H, as it is the case for the individual national fiscal authorities. We will again consider three different cases, depending on whether the initial-period target of the monetary authority implies that nominal rigidities never bind, bind both upon default or repayment, or upon repayment of debt, only.

First, consider the case when the target of the monetary authority leads to full employment in all countries in H independently on whether they default or repay debt. This is the case when

$$p_1^* \geq \max\left\{p_{1,D}^*, p_{1,R}^*\left(\bar{b}_{\widehat{NF},FE}\right)\right\}, \tag{59}$$

where $\bar{b}_{\widehat{NF},FE}$ denotes the threshold for the coalition of fiscal authorities in this circumstance. Under (59), the values to the coalition are again identical to those for the subnational fiscal authority. Hence, the default threshold $\bar{b}_{\widehat{NF},FE}$ is again given by (39):

$$\bar{b}_{\widehat{NF},FE} = \bar{b}_{SN}. \tag{60}$$

As it is the case for the individual national fiscal authority, when nominal rigidities do not bind, the optimal default decision of the subnational fiscal authority also holds for the coalition of national fiscal authorities. Corollary 4.1.1 implies that $p_{1,D}^* < p_{1,R}^*(\bar{b}_{\widehat{NF},FE})$, so that the inequality in (59) can be expressed as $p_1^* \geq p_{1,R}^*\left(\bar{b}_{\widehat{NF},FE}\right)$.

Second, when the price-level target of the monetary authority is sufficiently low, nominal rigidities bind in H independently of whether the coalesced fiscal authorities default or repay. Formally, this is the case when:

$$p_1^* < \min\left\{p_{1,D}^*, p_{1,R}^*\left(\bar{b}_{\widehat{NF},NR}\right)\right\}. \tag{61}$$

By comparing the two value functions in (57) and (58) we obtain the optimal default threshold, which can be expressed implicitly as follows:

$$\bar{b}_{\widehat{NF},NR} = (1+\beta\omega)\left[\left(y_{\mathfrak{L}}-\zeta_1\right)\left(y_{\mathfrak{H}}-\hat{\zeta}\right)^{\beta\omega}\left(\frac{p_{1,R}^*\left(\bar{b}_{\widehat{NF},NR}\right)}{p_{1,D}^*}\right)^{\frac{1-a}{a(\psi_F+a\psi_H)}}\right]^{\frac{1}{1+\beta\omega}} - (y_{\mathfrak{L}}+\beta\omega y_{\mathfrak{H}}). \tag{62}$$

The term in the ratio of price-level targets accounts for how the determination of prices in the monetary union equilibrium affects the coalition's default decision. The higher is the ratio $\frac{p_{1,R}^*}{p_{1,D}^*}$ the stronger is the expansionary effect of default on prices in the monetary union.[27] Hence, default is optimal for the coalition of fiscal authorities for a higher level

[27] It can be proven that this ratio is larger than unity, so that the coalition defaults for a higher level of assets than an individual national fiscal authority would. We present the proof in Appendix B.4, along

of initial assets, or for a lower level of debt. This is the case since the benefit of default rises when the coalition internalizes how its default would lead to higher prices in the monetary union, to less severe nominal rigidities and ultimately to lower unemployment in the countries in H. At the optimal default threshold, the price-level target that the monetary authority would have to set to guarantee full employment in H under repayment, $p^*_{1,R}\left(\bar{b}_{\widehat{NF},NR}\right)$ is larger than the corresponding one under default, $p^*_{1,D}$. Hence, the inequality in (61) reduces to $p^*_1 < p^*_{1,D}$.

Third, consider the case where the price-level target of the monetary authority is high enough for nominal rigidities not to bind in H conditional upon default, but low enough for them to bind upon repayment—i.e.:

$$p^*_1 \in \left[p^*_{1,D}, p^*_{1,R}(\bar{b}_{\widehat{NF},FE})\right) \tag{63}$$

The default threshold is given in this circumstance by:

$$\bar{b}_{\widehat{NF},FE-D}\left(p^*_1\right) = (1+\beta\omega)\left[(y_{\mathfrak{L}}-\zeta_1)\left(y_{\mathfrak{H}}-\hat{\zeta}\right)^{\beta\omega}\left(\frac{p^*_{1,R}\left(\bar{b}_{\widehat{NF},FE-D}\right)}{p^*_1}\right)^{\frac{1-a}{a(\psi_F+a\psi_H)}}\right]^{\frac{1}{1+\beta\omega}} - (y_{\mathfrak{L}}+\beta\omega y_{\mathfrak{H}}). \tag{64}$$

Again, the term in the ratio of price-level targets accounts for the effects of the union-wide equilibrium on the optimal default threshold. In this setting, given the relatively looser stance of monetary policy, the relative gain from defaulting due to the relaxation of nominal rigidities is weaker than in the setting considered in (62).

We can now define the optimal default threshold for the coalition of national fiscal authorities, as a function of the monetary authority's initial-period target for the price level.

Proposition 4.3 *The default threshold of the coalition of national fiscal authorities, as a function of target for the initial-period price level of the monetary authority is determined as the combination of the three thresholds that depend on the relative severity of nominal rigidities upon default and repayment:*

$$\bar{b}_{\widehat{NF}}\left(p^*_1\right) = \begin{cases} \bar{b}_{\widehat{NF},NR} & \textit{if } p^*_1 < p^*_{1,D} \\ \bar{b}_{\widehat{NF},FE-D}\left(p^*_1\right) & \textit{if } p^*_1 \in \left[p^*_{1,D}, p^*_{1,R}\left(\bar{b}_{\widehat{NF},FE}\right)\right) \\ \bar{b}_{\widehat{NF},FE} & \textit{if } p^*_1 \geq p^*_{1,R}\left(\bar{b}_{\widehat{NF},FE}\right). \end{cases} \tag{65}$$

Finally, having characterized the default threshold of the coalition of national fiscal authorities, we can show that it is optimal for the coalesced countries to default for lower levels of debt than for the individual national fiscal authorities.

with the proof of the results in Corollary 4.3.1.

Corollary 4.3.1 (Default threshold of the coalition of national fiscal authorities.) *The coalition of national fiscal authorities defaults for lower levels of debt than the individual national fiscal authorities, if nominal rigidities bind upon both default and repayment:*

$$\left|\bar{b}_{\widehat{NF},NR}\right| < \left|\bar{b}_{NF,NR}\right|. \tag{66}$$

The default thresholds of the coalition and of the individual national fiscal authorities are identical when nominal rigidities do not bind:

$$\left|\bar{b}_{\widehat{NF},FE}\right| = \left|\bar{b}_{NF,FE}\right| = \left|\bar{b}_{SN}\right|. \tag{67}$$

The two thresholds are also equal when nominal rigidities would only bind upon repayment and, at the threshold, all countries repay debt:

$$\left|\bar{b}_{\widehat{NF},FE-D}\left(p_1^*\right)\right| = \left|\bar{b}_{NF,FE-D}\left(p_{T,1,R}\right)\right|, \tag{68}$$

where $p_{T,1,R} = \left[p_1^{*\frac{1}{1-a}}\left(\frac{1}{\kappa}\right)^{(1-\psi)}\left(\frac{a}{1-a}\frac{l(1+\beta\omega)}{y_{\mathfrak{H}}+\beta\omega y_{\mathfrak{L}}-\bar{b}_{\widehat{NF},FE-D}}\right)^{\psi}\right]^{\frac{1-a}{\psi+(1-\psi)a}}$ *and* $p_1^* \in \left[p_{1,D}^*, p_{1,R}^*\left(\bar{b}_{\widehat{NF},FE}\right)\right)$.

Appendix B.4 presents the proof of these results, along with the more detailed expressions for the default thresholds.

4.1.4 Default and Monetary Policy under the Zero Lower Bound

We consider in this subsection the implications for countries' default policy of the zero lower bound on nominal interest rates, which acts as a constraint on the action of monetary policy. We discuss the conditions on parameters of the model economy under which the zero lower bound weakens the incentive to default on debt. We refer to Appendix A for an analysis of the equilibrium determination of the nominal interest rate and of the conditions under which the zero lower bound binds.

We showed in the previous subsection that the incentive to default of countries in H strengthens when they internalize the implications of their action on the conduct of monetary policy. One key feature of the model which leads to this result is the ability of the supranational monetary authority to conduct expansionary monetary policy in response to default by the countries in H. This result can be overturned when the zero lower bound hinders the ability of the central bank to conduct expansionary monetary policy. Proposition 4.4 below states our key result on the effect of the zero lower bound on the optimal default policy.

Proposition 4.4 *The zero lower bound weakens the incentive to default if the fall in N-good consumption induced in H by the zero lower bound is larger upon default than upon repayment:*

$$\frac{c_{N,H,R,Z}\left(\bar{b}_{\widehat{NF}}\right)}{c_{N,H,R,NZ}\left(\bar{b}_{\widehat{NF}}\right)} \geq \frac{c_{N,H,D,Z}}{c_{N,H,D,NZ}}. \tag{69}$$

We define $c_{N,H,R,NZ}\left(\bar{b}_{\widehat{NF}}\right)$ as the consumption of good N in countries in H in the absence of the zero lower bound, when they repay in the initial period an amount of debt equal to the default threshold $\bar{b}_{\widehat{NF}}$. We define analogously $c_{N,H,R,Z}\left(\bar{b}_{\widehat{NF}}\right)$ as the consumption of good N in H in the same circumstance, when the monetary authority is constrained by the zero lower bound. Consistently, we define $c_{N,H,D,NZ}$ as consumption of good N in countries in H in the absence of the zero lower bound, under initial-period default. Finally, we define $c_{N,H,D,Z}$ as consumption of good N in countries in H in the same circumstance, when the monetary authority is constrained by the zero lower bound.

