

# Dynamics of Secured and Unsecured Debt Over the Business Cycle

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**Abstract** Firms have heterogeneous debt structure. High-credit-quality firms rely almost exclusively on unsecured debt and borrow with lower leverage ratios than low-credit-quality firms. In this paper, we develop a tractable macroeconomic model featuring debt heterogeneity. Unsecured credit rests on the value that borrowers attach to a good credit track record. We argue that borrowers and lenders are more cautious in the unsecured debt market, so high-credit-quality firms have lower leverage ratios. Moreover, our model generates procyclical unsecured debt and acyclical secured debt, consistent with the US data. Our model with heterogeneous debt has a smaller amplification effect than a model featuring secured debt only.

**JEL Codes:** E32, E44, G32

**Key Words:** Secured debt, unsecured debt, corporate debt structure, financial accelerator

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## 1. Introduction

How do financial frictions affect business cycle fluctuations? Standard macro-finance models often assume a uniform debt structure where collateralized credit is the main channel that propagates and amplifies shocks. While this approach may help in building tractable theoretical models, it ignores the fact that firms are financed by different types of debt. In the cross section, [Rauh and Sufi \(2010\)](#) finds that a substantial fraction of a firm’s external debt financing is based on unsecured debt. Over the business cycle, [Azariadis, Kaas and Wen \(2016\)](#) find that while secured debt barely moves together with output, the unsecured part of firm debt is strongly procyclical.<sup>1</sup>

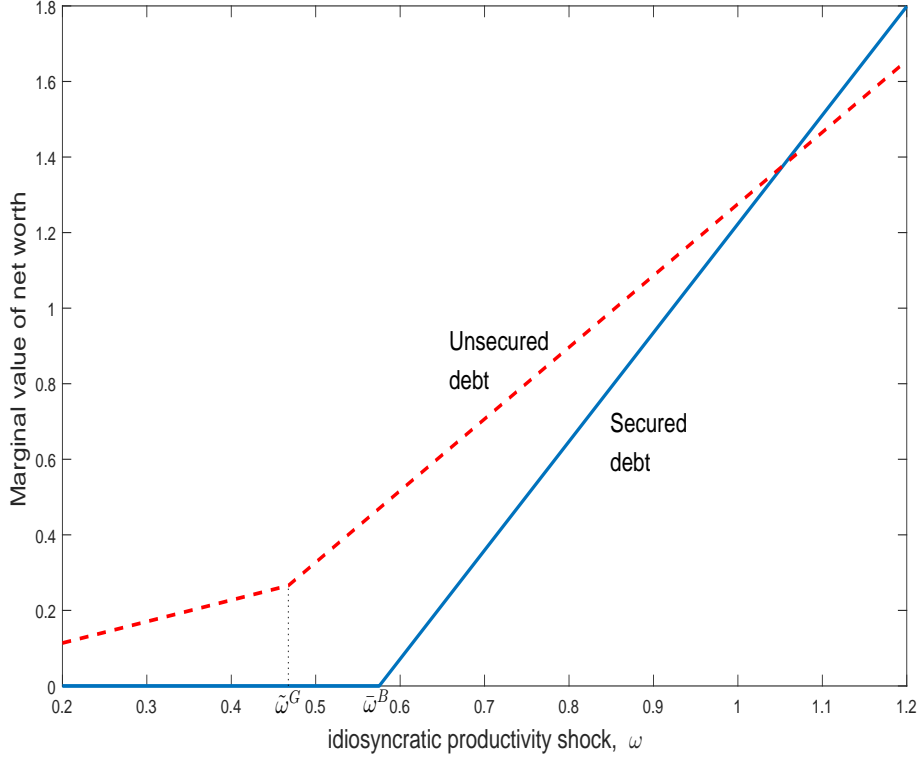
Given this evidence, the goal of this paper is to understand whether, and how, the conclusions of standard models of financial frictions change when firms have access to different debt instruments. We address two specific questions. First, what drives the cyclical behavior of firm’s secured and unsecured debt? Second, how do the amplification effects generated by financial frictions change when the inclusion of unsecured debt is considered.

To answer these questions, we cast debt heterogeneity into a tractable dynamic stochastic general equilibrium model. In the model, firms borrow secured or unsecured debt subject to an idiosyncratic productivity shock and a costly-state-verification problem similar to [Bernanke, Gertler and Gilchrist \(1999\)](#) (henceforth BGG). In secured borrowing, the creditor can recover a fraction of a firm’s assets in the event of default. In unsecured borrowing, the creditor receives no payment in the event of a default while the borrower may keep a fraction of revenue and keep operating. Lenders can observe firms’ track record which evolves endogenously, and a firm that has defaulted before is excluded from unsecured credit in the future. In equilibrium, firms with a good track record ( $G$  firms) borrow unsecured debt only and enjoy a higher franchise value than firms with a bad record ( $B$  firms).

We show that  $G$  firms operate with lower leverage than  $B$  firms, consistent with the US data. This is because both lenders and borrowers in the unsecured debt market are more cautious than in the secured debt market. Unsecured debt lenders are more cautious because they receive no payment when a borrower defaults. [Figure 1](#) illustrates why borrowers of unsecured debt are more cautious too. The blue solid (red dashed) line shows the marginal value of net worth of a secured (unsecured) debt borrower conditional on a realization of the idiosyncratic shock  $\omega$  in the optimal contract. For each contract, the borrower defaults when the realization of shock is below a threshold, as indicated by the flatter line segment in each line. When a secured debt borrower defaults, the borrower’s firm value is zero. But when a unsecured debt borrower defaults, its expected firm value remains positive because the borrower may keep some revenue and keep operating in subsequent periods. Borrowers

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<sup>1</sup>In this paper, we use ‘collateralized debt’ and ‘secured debt’ interchangeably.



**Fig. 1.** Plot of marginal value of net worth against the realized idiosyncratic shock  $\omega$ . Analysis is based on the steady state under the calibration in Section 5. The blue solid line shows the value function for bad ( $B$ ) firms which borrow secured debt. The red dashed line shows the value function for good ( $G$ ) firms which borrow unsecured debt.

in both contracts have a risk-shifting incentive because they enjoy the upside risk above the face value of their debt and leave the creditors to bear the downside risk. Since the red dashed line is less ‘convex’ than the blue solid line, borrowers of unsecured debt care more about the downside risk and less about the upside risk, so they are more cautious than borrowers of secured debt. But since lenders and borrowers in the unsecured debt market are more cautious, borrowers of unsecured debt ( $G$  firms) have a lower leverage and default less often.

Second, under reasonable parameterization, unsecured debt is more procyclical than secured debt in our model. Consider a negative TFP or financial shock which reduces the current stock of capital and increases the expected returns on capital. Since both lenders and borrowers of unsecured debt are more cautious, the shock induces a bigger increase in the leverage ratio in  $B$  firms than  $G$  firms. Moreover, since  $B$  firms increase their leverage ratio by more when the expected return is higher, the reputation of being a  $G$  firm becomes less valuable in a downturn, and, ceteris paribus, they default more often. As a result, lenders have an incentive to cut their lending disproportionately on unsecured debt to  $G$  firms. These effects lead to a more positive correlation between output and unsecured debt

than secured debt. Our simulation based on a calibrated model using US corporate debt data can generate strongly procyclical unsecured debt and weakly procyclical secured debt, and these results match our empirical findings based on actual US data.

An important implication of our paper is that the introduction of unsecured debt weakens the financial accelerator effect in BGG. The financial accelerator effect exists in both secured and unsecured contracts. Since the steady-state leverage ratio of unsecured debt borrowers is lower than that of secured debt, and unsecured debt borrowers default less often, an economy with a larger fraction of unsecured debt has lower aggregate leverage and less volatile macroeconomic fluctuations. Our simulation results suggest that this dampening effect is quantitatively important. For example, in response to a one standard deviation negative productivity shock, financial frictions in the BGG model amplify the fall in investment by about 56% (relative to a frictionless RBC model) one year after the shock, but our model with heterogeneous debt amplifies the fall only by 40%. Furthermore, the initial falls in aggregate net worth and debt in the BGG model are 44% and 47% larger than our model respectively. Overall, these results suggest that the standard one-sector BGG model may overstate the amplification effects of a financial accelerator mechanism.

Finally, we consider several extensions of the model with more realistic features in the firm sector. We allow for (1) positive recovery ratios for creditors of unsecured debt; (2) exogenous upgrading of credit ratings; (3) predetermined productivity differences in the firm sector; and (4) a mixed debt structure in low-credit-rating firms. Our key mechanism still exists and unsecured debt remains more procyclical than secured debt in each of these extensions.

Our paper is related to two strands of literature. First, this paper is related to a vast literature incorporating financial frictions into macroeconomic models. This paper adopts a costly state verification approach because it is straightforward to endogenize default. See [Carlstrom and Fuerst \(1997\)](#), [Bernanke, Gertler and Gilchrist \(1999\)](#), [Christiano, Motto and Rostagno \(2014\)](#) and [Nuno and Thomas \(2017\)](#). By contrast, default is eliminated as an equilibrium outcome in models in which financial frictions arise due to limited enforcement problems (see for example [Kiyotaki and Moore \(1997\)](#), [Meh and Moran \(2010\)](#), [Jermann and Quadrini \(2012\)](#) and [Gertler and Karadi \(2011\)](#)).

Second, there is a large theoretical literature on corporate debt structure, following [Diamond \(1991\)](#), [Besanko and Kanatas \(1993\)](#) and [Boot and Thakor \(1997\)](#). This literature focuses on the determinants of a firm's financing based on bank debt versus corporate bonds. For instance, [Diamond \(1991\)](#) argues that high credit quality firms have good reputations allowing them to avoid the additional costs of bank debt associated with monitoring. Our model is in this spirit. [Chemmanur and Fulghieri \(1994\)](#), [Bolton and Freixas \(2000\)](#) and [De Fiore and Uhlig \(2011\)](#) argue that banks have an information advantage about a firm's

profitability. Such information is particularly useful for assessing the risk of low-quality borrowers. Empirically, [Denis and Mihov \(2003\)](#) find that credit quality is a major determinant of a firm's debt structure, with higher credit quality firms choosing public debt and lower quality firms choosing bank loans. [Rauh and Sufi \(2010\)](#) show that high credit quality firms rely exclusively on unsecured debt; whereas low credit quality firms rely more on secured debt. This literature, however, does not study the macroeconomic effects of corporate debt structure.

A few papers discuss debt structure and its relation to the macroeconomy. [De Fiore and Uhlig \(2015\)](#) assume that bank monitoring yields useful information about relatively low productivity firms. They find that the flexibility in substituting alternative instruments by firms reduces macroeconomic volatility. In [Crouzet \(2017\)](#), firms borrow partly through banks because banks are more flexible in debt restructuring. The paper argues that since bond finance cannot be restructured in the future, firms switching from bank finance to bond finance will deleverage, which worsens the negative macroeconomics effects of a shock to the banking sector. Our paper addresses different aspects of debt choice by studying secured versus unsecured debt to explain the puzzle about the cyclicity of these debts. In terms of aggregate implications, we emphasize that due to different payoff structures, unsecured debt borrowers have lower leverage. Therefore, in an economy with a large fraction of unsecured debt, the amplification due to financial accelerator mechanism is weaker.

The work of [Azariadis, Kaas and Wen \(2016\)](#) is most relevant to ours. Their model features multiple equilibria brought by unsecured debt and relies on sunspot shocks to generate persistent and highly volatile dynamics of macroeconomic variables. They argue that fluctuations in unsecured debt, but not in secured debt, are driven by sunspot shocks, and that sunspot shocks account for around half of output volatility. In this paper, we show that the nature of secured and unsecured debt contracts implies that borrowers and lenders of unsecured debt are more cautious, and therefore the leverage ratios of unsecured debt borrowers are less volatile. Our simulation results demonstrate that even with only fundamental shocks, our endogenous mechanism can account for the relative procyclicality of unsecured debt observed in US data.

The rest of the paper is organized as follows. Section two provides empirical analysis. Section three describes the credit contracts. Section four embeds the debt contracts in a DSGE model. Section five describes calibration of the model. Section six discusses the model properties and quantitative results. Section seven compares the benchmark model with a standard BGG model. Section eight discuss four extensions to our benchmark models. Section nine concludes.

## 2. Empirical analysis

In this section, we present important stylized facts about firm capital and debt structures. Our main findings can be summarized as follows:

1. Debt structure is closely related with firm’s credit quality. High-credit-quality firms rely almost exclusively on unsecured debt while low-credit-quality firms have a substantial share of secured debt.
2. A firm’s leverage is countercyclical and there is huge heterogeneity among leverage ratios across credit quality distributions. In particular, high-credit-quality firms operate with relatively low leverage while low-credit-quality firm use higher leverage.
3. Unsecured and secured debt show different dynamics along the business cycle: unsecured debt is strongly procyclical, while secured debt is at best weakly procyclical.

We begin with the description of data and variables in our sample. The sampling universe includes public traded non-financial and non-utility U.S. firms included in Compustat with a long-term issuer credit rating in the last one year from 1981 to 2017.<sup>2</sup> There are 1142 rated firms in the sample. In line with [Azariadis, Kaas and Wen \(2016\)](#), we use the item “mortgages and other secured debt” to measure secured debt. We then attribute the difference between “long term debt + total current debt” and “mortgages and other secured debt” to unsecured debt. To clean the data, we remove those firm-year observations where any of the variables is missing, negative, or secured debt exceeds total debt. We also winsorized all firm-level variables at 1% and 99% levels to remove outliers.

We measure leverage as the sum of long-term debt and total current debt divided by total assets. Panel A of Table 1 shows summary statistics for the leverage of firms in the sample. Rated firm-year observations have a mean leverage ratio of 1.74 and a negative correlation with contemporaneous GDP -0.15. The dynamics of observed leverage for all observations over the business cycle is summarized in Column 4. The results show countercyclical dynamics for the average firm, with a correlation between leverage and GDP of -0.37, consistent with the findings of [Halling, Yu and Zechner \(2016\)](#). Panel B shows the leverage ratios across credit quality distributions. Interestingly, we observe that leverage stays low for firms with high credit ratings and jumps to more than 2.0 for firms rated CCC and below, implying a big difference in a firm’s financing choice and capital structure.

Next we focus on how debt structure varies across the credit-quality distribution. Figure 2 plots the time series of unsecured debt share by credit rating. On average, 75% of rated firms’

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<sup>2</sup>Coverage by Capital IQ is comprehensive only from 2001 onwards, therefore we restrict our main sample to Compustat. This allows us to have long enough sample periods to calculate correlations and other business cycle moments.

total debt financing comes from unsecured debt, implying a non-negligible role of unsecured debt in firm credit. Moreover, there is debt heterogeneity. Unsecured debt constitutes a substantial part of high credit quality firms' debt financing and is much lower for firms with low credit ratings. In particular, the share of unsecured debt for BBB and above rated firms ranges from 0.75 to 0.90. In contrast, it drops down to around 0.6 for BB+ and below rated firms. Note that the difference in unsecured debt share between high and low credit rating is smaller than what is found by [Rauh and Sufi \(2010\)](#). One reason for this is that Compustat is biased towards large public firms which have greater access to bond markets and other forms of unsecured debt financing. Therefore we explore the debt information for private firms in Capital IQ as well. [Figure 2](#) shows the time series of the unsecured debt share for samples obtained from Compustat and Capital IQ. Once private firms are included, the disparity in unsecured debt shares based on credit ratings increases substantially. For instance, the average unsecured debt share in Capital IQ for BBB firms is 0.81, which is higher than 0.49 for B- firms. Moreover, the differences in debt structure widens over time after 2000, represented by a sharply declining use of unsecured debt by low credit rating firms and a steadily increasing use of unsecured debt by high credit rating firms.

In line with the previous literature, the time series variation shows that unsecured debt plays a much stronger role in output dynamics than secured debt. We deflate the annual time series from Compustat by the gross value added index for business (a price index constructed by the Bureau of Economic Analysis), and detrend all series using HP filter (smoothing parameter = 100). As shown in [Table 2](#), the contemporaneous correlation between output and unsecured debt is 0.48 and but only 0.06 for secured debt. While our sample focuses on firms that are credit-rated, the vast majority of U.S. firms are not. To complement, we also compute the cyclical properties for all firms regardless of credit rating. The correlation between output and unsecured debt is 0.50 and s 0.15 for secured debt, similar to the result obtained from our main sample, suggesting that the results are robust.