The result in Proposition 4.4 follows from multiple steps. First, the presence of the zero lower bound affects welfare in H only through its implications on initial-period consumption of good N.[28]

Second, suppose that the level of initial assets in H is equal to the coalition's default threshold $\bar{b}_{\widehat{NF}}$ defined in (65). This threshold holds in the absence of the zero lower bound: when the monetary authority does not face constraints on its action, the coalition of national fiscal authorities would be indifferent between repayment and default on this amount of initial-period debt .

Third, consider now the presence of the zero lower bound. This constraint, if binding, induces a fall in N-good consumption in H, both upon repayment as well as upon default. This is the case since the zero lower bound prevents the monetary authority from reaching its objective for the price level, hence the zero lower bound implies low prices in the union: $p_{1,Z} \leq p_1^*$.[29] Therefore, the price of good T is also lower than in the absence of the zero lower bound: $p_{T,1,Z} \leq p_{T,1,NZ}$. The lower price of good T induces a fall in demand for good N, tightens the severity of nominal rigidities in H, and causes a fall in output of good N: $c_{N,H,D,Z} \leq c_{N,H,D,NZ}$ and $c_{N,H,R,Z} \leq c_{N,H,R,NZ}$.

Fourth, to establish our key result, we must compare whether the zero lower bound reduces welfare in H by more conditional upon default or upon repayment. At the level of assets $b_{H,1} = b_{\widehat{NF}}$, the coalition would be indifferent between default and repayment in the absence of the zero lower bound. If the zero lower bound causes a larger fall in welfare upon default, the coalition prefers to repay this amount of debt in the presence of the zero lower bound.

Fifth, we stated that the zero lower bound only affects welfare through initial-period consumption of good N. Hence, we must simply compare whether the zero lower bound induces a larger fall in consumption of this good upon default or upon repayment. Therefore, if condition (69) holds, the zero lower bound causes a larger welfare loss upon default, and the coalition of national fiscal authorities optimally chooses to repay levels of debt where it would instead default in the absence of the zero lower bound. In this sense, the zero lower bound acts as a device that enforces repayment of sovereign debt.

The condition on relative consumption (69) can be expressed as one on the nominal

[28]This follows from the fact that the severity of nominal rigidities has no implications for the intertemporal allocation of T-good consumption. In turn, this follows from the assumption of unitary intra-temporal and inter-temporal elasticities of substitution, as discussed in footnote 32.

[29]We define $p_{1,Z}$ as the average price level that prevails in the union when the zero lower bound prevents the monetary authority from achieving the objective $p_1 = p_1^*$. Analogously, we define $p_{T,1,Z}$ as the price of good T that prevails in the initial period in this circumstance, and $p_{T,1,NZ}$ as the one that would arise in the absence of constraints on monetary policy.

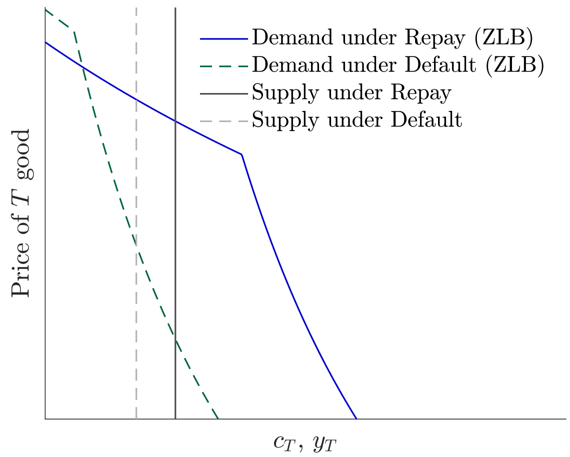

Figure 6: Aggregate demand and supply of good T, under initial-period default and repayment, considering the presence of the zero lower bound.

interest rate that would arise in the absence of the zero lower bound:[30]

$$\min\left\{\mathfrak{i}_{R,NZ}\left(\bar{b}_{\widehat{NF}}\right),0\right\} \geq \min\left\{\mathfrak{i}_{D,NZ},0\right\}. \tag{70}$$

where $\mathfrak{i}_{D,NZ}$ and $\mathfrak{i}_{R,NZ}\left(\bar{b}_{\widehat{NF}}\right)$ are the nominal interest rates that would hold in equilibrium in the absence of the zero lower bound under default and repayment, respectively. Intuitively the more stringent is the zero lower bound as a constraint on nominal interest rate, the more severe are its effects on demand in H. Hence, the stronger are its effects on welfare. If the nominal interest rate would have to be more negative conditional upon default, the zero lower bound acts as a repayment enforcement device.

The nominal interest rates that is consistent with the objective of policy is lower under default if relegation into autarky of countries in H leads to a large fall in the supply of assets in the monetary union. Formally this occurs when the terminal-period probability of repayment is sufficiently high:

$$\omega \geq \bar{\omega}, \tag{71}$$

where the threshold ω is defined in Appendix A. Intuitively, when ω is sufficiently high, the risky bonds issued by H allow F to effectively transfer resources to the terminal period. Upon default, H contracts the supply of such assets due to autarky, thereby putting downward pressure on interest rates. Hence, in this circumstance, the monetary authority is constrained the most in its action when countries in H default in the initial period.

We clarify the implications of the zero lower bound on aggregate demand and the equilibrium of the monetary union through the graphical framework that we introduced in Subsection 4.1.3. Figure 6 illustrates the result. The presence of the zero lower bound causes a fall in aggregate demand for good T. The fall in aggregate demand is larger upon default, if condition (92) holds. Hence, the initial period price of good T decreases by more upon default than upon repayment due to the zero lower bound. This reduction

[30]More precisely, this is the case under two additional assumptions that allow for the analytical characterization of this result. These are discussed in Appendix A.

in prices causes demand for good N to fall as the relative price of this good increases. Given the presence of nominal rigidities, this fall in demand is detrimental for welfare. The larger the fall in prices, the larger are the adverse welfare implications of the zero lower bound.

4.2 Monetary Policy

Monetary policy has important implications for welfare of households in the monetary union. Two key results emerge. First, savers and borrowers have diametrically opposite desires for how monetary policy should be conducted. Borrowers prefer an expansionary policy, to relax the severity of initial-period nominal rigidities. Savers would like, instead, the monetary authority to keep prices low, in order to allow for higher inflation the future, when their output will be lower. Second, savers also prefer the supranational monetary authority to conduct expansionary policy, if this induces debtors to repay them. We present these results in the next two subsections. Appendix A provides details on the nominal interest rate that the monetary authority sets to implement its price-stability objective, depending on consumption growth and inflation in the equilibrium of the world economy.

4.2.1 Heterogeneous Preferences over Monetary Policy

The two groups of countries have heterogeneous preferences over the conduct of monetary policy. Specifically, countries in H would prefer monetary policy to be expansionary in the initial period. Conversely, countries in F would prefer monetary policy to be contractionary in that instance. In other words, countries in H would like the monetary authority to target a high price level in the initial period, and countries in F would like it to target a low price level instead.

The driving force behind this result is the unsynchronized business cycle experienced by the two groups of countries: when H is in a bust phase of the cycle, F is in a boom. A relaxation of monetary policy in the initial period thus only benefits H. However, high prices in the initial period force policy to be tighter in the terminal period, for the monetary authority to achieve the inflation target. In turn, the tightening of future policy is detrimental to countries in F, who may face binding nominal rigidities in the terminal period.

We present the formal argument in what follows. First, we detail how monetary policy affects welfare in H. Second, we explain the welfare implications of monetary policy for countries in F.

Proposition 4.5 (Preferences for expansionary policy in H.) *Welfare in H is weakly increasing in the initial-period price-level target p_1^*.*

$$V_{HH,H}\left(p_{1,\mathfrak{H}}^*\right) \geq V_{HH,H}\left(p_{1,,\mathfrak{L}}^*\right), \quad \textit{where } p_{1,\mathfrak{H}}^* > p_{1,\mathfrak{L}}^*. \tag{72}$$

Instead, the conduct of monetary policy in the terminal period is irrelevant for welfare in these countries.

Consider the value function of a generic household in a country in H. This value function can be expressed as follows:

$$V_{HH,H} = a \log\left(c_{T,H,1}\right) + (1-a)\left(c_{N,H,1}\right) + \beta V_{HH,H,2}, \tag{73}$$

where, $c_{T,H,1}$ and $c_{N,H,1}$ denote the equilibrium values of initial-period consumption of the two goods, and $V_{HH,H,2}$ summarizes expected terminal-period welfare, which is independent of monetary policy.[31] In turn, consumption of good N is given by $c_{N,H,1} = \min\left\{\frac{1-a}{a}\frac{p_{T,1}}{\kappa}c_{T,H,1}, l\right\}$ depending on whether initial-period nominal rigidities bind in equilibrium or not. In the latter case, monetary policy does not have a direct impact on consumption and, therefore, neither it does on welfare. In the former case, instead, expansionary monetary policy is beneficial for welfare in H.[32] In equilibrium, the price of good T is increasing in the monetary policy target p_1^*:[33]

$$p_{T,1} = \left[\left(\frac{1-a}{a}\right)^{(a-1)\psi} \kappa^{(a-1)(1-\psi)}\ c_{T,F,1}^{(a-1)\psi} p_1^*\right]^{\frac{1}{a+(1-a)\psi}}. \tag{74}$$

Hence, since a higher price-level target p_1^* gives rise to a higher nominal price of good T, nominal rigidities becomes less severe, since the relative price of good N $\kappa/p_{T,1}$ falls. Therefore, initial-period consumption of good N rises in H, along with welfare.