The empirical findings above confirms [Azariadis, Kaas and Wen \(2016\)](#)'s key result that unsecured firm credit is more procyclical than secured credit. This finding suggests that macro-finance models should not only analyze secured credit, but also look at unsecured credit. In the next session, we build a model that features both secured and unsecured debt contracts. We show that by taking into account debt heterogeneity, the model can explain the stylized facts from US data.

**Table 1**

Summary statistics on leverage.

<i>Panel A: Sample Summary Statistics on Leverage</i>			
Rated Only		All Observations	
Mean	Correlation with GDP	Mean	Correlation with GDP
1.78	-0.15	1.83	-0.37

<i>Panel B: Leverage Ratios Across Quality Distribution</i>			
	Leverage Ratio		Leverage Ratio
AA and above	1.53	B- and below	1.95
BBB and above	1.62	CCC and below	2.13
BBB- and above	1.65	CC and below	2.31

This table reports summary statistics of firm leverage. Statistics are calculated for the Compustat sample of U.S. rated firms and all firms (both rated and non-rated) in Panel A. Panel B summarizes the leverage ratios across credit ratings.

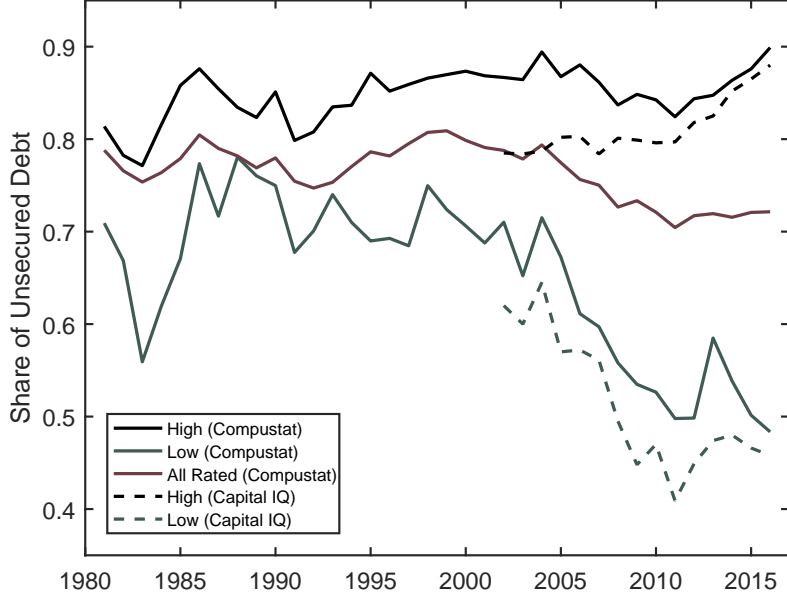
**Table 2**

Debt volatilities and correlations with GDP.

	Rated Only		All Observations	
	Std. Deviation	Corr. with GDP	Std. Deviation	Corr. with GDP
Secured Debt	10.16	0.06	10.07	0.15
Unsecured Debt	13.94	0.48	15.29	0.50

This table reports the standard deviations and contemporaneous correlations of debt with GDP. The left panel shows rated firms only. The right panel shows all firms (both rated and non-rated). GDP is deflated by the GDP deflator. Debt is deflated by business gross valued index. All series have been logged and HP filtered with  $\lambda = 100$ .





**Fig. 2.** This figure shows the share of unsecured debt for public and private U.S. firms by credit rating. (Compustat, 1981-2016 and Capital IQ, 2001-2016)

### 3. Model – credit contracts

In the firm sector, there is a unit measure of firms  $j \in [0, 1]$ . Each firm carries a publicly observed label  $i \in \{G, B\}$  which denotes high and low credit quality respectively.<sup>3</sup> The label may change over time and we discuss how the label determines a firm’s borrowing options later. Firms produce with the following Cobb-Douglas production function:

$$Y_{jt}^i = A_t(\omega_{jt}K_{jt-1}^i)^\alpha(L_{jt}^i)^{1-\alpha}, \quad (1)$$

where  $A_t$  denotes the TFP of the firm sector, and  $\omega_{jt}$  is an idiosyncratic shock to a firms’ capital quality. The idiosyncratic capital quality shock follows a log-normal distribution with mean 1 and variance  $\sigma_{t-1}^2$ , i.e.,  $\log(\omega_{jt}) \sim N(-\frac{1}{2}\sigma_{t-1}^2, \sigma_{t-1}^2)$ . The cumulative distribution function is  $F(\omega_t; \sigma_{t-1})$ . The idiosyncratic shock is independent across firms and time, and orthogonal to aggregate shocks.

In period  $t - 1$ , a firm with label  $i$  purchases capital  $K_{jt-1}^i$  at the price  $Q_{t-1}$ . At the beginning of period  $t$ , the firm faces an idiosyncratic productivity shock, so effective capital becomes  $\omega_{jt}K_{jt-1}^i$ . The firm then hires labor, produces and sells depreciated capital to capital

<sup>3</sup>Note that there is no intrinsic difference between firms with different credit quality, however. This assumption is relaxed in Section 8.3.

producing firms. The marginal product of capital  $r_t^K$  is defined such that  $r_t^K(\omega_{jt}K_{jt-1}^i) \equiv \max_{L_{jt}^i} \{Y_{jt}^i - w_t L_{jt}^i\}$ .<sup>4</sup> The optimal choice of labor requires  $w_t L_{jt}^i = (1 - \alpha)Y_{jt}^i$ , which implies that all firms have the same labor to output ratio. It is helpful to define the average return on capital of the firm sector as:

$$R_t^K \equiv \frac{r_t^K + (1 - \delta)Q_t}{Q_{t-1}}. \quad (2)$$

The return on capital of firm  $j$  is given by  $\omega_{jt}R_t^K$ .

A type- $i$  firm has net worth  $N_{jt-1}^i$  in period  $t - 1$ . It borrows  $B_{jt-1}^i$  from investors with one-period risky debt contracts to finance its purchase of capital. Each loan contract is subject to financial frictions because, as in BGG, lenders do not observe the realization of  $\omega_{jt}$ . However, lenders observe the label of the firms. So, financial contracts available to a firm depends on the firm's label. Consistent with stylized fact 1, we assume that  $G$  firms and  $B$  firms have different debt structures.  $G$  firms can issue secured and unsecured debt, but  $B$  firms can only issue secured debt.<sup>5</sup>

Let us describe secured debt contracts (issued by  $B$  firms) first. A secured debt contract is similar to that in BGG. If a  $B$  firm defaults, lenders have access to the firm's asset. Similar to [De Fiore and Uhlig \(2011\)](#), we assume that when a  $B$  firm borrows secured debt, lenders conduct initial monitoring of the firm which costs the firm a fraction  $\kappa$  of its net worth  $N_{jt-1}^B$ . So, lending to the firm is  $B_{jt-1}^B = Q_{t-1}K_{jt-1}^B - (1 - \kappa)N_{jt-1}^B$ . The optimal contract may be characterized by a gross non-default loan rate,  $Z_{jt}^B$ , and a default threshold,  $\bar{\omega}_{jt}^B$ , where

$$\bar{\omega}_{jt}^B R_t^K Q_{t-1} K_{jt-1}^B = Z_{jt}^B B_{jt-1}^B. \quad (3)$$

When  $\omega_{jt} \geq \bar{\omega}_{jt}^B$ , the firm repays the promised amount  $Z_{jt}^B B_{jt-1}^B$ . If  $\omega_{jt} < \bar{\omega}_{jt}^B$ , the firm goes bankrupt. The lender monitors the firm and the net receipt of the lender is  $(1 - \mu)\omega_{jt}R_t^K Q_{t-1} K_{jt-1}^B$ , where  $\mu$  is a linear default cost. The payoff structure of secured debt is summarized in [Table 3](#).

**Table 3**

Payoff structure of secured debt.

	Defaults: ( $\omega_{jt} < \bar{\omega}_{jt}^B$ )	Does not default: ( $\omega_{jt} \geq \bar{\omega}_{jt}^B$ )
$B$ firm	Goes bankrupt.	Repays debt and keeps profit.
Lender	Gets liquidation value of the firm.	Receives repayment.

We now turn to unsecured debt contracts. A  $G$  firm issues unsecured debt costlessly.

<sup>4</sup>This means that  $r_t^K \equiv \alpha A_t [(1 - \alpha)A_t/w_t]^{(1-\alpha)/\alpha}$ .

<sup>5</sup>In [Section 8.4](#), we consider an extension in which  $B$  firms use a mixed debt structure.

We restrict our attention to a case in which the cost advantage  $\kappa$  is large enough so that  $G$  firms issue unsecured debt only. The unsecured debt issued is given by  $B_{jt-1}^G = Q_{t-1}K_{jt-1}^G - N_{jt-1}^G$ . The firm promises a gross non-default loan rate  $Z_{jt}^G$ . We can similarly define a cutoff threshold  $\bar{\omega}_{jt}^G$ , where

$$\bar{\omega}_{jt}^G R_t^K Q_{t-1} K_{jt-1}^G = Z_{jt}^G B_{jt-1}^G. \quad (4)$$

When  $\omega_{jt} < \bar{\omega}_{jt}^G$ , the  $G$  firm defaults. When  $\omega_{jt} \geq \bar{\omega}_{jt}^G$ , a  $G$  firm may choose to repay the promised amount  $Z_{jt}^G B_{jt-1}^G$  or default. If a  $G$  firm defaults, it undergoes debt restructuring. With probability  $\zeta$ , debt restructuring is successful and the firm retains  $(1 - \mu)\omega_{jt}R_t^K Q_{t-1}K_{jt-1}^G$ , but it loses its  $G$  label and becomes a  $B$  firm in future.<sup>6</sup> With probability  $(1 - \zeta)$ , debt restructuring is unsuccessful, the firm shuts down and has nothing left. Whenever a  $G$  firm defaults, lenders do not receive anything.<sup>7</sup> Without loss of generality, assume that the  $G$  firm chooses to default when  $\omega_{jt} < \tilde{\omega}_{jt}^G$ , where  $\tilde{\omega}_{jt}^G \geq \bar{\omega}_{jt}^G$ , then the payoff structure of unsecured debt is summarized in Table 4.

**Table 4**

Payoff structure of unsecured debt.

	Defaults: ( $\omega_{jt} < \tilde{\omega}_{jt}^G$ )	Does not default: ( $\omega_{jt} \geq \tilde{\omega}_{jt}^G$ )
$G$ firm	With Prob= $\zeta$ , keeps assets and becomes $B$ firm; With Prob = $1 - \zeta$ , gets nothing.	Repays debt and keeps profit.
Lender	Gets nothing.	Receives repayment.

We discuss why the contracts above correspond to secured and unsecured contracts. We can interpret the BGG contract as a secured debt contract in which a  $B$  firm uses its entire stock of assets as collateral. Lenders give the firm a menu of options: if the firm borrows with a higher loan-to-value ratio, it faces a higher contractual interest rate. The firm will choose amongst these pairs of loan-to-value ratios and contractual rates to maximize its expected future value.<sup>8</sup> In the loan repayment phase, if the borrower fails to repay, lenders liquidate the collateral which is the remaining value of the firm. In unsecured debt contracts, the assumption that lenders receive no payment in a default event follows from [Azariadis, Kaas and Wen \(2016\)](#) and [Cui and Kaas \(2017\)](#). One interpretation is that a defaulting  $G$  firm liquidates its assets and the owner starts a new firm.

Perfectly-competitive investors lend in both secured and unsecured debt markets, and they break even in every state of the world as in BGG. For each firm with type  $i$  and net

<sup>6</sup>In the data, high credit quality is positively correlated with a firm's historical productivity. This is reflected in our model because the precedence of a credit downgrade implies that  $B$  firms on average have lower historical productivity.

<sup>7</sup>We relax this assumption in Section 8.2.

<sup>8</sup>[Kiyotaki and Moore \(1997\)](#) is a special case in which the loan-to-value ratio is inelastic.

worth  $N_{jt}^i$ , lenders offer a menu of debt and cutoff values (or contractual interest rates  $Z_{jt}^i$ ) which satisfies the lenders' break-even condition. The lenders' participation constraint in the secured debt market (for  $B$  firms) is:

$$R_t^K Q_{t-1} K_{jt-1}^B \left[ \int_{\bar{\omega}_{jt}^B} \bar{\omega}_{jt}^B dF_{t-1} + (1 - \mu) \int^{\bar{\omega}_{jt}^B} \omega dF_{t-1} \right] \geq R_{t-1} B_{jt-1}^B. \quad (5)$$

In this participation constraint, the first integral on the left hand side corresponds to borrowers who experience a shock  $\omega_{jt} \geq \bar{\omega}_{jt}^B$  and repay their debts. The second integral refers to borrowers who experience a shock  $\omega_{jt} < \bar{\omega}_{jt}^B$  and default. Therefore, the left hand side is lenders' average return. The right hand side is the risk-free return ( $R_{t-1}$ ) on loans.

The lenders' participation constraint in the unsecured debt market (for  $G$  firms) is:

$$R_t^K Q_{t-1} K_{jt-1}^G \left( \int_{\bar{\omega}_{jt}^G} \bar{\omega}_{jt}^G dF_{t-1} \right) \geq R_{t-1} B_{jt-1}^G, \quad (6)$$

Even although lenders cannot observe which  $G$  firms default strategically (i.e. default when the firm can repay), they take strategic default into account in the break-even condition. Moreover, we assume that in every state of the world, some  $G$  firms repay their debt, and so lenders always break even. In this paper we do not consider a potential bad equilibrium in which all  $G$  firms default and there is no reputation value of being a  $G$  firm.<sup>9</sup>

We assume that in each period, there is an exogenous probability  $(1 - \theta)$  that a firm exits.<sup>10</sup> When it exits the remaining wealth is transferred to the households. We denote the value of a firm with label  $i \in \{G, B\}$  and net worth  $N_{jt}^i$  as  $V_t^i(N_{jt}^i)$ . A  $B$  firm chooses  $(K_{jt}^B, \bar{\omega}_{jt+1}^B)$  to maximize the following value function:

$$\begin{aligned} & V_t^B(N_{jt}^B) \\ = & \max_{K_{jt}^B, \bar{\omega}_{jt+1}^B} E_t \Lambda_{t,t+1} \int_{\bar{\omega}_{jt+1}^B} \{ \theta V_{t+1}^B [(\omega - \bar{\omega}_{jt+1}^B) R_{t+1}^K Q_t K_{jt}^B] + (1 - \theta)(\omega - \bar{\omega}_{jt+1}^B) R_{t+1}^K Q_t K_{jt}^B \} d\mathbb{P}_t \end{aligned}$$

where  $\Lambda_{t,t+1}$  is the stochastic discount factor. In a given period, if a  $B$  firm draws  $\omega > \bar{\omega}_{jt+1}^B$ , it settles its debt repayment and has  $(\omega - \bar{\omega}_{jt+1}^B) R_{t+1}^K Q_t K_{jt}^B$  unit of net worth. If the firm exits (with probability  $(1 - \theta)$ ), the net worth is transferred to households. If the firm does not exit (with probability  $\theta$ ), it will operate in period  $t + 1$  with its net worth which has a value of  $V_{t+1}^B [(\omega - \bar{\omega}_{jt+1}^B) R_{t+1}^K Q_t K_{jt}^B]$ .

<sup>9</sup>This equilibrium is analyzed by Cui and Kaas (2017), Azariadis, Kaas and Wen (2016) and Gu, Mattesini, Monnet and Wright (2013).

<sup>10</sup>Following Carlstrom and Fuerst (1997), BGG and Gertler and Karadi (2011), this assumption prevents firms from growing out of their financial constraints.

A  $G$  firm chooses  $(K_{jt}^G, \bar{\omega}_{jt+1}^G, \tilde{\omega}_{jt+1}^G)$  to maximize the following value function:

$$V_t^G(N_{jt}^G) = \max_{K_{jt}^G, \bar{\omega}_{jt+1}^G} E_t \Lambda_{t,t+1} \int \max \{V_{jt+1}^{G,ND}, V_{jt+1}^{G,D}\} dF_t, \quad (8)$$

where  $V_{jt+1}^{G,ND}$  is the value of repaying the debt and  $V_{jt+1}^{G,D}$  is the value of defaulting. Here,  $V_{jt+1}^{G,ND}$  is given by:

$$V_{jt}^{G,ND} = \theta V_t^G [(\omega - \bar{\omega}_{jt}^G) R_t^K Q_{t-1} K_{jt-1}^G] + (1 - \theta)(\omega - \bar{\omega}_{jt}^G) R_t^K Q_{t-1} K_{jt-1}^G, \quad (9)$$

where the first term corresponds to the value of the firm if it keeps operating, and the second term corresponds to remaining assets of an exiting firm. If a firm chooses to default, it pays a default cost  $\mu$ , and learns whether it can keep operating (with probability  $\zeta$ ). Hence,  $V_{jt+1}^{G,D}$  is given by:

$$V_{jt}^{G,D} = \theta \zeta V_t^B [(1 - \mu) \omega R_t^K Q_{t-1} K_{jt-1}^G] + (1 - \theta) \zeta (1 - \mu) \omega R_t^K Q_{t-1} K_{jt-1}^G. \quad (10)$$

To summarize, a  $B$  firm maximizes its value (7) subject to the participation constraint (5) in the secured debt market. A  $G$  firm maximizes its value (8) subject to the participation constraint (6) in the unsecured debt market.

We guess the value functions are given by  $V_t^i(N_{jt}^i) = \lambda_t^i N_{jt}^i$  for  $i \in \{G, B\}$ , where  $\lambda_t^G, \lambda_t^B$  are the marginal values of net worth in a  $G$  firm and a  $B$  firm respectively. We require that  $\lambda_t^G > \lambda_t^B > 1$  for all  $t$ . The first equality ensures that  $G$  firms have no incentives to borrow in the secured debt market, and the second ensures that firms prefer operating until they quit by default or exit.<sup>11</sup>

The following proposition states the solution of the optimal financial contracting problem.<sup>12</sup>

**Proposition 1.** *Suppose initial monitoring costs  $\kappa$  are such that  $\lambda^G > \lambda^B > 1$ , where  $\lambda^G, \lambda^B$  are the steady-state values of  $\lambda_t^G, \lambda_t^B$  respectively.<sup>13</sup> The equilibrium dynamics of the credit contracts in the neighborhood of the deterministic steady state is characterized by the following features:*

<sup>11</sup>We check that these conditions are satisfied in our numerical exercise.

<sup>12</sup>All proofs are given in [Appendix B](#).

<sup>13</sup>To be precise, we require:

$$1 - \left[ \frac{1 - F(\bar{\omega}^B) - \mu \bar{\omega}^B f(\bar{\omega}^B)}{1 - F(\bar{\omega}^B)} \right] > \kappa > 1 - \left[ \frac{1 - F(\tilde{\omega}^G)}{1 - F(\tilde{\omega}^G) - \tilde{\omega}^G f(\tilde{\omega}^G)} \right] \left[ \frac{1 - F(\bar{\omega}^B) - \mu \bar{\omega}^B f(\bar{\omega}^B)}{1 - F(\bar{\omega}^B)} \right],$$

where  $\tilde{\omega}^G, \bar{\omega}^B$  are the steady-state values of  $\tilde{\omega}_t^G, \bar{\omega}_t^B$  respectively, and  $f(\cdot)$  is the probability density function of  $\omega_t$ . If  $\kappa$  is too small  $\lambda_t^B$  may exceed  $\lambda_t^G$ , and if  $\kappa$  is too large,  $\lambda_t^B$  may be smaller than unity.

1. All  $i \in \{G, B\}$  firms choose the same cutoff value  $\bar{\omega}_t^i = \bar{\omega}_{jt}^i$  and leverage  $\phi_t^i$  given by:

$$\phi_t^B \equiv \frac{Q_t K_{jt}^B}{(1-\kappa)N_{jt}^B}, \quad \phi_t^G \equiv \frac{Q_t K_{jt}^G}{N_{jt}^G}, \quad (11)$$

2. A  $G$  firm defaults when  $\omega_{jt} < \tilde{\omega}_t^G$ , where  $\tilde{\omega}_t^G$  is given by:

$$\tilde{\omega}_t^G = \xi_t^{-1} \bar{\omega}_t^G, \quad (12)$$

and  $\xi_t$  is defined as:

$$\xi_t \equiv 1 - \zeta(1-\mu) \frac{\Omega_t^B}{\Omega_t^G} \leq 1, \quad (13)$$

where  $\Omega_t^i \equiv \theta \lambda_t^i + 1 - \theta$  for  $i \in \{B, G\}$ .

3. The marginal values of net worth for a  $G$  firm and a  $B$  firm evolve as follows:

$$\lambda_t^B = (1-\kappa) \phi_t^B E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K \int_{\bar{\omega}_{t+1}^B} (\omega - \bar{\omega}_{t+1}^B) dF_t, \quad (14)$$

$$\lambda_t^G = \phi_t^G E_t \Lambda_{t,t+1} \Omega_{t+1}^G R_{t+1}^K \left[ (1 - \xi_{t+1}) \int^{\tilde{\omega}_{t+1}^G} \omega dF_t + \int_{\tilde{\omega}_{t+1}^G} (\omega - \bar{\omega}_{t+1}^G) dF_t \right]. \quad (15)$$

4. The optimal cutoff values satisfy:

$$\lambda_t^B = \frac{(1-\kappa) E_t \Lambda_{t+1} \Omega_{t+1}^B R_{t+1}^K [1 - F(\bar{\omega}_{t+1}^B)]}{E_t \frac{R_{t+1}^K}{R_t} [1 - F(\bar{\omega}_{t+1}^B) - \mu \bar{\omega}_{t+1}^B f(\bar{\omega}_{t+1}^B)]}, \quad (16)$$

$$\lambda_t^G = \frac{E_t \Lambda_{t+1} R_{t+1}^K \Omega_{t+1}^G \xi_{t+1} (1 - F(\tilde{\omega}_{t+1}^G))}{E_t \frac{R_{t+1}^K}{R_t} \xi_{t+1} [1 - F(\tilde{\omega}_{t+1}^G) - \tilde{\omega}_{t+1}^G f(\tilde{\omega}_{t+1}^G)]}. \quad (17)$$

5. The participation constraints hold with equality:

$$\phi_{t-1}^B = PC^B \left( \bar{\omega}_t^B, \frac{R_t^K}{R_{t-1}}; \sigma_{t-1} \right) \equiv \left\{ 1 - \frac{R_t^K}{R_{t-1}} \left[ \int_{\bar{\omega}_t^B} \bar{\omega}_t^B dF_{t-1} + (1-\mu) \int^{\bar{\omega}_t^B} \omega dF_{t-1} \right] \right\}^{-1} \quad (18)$$

$$\phi_{t-1}^G = PC^G \left( \tilde{\omega}_t^G, \xi_t, \frac{R_t^K}{R_{t-1}}; \sigma_{t-1} \right) \equiv \left[ 1 - \frac{R_t^K}{R_{t-1}} \left( \int_{\tilde{\omega}_t^G} \bar{\omega}_t^G dF_{t-1} \right) \right]^{-1}. \quad (19)$$

Equation (12) introduces a default threshold  $\tilde{\omega}_t^G$  for  $G$  firms. It states that a  $G$  firm which draws  $\omega_{jt} \in [\bar{\omega}_t^G, \tilde{\omega}_t^G]$  defaults strategically. A  $G$  firm trades off the benefits of reneging on its unsecured debt and its reputation costs. In future, any borrowing of secured debt by this firm is subject to costly initial monitoring  $\kappa$ . This substitution between reputation and monitoring is similar to [Diamond \(1991\)](#)'s theory of loan demand.

Equation (13) describes the way this tradeoff evolves over the business cycle. A  $G$  firm's reputation value is characterized by  $\xi_t$ : if  $\xi_t$  increases, the distance between  $\bar{\omega}_t^G$  and  $\tilde{\omega}_t^G$  shrinks, and the firm is less likely to default strategically. The reputation value  $\xi_t$  only depends on macroeconomic conditions. In particular, it is an increasing function of  $\Omega_t^G/\Omega_t^B \approx \lambda_t^G/\lambda_t^B$ , which is the ratio of marginal values of  $G$  and  $B$  firms. When a shock increases  $\lambda_t^B$  more than  $\lambda_t^G$ , the reputation value of being a  $G$  firm is low,  $\xi_t$  falls and more strategic default arises.

Equations (14) and (15) express the value of  $G$  firm and  $B$  firms in terms of their future value. Consider (14) for instance. Conditional on a given shock realization  $\omega$ , a unit of net worth in a  $B$  firm is leveraged up by  $(1 - \kappa)\phi_t^B$  times, yields an aggregate return  $R_{t+1}^K$  and appropriately discounted by  $\Lambda_{t,t+1}\Omega_{t+1}^B$ , where  $\Omega_{t+1}^B$  is a probability weighted average of the marginal values of net worth to exiting and continuing firms at  $t + 1$ . If  $\omega < \bar{\omega}_{t+1}^B$  the firm defaults and the borrower's value is 0. If  $\omega \geq \bar{\omega}_{t+1}^B$  the firm receives a share  $(\omega - \bar{\omega}_{t+1}^B)$  of the revenue. This schedule is represented by the blue solid line in Figure 1. We integrate with respect to the distribution  $F(\omega)$  to obtain the unconditional value  $\lambda_t^B$ . The value of a  $G$  firm, (15), can be understood similarly.

Equations (16) and (17) determine the optimal default thresholds for  $B$  and  $G$  firms,  $\bar{\omega}_{t+1}^B, \bar{\omega}_{t+1}^G$ . The following two propositions explain their determinants.

**Proposition 2.** *Up to a first order approximation, the cutoff value for the secured debt contract,  $\bar{\omega}_t^B$ , satisfies:*

$$E_t \left( \frac{R_{t+1}^K}{R_t} \right) = E_t \rho^B(\bar{\omega}_{t+1}^B; \sigma_t) \geq 1, \quad (20)$$

where the credit demand function  $\rho^B(\bar{\omega}_{t+1}^B; \sigma_t)$ , defined in [Appendix B](#), is increasing in the cutoff value  $\bar{\omega}_{t+1}^B$ , and increasing in the cross-sectional dispersion of the idiosyncratic shock  $\sigma_t$ . Furthermore,

$$\lim_{\bar{\omega}_{t+1}^B \rightarrow 0} \rho^B(\bar{\omega}_{t+1}^B; \sigma_t) = 1.$$

This optimality condition for secured contracts states that the cutoff value  $\bar{\omega}_{t+1}^B$  is increasing in the external finance premium, defined as  $E_t(R_{t+1}^K)/R_t$ . A  $B$  firm cares about its upside profit when it does not default. If the external finance premium is higher, the firm chooses to borrow more and defaults more often, so  $\bar{\omega}_{t+1}^B$  rises. In equilibrium, the external finance premium is weakly greater than unity because lenders expect resources to be lost through monitoring, which has to be compensated by the external finance premium. Moreover,  $\rho_\sigma^B > 0$  because a more spread-out distribution of idiosyncratic shock means more expected defaults and a higher premium. When  $\bar{\omega}_{t+1}^B$  approaches 0, there is no default and no monitoring, so  $\rho^B = 1$ .

**Proposition 3.** *Up to a first order approximation, the cutoff value for the unsecured debt contract,  $\tilde{\omega}_t^G$ , satisfies:*

$$E_t \left( \frac{R_{t+1}^K}{R_t} \right) = E_t \rho^G(\tilde{\omega}_{t+1}^G, \xi_{t+1}; \sigma_t) \geq 1, \quad (21)$$

where the credit demand function  $\rho^G(\tilde{\omega}_{t+1}^G, \xi_{t+1}; \sigma_t)$ , defined in [Appendix B](#), is increasing in the cutoff value  $\tilde{\omega}_{t+1}^G$ , decreasing in  $\xi_{t+1}$ , and increasing in the cross-sectional dispersion of the idiosyncratic shock  $\sigma_t$ . Furthermore,

$$\lim_{\tilde{\omega}_{t+1}^G \rightarrow 0} \rho^G(\tilde{\omega}_{t+1}^G, \xi_{t+1}; \sigma_t) = 1.$$

A novel feature of the unsecured debt contract is that the default threshold depends on  $\xi$ . The intuition for  $\rho_\xi^G < 0$  is as follows. For a given  $\tilde{\omega}_{t+1}^G$ , if  $\xi_{t+1}$  is smaller (i.e.  $G$  firms have lower reputation value),  $\bar{\omega}_{t+1}^G = \xi_{t+1} \tilde{\omega}_{t+1}^G$  is lower. According to Equation (4) the contractual interest rate  $Z_t^G$  is lower. From the lenders' perspective, a  $G$  firm's default threshold is unchanged, but the contractual interest rate is lower, so lenders cannot break even with the same contract. To break even, lenders must require a higher external finance premium. This is why  $\rho_\xi^G < 0$ .

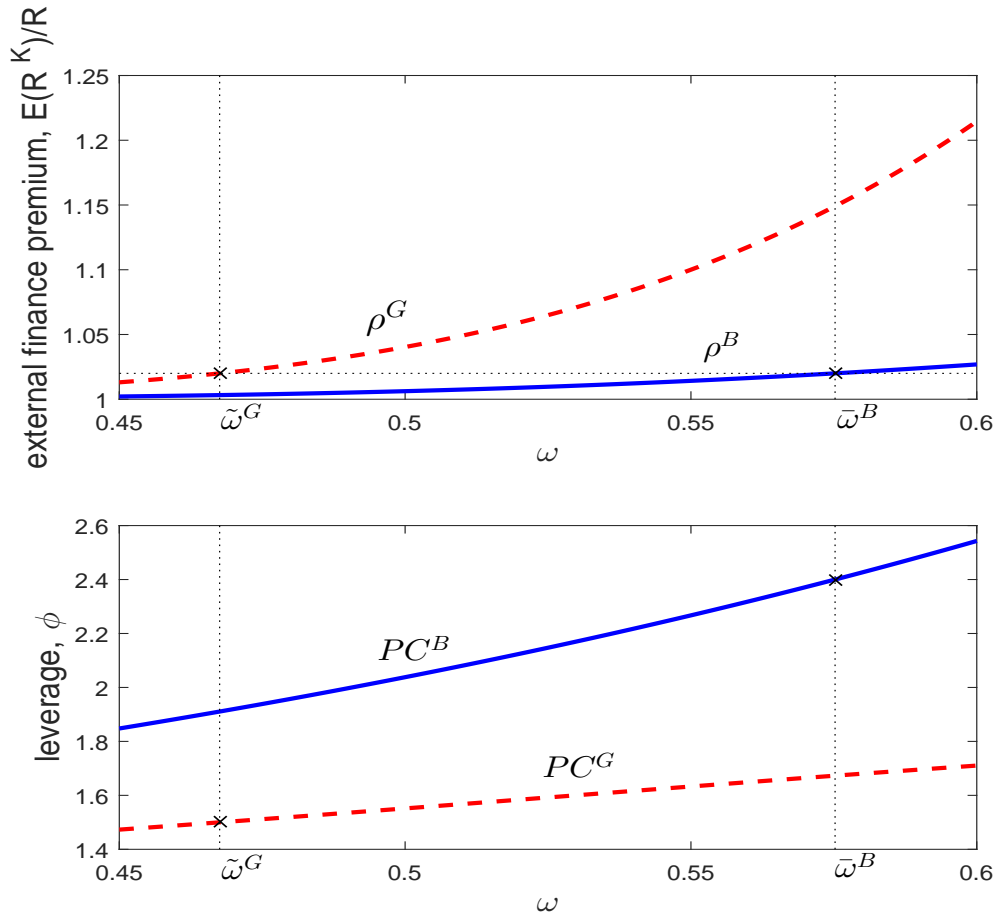
The following two propositions describe the relative cautiousness of borrowers and lenders in the two debt markets:

**Proposition 4.** *For any  $\bar{\omega}_t > 0$ ,  $1 > \xi_t > \mu$  and  $\sigma_{t-1} > 0$ ,*

$$\frac{\partial \rho^G(\bar{\omega}_t, \xi_t; \sigma_{t-1})}{\partial \bar{\omega}_t} > \frac{\partial \rho^B(\bar{\omega}_t; \sigma_{t-1})}{\partial \bar{\omega}_t}. \quad (22)$$

This proposition states that borrowers are more cautious in the unsecured debt market than in the secured debt market. As shown in [Figure 1](#), borrowers in the secured debt market care less about downside risks, because when they default their assets are transferred to the lenders. Consider the following thought experiment. Suppose both secured and unsecured debt contracts have the same cutoff value  $\bar{\omega}_t$  initially. A marginal increase in the external finance premium would induce a bigger rise in the cutoff value in the secured debt contract relative to the cutoff value in the unsecured debt contract. In other words, for a given cutoff value, the slope of the credit demand function  $\rho^B$  is less steep compared with the slope of credit demand function  $\rho^G$ . The top panel of [Figure 3](#) plots the credit demand functions for  $B$  firms and  $G$  firms using the actual contract calibrated for our model in the steady state, keeping  $\xi$  fixed at its steady-state value. Clearly,  $\rho^G$  is steeper than  $\rho^B$  for any given cutoff value  $\bar{\omega}$ .