Proposition 4.6 (Preferences for contractionary policy in F.) *Welfare in F is weakly decreasing in the initial-period price-level target p_1^*, conditional on the default-repayment decision in H:*

$$V_{HH,F}\left(p^*_{1,\mathfrak{H}}\right) \leq V_{HH,F}\left(p^*_{1,,\mathfrak{L}}\right), \text{ where } p^*_{1,\mathfrak{H}} > p^*_{1,\mathfrak{L}} \text{ and } D_{H,1}\left(p^*_{1,\mathfrak{H}}\right) = D_{H,1}\left(p^*_{1,\mathfrak{L}}\right) \tag{75}$$

Even if nominal rigidities never bind in the initial period in countries in F, the conduct of monetary policy in this period affects their welfare nonetheless. Consider the value function of a household in F:

$$V_F = a\log\left(c_{T,F,1}\right) + (1-a)\log(l) + \beta E_1\left[a\log\left(c_{T,F,2}\right) + (1-a)\left(c_{N,F,2}\right)\right] \tag{76}$$

where, again, $c_{T,F,t}$ and $c_{N,F,t}$ denote the equilibrium values of consumption of the two goods at time t, and where we already imposed the equilibrium condition $c_{N,F,1} = l$. We

[31]The equilibrium values of consumption may differ depending on what decision-maker chooses the default policy. Hence, the value $V_{HH,H}$ may correspond alternatively to $V_{SN,H}$, $V_{NF,H}$ or to $V_{\widehat{NF},H}$.

[32] Note that the equilibrium amount of c_T is independent of the equilibrium amount of c_N, conditional on the debt repayment decision, since both the intra-temporal elasticity of substitution between c_T and c_N and the inter-temporal one between c_1 and c_2 equal unity. Hence, the inter-temporal allocation of c_T is also independent of the stance of monetary policy. This parametrization is similar to the one in Cole and Obstfeld (1991).

[33]The combination of the intratemporal allocation of consumption in H and F, (6) and the initial-period price-level target (13) yields this result.

focus attention on the case where changes in the price target p_1^* do not affect the default decision by H.[34]

In the terminal period, if the downward wage rigidities bind, consumption of N-good in countries in F is increasing in the good-T price inflation. Intuitively, households demand more of good N if good T becomes relatively more expensive.[35] Indeed, $c_{N,F,2} = \min\left\{l, \kappa\pi_{T,2}\frac{c_{T,F,2}}{c_{T,F,1}}\right\}$, depending on whether nominal rigidities bind or not. When nominal rigidities bind, output is demand-determined. Hence, the increase in the demand of good N leads to an increase in the production of this good, and it has a beneficial effect on the welfare of households in F. In equilibrium, inflation in the price of good T is determined as follows:[36]

$$\pi_{T,2} = \left[\pi_2^* \; \kappa^{(a-1)(\psi)} \; \left(\frac{c_{T,H,2}}{c_{T,H,1}} c_{N,H,1}\right)^{(a-1)(1-\psi)}\right]^{\frac{1}{a+(1-a)\psi}}. \tag{77}$$

For a given inflation target π_2^*, inflation in the price of good T is decreasing in $c_{N,H,1}$. In countries in H, prices adjust sluggishly in the initial period, due to the presence of nominal rigidities. The adjustment falls on the quantities, instead. Hence, an increase in the price target p_1^* would give rise to a delayed increase in inflation $\frac{p_{N,H,2}}{p_{N,H,1}}$, as well as to the increase in $c_{N,H,1}$. The increase in inflation would not be consistent with the target of the monetary authority, who thus reacts by tightening T-good inflation $\pi_{T,2}$. Hence, $\pi_{T,2}$ is decreasing in initial-period N-good consumption in H. Hence, $c_{N,F,2}$ is weakly decreasing in p_1^* and an expansionary stance of monetary policy in the initial period is detrimental for welfare in F.

4.2.2 Default and Monetary Policy Preferences

The presence of sovereign default risk induces countries in F to support a more expansionary stance of monetary policy, in order to avoid default on their external assets.

Proposition 4.7 (Default broadens support for expansionary monetary policy.) *Welfare in F marginally increases in the initial-period price-level target p_1^*, when the change in the target induces H to repay debt.*

$$V_{HH,F}\left(p_{1,\mathfrak{H}}^*\right) > V_{HH,F}\left(p_{1,,\mathfrak{L}}^*\right), \quad \textit{where } p_{1,\mathfrak{H}}^* > p_{1,\mathfrak{L}}^* \quad \textit{and } D_{H,1}\left(p_{1,\mathfrak{H}}^*\right) < D_{H,1}\left(p_{1,\mathfrak{L}}^*\right) \tag{78}$$

First, consider the situation where the coalition of fiscal authorities in H is exactly indifferent between default and repayment, given their external position $b_{H,1}$ and the monetary authority's target p_i^*. Given this indifference, it is possible to assume that these countries default on their debt.

[34] We analyze below, in Section 4.2.2 how preferences in F for the conduct of monetary policy can be overturned by the possibility to influence the choice by H to repay debt.

[35] Terminal-period good-T price inflation is intuitively defined as $\pi_{T,2} = \frac{p_{T,2}}{p_{T,1}}$.

[36] The combination of the intratemporal allocation of consumption in H and F (6) and the terminal-period inflation level target (15) yields this result.

Second, consider the intuitive setting where countries in F are better off when countries in H repay debt, rather than defaulting on it.[37] When countries in H repay, countries in F enjoy higher consumption of good T, as well as higher consumption of good N in the terminal period, if countries in H then repay debt, as well.

A marginal increase in the nominal target of the monetary authority is beneficial for all countries in the monetary union, in this circumstance. First, consider countries in H. There, expansionary policy in the initial period is welfare-increasing, as stated in Proposition 4.5. In addition, the higher target implies that the level of debt falls below the default threshold, and it is then optimal to repay external debt. Second, consider countries in F. Their welfare would marginally fall due to the expansionary monetary policy. However, the fact that countries in H repay debt generates a discrete rise in welfare in F. The gains from debt repayment by H thus induce countries in F to support the marginal change in monetary policy that delivers repayment itself.

5 Concluding Remarks

We develop a model of sovereign default in a monetary union composed of heterogeneous countries, which includes savers and borrowers alike.

We show that the borrowers' and savers' shared memebership of the monetary union has important implications for the optimal default decision of the former. In particular, their incentive to default strengthens if, in the union, expansionary monetary policy follows a default episode. Conversely, default risk is reduced when the zero lower bound impedes expansionary monetary policy and when default would lead to a fall in the natural rate of interest. In addition, we find that the possibility to avert default induces savers to support expansionary monetary policy, even if they would otherwise favour a tightening in the nominal objective of monetary policy.

We obtain our results analytically, in a relatively simple model where agents' time horizon is finite and where there is no uncertainty over the path of output.[38] We plan to relax these restrictive assumption in a quantitative model in upcoming work.

[37] A relatively high-probability of terminal-period repayment ω ensures this result. Appendix B.5 formally details this condition.

[38] We allow, however, for stochastic default risk.

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A The Nominal Interest Rate in the Monetary Union

In the first period, the supranational monetary authority can reach its price-stabilization objective if the nominal interest rate consistent with this target is positive. Otherwise, the monetary authority sets the nominal interest rate to zero and it allows prices to adjust accordingly. In this instance, the average price of consumption goods in the union will fall below the target. We study in this appendix the determination of the nominal interest rate in the equilibrium of the monetary union. In addition, we detail the conditions under which the zero lower bound binds and the monetary authority fails to achieve its nominal target for the initial period.

A.1 The Nominal Interest Rate in the Absence of the Zero Lower Bound

In equilibrium, the interest rate on nominal, risk-free assets is priced according to intertemporal optimization of households in F, (7):

$$\frac{1}{(1+\mathfrak{i})} = \beta \mathbb{E}\left[\frac{p_{T,1}}{p_{T,2}} \frac{c_{T,F,1}}{c_{T,F,2}}\right]. \tag{79}$$

Households in H have no role in pricing this key interest rate. These households do not hold nominal assets in the initial period, since they have a desire to borrow due to their back-loaded endowment path.[39] On the other hand, households in F have a desire to save, and they would be happy to buy nominal assets issued by the monetary authority. Hence, these households are the ones whose inter-temporal consumption pattern determines the interest rate that the monetary authority must set to achieve its objectives.

It is useful to define the implicit, gross real interest rate on risk-free assets as the inverse of $\beta \mathbb{E}\left[\frac{c_{T,F,1}}{c_{T,F,2}}\right]$. Three main forces determine the level of the nominal interest rate: the real interest rate, expected inflation in the price of good T, and the correlation between the real interest rate and inflation. We analyze below the determination of all three.

[39] Note that in the households' problem (5), a borrowing constraint prevents households from borrowing by issuing nominal assets, which can only be issued by the monetary authority. In equilibrium, the total supply of these assets is thus equal to zero.

First, the real interest rate is low if T-good consumption growth in F is low. Two key reasons, both related to default, may give rise to low expected consumption growth in F. Initial-period default by countries in H and a high risk of terminal-period default both cause to negative expected consumption growth. If countries in H default in the initial period, households in F lose the ability to save, hence, they are forced into financial autarky. Their consumption path thus equals their endowment path, which is front-loaded: $\frac{c_{T,F,1}}{c_{T,F,2}} = \frac{y_{\mathfrak{H}}}{y_{\mathfrak{L}}} > 1$. If, instead, countries in H repay debt in the initial period, but are unlikely to repay in the terminal one, expected consumption growth in F is also negative. Households in F equalize their consumption across the initial period and the terminal-period state of the world where countries in H repay debt. However, with probability $(1-\omega)$ countries in H default on debt in the terminal period. In that state of the world, F-households' consumption falls. Hence, expected consumption growth can be expressed as $\mathbb{E}\left[\frac{c_{T,F,1}}{c_{T,F,2}}\right] = \frac{1}{\omega+(1-\omega)\left(c_{T,F,1}/y_{\mathfrak{L}}\right)} > 1$, which is increasing in terminal-period default risk $(1-\omega)$.