**Fig. 3.** Comparative static analysis of credit demand functions ( $\rho^G, \rho^B$ ), and participation constraints ( $PC^G, PC^B$ ) based on steady-state calibrations. The top panel plots the external finance premium with respect to cutoff value, and the bottom panel plots the leverage with respect to cutoff value.  $\times$  denotes the steady state values of the external finance premium and leverage obtained by benchmark calibration in Section 5.

**Proposition 5.** For any  $\bar{\omega}_t > 0$ ,  $1 > \xi_t > \mu$  and  $\sigma_{t-1} > 0$ ,

$$\frac{\partial PC^B \left( \bar{\omega}_t, \frac{R_t^K}{R_{t-1}}; \sigma_{t-1} \right)}{\partial \bar{\omega}_t} > \frac{\partial PC^G \left( \bar{\omega}_t, \frac{R_t^K}{R_{t-1}}, \xi_t; \sigma_{t-1} \right)}{\partial \bar{\omega}_t} > 0. \quad (23)$$

This proposition states that lenders in the unsecured debt market are more cautious than lenders in the secured debt market. Consider the following thought experiment. Suppose both debt contracts have the same cutoff value  $\bar{\omega}_t$  initially. If a borrower of unsecured debt asks for an additional unit of loan, and if it turns out that the borrower defaults, lenders cannot get anything back. In the secured debt market, however, lenders can retrieve the remaining value in the firm after monitoring. Therefore, to increase lending by one unit, lenders need to be compensated by a bigger increase in the cutoff value in the unsecured debt market. In other words, for a given cutoff value, the slope of the participation constraint is steeper for  $B$  firms than for  $G$  firms. The bottom panel of Figure 3 plots the participation constraints associated with  $B$  firms and  $G$  firms, with  $R^K/R$  and  $\xi$  fixed at their steady-state values. For any given cutoff value  $\bar{\omega}$ ,  $PC^B$  is steeper than  $PC^G$ .

The following proposition is our first main result:

**Proposition 6.** The leverage ratio of  $G$  firms is lower than the leverage ratio of  $B$  firms. That is  $\phi_t^B > \phi_t^G$ .

The intuition can be illustrated using Figure 3. In the top panel, borrowers in both secured and unsecured debt contracts face the same external finance premium  $E(R_{t+1}^K)/R_t$ , but since  $\rho^G$  is steeper than  $\rho^B$  (See Proposition 4), borrowers of unsecured debt choose a lower cutoff value  $\tilde{\omega}_{t+1}^G$  than the cutoff value for secured debt,  $\bar{\omega}_{t+1}^B$ , which means that they default less often. In the bottom panel, as the participation constraint for  $B$  firms is steeper than the participation constraint for  $G$  firms (See Proposition 5), and  $\bar{\omega}_{t+1}^B > \tilde{\omega}_{t+1}^G$ , we must have  $\phi_t^B > \phi_t^G$ . Our result that  $G$  firms have lower leverage than  $B$  firms is consistent with stylized fact 2.

Moreover, our model is consistent with stylized fact 3, that is unsecured debt has a higher correlation with output than secured debt. This is the second main result of this paper. We provide the intuitions for this in the rest of this section and support the intuition with numerical simulations in the following sections.

Consider a negative TFP shock. In a standard one-sector financial accelerator model with secured debt as in BGG, the external finance premium rises in equilibrium. This is shown in the top-left panel in Figure 4. As a result, the cutoff value of the financial contract increases. The lower-left panel shows the participation constraint of this contract, written as the leverage ratio as a function of the cutoff value of the contract. A rise in the external

finance premium increases lenders' revenue, so the participation constraint shifts up, which, together with the rise in the cutoff value, leads to a sharp increase in the leverage ratio.

In our model, the rise in the external finance premium affects secured and unsecured contracts differently. In Figure 4, the left (right) panel represents the secured (unsecured) debt market. Propositions 4 and 5 state that  $\rho^i$  and  $PC^i$  have different slopes in the two markets  $i \in \{B, G\}$ . In particular, when the external finance premium rises, the fact that  $\rho^B$  is steeper than  $\rho^G$  means that  $\tilde{\omega}_{t+1}^G$  shifts to the right by less than  $\bar{\omega}_{t+1}^B$ . The fact that  $PC^B$  is steeper than  $PC^G$  implies that  $\phi_t^G$  goes up by less than  $\phi_t^B$ .

Moreover, there is an effect arising from strategic defaults. A rise in the external finance premium increases both  $\lambda_t^G$  and  $\lambda_t^B$ . Moreover, as the leverage ratio in  $B$  firms rises more than the leverage ratio of  $G$  firms,  $\lambda_t^B$  increases more than  $\lambda_t^G$ , and (13) implies that  $\xi_t$  falls and there is an increase in strategic defaults. The top-right panel shows that a fall in  $\xi_t$  further shifts the  $\rho^G$  curve up, leading to a smaller increase in the default threshold  $\tilde{\omega}_t^G$ . Furthermore, more strategic defaults shift the participation constraint down, as in the bottom-right panel. As a result, the leverage ratio of the  $G$  firms increases by less than the leverage ratio of  $B$  firms. This is crucial in understanding why unsecured debt is more procyclical than secured debt in the general equilibrium.

#### 4. The rest of the model

This section embeds our financial contract into a standard real business cycle framework. Besides the firm sector described above, there are three other types of agents, namely homogeneous households, investors and capital producers, which is standard in the literature.

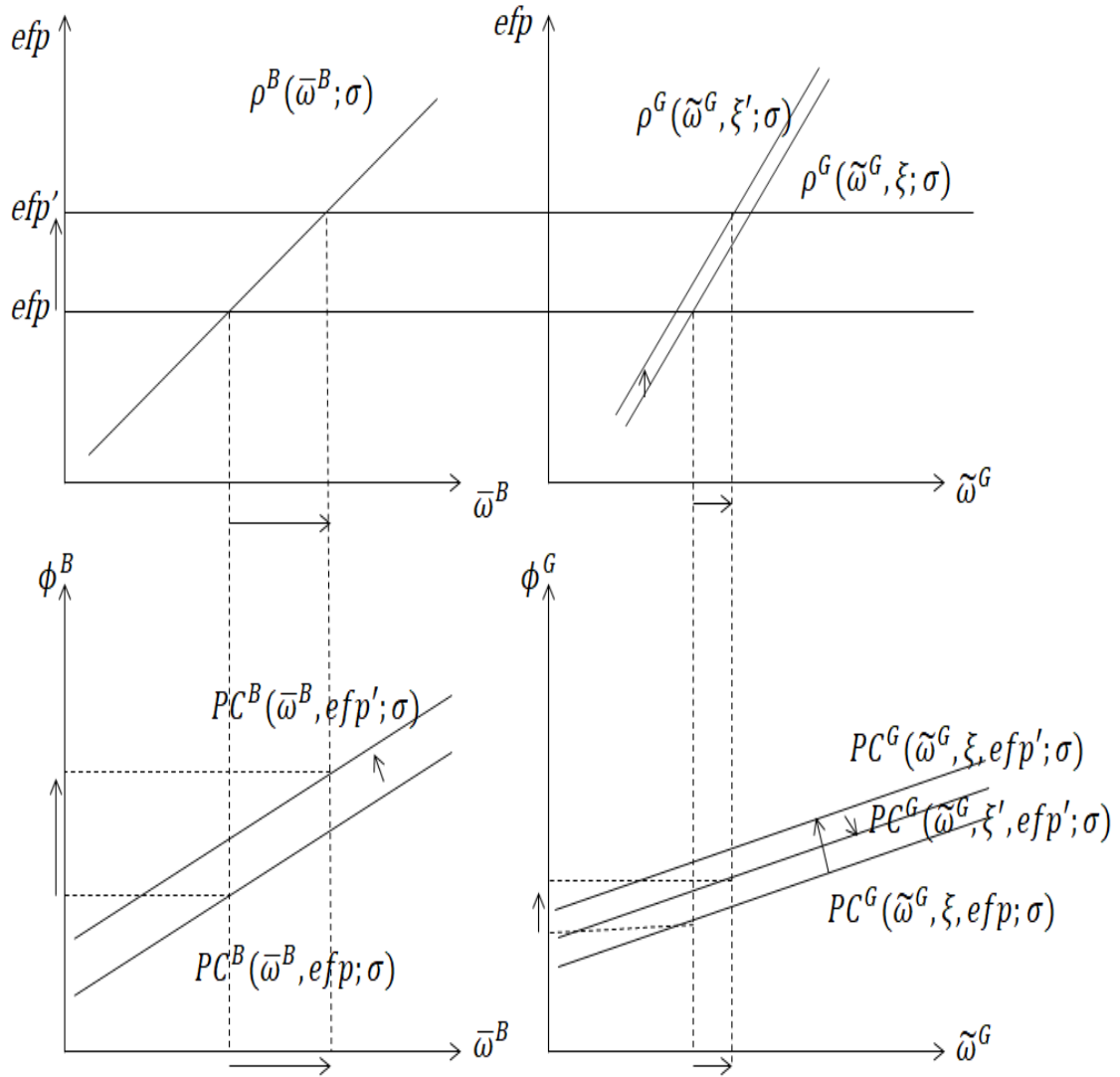
##### 4.1. Households

Infinite-lived representative households derive utility from consumption and disutility from supplying labor. The preferences of the representative household are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln(C_t - hC_{t-1}) - \chi \frac{L_t^{1+\varphi}}{1+\varphi} \right], \quad (24)$$

where the parameter  $\chi$  is the weight on labor disutility,  $h < 1$  is a parameter which captures habit persistence in consumption and  $\varphi$  is the inverse of Frisch labor elasticity.

In each period, a representative household supplies labor and receives wage income, makes deposits and consumes.  $R_t$  is the risk-free interest rate.  $\Pi_t^K$  denotes profits from capital producing firms. The transfer term  $tr_t$  includes startup funds paid to new firms and revenues remitted from old firms. To sum up, a representative household faces the following



**Fig. 4.** This figure illustrates the relationships among cutoff values, the external finance premium, and leverage.

budget constraint:

$$w_t L_t + R_{t-1} D_{t-1} = C_t + D_t + tr_t. \quad (25)$$

The consumption Euler equation and labor supply conditions are:

$$1 = R_t E_t(\Lambda_{t,t+1}), \quad (26)$$

$$w_t = \chi L_t^\varphi U_{C_t}^{-1}. \quad (27)$$

where the stochastic discount factor is given by  $\Lambda_{t-1,t} = \beta U_{C_t} / U_{C_{t-1}}$ , and  $U_{C_t} = (C_t - hC_{t-1})^{-1} - \beta h E_t(C_{t+1} - hC_t)^{-1}$ .

#### 4.2. Investors

Investors collect deposits from households and lend to firms. They observe the credit quality of each firm and issue unsecured debt to  $G$  firms and secured debt to  $B$  firms. Investors require a risk-free return  $R_t$  in every state of the world for each of these loans. Investors do not play a meaningful role in the model other than making sure households hold a diversified loan portfolio across firms.

#### 4.3. Capital goods producers

A representative capital goods producer buys previously installed capital and combines it with investment good  $I_t$  to produce new capital. Newly produced capital is sold back to the firms within the same period. Production of new capital is subject to convex investment adjustment costs  $Adj_t = 0.5\Psi^I (I_t/I_{t-1} - 1)^2$ . The evolution of aggregate capital  $K_t$  is given by:

$$K_t = (1 - \delta)K_{t-1} + (1 - Adj_t)I_t. \quad (28)$$

Capital goods producers maximize discounted sum of expected future profits,  $E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \Pi_{t+s}^K$ , where  $\Pi_t^K = Q_t[K_t - (1 - \delta)K_{t-1}] - I_t$ . The first order condition for the optimal investment choice is:

$$1 = Q_t \left[ 1 - Adj_t - \Psi^I \frac{I_t}{I_{t-1}} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right] + E_t \left[ \Lambda_{t,t+1} Q_{t+1} \Psi^I \left( \frac{I_{t+1}}{I_t} \right)^2 \left( \frac{I_{t+1}}{I_t} - 1 \right) \right] \quad (29)$$

#### 4.4. Aggregation and accumulation of net worth

Since each type of firms has the same capital to labor ratio and leverage ratio, we only need to keep track of sector-level quantities. For  $X \in \{Y, K, L, N, B\}$ , we define  $X_t^i \equiv \int_i X_{jt}^i dj$ , where  $i \in \{G, B\}$ , and we also define economy-wide variables  $X_t \equiv X_t^G + X_t^B$ .

Since the leverage ratio is the same for each type of firm, we have:

$$N_t^G \phi_t^G = Q_t K_t^G, \quad (30)$$

$$(1 - \kappa) N_t^B \phi_t^B = Q_t K_t^B. \quad (31)$$

It is helpful to define the average leverage ratio of the economy as  $\phi_t \equiv Q_t K_t / N_t$ .

We write down the evolution of net worth for  $G$  and  $B$  firms. We assume that in each period, new firms enter to keep the number of firms of each credit rating in the economy constant. We assume households transfer to a new firm a small fraction  $\tau$  of the net worth of the average firm with the same credit rating. These initial funds are one-time lump-sum transfer from households.  $G$  firms' net worth evolves as follow:

$$N_t^G = \theta \int_{\bar{\omega}_t^G} (\omega - \bar{\omega}_t^G) R_t^K Q_{t-1} K_{t-1}^G dF_{t-1} + \tau N_{t-1}^G, \quad (32)$$

where the first term represents the firms which are  $G$  firms in period  $t - 1$  and remain  $G$  firms in period  $t$ . The second term denotes the transfer to new entrants.

Net worth of  $B$  firms evolves as follow:

$$N_t^B = \theta \int_{\bar{\omega}_t^G} \zeta (1 - \mu) \omega R_t^K Q_{t-1} K_{t-1}^G dF_{t-1} + \theta \int_{\bar{\omega}_t^B} (\omega - \bar{\omega}_t^B) R_t^K Q_{t-1} K_{t-1}^B dF_{t-1} + \tau N_{t-1}^B \quad (33)$$

The first term represents  $G$  firms who default in the last period and therefore becomes  $B$  firms in period  $t$ . The second term refers to  $B$  firms in period  $t - 1$  who remain  $B$  firms in period  $t$ . The last term is the transfer to new entrants.

The market clearing condition is given by:

$$Y_t = C_t + I_t + [\mu + (1 - \mu)(1 - \zeta)] \int_{\bar{\omega}_t^G} \omega dF_{t-1} R_t^K Q_{t-1} K_{t-1}^G + \mu \int_{\bar{\omega}_t^B} \omega dF_{t-1} R_t^K Q_{t-1} K_{t-1}^B + \kappa N_t^B. \quad (34)$$

The expenditure side consists of consumption, investment, resources lost in defaulting  $G$  and  $B$  firms and initial monitoring costs for  $B$  firms.

#### 4.5. Shocks

There are two shocks in the economy, namely a TFP shock and a shock to  $\sigma_t$ , the cross-sectional variance of the idiosyncratic shock. [Christiano, Motto and Rostagno \(2014\)](#) interprets  $\sigma_t$  as a risk shock and shows that it is important in explaining the US business

cycle. We assume that these shocks following exogenous AR(1) processes as follows:

$$\ln A_t = \rho_A \ln A_{t-1} + \epsilon_{At}, \quad \epsilon_{At} \sim N(0, s_A^2) \quad (35)$$

$$\ln \sigma_t = (1 - \rho_\sigma) \ln s + \rho_\sigma \ln \sigma_{t-1} + \epsilon_{\sigma t}, \quad \epsilon_{\sigma t} \sim N(0, s_\sigma^2) \quad (36)$$

The innovations of all shocks are assumed to be i.i.d, uncorrelated over time and with each other.

This completes the description of the model. [Appendix A](#) shows the equations of the full system.

## 5. Calibration

In the following we solve and simulate the model numerically by log-linearizing the system around its non-stochastic steady state. This section discusses our calibration.

Each period is a year. The parameters in production and household sectors are relatively standard in the macroeconomic literature and are given in [Table 5](#). We set  $\beta = 0.96$ , which corresponds to around 4% steady-state interest rate. We set  $\Psi^L = 5$ , so households devote 41 percent of their time to work. The parameter that governs the Frisch elasticity of labor supply is set to  $\chi = 1$ . For production, the capital share is set to  $\alpha = 0.33$ , and the depreciation rate to  $\delta = 0.08$ . The curvature of investment adjustment costs  $\Psi^I$  is set to 1 and the consumption habit parameter  $h$  is set to 0.4. These parameter values are within an acceptable range in the literature.

Our calibration strategy for financial parameters is as follows. We set the survival rate of firms to  $\theta = 0.87$  so an average firm exits in 7.7 years. We set the default costs to 0.2, following the estimation by [Davydenko, Strebulaev and Zhao \(2012\)](#). The default costs is between 0.25 used in [Carlstrom and Fuerst \(1997\)](#) and 0.12 in BGG. We calibrate the remaining parameters to hit four targets. First, the external finance premium  $R^K/R$  is 2%, based on [Gilchrist and Zakrajsek \(2012\)](#). Second, we target an unsecured debt to total debt ratio  $B^G/B = 0.75$ , to match the Compustat data in [Figure 2](#). Third, we target a steady-state leverage ratio of  $B$  firms to  $\phi^B = 2.4$ . Fourth, we target a steady-state leverage ratio of  $G$  firms to  $\phi^G = 1.5$ . These leverage ratios are close to the leverage ratios of firms with credit quality ‘AA and above’ and ‘CC or below’ in our dataset and are close to what is found in [Rauh and Sufi \(2010\)](#). They imply the aggregate leverage of the firm sector is 1.59, which is in between 2 used in BGG and 1.43 found in [De Fiore and Uhlig \(2011\)](#) for the period 1999-2007. These conditions pin down  $\{\sigma, \zeta, \kappa, \tau\}$ . We find that the initial monitoring costs for secured debt are  $\kappa = 0.017$ , which are large enough so that  $\lambda^G = 1.28 > 1.23 = \lambda^B$  in

**Table 5**  
Calibrated parameters.

Parameter	Value	Meaning
$\beta$	0.96	Subjective discount factor
$\alpha$	0.33	Capital share in production
$\delta$	0.08	Capital depreciation rate
$\Psi^L$	5	Labor disutility
$\varphi$	1	Inverse of Frisch labor elasticity
$\Psi^I$	1	Convexity of investment adjustment costs
$h$	0.4	Consumption habit
$\theta$	0.87	Firm survival probability
$\kappa$	0.017	Initial monitoring cost for secured debt
$\mu$	0.2	Default costs
$\zeta$	0.388	Debt restructuring success rate
$\bar{\sigma}$	0.257	Std. dev of idiosyncratic shock
$\tau$	0.068	Firm initial transfer
$\rho_A$	0.56	Persistence of TFP shock
$\rho_\sigma$	0.85	Persistence of financial shock
$s_A$	0.023	Std. dev of TFP shock innovation
$s_\sigma$	0.026	Std. dev of financial shock innovation

the steady state, but are not too large so that  $\lambda_t^B > 1$  around the steady state.<sup>14</sup>

The shock parameters are calibrated as follows. We calibrate the persistence and standard deviation of the cross-sectional volatility shock using annual industry-level TFP data in 1983-2011 by the National Bureau of Economic Research (NBER) and the Center for Economic Studies (CES). We linearly detrend each industry-level TFP series and compute the cross-sectional variance at each point in time. We fit an AR(1) process and obtain  $\rho_\sigma = 0.85$ ,  $s_\sigma = 0.026$ . This procedure follows [Nuno and Thomas \(2017\)](#). For the TFP shock we use the annual TFP series in 1983-2011 constructed by the CSIP at the Federal Reserve Bank of San Francisco. The log-TFP series is HP-filtered (smoothing parameter =100) fitted with an AR(1) process. We get  $\rho_A = 0.56$  and  $s_A = 0.023$ .

## 6. Model results

### 6.1. Impulse responses

Figures 5 and 6 show the response of macroeconomic and financial variables to a one standard deviation fall in TFP and increase in cross-sectional volatility respectively. All

<sup>14</sup>[Appendix C](#) shows the details of our calibration.



variables are presented as percentage deviations from steady-state values. For sectoral variables, blue solid lines denote  $G$  firms which borrow in the unsecured debt market and red dashed lines denote  $B$  firms which borrow in the secured debt market.

In Figure 5, a bad TFP shock reduces the realized return on capital. This reduces the net worth of all firms in the economy and limits their ability to borrow in subsequent periods. Investment demand drops, the price of capital  $Q$  falls and the external finance premium  $E_t(R_{t+1}^K)/R_t$  rises. A fall in the price of capital further reduces the realized return on capital, increasing the break-even contractual interest rate, so the cutoff values rise. This aggravates the initial fall in net worth of the firms through the financial accelerator effect discussed in BGG. This effect leads to a large and persistent fall in output and investment.

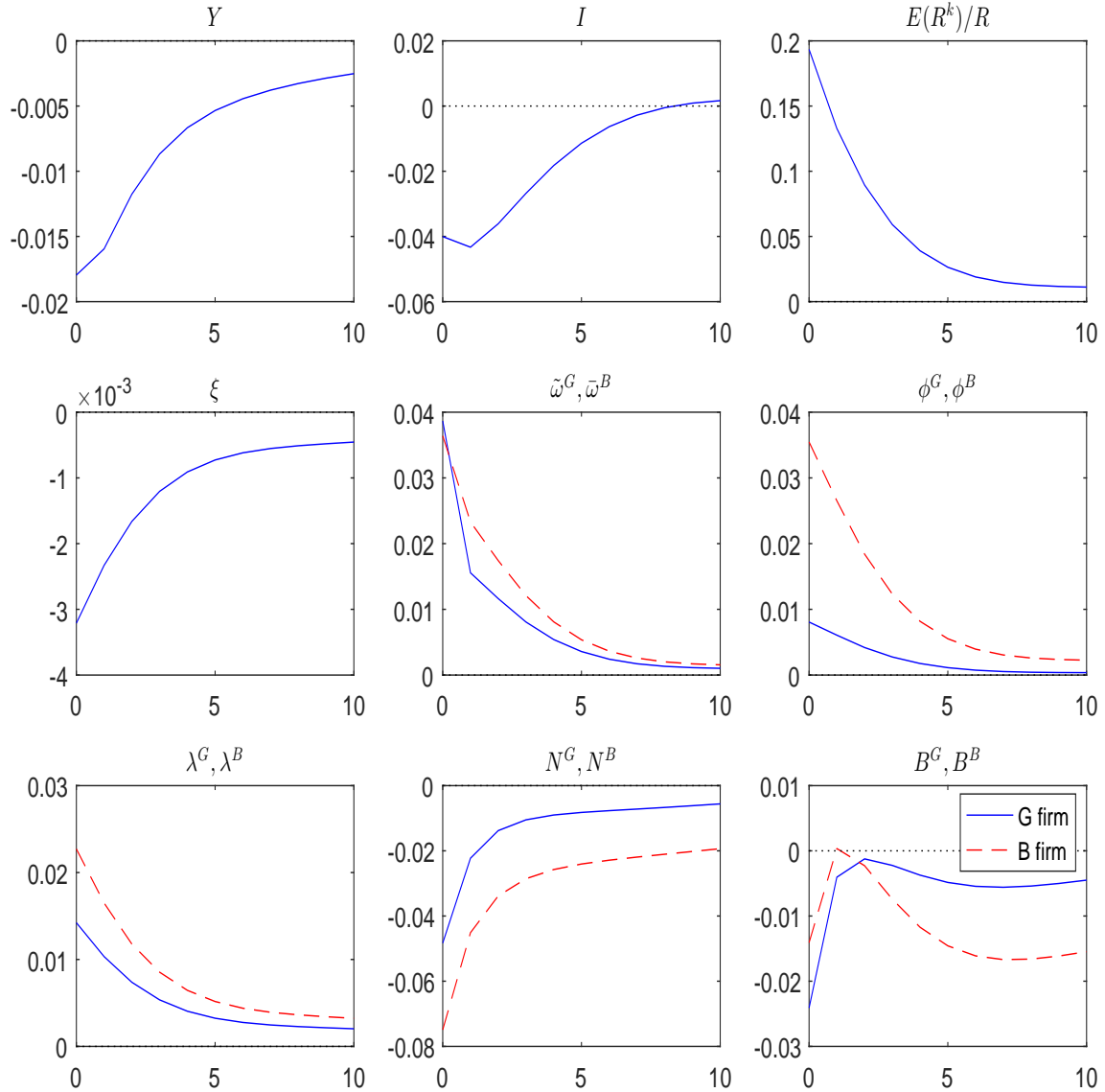
We are interested the quantity of unsecured and secured debt in the economy in response to shocks. We can rewrite  $B_t^G, B_t^B$  in terms of their net worth and leverage ratios as follows:

$$B_t^G = (\phi_t^G - 1)N_t^G, \quad B_t^B = (1 - \kappa)(\phi_t^B - 1)N_t^B.$$

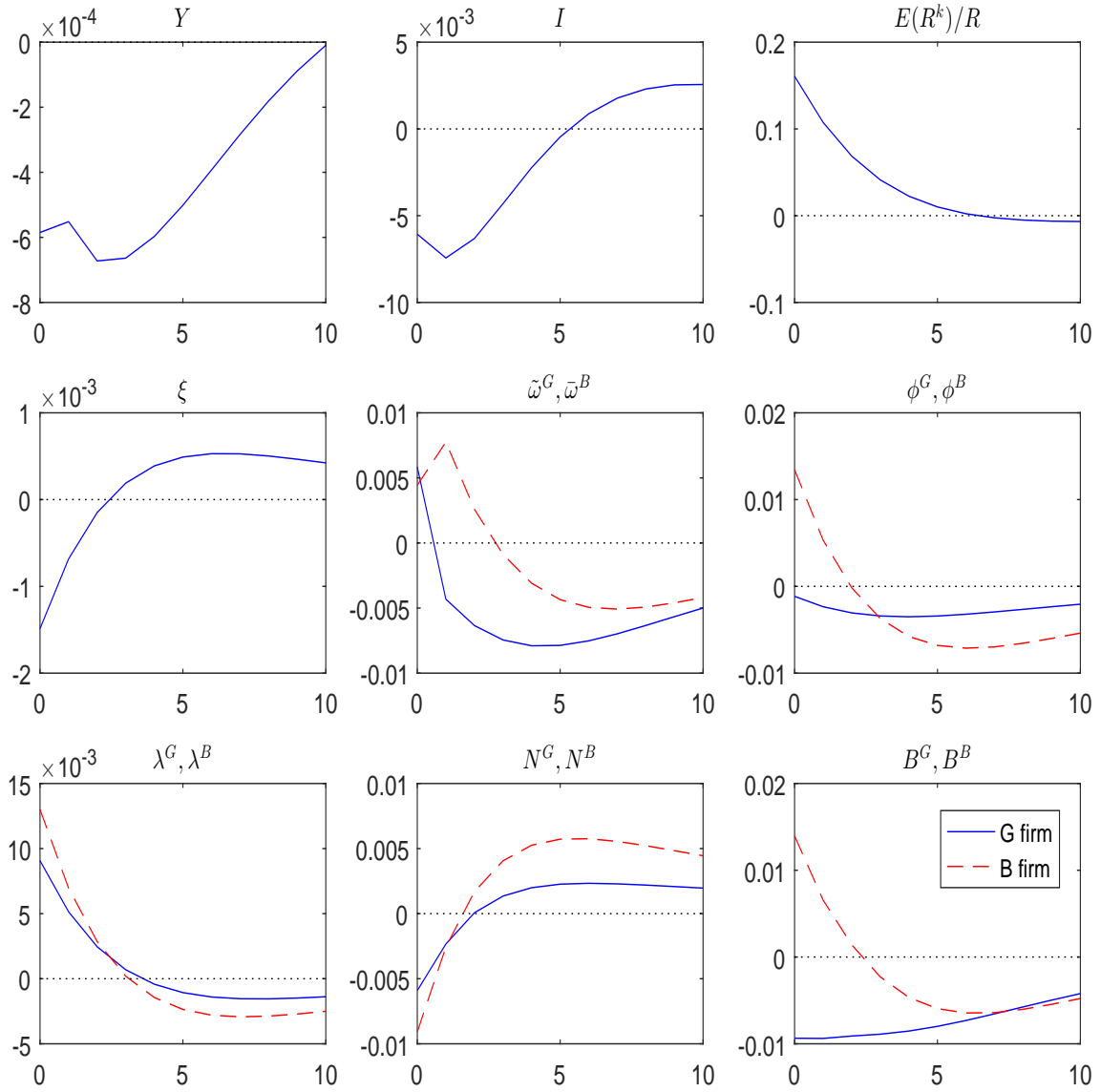
These equations state that the evolution of unsecured and secured debt is determined by net worth and the leverage ratios of the respective firms. Figure 5 shows that, as a negative TFP shock hits,  $N^B$  falls by more than 7% whereas  $N^G$  falls by around 5%. This is because  $B$  firms borrow with a higher steady-state leverage ratio than  $G$  firms, and their net worth is more volatile. On the other hand, the leverage ratio of  $B$  firms rises by about 3.5%, which is more than three times the rise in leverage ratio of  $G$  firms (0.8%). The mechanism behind the differentiated response in the leverage ratios is explained in Section 3 (See Figure 4): as borrowers and lenders in the unsecured debt market are more cautious, and the reputation value of a  $G$  firm falls in bad times,  $\tilde{\omega}_{t+1}^G$  shifts to the right by less than  $\tilde{\omega}_{t+1}^B$  (from period 1 onwards), and the leverage ratio of  $G$  firms rises less than the leverage ratio of  $B$  firms. A fall in net worth combined with muted response in the leverage ratio of  $G$  firms mean that unsecured debt falls strongly initially and is highly procyclical. By contrast, a sharp increase in the leverage ratio of  $B$  firms mitigates the fall in the quantity of secured debt in the initial periods.

Figure 6 shows the response to a rise in the cross-sectional volatility of firms' productivity. This shock increases the default probability of the firms, thus requiring higher cutoff values for the lenders to break even, which reduces net worth and the price of capital, triggering the financial accelerator mechanism.

Again, the volatility shock affects the quantity of unsecured and secured borrowing through net worth and leverage ratios. A volatility shock is mean-preserving, so its effect on the price of capital and the net worth of the firms is smaller than a TFP shock. The shock affects the leverage ratios through multiple channels. First, a volatility shock increases the



**Fig. 5.** Impulse response to a negative TFP shock. *Note:* The impulse response functions measure the response to a one standard deviation negative shock to the innovations of TFP as the percent deviation from the steady state.