Second, inflation in the price of good T depends on relative consumption growth for goods T and N in all countries, given the inflation target π^*. In equilibrium, given intratemporal optimality in all countries (6) and the inflation target (15), inflation equals a weighted average of relative consumption growth in all countries, with weights ψ and $1-\psi$ as in the monetary policy objective:

$$\pi_{T,2} = \frac{p_{T,2}}{p_{T,1}} = \pi^* \left[\left(\frac{c_{T,F,1}}{c_{T,F,2}} \frac{c_{N,F,2}}{c_{N,F,1}} \right)^{\psi} \left(\frac{c_{T,H,1}}{c_{T,H,2}} \frac{c_{N,H,2}}{c_{N,H,1}} \right)^{(1-\psi)} \right]^{(1-a)} . \tag{80}$$

If good T becomes relatively less abundant over time in the union, on average, its price relative to that of good N falls over time. Low relative inflation in the price of good T, in turn, implies that absolute inflation is low, as well. This is the case since the target of the monetary authority pins down the average inflation in the prices of the two goods. The implication of consumption growth on the nominal interest rate through inflation thus depends on the good that is considered. A rise in average consumption growth of good T leads to a fall in inflation and thus to a fall in the nominal interest rate, for a given real interest rate. Conversely, a tightening of initial-period nominal rigidities increases the nominal interest rate and good-T inflation, as it is equivalent to an increase in average consumption growth of good N.

Finally, the correlation between T-good inflation and consumption growth in F depends crucially on countries' weights in the target of the monetary authority. Suppose that the monetary authority gives a predominant weight to countries in F, i.e. $\psi \approx 1$. When this is the case, high growth of good-T consumption in F induces the monetary authority to engineer low inflation in the price of the same good in the union, according to (80). Hence, the correlation between the real interest rate and inflation is positive, leading to a higher nominal interest rate, ceteris paribus. Suppose instead that the monetary authority mostly targets inflation in countries in H, $\psi \approx 0$. When this is the case, high consumption growth in F implies low consumption growth in H and, through (80), high inflation in the price of good T. Thus, the correlation between the real interest rate and inflation is negative, causing a lower nominal interest rate. Finally, given continuity in this relationship, there exists a weight $\tilde{\psi}$ such that this correlation equals zero. We now turn to the analysis of the conditions under which the zero lower bound binds.

A.2 The Zero Lower Bound as a Constraint on Monetary Policy

We study here under what conditions on endowments, parameters, and policy targets the zero lower bound constrains the action of the monetary policy authority in the monetary union.

The key equation that determines the presence, or absence, of the zero lower bound is the inter-temporal optimality condition for nominal assets in countries in F:

$$\frac{1}{(1+\mathfrak{i})} = \beta E\left(\frac{c_{T,F,1}}{c_{T,F,2}}\frac{p_{T,1}}{p_{T,2}}\right) \lesseqgtr 1.$$

We study in the rest of the section the sign of the above inequality depending on whether countries in H default or repay initial-period debt, and on whether nominal rigidities bind or not. We analyze this condition under one additional assumption, which simplifies the characterization of terminal-period prices:

Assumption 1 *We restrict our analysis to a setting in which nominal wage rigidities never bind in F in the terminal period.*

We consider two different cases, depending on whether countries in H default or repay debt in the initial period. First, suppose that countries in H default in the initial period:

Proposition A.1 *The zero lower bound on the nominal interest rate binds upon initial-period default if the inflation target of the monetary authority is below the endogenous threshold $\bar{\pi}^*_D$:*

$$\pi^* < \bar{\pi}^*_D \equiv \bar{\pi}^*_{D,FE}\left(\frac{c_{N,H,1}}{l}\right)^{(1-a)(1-\psi)}, \tag{81}$$

where

$$\bar{\pi}^*_{D,FE} \equiv \beta\left(\frac{y_{\mathfrak{H}}}{y_{\mathfrak{L}}}\right)^{1-(1-a)\psi}\frac{\omega\left(y_{\mathfrak{H}}-\hat{\zeta}\right)^{(1-a)(1-\psi)}+(1-\omega)\left(y_{\mathfrak{H}}\right)^{(1-a)(1-\psi)}}{\left(y_{\mathfrak{L}}-\zeta_1\right)^{(1-a)(1-\psi)}}. \tag{82}$$

The threshold for the inflation target under which the zero lower bound binds is the combination of two terms. The first is given by the threshold that would apply in the absence of binding nominal rigidities, $\bar{\pi}^*_{D,FE}$. This term depends on the relative endowment growth in H and F, as the consumption profiles of the two countries in autarky determine the real interest rate and the expected inflation rate. The second term is a function of the severity of nominal rigidities in H in the initial period. When nominal rigidities bind in the initial period, the terminal-period T-good price inflation associated with a given target π^* is higher. Hence, the equilibrium nominal interest rate is higher as well. Therefore, even if the zero lower bound binds when nominal rigidities are not present, nominal rigidities may imply that the monetary authority is unconstrained in its action. This is the case since, in equilibrium, a higher T-good price inflation causes an increase in the nominal interest rate consistent with the target of the monetary authority.

Second, consider now the case where countries in H repay debt in the initial period:

Proposition A.2 *The zero lower bound on the nominal interest rate binds upon initial-period repayment if the inflation target of the monetary authority is below the endogenous threshold* $\pi^*_R(b_{H,1})$*:*

$$\pi^* < \bar{\pi}^*_R(b_{H,1}) \equiv \bar{\pi}^*_{R,FE}(b_{H,1}) \left(\frac{c_{N,H,1}(b_{H,1})}{y_N}\right)^{(1-a)(1-\psi)}, \tag{83}$$

where

$$\bar{\pi}^*_{R,FE}(b_{H,1}) = \beta \left[\omega + (1-\omega)\left(\frac{y_{\mathfrak{L}}}{c_{T,F,1}(b_{H,1})}\right)^{\psi(1-a)-1}\left(\frac{y_{\mathfrak{H}}}{c_{T,H,1}(b_{H,1})}\right)^{(1-\psi)(1-a)}\right]. \tag{84}$$

Again, two terms compose the threshold for the inflation target under which the zero lower bound binds. The first term is the relevant threshold in the absence of nominal rigidities, $\bar{\pi}^*_{R,FE}(b_{H,1})$. This term depends on the implicit real interest rate on risk-free assets and on expected T-good price inflation in this setting. In turn, the risk-free real interest rate is decreasing in the quantity of terminal-period default risk $1-\omega$ and in the severity of such risk, given by the fall in consumption accruing to F, $c_{T,F,1}(b_{H,1})/y_{\mathfrak{L}}$. Inflation in the price of good T is instead decreasing in average growth of consumption of good T in H and F. The second term is identical to the one already discussed in reference to the setting under initial-period default. The presence of nominal rigidities in the initial period increases the nominal interest rate and thus it relaxes the severity of the zero lower bound as a constraint on the monetary authority.

A.3 The Zero Lower Bound and Debt Repayment

This appendix clarifies the conditions under which the zero lower bound enforces repayment of sovereign debt in the initial period. Subsection 4.1.4 presents our main result. Here we show what are the conditions on parameters under which the result holds.

Conditions on the Parameters for the Zero Lower Bound to Enforce Repayment. We introduce one additional assumption that, in conjunction with Assumption 1, allows us to describe analytically the conditions on parameters under which equation (69) holds.

Assumption 2 *Income growth in F is negative and sufficiently low:*

$$\left(\frac{y_{\mathfrak{L}}}{y_{\mathfrak{H}}}\right) \leq \left(\frac{y_{\mathfrak{H}-\hat{\zeta}}}{y_{\mathfrak{L}} - \zeta_1}\right)^{\frac{(1-a)(1-\psi)}{1-(1-a)\psi}}. \tag{85}$$

This assumption ensures that consumption growth in F is relatively low conditional on initial period default, so that the real interest rate on risk-free assets is low as well. In addition, Assumption 1 allows us to conveniently characterize consumption of good N as a log-linear function of the nominal interest rate.

From the Euler equation for nominal risk-free assets in F, (7), we can derive the log-linear function describing N-good consumption in H as a function of the nominal interest rate. The expressions are as follows:

$$c_{N,H,D} = A_D \left(1 + \mathfrak{i}_D\right)^{-\frac{1}{((1-a)(1-\psi))}}, \tag{86}$$

$$c_{N,H,R}\left(\bar{b}_{\widehat{NF}}\right) = A_R\left(\bar{b}_{\widehat{NF}}\right)\left(1 + \mathfrak{i}_R\left(\bar{b}_{\widehat{NF}}\right)\right)^{-\frac{1}{((1-a)(1-\psi))}}, \tag{87}$$

where we denote by $\mathfrak{i}_D$ and $\mathfrak{i}_R\left(\bar{b}_{\widehat{NF}}\right)$ the equilibrium nominal interest rates that hold upon initial period default and repayment, respectively, and

$$A_D \equiv \frac{y_{\mathfrak{H}} - \hat{\zeta}}{y_{\mathfrak{L}-\zeta_1}} l \left\{ \frac{\beta}{\pi^*} \left(\frac{y_{\mathfrak{H}}}{y_{\mathfrak{L}}}\right)^{1-(1-a)\psi} \left[\omega + (1-\omega)\left(\frac{y_{\mathfrak{H}}}{y_H - \hat{\zeta}}\right)^{(1-a)(1-\psi)}\right]\right\}^{\frac{1}{(1-a)(1-\psi)}} \tag{88}$$

and

$$A_R\left(\bar{b}_{\widehat{NF}}\right) \equiv l \left(\frac{\beta}{\pi^*}\right)^{\frac{1}{(1-a)(1-\psi)}}$$
$$\left[\omega + (1-\omega)\left(\frac{y_{\mathfrak{H}}}{c_{T,H,R,NZ}\left(\bar{b}_{\widehat{NF}}\right)}\right)^{(1-a)(1-\psi)} \left(\frac{c_{T,F,R,NZ}\left(\bar{b}_{\widehat{NF}}\right)}{y_{\mathfrak{H}}}\right)^{1-(1-a)\psi}\right]^{\frac{1}{(1-a)(1-\psi)}}. \tag{89}$$