**Fig. 6.** Impulse response to a negative financial shock. *Note:* The impulse response functions measure the response to a one standard deviation increase in the innovations of cross-sectional variance of the idiosyncratic shock as the percent deviation from the steady state.

external finance premium  $E_t(R_{t+1}^K)/R_t$ . The secured and unsecured debt markets respond to this differently because the credit demand functions  $(\rho^G, \rho^B)$  and the participation constraints  $(PC^G, PC^B)$  in the two markets have different slopes in a way described by Figure 4. Second, a volatility shock shifts up both  $\rho^G$  and  $\rho^B$  because  $\frac{\partial \rho^G}{\partial \sigma} > 0$ ,  $\frac{\partial \rho^B}{\partial \sigma} > 0$ . Intuitively, when there is more cross-sectional risk, firms borrow more cautiously and default less for a given external finance premium. In equilibrium, the upward shift of  $\rho^G$  and  $\rho^B$  reduces the initial jump of the default thresholds  $\tilde{\omega}_{t+1}^G$  and  $\tilde{\omega}_{t+1}^B$ . Third, a volatility shock shifts the participation constraints  $PC^G, PC^B$  downwards because lenders break even by lending at lower leverage ratios. So the leverage ratios  $\phi_t^G, \phi_t^B$  jump up by less initially. Figure 6 shows that, in equilibrium, the leverage ratio in  $B$  firms increases whereas the leverage ratio in  $G$  firms actually falls on impact. As a result, unsecured debt falls by about 1% and secured debt rises by more than 1% initially.

## 6.2. Business Cycle Moments

Table 6 presents the model's performance along with the empirical moments. Panel A shows the standard deviation of output produced by the benchmark model, while Panel B and C report the relative standard deviation and correlation of other variables with output. The most important result that emanates from Table 6 is that the model is able to reproduce the cyclicity of secured and unsecured debt. Unsecured debt is highly procyclical with  $Corr(B^G, Y) = 0.64$ , whereas secured debt is only slightly procyclical  $Corr(B^B, Y) = 0.09$ . They are close to the corresponding empirical moments: 0.48(0.50) for unsecured debt and 0.06(0.15) for secured debt in US rated (rated and non-rated) firms. Simultaneously, the model performs well in terms of matching the other moments characterizing the business cycle. In particular, it is able to generate output and investment volatility similar to that observed in the actual data. As in the data, consumption in the model is less volatile than output, although a bit less than its empirical counterpart. The procyclicality of consumption in the model is also consistent with the data. Lastly, the model is able to reproduce the correlations of total debt with output.

The model performs slightly worse in terms of the comovement of investment and output. In the model, the correlation is as high as 0.96, compared to 0.87 in the data. Finally, the model underestimates the volatility of secured, unsecured, and total debt compared to the data. Although the model does not perform well in this dimension, a standard BGG model is not able to generate large fluctuations in debt either.<sup>15</sup> Rannenberg (2016) compare moments generated by different types of financial frictions models and show that a Gertler and Karadi (2011) type model with financial frictions in the banking sector can better

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<sup>15</sup>Using our calibrated parameters, a standard BGG model yields a standard deviation of total debt of 1.79 times to standard deviation of output, which is still far smaller than the US data.

**Table 6**  
Moments.

	U.S. Data	Benchmark Model
<i>Panel A: Standard Deviation</i>		
Output ( $Y$ )	1.81	1.79
<i>Panel B: Standard Deviation/ std.(Y)</i>		
Consumption ( $C$ )	0.90	0.71
Investment ( $I$ )	3.18	2.73
Unsecured Debt ( $B^G$ )	7.68	1.30
Secured Debt ( $B^B$ )	5.60	1.19
Total Debt ( $B$ )	4.39	1.10
<i>Panel C: Correlation with Output</i>		
Consumption ( $C$ )	0.94	0.97
Investment ( $I$ )	0.87	0.96
Unsecured Debt ( $B^G$ )	<b>0.48</b>	<b>0.64</b>
Secured Debt ( $B^B$ )	<b>0.06</b>	<b>0.09</b>
Total Debt ( $B$ )	0.53	0.59

Moments of U.S. are computed by using annual data from 1981 to 2016. The numbers from the model are theoretical moments based on the benchmark calibration. Panel A reports the standard deviation of output. Panel B reports the relative standard deviations with respect to output. Panel C reports the contemporaneous correlations with output.

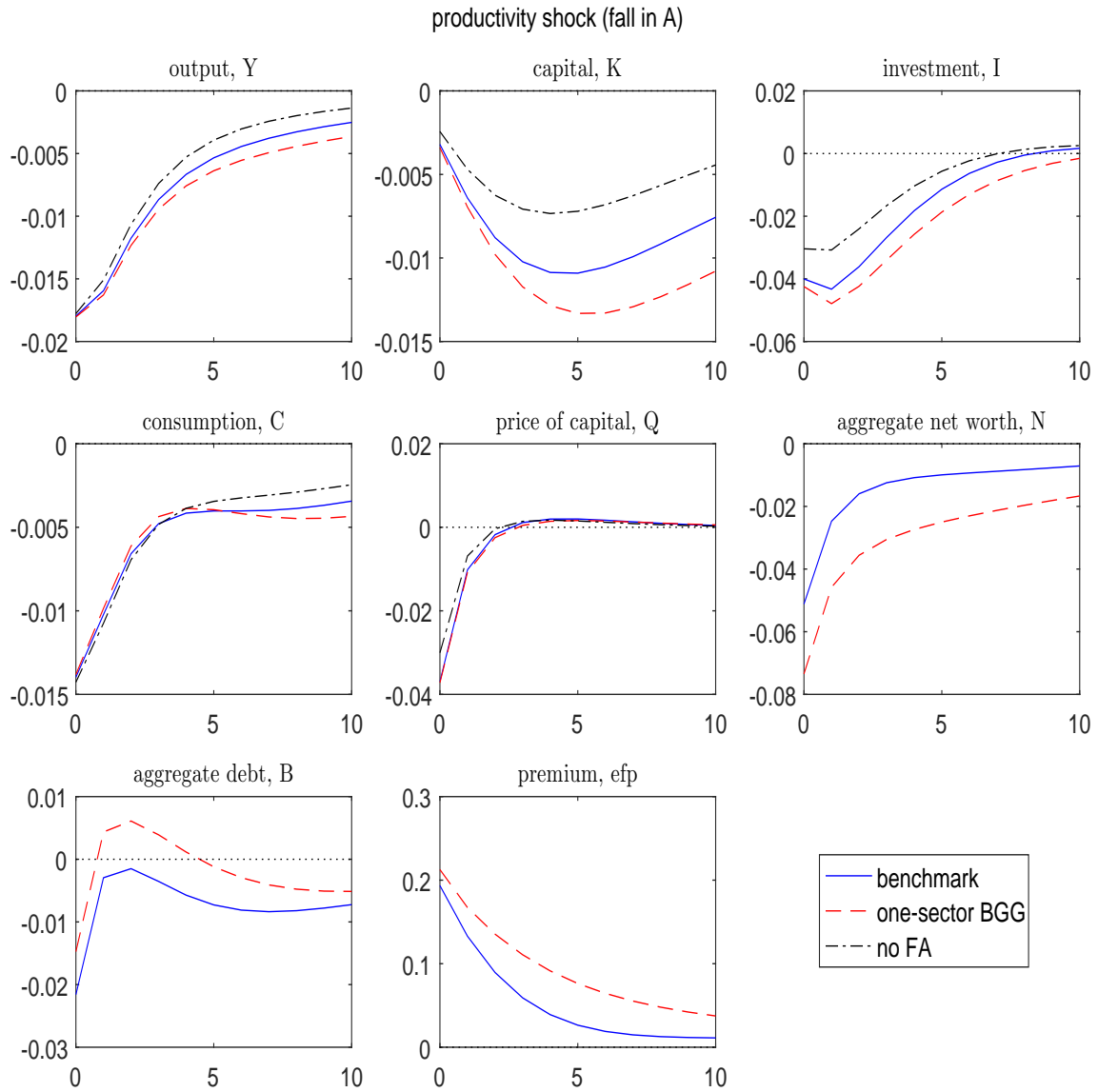
match the standard deviation of loans relative to GDP than a BGG-type model. However, our model is able to capture the relative size of the volatility of secured, unsecured, and total debt, with unsecured debt the most volatile series and total debt the least.

## 7. Macroeconomic implications

What are the macroeconomic implications of using a model with both secured and unsecured debt? To answer this question, we compare our model with a standard one-sector BGG model and a real business cycle model without financial frictions. We describe the details of the one-sector BGG system and the RBC model in the Appendix.

To make the comparison fair, we use the same parameters as in our benchmark model with one exception. We assume in the one-sector BGG model that the initial monitoring cost is given by  $\tilde{\kappa} = \kappa \bar{N}^B / \bar{N}$ , where  $\bar{N}^B$  are  $\bar{N}$  are the steady-state value of  $N_t^B$  and  $N_t$  in the benchmark model. This means that the initial monitoring costs are now shared evenly by every firm.

Figure 7 shows the impulse responses to a negative TFP shock. Blue solid lines correspond to our benchmark model, red dashed lines to the one-sector BGG model, and black



**Fig. 7.** Impulse response to a negative TFP shock. *Note:* The impulse response functions measure the response to a one standard deviation negative shock to the innovations of TFP as the percent deviation from the steady state.

**Table 7**  
Steady state values.

SS Values	Benchmark Model	One-Sector BGG model
$K$	1.44	1.54
$N$	0.91	0.68
$B$	0.54	0.86
$\phi$	1.59	2.27
$Y$	0.62	0.64

Steady state values of key variables based on benchmark calibration for benchmark model and standard BGG model respectively.

dash-dotted lines to the real business cycle model without financial frictions. The one-sector BGG model has the biggest amplification effects. For instance, the one-sector BGG model amplifies the fall in investment by 56% relative to the RBC model one year after the shock; whereas our benchmark model amplifies the fall only by 40%. Moreover, the initial falls in aggregate net worth and debt in the BGG model are 44% and 47% larger than our model respectively. The bottom line is that our model with heterogeneous debt has a financial accelerator effect, but the effect is smaller than in a conventional BGG model.

There are two key reasons behind the dampening effect of the financial accelerator. First, the one-sector BGG model has a higher steady-state leverage, given the same set of parameters. Table 7 reports key steady-state values in the two models and shows that the leverage in the one-sector BGG model is  $K/N = 2.27$ , much higher than 1.59 in the benchmark model. This is because cautious borrowers and lenders in the unsecured debt market choose a lower leverage. When the economy has a bigger fraction of unsecured lending the aggregate leverage ratio is lower.

Second, the presence of unsecured debt dampens the dynamics of the system because unsecured debt borrowers default less often than secured debt borrowers. When an economy has a larger proportion of unsecured debt in the steady state, a negative shock to the aggregate economy induces a smaller increase in default. This means that firms are able to retain a bigger fraction of their revenues, so the fall in net worth in the firm sector is mitigated, and firm leverage is less volatile. It requires a shorter time for firm net worth to recover, resulting in a less deep and persistent recession.

## 8. Model extensions

In this section we discuss four model extensions. The purpose is to show that our key mechanism holds under a more general environment. We outline each of the extensions below and report our key moments (i.e. the correlations of secured and unsecured debt with

output) in Table 8.<sup>16</sup>

### 8.1. Credit upgrade

In the benchmark model, firms which are downgraded to  $B$  firms will not become  $G$  firms in any future periods. In reality some firms do regain high credit ratings and favorable terms with creditors. We allow for this in the current extension. Following Cui and Kaas (2017), we assume there is an exogenous probability  $\gamma^{up}$  that a  $B$  firm becomes a  $G$  firm in a given period. We also assume an exogenous probability  $\gamma^{down}$  that a  $G$  firm becomes a  $B$  firm in the next period. To implement this, the future values of marginal net worth are modified to:

$$\Omega_t^G = \theta[(1 - \gamma^{down})\lambda_t^G + \gamma^{down}\lambda_t^B] + 1 - \theta, \quad (37)$$

$$\Omega_t^B = \theta[(1 - \gamma^{up})\lambda_t^B + \gamma^{up}\lambda_t^G] + 1 - \theta. \quad (38)$$

The rest of the credit contract equations remain unchanged. For small values of  $\gamma^{up}$  and  $\gamma^{down}$ , all of our analytical results remain valid. To simulate this model, we set the credit upgrade parameter to  $\gamma^{up} = 0.1$  to corresponds to a firm staying at a  $B$  rating for 10 years on average.<sup>17</sup> Exogenous downgrade is set to  $\gamma^{down} = 0.015$  which has little effect on dynamics.

### 8.2. Positive recovery ratio in unsecured debt

We assume in the benchmark model that creditors of unsecured debt do not have access to the firm's asset when it defaults. Here we relax this assumption and assume that if a unsecured debt borrower defaults, a fraction  $\mu$  is lost. Unsecured debt lenders get a fraction  $\rho < 1 - \mu$  of the remaining value in the firm. The borrower has a probability  $\zeta$  of retaining the remaining  $(1 - \mu - \rho)$  fraction of net worth and becomes a  $B$  firm. With probability  $(1 - \zeta)$  the borrower gets nothing.<sup>18</sup>

Data on recovery rates is available in Mora (2012). The recovery rate measures the extent to which the creditor recovers the principal and accrued interest due on a defaulted debt. According to Moody's Default Risk Service (DRS) data 1970-2008, the mean recovery rate is 39%, and the mode is smaller than 10% (These figures haven't accounted for the discounting of delayed repayments due to litigation or other reasons). However, for senior unsecured debt, the median recovery rate is only 26%. We thus set  $\rho = 0.3$  in our simulations.

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<sup>16</sup>Detailed descriptions of each of these extensions are discussed in an additional appendix available from the authors.

<sup>17</sup>This calibration corresponds to the bankruptcy flag for sole proprietors filing for bankruptcy under Chapter 7 of the US Bankruptcy Code.

<sup>18</sup>The benchmark model is a special case in which  $\rho = 0$ .



**Table 8**

Model extensions.

	Correlations with output	Unsecured Debt	Secured Debt
Data	Rated firms	0.48	0.06
	All firms	0.50	0.15
Model	Benchmark	0.64	0.09
	Credit upgrade	0.64	0.13
	Positive recovery ratio	0.66	0.18
	Different avg. productivity	0.61	0.18
	Mixed debt	0.76	0.26

Note: This table reports the contemporaneous correlations with output.

### 8.3. Different average productivity

The benchmark model does not allow for differences in aggregate productivity. As a result, all firms face the same expected future return on capital  $E_t R_{t+1}^K$ . In this extension we relax this assumption. Specifically, in each period a fraction  $\pi$  of firms have high productivity  $A^H$ , and the remaining  $(1 - \pi)$  fraction has low productivity  $A^L$  such that  $A^H > A^L$ . For simplicity, productivity in each period is uncorrelated. Firms produce with the following Cobb-Douglas production function:

$$Y_{jt}^{m,i} = A_t A^m (\omega_{jt} K_{jt-1}^{m,i})^\alpha (L_{jt}^{m,i})^{1-\alpha}, \quad (39)$$

where  $A_t$  denotes the TFP of the economy,  $A^m$  where  $m \in \{H, L\}$  is the firm's productivity type such that  $A^H > A^L$ , and  $\omega_{jt}$  is an idiosyncratic shock to a firm's capital quality.  $A^m$  has an i.i.d two point distribution with  $Pr(A^H) = \pi$  and mean 1. Its realization is observed by lenders when the loan contracts are decided.

Now, the average return on capital of the firm whose current productivity is  $A^m$  is given by:

$$R_t^{m,K} \equiv \frac{r_t^{m,K} + (1 - \delta)Q_t}{Q_{t-1}}. \quad (40)$$

where  $r_t^{m,K} \equiv \alpha A_t A^m \left( \frac{(1-\alpha)A_t A^m}{w_t} \right)^{\frac{1-\alpha}{\alpha}}$ . Clearly  $R_t^{H,K} > R_t^{L,K}$ .

For each average productivity  $\{A^H, A^L\}$ , there are  $G$  and  $B$  firms. More importantly, all of our analytical results hold for  $G$  and  $B$  firms with the same average productivity. But firms with average productivity  $A^H$  face higher expected return and external finance premium than firms with low average productivity  $A^L$ .

In the simulation exercise, we set the fraction of productive firms to be  $\pi = 20\%$ , which is common in the literature. We choose  $A^H = (1.15)^\alpha$ , and  $A^L = (1 - \pi A^H)/(1 - \pi)$  so that the unconditional productivity is 1.

#### 8.4. Mixed debt in low-credit-quality firms

In the data, low credit quality firms usually have a multi-tier debt structure, borrowing both secured and unsecured debt. In this extension, we assume that  $B$  firms borrow a fixed fraction  $(1 - \nu)$  of unsecured debt and the remaining fraction  $\nu$  of secured debt. For simplicity, assume that a firm either repays or defaults all its debt obligations. In the case of default, the secured debt lender is entitled to  $\nu(1 - \mu)\omega_{jt+1}R_{t+1}^K Q_t K_{jt}^B$  fraction of assets after monitoring. The default  $B$  firm undergoes debt restructuring. With probability  $\zeta$ , debt restructuring is successful and the firm retains  $(1 - \mu)(1 - \nu)\omega_{jt+1}R_{t+1}^K Q_t K_{jt}^B$ , but it loses its  $B$  label and is excluded from *any* loans in future. With probability  $(1 - \zeta)$ , debt restructuring is unsuccessful, the firm shuts down and has nothing left.

We show that it is optimal for a  $B$  firm to choose to default when  $\omega < \tilde{\omega}_t^B$ , where  $\tilde{\omega}_t^B = (\xi_t^B)^{-1}\bar{\omega}_t^B$ , and  $\xi_t^B \leq 1$  is the reputation value of being a  $B$  firm. Furthermore, the value of a firm is still given by  $V_t^i(N_{jt}^i) = \lambda_t^i N_{jt}^i$  for  $i \in \{G, B, X\}$ , where  $\lambda_t^G > \lambda_t^B > \lambda_t^X > 1$  for all  $t$ , and ‘ $X$ ’ is the label for a firm which is excluded from the financial market.

We use the same calibration strategy for financial parameters, targeting a 75% unsecured debt to total debt ratio  $[B^G + (1 - \nu)B^B]/(B^G + B^B)$ , and we assume  $B$  firms use  $\nu = 80\%$  secured debt.

## 9. Conclusion

In this paper, we study the important features of firms’ debt structure. We find that firms with a high-credit-rating rely almost exclusively on unsecured debt, while those with a low-credit-quality use a multi-tiered debt structure often consisting of a large share of secured debt. We show that debt heterogeneity is a first-order aspect of firm capital structure, and is essential to the understanding of debt dynamics and cyclical fluctuations.

We embed secured and unsecured debt in a dynamic stochastic general equilibrium model featuring costly state verification. In our model, unsecured debt borrowers may default and still keep their assets, which allows them to strategically default on their borrowing and run the risk of losing their high credit rating. Under this contractual arrangement, market participants of unsecured debt are relatively cautious, relative to participants in the secured debt market. This accounts for low leverage ratios in high-credit-rating firms. This effect implies that lenders cut lending disproportionately on unsecured debt in a recession, thus leading to a higher correlation between output and unsecured debt than for secured debt.

A calibrated version of our economy matches well with the observed volatility and correlations of output, firm credit, and investment. We find that the amplification effect of an economic shock in our model is smaller than that generated by a model featuring secured

debt only. Our results show that too much investment volatility would be incorrectly predicted by frictions in the secured firm debt market – a standard result in the literature. We conclude that unsecured debt and its dynamics are important to a better understanding of fluctuations in business cycles.

## **10. Acknowledgments**

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## Appendix A. Full system

The full system has a macroeconomic part and a credit contract part. The macroeconomic part is given by:

$$\Lambda_{t-1,t} = \beta \frac{C_{t-1}}{C_t} \quad (\text{A.1})$$

$$1 = R_t E_t(\Lambda_{t,t+1}) \quad (\text{A.2})$$

$$w_t = \chi L_t^\varphi U_{C_t}^{-1} \quad (\text{A.3})$$

$$w_t L_t = (1 - \alpha) Y_t \quad (\text{A.4})$$

$$Y_t = A_t K_{t-1}^\alpha L_t^{1-\alpha} \quad (\text{A.5})$$

$$K_t = (1 - \delta) K_{t-1} + \left[ 1 - \frac{\psi^I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t \quad (\text{A.6})$$

$$Y_t = C_t + I_t + [\mu + (1 - \mu)(1 - \zeta)] G(\tilde{\omega}_t^G) R_t^K Q_{t-1} K_{t-1}^G + \mu G(\bar{\omega}_t^B) R_t^K Q_{t-1} K_{t-1}^B + \kappa \mathcal{N}_t^B \quad (\text{A.7})$$

$$1 = Q_t \left[ 1 - \frac{\psi^I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - \psi^I \frac{I_t}{I_{t-1}} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right] \\ + E_t \left[ \Lambda_{t,t+1} Q_{t+1} \psi^I \left( \frac{I_{t+1}}{I_t} \right)^2 \left( \frac{I_{t+1}}{I_t} - 1 \right) \right] \quad (\text{A.8})$$

$$R_t^K = \frac{\alpha \frac{Y_t}{K_{t-1}} + (1 - \delta) Q_t}{Q_{t-1}} \quad (\text{A.9})$$

The credit contract part:

$$\lambda_t^G = \phi_t^G E_t \Lambda_{t,t+1} \Omega_{t+1}^G R_{t+1}^K \{1 - \xi_{t+1} [G(\tilde{\omega}_{t+1}^G) + \tilde{\omega}_{t+1}^G (1 - F(\tilde{\omega}_{t+1}^G))]\} \quad (\text{A.10})$$

$$1 - \frac{1}{\phi_{t-1}^G} = \frac{R_t^K}{R_{t-1}} \xi_t \tilde{\omega}_t^G [1 - F(\tilde{\omega}_t^G)] \quad (\text{A.11})$$

$$\lambda_t^G = \frac{E_t \Lambda_{t+1} R_{t+1}^K \Omega_{t+1}^G \xi_{t+1} (1 - F(\tilde{\omega}_{t+1}^G))}{E_t \frac{R_{t+1}^K}{R_t} \xi_{t+1} [1 - F(\tilde{\omega}_{t+1}^G) - \tilde{\omega}_{t+1}^G f(\tilde{\omega}_{t+1}^G)]} \quad (\text{A.12})$$

$$\bar{\omega}_t^G = \xi_t \tilde{\omega}_t^G \quad (\text{A.13})$$

$$\xi_t = 1 - \frac{\zeta(1 - \mu)(\theta \lambda_t^B + 1 - \theta)}{\Omega_t^G} \quad (\text{A.14})$$

$$\lambda_t^B = (1 - \kappa) \phi_t^B E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K [1 - G(\bar{\omega}_{t+1}^B) - \bar{\omega}_{t+1}^B (1 - F(\bar{\omega}_{t+1}^B))] \quad (\text{A.15})$$

$$1 - \frac{1}{\phi_{t-1}^B} = \frac{R_t^K}{R_{t-1}} \{ \bar{\omega}_t^B [1 - F(\bar{\omega}_t^B)] + (1 - \mu) G(\bar{\omega}_t^B) \} \quad (\text{A.16})$$

$$\lambda_t^B = \frac{(1 - \kappa) E_t \Lambda_{t+1} \Omega_{t+1}^B R_{t+1}^K [1 - F(\bar{\omega}_{t+1}^B)]}{E_t \frac{R_{t+1}^K}{R_t} [1 - F(\bar{\omega}_{t+1}^B) - \mu \bar{\omega}_{t+1}^B f(\bar{\omega}_{t+1}^B)]} \quad (\text{A.17})$$

$$K_t = K_t^G + K_t^B \quad (\text{A.18})$$

$$Q_t K_t^G = N_t^G \phi_t^G \quad (\text{A.19})$$

$$Q_t K_t^B = (1 - \kappa) N_t^B \phi_t^B \quad (\text{A.20})$$

$$N_t^G = (\theta R_t^K \phi_{t-1}^G \{1 - G(\tilde{\omega}_t^G) - \bar{\omega}_t^G [1 - F(\tilde{\omega}_t^G)]\} + \tau) N_{t-1}^G \quad (\text{A.21})$$

$$N_t^B = \zeta(1 - \mu) G(\tilde{\omega}_t^G) \theta R_t^K \phi_{t-1}^G N_{t-1}^G + (1 - \kappa) \theta \{1 - G(\bar{\omega}_t^B) - \bar{\omega}_t^B [1 - F(\bar{\omega}_t^B)]\} R_t^K \phi_{t-1}^B N_{t-1}^B + \tau N_{t-1}^B \quad (\text{A.22})$$

$$\Omega_t^B = \theta \lambda_t^B + 1 - \theta \quad (\text{A.23})$$

$$\Omega_t^G = \theta \lambda_t^G + 1 - \theta \quad (\text{A.24})$$

where  $f(\bar{\omega}_t; \sigma_{t-1}) \equiv \frac{\partial}{\partial \bar{\omega}_t} F(\bar{\omega}_t; \sigma_{t-1})$  is the probability density function of  $\bar{\omega}_t$ , and  $G(\bar{\omega}_t; \sigma_{t-1}) \equiv \int^{\bar{\omega}_t} \omega dF(\omega, \sigma_{t-1})$ . The above 24 equations solve the following 24 variables

$$\{\Lambda_{t-1,t}, C_t, w_t, L_t, Y_t, K_t, I_t, Q_t, R_t^K, R_t, \lambda_t^G, \phi_t^G, \tilde{\omega}_t^G, \bar{\omega}_t^G, \xi_t, N_t^G, K_t^G, \Omega_t^G, \lambda_t^B, \phi_t^B, \bar{\omega}_t^B, N_t^B, K_t^B, \Omega_t^B\}.$$

### Appendix A.1. BGG system

This appendix presents the BGG system. The macroeconomic part is identical to our benchmark model, except that the goods market clearing condition is now given by:

$$Y_t = C_t + I_t + \mu G(\bar{\omega}_t) R_t^K Q_{t-1} K_{t-1} + \tilde{\kappa} N_t \quad (\text{A.25})$$

The credit contract part is:

$$\lambda_t = (1 - \tilde{\kappa})\phi_t E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1}^K [1 - G(\bar{\omega}_{t+1}) - \bar{\omega}_{t+1}(1 - F(\bar{\omega}_{t+1}))] \quad (\text{A.26})$$

$$1 - \frac{1}{\phi_{t-1}} = \frac{R_t^K}{R_{t-1}} \{ \bar{\omega}_t [1 - F(\bar{\omega}_t)] + (1 - \mu)G(\bar{\omega}_t) \} \quad (\text{A.27})$$

$$\lambda_t = \frac{(1 - \tilde{\kappa})E_t \Lambda_{t+1} \Omega_{t+1} R_{t+1}^K [1 - F(\bar{\omega}_{t+1})]}{E_t \frac{R_{t+1}^K}{R_t} [1 - F(\bar{\omega}_{t+1}) - \mu \bar{\omega}_{t+1} f(\bar{\omega}_{t+1})]} \quad (\text{A.28})$$

$$Q_t K_t = (1 - \tilde{\kappa}) N_t \phi_t \quad (\text{A.29})$$

$$N_t = (1 - \tilde{\kappa})\theta \{ 1 - G(\bar{\omega}_t) - \bar{\omega}_t [1 - F(\bar{\omega}_t)] \} R_t^K \phi_{t-1} N_{t-1} + \tau N_{t-1} \quad (\text{A.30})$$

$$\Omega_t = \theta \lambda_t + 1 - \theta \quad (\text{A.31})$$

The 15-equation system solves the following 15 variables:

$$\{ \Lambda_{t-1,t}, C_t, w_t, L_t, Y_t, K_t, I_t, Q_t, R_t^K, R_t, \lambda_t, \phi_t, \bar{\omega}_t, N_t, \Omega_t \}.$$

### Appendix A.2. The simple RBC system

The simple RBC system has a macroeconomic system similar to our benchmark model, except that the goods market clearing condition is now given by:

$$Y_t = C_t + I_t, \quad (\text{A.32})$$

and the return on capital is equal to the risk-free rate:

$$1 = E_t \left[ \Lambda_{t,t+1} \frac{\alpha \frac{Y_{t+1}}{K_t} + (1 - \delta) Q_{t+1}}{Q_t} \right]. \quad (\text{A.33})$$

The system solves the following 9 variables:

$$\{ \Lambda_{t-1,t}, C_t, w_t, L_t, Y_t, K_t, I_t, Q_t, R_t \}.$$

## Appendix B. Proofs

### Proof of proposition 1

With perfect competition, the participation constraints hold with equality. We begin by solving the problem for the  $B$  firms. We substitute the guess into the objective function. The objective function can be rewritten as:

$$V_t^B(N_{jt}^B) = \max E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K Q_t K_{jt}^B \int_{\bar{\omega}_{jt+1}^B} (\omega - \bar{\omega}_{jt+1}^B) dF_t, \quad (\text{B.1})$$

where  $\Omega_t^B \equiv \theta \lambda_t^B + 1 - \theta$ .

We write down the Lagrangian as

$$V_t^B(N_{jt}^B) = \max E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K Q_t K_{jt}^B \int_{\bar{\omega}_{jt+1}^B} (\omega - \bar{\omega}_{jt+1}^B) dF_t \\ + lm_{jt}^B \left[ \frac{R_{t+1}^K}{R_t} \frac{Q_t K_{jt}^B}{(1-\kappa)} \left( \int_{\bar{\omega}_{jt+1}^B} \bar{\omega}_{jt+1}^B dF_t + (1-\mu) \int^{\bar{\omega}_{jt+1}^B} \omega dF_t \right) - \frac{Q_t K_{jt}^B}{(1-\kappa)} + N_{jt}^B \right],$$

where  $lm_{jt}^B$  is the Lagrange multiplier. The envelope condition says that  $\lambda_t^B = lm_{jt}^B$ . The first order condition for  $K_{jt}^B$  is:

$$K_{jt}^B : 0 = E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K \left( \int_{\bar{\omega}_{jt+1}^B} (\omega - \bar{\omega}_{jt+1}^B) dF_t \right) \\ + \lambda_t^B \left[ \frac{R_{t+1}^K}{R_t} \frac{1}{(1-\kappa)} \left( \int_{\bar{\omega}_{jt+1}^B} \bar{\omega}_{jt+1}^B dF_t + (1-\mu) \int^{\bar{\omega}_{jt+1}^B} \omega dF_t \right) - \frac{1}{(1-\kappa)} \right]. \quad (\text{B.3})$$

In this equation,  $\bar{\omega}_{jt}^B$  is the only firm-specific variable. This implies that every firm chooses the same cutoff value  $\bar{\omega}_t^B$ . The participation constraint implies every firm chooses the same leverage ratio:

$$\frac{R_{t+1}^K}{R_t} \left( \int_{\bar{\omega}_{t+1}^B} \bar{\omega}_{t+1}^B dF_t + (1-\mu) \int^{\bar{\omega}_{t+1}^B} \omega dF_t \right) = 1 - \frac{1}{\phi_t^B}, \quad (\text{B.4})$$

where  $\phi_t^B \equiv Q_t K_{jt}^B / [(1-\kappa) N_{jt}^B]$ . Rearranging the first order condition for  $K_{jt}^B$ , we obtain:

$$\lambda_t^B = (1-\kappa) \phi_t^B E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K \int_{\bar{\omega}_{t+1}^B} (\omega - \bar{\omega}_{t+1}^B) dF_t. \quad (\text{B.5})$$

Using the results that  $V_t^B(N_{jt}^B) = \lambda_t^B N_{jt}^B$  and  $\phi_t^B = Q_t K_{jt}^B / [(1-\kappa) N_{jt}^B]$ , the objective function can be expressed as:

$$V_t^B(N_{jt}^B) = E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K Q_t K_{jt}^B \int_{\bar{\omega}_{t+1}^B} (\omega - \bar{\omega}_{t+1}^B) dF_t \\ \lambda_t^B = (1-\kappa) \phi_t^B E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K \int_{\bar{\omega}_{t+1}^B} (\omega - \bar{\omega}_{t+1}^B) dF_t. \quad (\text{B.6})$$

This is the same as the first order condition for  $K_{jt}^B$ . Our guess is verified.