When the zero lower bound binds, the nominal interest rate equals zero. The ratio of consumption of good N in the presence and in the absence of the zero lower bound can be written as:

$$\frac{c_{N,H,D,Z}}{c_{N,H,D,NZ}} = \min\left\{\left(1 + \mathfrak{i}_{D,NZ}\right)^{\frac{1}{(1-a)(1-\psi)}}, 1\right\}, \tag{90}$$

$$\frac{c_{N,H,R,Z}\left(\bar{b}_{\widehat{NF}}\right)}{c_{N,H,R,NZ}\left(\bar{b}_{\widehat{NF}}\right)} = \min\left\{\left(1 + \mathfrak{i}_{R,NZ}\left(\bar{b}_{\widehat{NF}}\right)\right)^{\frac{1}{(1-a)(1-\psi)}}, 1\right\}, \tag{91}$$

where $\mathfrak{i}_{D,NZ}$ and $\mathfrak{i}_{R,NZ}\left(\bar{b}_{\widehat{NF}}\right)$ are the nominal interest rates that would hold in equilibrium in the absence of the zero lower bound. Hence, condition (69) can be expressed as a condition on nominal interest rates in the absence of the zero lower bound:

$$\min\left\{\mathfrak{i}_{R,NZ}\left(\bar{b}_{\widehat{NF}}\right), 0\right\} \geq \min\left\{\mathfrak{i}_{D,NZ}, 0\right\}. \tag{92}$$

To express the condition on the nominal interest rates as one on parameters of the model economy, it is convenient to define the following combinations of equilibrium quantities and parameters:

$$\tilde{\alpha} \equiv \left(\frac{c_{N,R,NLZB}\left(\bar{b}_{\widehat{NF}}\right)}{c_{N,D,NZLB}}\right)^{(1-a)(1-\psi)} \left(\frac{y_{\mathfrak{L}}}{y_{\mathfrak{H}}}\right)^{1-(1-a)\psi} \left(\frac{y_{\mathfrak{L}}-\zeta_1}{y_{\mathfrak{H}}-\hat{\zeta}}\right)^{(1-a)(1-\psi)} < 1 \tag{93}$$

$$\tilde{\beta} \equiv \left(\frac{c_{T,F,R,NZ}\left(\bar{b}_{\widehat{NF}}\right)}{y_{\mathfrak{L}}}\right)^{1-(1-a)\psi} \left(\frac{y_{\mathfrak{H}}}{c_{T,H,R,NZ}\left(\bar{b}_{\widehat{NF}}\right)}\right)^{(1-a)(1-\psi)} \tag{94}$$

$$\tilde{\gamma} \equiv \left(\frac{y_{\mathfrak{H}}}{y_{\mathfrak{H}}-\hat{\zeta}}\right)^{(1-a)(1-\psi)} \tag{95}$$

Condition (92) is satisfied when the terminal period repayment probability ω is sufficiently high:

$$\omega \geq \bar{\omega} \equiv \frac{\tilde{\alpha}\tilde{\beta}-\tilde{\gamma}}{\tilde{\alpha}\tilde{\beta}-\tilde{\gamma}+1-\tilde{\alpha}}, \tag{96}$$

where $\tilde{\alpha}\tilde{\beta}-\tilde{\gamma}>0$ and $\tilde{\alpha}<1$, meaning that $\bar{\omega}\in(0,1)$.[40]

B Appendix on Optimal Sovereign Default

B.1 Terminal-Period Default Problem

The terminal-period default decision of the subnational fiscal authorities is formally defined as follows:

$$\max_{D_2} c_{T,i,j,2} = y_{T,i,2} - \left[D_2\zeta_2 - (1-D_2)\, b_{i,j,2}\right] + \frac{1}{p_{T,2}}\left(b_{M,i,j,2} + w_{i,2}l_{i,2} + p_{N,i,2}c_{N,i,j,2}\right), \tag{99}$$

[40] It is possible to show that $\tilde{\alpha}\tilde{\beta}-\tilde{\gamma}>0$. This combination of parameters can be expressed as follows:

$$\tilde{\alpha}\tilde{\beta}-\tilde{\gamma} = \left(\frac{y_{\mathfrak{H}}}{y_{\mathfrak{H}}-\hat{\zeta}}\right)^{(1-a)(1-\psi)} \left[\left(\frac{c_{N,H,R,NZ}\left(\bar{b}_{\widehat{NF}}\right)}{c_{N,H,D,NZ}}\right)^{(1-a)(1-\psi)} \left(\frac{y_{\mathfrak{L}-\zeta_1}}{c_{T,H,R,NZ}\left(\bar{b}_{\widehat{NF}}\right)}\right)^{(1-a)(1-\psi)} \left(\frac{c_{T,F,R,NZ}\left(\bar{b}_{\widehat{NF}}\right)}{y_{\mathfrak{H}}}\right)^{1-(1-a)\psi} - 1\right]. \tag{97}$$

The sign of this expression depends exclusively on the sign of the term in the square parentheses. This term equals:

$$\begin{cases} \left[\left(\frac{y_{\mathfrak{L}-\zeta_1}}{c_{T,H,R,NZ}\left(\bar{b}_{\widehat{NF}}\right)}\right)^{(1-a)(1-\psi)} \left(\frac{c_{T,F,R,NZ}\left(\bar{b}_{\widehat{NF}}\right)}{y_{\mathfrak{H}}}\right)^{1-(1-a)\psi} - 1\right] > 0 & \text{if nominal rigidities never bind,} \\ \left(\frac{c_{T,F,R,NZ}\left(\bar{b}_{\widehat{NF}}\right)}{y_{\mathfrak{H}}}\right)^{\frac{a}{(1-a)\psi+a}} - 1 > 0 & \text{if they bind under default and repayment,} \\ \left(\frac{p_{N,D}}{\kappa}\right)^{\frac{(1-a)(1-\psi)}{a+(1-a)\psi}} \left(\frac{c_{T,F,R,NZ}\left(\bar{b}_{\widehat{NF}}\right)}{y_{\mathfrak{H}}}\right)^{\frac{a}{(1-a)\psi+a}} - 1 > 0 & \text{otherwise.} \end{cases} \tag{98}$$

where D_2 is an indicator policy that takes the value of unity in the event of default.

The default policy function associated with this problem is formally defined as follows:

$$D_2 = \begin{cases} 1 \text{ if } \zeta_2 < -b_{i,j,2}, \\ 0 \text{ otherwise.} \end{cases} \tag{100}$$

The subnational fiscal authority finds it optimal to default when debt is higher than the output cost of default.

Given the process for default costs (18), default occurs with certainty if debt issued is above the highest possible realization of the process: $b_{i,j,2} < -\hat{\zeta}$. Only if the subnational fiscal authority carries positive assets $b_{i,j,2} \geq 0$ into the terminal period, default never occurs. For debt in the range $b_{i,j,2} \in \left[-\hat{\zeta}, 0\right)$ default occurs with probability $1-\omega$ in the terminal period, when default costs are low: $\zeta_2 = 0$.

The interest rate associated with debt issued by the subnational fiscal authority in the initial period compensates lenders in F for default risk. The interest rate is determined as follows

$$\frac{1}{1+\mathfrak{r}(b_{i,j,2})} = \begin{cases} 0 & \text{if } b_{i,j,2} < -\hat{\zeta} \\ \beta\omega \frac{c_{T,F,1}}{c_{T,F,2,\hat{\zeta}}} & \text{if } b_{i,j,2} \in \left[-\hat{\zeta}, 0\right) \\ \beta\left[\omega \frac{c_{T,F,1}}{c_{T,F,2,\hat{\zeta}}} + (1-\omega)\frac{c_{T,F,1}}{c_{T,F,2,0}}\right] & \text{if } b_{i,j,2} \geq 0 \end{cases} \tag{101}$$

where $c_{T,F,2,\hat{\zeta}}$ and $c_{T,F,2,0}$ denote consumption of good T in countries in F in the two states of the world where default costs equal $\hat{\zeta}$ or 0, respectively. In equilibrium, $c_{T,F,2,\hat{\zeta}} = c_{T,F,1}$ and $c_{T,F,2,0} = y_{\mathfrak{L}}$ so that $\frac{1}{1+\mathfrak{r}(b_{i,j,2})} = \beta\omega$ when $b_{i,j,2} \in \left[-\hat{\zeta}, 0\right)$.

B.2 Subnational Fiscal Authority

Default Threshold of Subnational Fiscal Authority. The comparison of the two values in (40) and (41) implies that the default thresholdof the subnational fiscal authority is implicitly defined as follows:

$$(1+\beta\omega)\log\left(\frac{y_{\mathfrak{L}} + \beta\omega y_{\mathfrak{H}} + \bar{b}_{SN}}{1+\beta\omega}\right) = \log\left(y_{\mathfrak{L}} - \zeta_1\right) + \beta\omega\log\left(y_{\mathfrak{H}} - \hat{\zeta}\right) \tag{102}$$

The expression in (102) yields the threshold (39) of Proposition 4.1.

In addition, this expression implies that T-good consumption under repayment is equal to the geometric average of consumption under default, when the subnational fiscal authority is indifferent between default and repayment:

$$c_{T,H,R} = \frac{y_{\mathfrak{L}} + \beta\omega y_{\mathfrak{H}} + \bar{b}_{SN}}{1+\beta\omega} = \left(y_{\mathfrak{L}} - \zeta_1\right)^{\frac{1}{1+\beta\omega}}\left(y_{\mathfrak{H}} - \hat{\zeta}\right)^{\frac{\beta\omega}{1+\beta\omega}} \tag{103}$$

Note that terminal-period consumption of T good in the low default-cost state can be ignored, as consumption in that state is identical independently of whether initial period debt is repaid or defaulted upon. In the former case, default occurs in the terminal period in that state. In the latter case, households suffer from the zero default cost because of the initial-period default.

B.3 National Fiscal Authority

B.3.1 Values and Default Threshold

Values of Default and Repayment. The value associated with default by the national fiscal authority in H, (26), can be expressed as follows, after imposing that consumption of good T equals endowments net of default costs (32), the intratemporal choice by households (6), and equilibrium in the market for good N in both periods (33):

$$\begin{aligned} V^D_{NF,H} =& a\left[\log\left(y_{\mathfrak{L}}-\zeta_1\right)+\beta\omega\log\left(y_{\mathfrak{H}}-\hat{\zeta}\right)+\beta\left(1-\omega\right)\log\left(y_{\mathfrak{H}}\right)\right]+ \\ & (1-a)\left[\log\left(c_{N,H,1}\right)+\beta\log\left(c_{N,H,2}\right)\right] \\ \text{s.t. } & c_{N,H,1}=\min\left\{\frac{1-a}{a}\frac{p_{T,1}}{\kappa}\left(y_{\mathfrak{L}}-\zeta_1\right),l\right\}, \\ & c_{N,H,2}=y_{N,H,2}=l. \end{aligned} \tag{104}$$

Nominal rigidities never bind in H in the terminal period. In the initial period, they bind when the price of good T is low, so that demand for good N is also low: $p_{T,1}<\tilde{p}_{T,D}=\frac{l}{y_{\mathfrak{L}}-\zeta_1}\frac{a}{1-a}\kappa$. When this is the case, the value of default can be expressed as follows:

$$\begin{aligned} V^D_{NF,H} =& \log\left(y_{\mathfrak{L}}-\zeta_1\right)+a\left[\beta\omega\log\left(y_{\mathfrak{H}}-\hat{\zeta}\right)+\beta\left(1-\omega\right)\log\left(y_{\mathfrak{H}}\right)\right]+ \\ & (1-a)\left[\log\left(\frac{1-a}{a}\frac{p_{T,1}}{\kappa}\right)+\beta\log\left(l\right)\right]. \end{aligned} \tag{105}$$

When nominal rigidities do not bind in H upon default, i.e. when $p_{T,1}\geq\tilde{p}_{T,D}$, the value of default is:

$$\begin{aligned} V^D_{NF,H} =& a\left[\log\left(y_{\mathfrak{L}}-\zeta_1\right)+\beta\omega\log\left(y_{\mathfrak{H}}-\hat{\zeta}\right)+\beta\left(1-\omega\right)\log\left(y_{\mathfrak{H}}\right)\right]+ \\ & (1-a)\left(1+\beta\right)\log\left(l\right). \end{aligned} \tag{106}$$

This value is identical to the one for the subnational fiscal authority under full employment, $V^D_{SN,FE}$, which follows from (40) after imposing $l_{H,1}=l_{H,2}=l$. Given this value, it is easy to derive the expression in (47).

The value associated with repayment by the national fiscal authority defined in (27) can also be expressed as a function of equilibrium quantities and prices after imposing that consumption of good T follows from the intertemporal allocation (22), optimal terminal-period default and, again, the intratemporal choice by households (6), and equilibrium in the market for good N in both periods (33):

$$\begin{aligned} V^R_{NF,H} =& a\left[(1+\beta\omega)\log\left(\frac{y_{\mathfrak{L}}+\beta\omega y_{\mathfrak{H}}-b_{H,1}}{1+\beta\omega}\right)+\beta(1-\omega)\log(y_{\mathfrak{H}})\right]+ \\ & (1-a)\left[\log(c_{N,H,1})+\beta\log(c_{N,H,2})\right] \\ \text{s.t. } & c_{N,H,1}=\min\left\{\frac{1-a}{a}\frac{p_{T,1}}{\kappa}\frac{y_{\mathfrak{L}}+\beta\omega y_{\mathfrak{H}}-b_{H,1}}{1+\beta\omega},l\right\}, \\ & c_{N,H,2}=y_{N,H,2}=l. \end{aligned} \tag{107}$$

When nominal rigidities do not bind— i.e. when $p_{T,1}\geq\tilde{p}_{T,R}(b_{H,1})$, the economy is at full employment, $c_{N,H,1}=l$ and the value is identical to the of the subnational fiscal authority $V^R_{SN,FE}(b_{H,1})$, after imposing $l_{H,1}=l_{H,2}=l$. Otherwise, when $p_{T,1}<\tilde{p}_{T,R}(b_{H,1})$, the ratio of $c_{N,H,1}$ to l can be expressed as the ratio $p_{T,1}/\tilde{p}_{T,R}(b_{H,1})$. From these results, it is again easy to derive the expression for the value of repayment (48).

Default Threshold of National Fiscal Authority. The default threshold for the national fiscal authority when nominal rigidities bind both under default and under repayment (52) can be expressed explicitly as:

$$\bar{b}_{NF,NR}=(1+\beta\omega)\left[(y_{\mathfrak{L}}-\zeta_1)^{\frac{1}{1+a\beta\omega}}\left(y_{\mathfrak{H}}-\hat{\zeta}\right)^{\frac{a\beta\omega}{1+a\beta\omega}}\right]-(y_{\mathfrak{L}}+\beta\omega y_{\mathfrak{H}}). \tag{108}$$

This expression follows from (52) after imposing the definition of $\tilde{p}_{T,R}\left(\bar{b}_{NF,NR}\right)$ and $\tilde{p}_{T,D}$. The key difference between this default threshold and the one for the subnational fiscal authority (39) lies in the weights in the geometric average term in square brackets. In this instance, the national fiscal authority places a higher weight on initial-period consumption. This is the case because consumption of good T not only delivers welfare directly, but also by expanding demand for good N and, in the presence of nominal rigidities, its production. Given the assumption in (23), at this threshold, initial-period consumption upon repayment is lower than upon default, as Corollary 4.1.1 states for the subnational fiscal authority. Hence, $\tilde{p}_{T,R}\left(\bar{b}_{NF,NR}\right)>\tilde{p}_{T,D}$ follows, and $\bar{b}_{NF,NR}>\bar{b}_{SN}$.

When nominal rigidities only bind upon repayment, the default threshold (109) can be expressed explicitly as follows, after imposing the definition of $\tilde{p}_{T,R}\left(\bar{b}_{NF,FE-D}\right)$:

$$\bar{b}_{NF,FE-D}(p_{T,1})=(1+\beta\omega)\left[(y_{\mathfrak{L}}-\zeta_1)^{\frac{a}{1+a\beta\omega}}\left(\frac{a}{1-a}\frac{\kappa l}{p_{T,1}}\right)^{\frac{1-a}{1+a\beta\omega}}\left(y_{\mathfrak{H}}-\hat{\zeta}\right)^{\frac{a\beta\omega}{1+a\beta\omega}}\right]-(y_{\mathfrak{L}}+\beta\omega y_{\mathfrak{H}}). \tag{109}$$

The second term in the geometric average shows that the relative value of default is decreasing in the price of good T. The higher this price, the lower is the severity of nominal rigidities that bind under repayment. Hence, the lower is the additional gain from defaulting that comes from relaxing nominal rigidities. Finally, given the assumption in (23) and given $p_{T,1}<\tilde{p}_{T,D}$, default takes place at lower levels of debt than for the subnational fiscal authority: $\bar{b}_{NF,FE-D}(p_{T,1})>\bar{b}_{SN}$.

B.3.2 Multiplicity and Absence of Equilibria

When the national fiscal authorities of the countries in H take the decision to default or repay external debt, multiple equilibria or no equilibria may arise for some levels of initial assets, in certain regions of the parameter space. Two key forces lie behind the result. First, the binary nature of the default-repayment decision implies that the equilibrium price of good T may differ depending on whether countries in H default or repay, given the objective of the monetary authority. Second, the decision to default or to repay of national fiscal authorities depends on the price of good T in the initial period. In turn this price depends on the action of the other countries in H, giving rise to the possibility of multiple equilibria in the presence of strategic complementarity, and of absence of equilibria if the actions of the countries in H are strategic substitutes.

First, we analyze the implications of default and repayment on the equilibrium price of good T. We focus attention on the setting where nominal rigidities would bind in H upon repayment of debt, but not upon default. In addition, we will consider two extreme cases for the objective of monetary policy. First, suppose that the monetary policy authority only cares about prices in countries in H in its objective—i.e. $\psi = 0$. When this is the case, the equilibrium price of good T in the initial period is higher when countries in H repay debt than when they default, for a given target p_1^*. To see this, note that when $\psi = 0$:

$$p_1^* = p_{T,1}^a p_{NH,1}^{1-a} \tag{110}$$

and that upon default $p_{NH,1} > \kappa$, while $p_{NH,1} = \kappa$, upon repayment. Second, suppose instead that he monetary policy authority only cares about prices in countries in F in its objective—i.e. $\psi = 1$. In this instance the equilibrium price of good T in the initial period is higher when countries in H default on debt, for a given target p_1^*. The objective of monetary policy reduces to

$$p_1^* = p_{T,1} \left(\frac{1-a}{a} \frac{c_{TF,1}}{l} \right)^{1-a} \tag{111}$$

Since default by countries in H reduces T-good consumption in F, the monetary authority implements a higher price of good T to achieve its objective.

Second, consider the interaction between the implications of default on prices and the optimal decision to default by an individual country in H. Consider first the case where $\psi = 1$. Suppose H-countries' assets $b_{H,1}$ are such that they are indifferent between default and repayment, given the target of monetary policy p_1^*, and given the price of T good that would arise if they all defaulted. Formally:

$$b_{H,1} = \bar{b}_{NF,FE-D} \left(p_{T,1,D} \right) \text{ and } p_{T,1,D} = p_1^* \left(\frac{1-a}{a} \frac{y_{\mathfrak{H}}}{l} \right)^{-(1-a)} \tag{112}$$

Suppose now that one individual country's assets were just above this threshold. Given the target of monetary policy, if all other countries defaulted, it would be optimal for this country to repay. If all countries had this level of assets, they would thus all repay debt. However, the price of good T would be lower than $p_{T,1,D}$ and it would no longer be optimal to repay. Countries would thus default instead. Given the monetary policy objective, the actions of the identical countries in H are strategic substitutes, and given the binary nature of the default decision, no equilibrium exists in a region of initial-period asset levels.

Consider now the case where the monetary policy authority only cares about countries in H, $\psi = 0$, and again, H-countries' assets $b_{H,1}$ are such that they are indifferent between default and repayment, given the target of monetary policy p_1^*, and given the price of T good that would arise if they all defaulted. Again, $b_{H,1} = \bar{b}_{NF,FE-D}(p_{T,1,D})$, but

$$p_{T,1,D} = p_1^* \left(\frac{1-a}{a} \frac{y_{\mathfrak{L}-\zeta_1}}{l} \right)^{-(1-a)}. \tag{113}$$

Suppose that one individual country's assets were just below the threshold $\bar{b}_{NF,FE-D}(p_{T,1,D})$. Given the target of monetary policy, if all other countries defaulted, it would be optimal for this country to default as well. However, if all other countries repaid debt instead, the price of good T would be higher, and it would be optimal for this country to repay as well. Given the monetary policy objective, the actions of the identical countries in H are strategic complements, and multiple equilibria exist in a region of initial-period asset levels.

The region of asset levels where multiple or no equilibria arise depends on the weight ψ in the monetary policy objective. Whenever this weight ψ implies that the price of good T differs depending on default or repayment by countries in H, no equilibria or multiple equilibria can emerge, depending on whether the price of good T is higher upon default or repayment, respectively. This is only the case when prices imply that nominal rigidities bind in H upon repayment but not upon default. When nominal rigidities bind in both cases, or in neither cases, a unique equilibrium emerges.

B.4 Coalition of National Fiscal Authorities

B.4.1 Values of Default and Repayment

We evaluate the value associated with default by the coalition of national fiscal authorities (30) after imposing, crucially, the equilibrium determination of prices given the monetary authority objective (13) and market clearing for good T, (32), i.e. that consumption of good T equals endowments net of default costs, the intratemporal choice by households

(6), and equilibrium in the market for good N (33) in both periods:

$$
\begin{aligned}
V^D_{\widehat{NF},H} =& a\left[\log\left(y_{\mathfrak{L}}-\zeta_1\right)+\beta\omega\log\left(y_{\mathfrak{H}}-\hat{\zeta}\right)+\beta\left(1-\omega\right)\log\left(y_{\mathfrak{H}}\right)\right]+\\
&(1-a)\left[\log\left(c_{N,H,1}\right)+\beta\log\left(c_{N,H,2}\right)\right]\\
\text{s.t. } & c_{N,H,1}=\min\left\{\frac{1-a}{a}\frac{p_{T,1}}{\kappa}\left(y_{\mathfrak{L}}-\zeta_1\right),l\right\},\\
& c_{N,H,2}=y_{N,H,2}=l,\\
& p_{T,1}=\left[p_1^*\left(\frac{1}{\kappa}\right)^{(1-\psi)(1-a)}\left(\frac{a}{1-a}\frac{l}{c_{T,F,1}}\right)^{\psi(1-a)}\right]^{\frac{1}{a+\psi(1-a)}},\\
& c_{T,F,1}=y_{\mathfrak{H}}.
\end{aligned}
\tag{114}
$$

Nominal rigidities bind in H when the monetary authority targets a low average price in the monetary union: $p_1^*<p_{1,D}^*$. When this is the case, the value of default can be expressed as follows:

$$
\begin{aligned}
V^D_{\widehat{NF},H} =& \log\left(y_{\mathfrak{L}}-\zeta_1\right)+a\left[\beta\omega\log\left(y_{\mathfrak{H}}-\hat{\zeta}\right)+\beta\left(1-\omega\right)\log\left(y_{\mathfrak{H}}\right)\right]+\\
&(1-a)\left[\log\left(\frac{1-a}{a}\frac{p_{T,1}}{\kappa}\right)+\beta\log\left(l\right)\right]\\
\text{s.t. } & p_{T,1}=\left[p_1^*\left(\frac{1}{\kappa}\right)^{(1-\psi)(1-a)}\left(\frac{a}{1-a}\frac{l}{y_{\mathfrak{H}}}\right)^{\psi(1-a)}\right]^{\frac{1}{a+\psi(1-a)}}.
\end{aligned}
\tag{115}
$$

When nominal rigidities do not bind and the coalition of national fiscal authorities defaults on debt, the value is given by $V^D_{SN,FE}$. Given these results, we can easily derive the expression in (58).

We can also express the value of repayment for the coalition of national fiscal authorities (29) as a function of equilibrium quantities and prices after imposing the equilibrium determination of prices given T-good market clearing (32), that consumption of good T follows from the intertemporal allocation (22), optimal terminal-period default (99), the intratemporal choice by households (6), and equilibrium in the market for good N in both periods (33)

$$
\begin{aligned}
V^{R}_{\widehat{NF},H} =& a\left[(1+\beta\omega)\log\left(\frac{y_{\mathfrak{L}}+\beta\omega y_{\mathfrak{H}}+b_{H,1}}{1+\beta\omega}\right)+\beta(1-\omega)\log(y_{\mathfrak{H}})\right]+\\
&(1-a)\left[\log(c_{N,H,1})+\beta\log(c_{N,H,2})\right]\\
\text{s.t. } & c_{N,H,1}=\min\left\{\frac{1-a}{a}\frac{p_{T,1}}{\kappa}\frac{y_{\mathfrak{L}}+\beta\omega y_{\mathfrak{H}}+b_{H,1}}{1+\beta\omega},l\right\},\\
& c_{N,H,2}=y_{N,H,2}=l,\\
& p_{T,1}=\left[p_1^*\left(\frac{1}{\kappa}\right)^{(1-\psi)(1-a)}\left(\frac{a}{1-a}\frac{l}{c_{T,F,1}}\right)^{\psi(1-a)}\right]^{\frac{1}{a+\psi(1-a)}},\\
& c_{T,F,1}=\frac{y_{\mathfrak{H}}+\beta\omega y_{\mathfrak{L}}-b_{H,1}}{1+\beta\omega}.
\end{aligned}
\tag{116}
$$

Nominal rigidities do not bind in H, conditionally on repayment of debt, when the price-level target of the monetary authority is sufficiently high: $p_1^*>p_{1,R}^*(b_{H,1})$. In this instance, the value of repayment to the coalition is given by the value for the subnational fiscal authority at full employment, $V^{R}_{SN,FE}$. Otherwise, when nominal rigidities bind in H, the value of repayment can be expressed as:

$$
\begin{aligned}
V^{R}_{\widehat{NF},H} =& (1+a\beta\omega)\log\left(\frac{y_{\mathfrak{L}}+\beta\omega y_{\mathfrak{H}}+b_{H,1}}{1+\beta\omega}\right)+a\beta(1-\omega)\log(y_{\mathfrak{H}})+\\
&(1-a)\left[\log\left(\frac{1-a}{a}\frac{p_{T,1}}{\kappa}\right)+\beta\log(l)\right]\\
\text{s.t. } p_{T,1}=&\left[p_1^*\left(\frac{1}{\kappa}\right)^{(1-\psi)(1-a)}\left(\frac{a}{1-a}\frac{l(1+\beta\omega)}{y_{\mathfrak{H}}+\beta\omega y_{\mathfrak{L}}-b_{H,1}}\right)^{\psi(1-a)}\right]^{\frac{1}{a+\psi(1-a)}}.
\end{aligned}
\tag{117}
$$

Hence, given these results, we derive the expression in (58).

B.4.2 Default Threshold of Coalition of National Fiscal Authorities

When nominal rigidities bind both under repayment and under default, the optimal default decision of the coalition follows from the comparison of the values of default and repayment (57) and (58), respectively. The threshold that summarizes this default decision is given by (62), and it can alternatively be expressed as:

$$
\begin{aligned}
\bar{b}_{\widehat{NF},NR} =& (1+\beta\omega)\left[(y_{\mathfrak{L}}-\zeta_1)\left(y_{\mathfrak{H}}-\hat{\zeta}\right)^{a\beta\omega}\left(\frac{y_{\mathfrak{H}}+\beta\omega y_{\mathfrak{L}}-\bar{b}_{\widehat{NF},NR}}{y_{\mathfrak{H}}(1+\beta\omega)}\right)^{\frac{\psi(1-a)^2}{a+\psi(1-a)}}\right]^{\frac{1}{1+a\beta\omega}}\\
&-(y_{\mathfrak{L}}+\beta\omega y_{\mathfrak{H}}).
\end{aligned}
\tag{118}
$$

The term $\left(\frac{y_{\mathfrak{H}}+\beta\omega y_{\mathfrak{L}}-\bar{b}_{\widehat{NF},NR}}{y_{\mathfrak{H}}(1+\beta\omega)}\right)$ is the ratio of consumption of good T in F across the two cases of repayment and default by the countries in H, respectively. The term enters the definition of the default threshold due to its effects on the nominal price of good T in the world economy. Consumption in F is higher under repayment, so that this ratio is larger than unity.[41] Hence, when H defaults it engineers a fall in consumption in the countries in F, where nominal rigidities do not bind. This consumption fall is deflationary, and it induces the monetary authority to aim for a higher price of good T to achieve its price-level target. In turn, the higher price of good T is beneficial for countries in H, as it helps relaxing nominal rigidities. Hence, when countries in H take the reaction of monetary policy into account, they perceive default to be relatively more attractive, because of its beneficial expansionary effect on monetary policy. Note that for monetary policy to be able to have an expansionary reaction to default, it must be the case that the monetary authority is not constrained by limits to its action such as the zero lower bound.

We prove here that the threshold $\bar{b}_{\widehat{NF},NR}$ defined by (118) is lareger (lower in absolute value) than the one of the individual national fiscal authority, $\bar{b}_{NF,NR}$, defined by (108). From comparing the two definitions, the following relationship holds:

$$\bar{b}_{\widehat{NF},NR} > \bar{b}_{NF,NR} \leftrightarrow \frac{y_{\mathfrak{H}}+\beta\omega y_{\mathfrak{L}}-\bar{b}_{\widehat{NF},NR}}{(1+\beta\omega)} > y_{\mathfrak{H}}. \tag{119}$$

The condition $\frac{y_{\mathfrak{H}}+\beta\omega y_{\mathfrak{L}}-\bar{b}_{\widehat{NF},NR}}{(1+\beta\omega)} > y_{\mathfrak{H}}$ implies that T-good consumption in F would be higher if countries in H repaid debt at the threshold rather than if they defaulted. Hence, it implies that implies that consumption in H would be lower than the endowment $y_{\mathfrak{L}}$ if these countries repaid debt:

$$\frac{y_{\mathfrak{L}}+\beta\omega y_{\mathfrak{H}}+\bar{b}_{\widehat{NF},NR}}{(1+\beta\omega)} < y_{\mathfrak{L}} \tag{120}$$

We prove that (120) holds by contradiction. Suppose that the condition does not hold. The assumption in (23) implies that $(y_{\mathfrak{L}}-\zeta_1)^{\frac{1}{1+a\beta\omega}}\left(y_{\mathfrak{H}}-\hat{\zeta}\right)^{\frac{a\beta\omega}{1+a\beta\omega}} < y_{\mathfrak{L}}$. Hence, if (120) is violated, it must be the case that $\left(\frac{y_{\mathfrak{H}}+\beta\omega y_{\mathfrak{L}}-\bar{b}_{\widehat{NF},NR}}{y_{\mathfrak{H}}(1+\beta\omega)}\right) > 1$. But this is only the case if (120) holds, which is a contradiction. Hence, $\bar{b}_{\widehat{NF},NR} > \bar{b}_{NF,NR}$.

Finally, consider the case where nominal rigidities only bind under repayment. In this setting, the default threshold is given by (64) and it can alternatively be expressed as

[41] The assumption in (23) ensures this result, as stated in Corollary 4.1.1.

follows:

$$\bar{b}_{\widehat{NF},FE-D} = (1+\beta\omega)\cdot\left[(y_{\mathfrak{L}}-\zeta_1)^a\left(y_{\mathfrak{H}}-\hat{\zeta}\right)^{a\beta\omega}\left(l\frac{a}{1-a}\frac{\kappa}{p_{T,1,R}}\right)^{1-a}\right]^{\frac{1}{1+a\beta\omega}}$$
$$-\left(y_{\mathfrak{L}}+\beta\omega y_{\mathfrak{H}}\right),$$
$$\text{where } p_{T,1,R} = \left[p_1^*\left(\frac{1}{\kappa}\right)^{(1-\psi)(1-a)}\left(\frac{a}{1-a}\frac{l\,(1+\beta\omega)}{y_{\mathfrak{H}}+\beta\omega y_{\mathfrak{L}}-\bar{b}_{\widehat{NF},FE-D}}\right)^{\psi(1-a)}\right]^{\frac{1}{a+\psi(1-a)}}. \tag{121}$$

The price $p_{T,1,R}$ is defined as the price that arises in equilibrium when all countries in H repay debt $\left|\bar{b}_{\widehat{NF},FE-D}\right|$ in the initial period, and the target of the monetary authority is given by p_1^*. This threshold is analogous to the one under always binding nominal rigidities, (118) with the key difference that the benefits from reducing consumption in F to inflate the price of good T are limited when nominal rigidities do not bind upon default. Hence, default occurs for a higher level of debt in this instance. It is easy to show that this threshold is identical to the one that holds for the individual national fiscal authority when nominal rigidities only bind upon repayment, (109). When all countries in H repay debt $\left|\bar{b}_{\widehat{NF},FE-D}\right|$, the price $p_{T,1}$ in (109) is given by $p_{T,1,R}$ in (121). Hence, the thresholds of individual and coalesced national fiscal authorities are equal.

B.5 Welfare in F under Repayment and Default

This appendix details the conditions on parameters under which countries in F prefer H to repay debt rather than default on it, in the initial period. While it may seem intuitive that F prefers H to repay, this may not the case when repayment by H in the initial period, followed by default in the terminal period generates detrimental inflation dynamics that lead to a large fall in N-good consumption in F. We can thus show that a relatively high probability of terminal-period repayment ω ensures that welfare of F is higher under repayment by H.

We compare welfare in F across the two situations where H defaults or repays in the initial period. Initial-period consumption of good N and terminal period, low-default-cost consumption of good T are identical across the two cases. For simplicity, we consider welfare for a given target p_1^*. Welfare in either case would be negligibly lower for a marginally higher price target.

Upon default by H, consumption of good T in the initial period and terminal-period, high-default cost state are given by the endowments $y_{\mathfrak{L}}$ and $y_{\mathfrak{H}}$, respectively. Upon repayment, they are instead equal, and given by $c_{F,R}$, which depends on the amount of assets held by F.

Consider a setting where nominal rigidities are always binding in F in the terminal period. Consumption of good N in the terminal period crucially depends on the dynamics of good-T inflation:

$$c_{N,F,2,RR} = \frac{l}{\kappa}\pi_{T,RR},\ c_{N,F,2,RD} = \frac{l}{\kappa}\pi_{T,RR}\frac{y_{\mathfrak{L}}}{c_{F,R}},$$
$$c_{N,F,2,DR} = \frac{l}{\kappa}\pi_{T,DR}\frac{y_{\mathfrak{L}}}{y_{\mathfrak{H}}},\ c_{N,F,2,DD} = \frac{l}{\kappa}\pi_{T,DD}\frac{y_{\mathfrak{L}}}{y_{\mathfrak{H}}}.$$

where the four subscripts RR,RD,DR,DD indicate initial-period repayment/default and terminal period high-low default-cost states.

The comparison in welfare between default and repayment can be expressed as follows:

$$\begin{aligned} V_{D,F} - V_{R,F} = &\{a\left[\log\left(y_{\mathfrak{H}}\right) + \beta\omega\log\left(y_{\mathfrak{L}}\right)\right] \\ &+\beta\left(1-a\right)\left[\log\left(\frac{y_{\mathfrak{L}}}{y_{\mathfrak{H}}}\right) + \omega\log\left(\pi_{T,DR}\right) + \left(1-\omega\right)\log\left(\pi_{T,DD}\right)\right]\Bigg\} \\ &-\left\{a\left[\log\left(c_{F,R}\right)\left(1+\beta\omega\right)\right]\right. \\ &+\beta\left(1-a\right)\left[\omega\log\left(\pi_{T,RR}\right) + \left(1-\omega\right)\log\left(\pi_{T,RD}\frac{y_{\mathfrak{L}}}{c_{F,R}}\right)\right]\Bigg\}. \end{aligned}$$

Separating welfare derived from T and N-goods:

$$\begin{aligned} V_{D,F} - V_{R,F} = &a\left\{\log\left(y_{\mathfrak{H}}\right) + \beta\omega\log\left(y_{\mathfrak{L}}\right) - \log\left(c_{F,R}\right)\left(1+\beta\omega\right)\right\} \\ &+\beta\left(1-a\right)\left\{\omega\log\left(\frac{\pi_{T,DR}}{\pi_{T,RR}}\right) + \left(1-\omega\right)\log\left(\frac{\pi_{T,DD}}{\pi_{T,RD}}\right)\right. \\ &\left.\omega\log\left(y_{\mathfrak{L}}\right) - \log\left(y_{\mathfrak{H}}\right) + \left(1-\omega\right)\log\left(c_{F,R}\right)\right\}. \end{aligned}$$

It can be shown that, in equilibrium, the two inflation ratios are identical and given by:

$$\log\left(\frac{\pi_{T,DR}}{\pi_{T,RR}}\right) = \log\left(\frac{\pi_{T,DD}}{\pi_{T,RD}}\right) = x\log\left(\frac{y_{\mathfrak{H}}}{c_{F,R}}\right),$$

where x is a function of parameters:

$$x = \frac{\left(1-a\right)^2\psi_H\psi_F}{\left(a+\left(1-a\right)\psi_F\right)\left(a+\left(1-a\right)\psi_H\right)}.$$

Finally, this welfare comparison can be simplified as:

$$\begin{aligned} V_{D,F} - V_{R,F} = &a\left\{\left(1+\beta\omega\right)\log\left(\frac{y_{\mathfrak{H}}}{c_{F,R}}\right) + \beta\omega\log\left(\frac{y_{\mathfrak{L}}}{y_{\mathfrak{H}}}\right)\right\} \\ &+\beta\left(1-a\right)\left\{\left(x-\left(1-\omega\right)\right)\log\left(\frac{y_{\mathfrak{H}}}{c_{F,R}}\right) + \omega\log\left(\frac{y_{\mathfrak{L}}}{y_{\mathfrak{H}}}\right)\right\}. \end{aligned}$$

Since, in equilibrium, $y_{\mathfrak{H}} < c_{F,R}$, $V_{D,F} < V_{R,F}$ holds whenever $\omega \geq 1-x$. This is a sufficient condition, and weaker conditions on parameters would also ensure the result. Hence, welfare in F is higher when countries in H repay debt, when the probability of terminal-period repayment ω is sufficiently high.