The first order condition for  $\bar{\omega}_{t+1}^B$  is given by:

$$\lambda_t^B = \frac{(1 - \kappa)E_t\Lambda_{t+1}\Omega_{t+1}^B R_{t+1}^K [1 - F(\bar{\omega}_{t+1}^B)]}{E_t \frac{R_{t+1}^K}{R_t} [1 - F(\bar{\omega}_{t+1}^B) - \mu \bar{\omega}_{t+1}^B f(\bar{\omega}_{t+1}^B)]}. \quad (\text{B.7})$$

In the steady state

$$\frac{\lambda^B}{\theta \lambda^B + 1 - \theta} = \frac{(1 - \kappa)[1 - F(\bar{\omega}^B)]}{[1 - F(\bar{\omega}^B) - \mu \bar{\omega}^B f(\bar{\omega}^B)]}. \quad (\text{B.8})$$

We need  $\lambda^B > 1$  in the steady state, which requires that

$$\frac{(1 - \kappa)(1 - F(\bar{\omega}^B))}{(1 - F(\bar{\omega}^B) - \mu \bar{\omega}^B f(\bar{\omega}^B))} > 1.$$

We turn to the problem of  $G$  firms. We substitute  $V_t^G(N_{jt}^G) = \lambda_t^G N_{jt}^G$ ,  $V_t^B(N_{jt}^B) = \lambda_t^B N_{jt}^B$  into the objective function. The maximization problem in the integral becomes:

$$\max\{\Omega_{t+1}^G(\omega - \bar{\omega}_{jt+1}^G), \zeta(1 - \mu)\Omega_{t+1}^B\omega\}, \quad (\text{B.9})$$

where  $\Omega_t^i \equiv \theta \lambda_t^i + 1 - \theta$ , for  $i \in \{B, G\}$ . This means that default is chosen when  $\omega < \bar{\omega}_{jt}^G$ , where  $\bar{\omega}_{jt}^G \in [\bar{\omega}_{jt}^G, \infty)$  (because we rule out the case that all  $G$  firms default) and is given by:

$$\bar{\omega}_t^G = \xi_t^{-1} \bar{\omega}_t^G, \quad \xi_t \equiv 1 - \frac{\zeta(1 - \mu)\Omega_t^B}{\Omega_t^G}. \quad (\text{B.10})$$

These mean that we can rewrite the objective function as:

$$V_t^G(N_{jt}^G) = \max E_t \Lambda_{t,t+1} \Omega_{t+1}^G R_{t+1}^K Q_t K_{jt}^G \left( (1 - \xi_{t+1}) \int^{\bar{\omega}_{jt+1}^G} \omega dF_t + \int_{\bar{\omega}_{jt+1}^G} (\omega - \bar{\omega}_{jt+1}^G) dF_t \right) \quad (\text{B.11})$$

and the participation constraint as:

$$R_{t+1}^K Q_t K_{jt}^G \left( \int_{\bar{\omega}_{jt+1}^G} \bar{\omega}_{jt+1}^G dF_t \right) = R_t (Q_t K_{jt}^G - N_{jt}^G). \quad (\text{B.12})$$

We write down the Lagrangian as

$$\begin{aligned} V_t^G(N_{jt}^G) = & \max E_t \Lambda_{t,t+1} \Omega_{t+1}^G R_{t+1}^K Q_t K_{jt}^G \left( (1 - \xi_{t+1}) \int^{\bar{\omega}_{jt+1}^G} \omega dF_t + \int_{\bar{\omega}_{jt+1}^G} (\omega - \bar{\omega}_{jt+1}^G) dF_t \right) \\ & + l m_{jt}^G \left[ \frac{R_{t+1}^K}{R_t} Q_t K_{jt}^G \left( \int_{\bar{\omega}_{jt+1}^G} \bar{\omega}_{jt+1}^G dF_t \right) - Q_t K_{jt}^G + N_{jt}^G \right], \end{aligned} \quad (\text{B.13})$$



where  $lm_{jt}^G$  is the Lagrange multiplier. The envelope conditions says that  $\lambda_t^G = lm_{jt}^G$ . The first order condition for  $K_{jt}^G$  is:

$$K_{jt}^G : 0 = E_t \Lambda_{t,t+1} \Omega_{t+1}^G R_{t+1}^K \left( (1 - \xi_{t+1}) \int^{\tilde{\omega}_{jt+1}^G} \omega dF_t + \int_{\tilde{\omega}_{jt+1}^G} (\omega - \bar{\omega}_{jt+1}^G) dF_t \right) + \lambda_t^G \left[ \frac{R_{t+1}^K}{R_t} \left( \int_{\tilde{\omega}_{jt+1}^G} \bar{\omega}_{jt+1}^G dF_t \right) - 1 \right]. \quad (\text{B.14})$$

In this equation,  $\bar{\omega}_{jt}^G$  is the only firm-specific variable. This implies that every firm chooses the same cutoff value  $\bar{\omega}_t^G$ . Then the participation constraint implies every firm chooses the same leverage:

$$1 - \frac{1}{\phi_t^G} = \frac{R_{t+1}^K}{R_t} \left( \int_{\bar{\omega}_{t+1}^G} \bar{\omega}_{t+1}^G dF_t \right), \quad (\text{B.15})$$

where  $\phi_t^G \equiv Q_t K_{jt}^G / N_{jt}^G$ . Rearranging the first order condition for  $K_{jt}^G$ , we obtain:

$$\lambda_t^G = \phi_t^G E_t \Lambda_{t,t+1} \Omega_{t+1}^G R_{t+1}^K \left( (1 - \xi_{t+1}) \int^{\tilde{\omega}_{t+1}^G} \omega dF_t + \int_{\tilde{\omega}_{t+1}^G} (\omega - \bar{\omega}_{t+1}^G) dF_t \right). \quad (\text{B.16})$$

We can substitute these results back to the objective function to verify the guess  $V_t^G(N_{jt}^G) = \lambda_t^G N_{jt}^G$  is indeed correct:

$$V_t^G(N_{jt}^G) = E_t \Lambda_{t,t+1} \Omega_{t+1}^G R_{t+1}^K Q_t K_{jt}^G \left[ (1 - \xi_{t+1}) \int^{\tilde{\omega}_{t+1}^G} \omega dF_t + \int_{\tilde{\omega}_{t+1}^G} (\omega - \bar{\omega}_{t+1}^G) dF_t \right] \\ \lambda_t^G = \phi_t^G E_t \Lambda_{t,t+1} \Omega_{t+1}^G R_{t+1}^K \left[ (1 - \xi_{t+1}) \int^{\tilde{\omega}_{t+1}^G} \omega dF_t + \int_{\tilde{\omega}_{t+1}^G} (\omega - \bar{\omega}_{t+1}^G) dF_t \right]. \quad (\text{B.17})$$

This is the same as the first order condition for  $K_{jt}^G$ . Our guess is verified.

The first order condition for  $\tilde{\omega}_{t+1}^G$  is given by:

$$\lambda_t^G = \frac{E_t \Lambda_{t+1} R_{t+1}^K \Omega_{t+1}^G \xi_{t+1} [1 - F(\tilde{\omega}_{t+1}^G)]}{E_t \frac{R_{t+1}^K}{R_t} \xi_{t+1} [1 - F(\tilde{\omega}_{t+1}^G) - \tilde{\omega}_{t+1}^G f(\tilde{\omega}_{t+1}^G)]}. \quad (\text{B.18})$$

In the steady state, this implies:

$$\frac{\lambda^G}{\theta \lambda^G + 1 - \theta} = \frac{1 - F(\tilde{\omega}^G)}{1 - F(\tilde{\omega}^G) - \tilde{\omega}^G f(\tilde{\omega}^G)} > 1. \quad (\text{B.19})$$

We derive the condition under which  $\lambda^G > \lambda^B$ . Using (B.8) and (B.19), we show that:

$$\begin{aligned} \lambda^G &> \lambda^B \\ \frac{\lambda^G}{\theta\lambda^G + 1 - \theta} &> \frac{\lambda^B}{\theta\lambda^B + 1 - \theta} \\ \frac{1 - F(\tilde{\omega}^G)}{1 - F(\tilde{\omega}^G) - \tilde{\omega}^G f(\tilde{\omega}^G)} &> (1 - \kappa) \left[ \frac{1 - F(\bar{\omega}^B)}{1 - F(\bar{\omega}^B) - \mu\bar{\omega}^B f(\bar{\omega}^B)} \right] \\ \kappa &> 1 - \left[ \frac{1 - F(\tilde{\omega}^G)}{1 - F(\tilde{\omega}^G) - \tilde{\omega}^G f(\tilde{\omega}^G)} \right] / \left[ \frac{1 - F(\bar{\omega}^B)}{1 - F(\bar{\omega}^B) - \mu\bar{\omega}^B f(\bar{\omega}^B)} \right] \end{aligned} \quad (\text{B.20})$$

### Proof of propositions 2, 3

We prove some important properties of  $\rho^B(\bar{\omega}^B; \sigma)$  and  $\rho^G(\tilde{\omega}^G, \xi; \sigma)$ . Define:

$$\begin{aligned} G(\bar{\omega}_t; \sigma_{t-1}) &\equiv \int^{\bar{\omega}_t} \omega dF(\omega, \sigma_{t-1}), \\ \Gamma(\bar{\omega}_t; \sigma_{t-1}) &\equiv G(\bar{\omega}_t; \sigma_{t-1}) + \bar{\omega}[1 - F(\bar{\omega}_t; \sigma_{t-1})]. \end{aligned}$$

The function  $G$  denotes the mean of the idiosyncratic shock conditional on the shock below a given threshold  $\bar{\omega}$ . The function  $\Gamma$  adds the function  $G$  and a constant return  $\bar{\omega}$  if the realization of idiosyncratic shock is above the threshold. This function is the share of revenue transferred to lenders (before monitoring) in the secured debt contract. We denote  $G_\omega, \Gamma_\omega$  the first derivatives of  $G$  and  $\Gamma$  with respect to  $\bar{\omega}$ , and denote  $G_\sigma, \Gamma_\sigma$  the first derivatives of  $G$  and  $\Gamma$  with respect to  $\sigma$ , and so on. In the following, we suppress the arguments of the functions when this does not cause any confusions.

To derive the function  $\rho^B$ , we first note that the evolution of  $\lambda_t^B$ , the optimal threshold  $\bar{\omega}_{t+1}^B$  and the participation constraint can be written as:

$$\lambda_t^B = (1 - \kappa)\phi_t^B E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K [1 - \Gamma(\bar{\omega}_{t+1}^B)], \quad (\text{B.21})$$

$$\lambda_t^B = \frac{(1 - \kappa) E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K \Gamma_\omega(\bar{\omega}_t^B)}{E_t \frac{R_{t+1}^K}{R_t} [\Gamma_\omega(\bar{\omega}_t^B) - \mu G_\omega(\bar{\omega}_t^B)]}, \quad (\text{B.22})$$

$$1 - \frac{1}{\phi_{t-1}^B} = \frac{R_t^K}{R_{t-1}} [\Gamma(\bar{\omega}_t^B) - \mu G(\bar{\omega}_t^B)]. \quad (\text{B.23})$$

We roll the participation constraint one period forward, rearrange these three equations to eliminate the leverage ratio and the marginal value  $\lambda_t^B$  to get, up to a first order approximation:

$$E_t \left( \frac{R_{t+1}^K}{R_t} \right) = E_t \rho^B(\bar{\omega}_{t+1}^B; \sigma_t), \quad (\text{B.24})$$

where

$$\rho^B(\bar{\omega}_{t+1}^B) \equiv \frac{\Gamma_\omega(\bar{\omega}_{t+1}^B)}{[1 - \Gamma(\bar{\omega}_{t+1}^B)][\Gamma_\omega(\bar{\omega}_{t+1}^B) - \mu G_\omega(\bar{\omega}_{t+1}^B)] + [\Gamma(\bar{\omega}_{t+1}^B) - \mu G(\bar{\omega}_{t+1}^B)]\Gamma_\omega(\bar{\omega}_{t+1}^B)}. \quad (\text{B.25})$$

Following the same procedures, we can show that for the unsecured debt contract, up to a first-order approximation,

$$E_t \left( \frac{R_{t+1}^K}{R_t} \right) = E_t \rho^G(\tilde{\omega}_{t+1}^G, \xi_{t+1}; \sigma_t), \quad (\text{B.26})$$

where

$$\rho^G(\tilde{\omega}_{t+1}^G, \xi_{t+1}) \equiv \frac{\Gamma_\omega(\tilde{\omega}_{t+1}^G)}{[1 - \xi \Gamma(\tilde{\omega}_{t+1}^G)][\Gamma_\omega(\tilde{\omega}_{t+1}^G) - G_\omega(\tilde{\omega}_{t+1}^G)] + \xi [\Gamma(\tilde{\omega}_{t+1}^G) - G(\tilde{\omega}_{t+1}^G)]\Gamma_\omega(\tilde{\omega}_{t+1}^G)}. \quad (\text{B.27})$$

We now analyze the properties of  $\rho^B, \rho^G$ . First, it is straightforward to show that:

$$F(\bar{\omega}; \sigma) = \Phi \left( \frac{\log \bar{\omega} + 0.5\sigma^2}{\sigma} \right) > 0, \quad G(\bar{\omega}; \sigma) = \Phi \left( \frac{\log \bar{\omega} - 0.5\sigma^2}{\sigma} \right) > 0.$$

where we define  $\Phi(\cdot), \phi(\cdot)$  as the cdf and pdf of a standard normal distribution.

The first derivatives are:

$$\begin{aligned} F_\omega &= \frac{1}{\sigma \bar{\omega}} \phi \left( \frac{\log \bar{\omega} + 0.5\sigma^2}{\sigma} \right) > 0, \\ F_\sigma &= -\frac{1}{\sigma} \phi \left( \frac{\log \bar{\omega} + 0.5\sigma^2}{\sigma} \right) \left( \frac{\log \bar{\omega} - 0.5\sigma^2}{\sigma} \right) > 0, \\ G_\omega &= \frac{1}{\sigma \bar{\omega}} \phi \left( \frac{\log \bar{\omega} - 0.5\sigma^2}{\sigma} \right) > 0, \\ G_\sigma &= -\frac{1}{\sigma} \phi \left( \frac{\log \bar{\omega} - 0.5\sigma^2}{\sigma} \right) \left( \frac{\log \bar{\omega} + 0.5\sigma^2}{\sigma} \right) > 0, \\ \Gamma_\omega &= 1 - F > 0, \\ \Gamma_\sigma &= G_\sigma - \bar{\omega} F_\sigma = -\phi \left( \frac{\log \bar{\omega} - 0.5\sigma^2}{\sigma} \right) < 0, \end{aligned}$$

where  $\left( \frac{\log \bar{\omega} - 0.5\sigma^2}{\sigma} \right) < \left( \frac{\log \bar{\omega} + 0.5\sigma^2}{\sigma} \right) < 0$  because the default probability is small in economi-

cally relevant cases.<sup>19</sup>

The following second derivatives are useful:

$$\begin{aligned}
G_{\omega\omega} &= -\frac{1}{\sigma\bar{\omega}^2}\phi\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right) - \frac{1}{\sigma^2\bar{\omega}^2}\phi\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right) \\
&= -\frac{1}{\sigma\bar{\omega}^2}\phi\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\left(\frac{\log\bar{\omega}+0.5\sigma^2}{\sigma}\right) > 0, \\
G_{\omega\sigma} &= -\frac{1}{\sigma^2\bar{\omega}}\phi\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right) - \frac{1}{\sigma^2\bar{\omega}}\phi'\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\left(\frac{\log\bar{\omega}+0.5\sigma^2}{\sigma}\right) \\
&= \frac{1}{\sigma^2\bar{\omega}}\phi\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\left[\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\left(\frac{\log\bar{\omega}+0.5\sigma^2}{\sigma}\right) - 1\right] > 0, \\
\Gamma_{\omega\omega} &= -F_\omega < 0, \\
\Gamma_{\omega\sigma} &= -F_\sigma < 0.
\end{aligned}$$

we have used  $\phi'(x) = -x\phi(x)$ .

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<sup>19</sup>To derive the expression for  $\Gamma_\sigma$  we note that:

$$\begin{aligned}
\bar{\omega}F_\sigma &= -\frac{\bar{\omega}}{\sigma}\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\phi\left(\frac{\log\bar{\omega}+0.5\sigma^2}{\sigma}\right) \\
&= -\frac{\bar{\omega}}{\sigma}\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\frac{[(\log\bar{\omega}-0.5\sigma^2)+\sigma^2]^2}{\sigma^2}\right\} \\
&= -\frac{\bar{\omega}}{\sigma}\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\frac{(\log\bar{\omega}-0.5\sigma^2)^2+2(\log\bar{\omega}-0.5\sigma^2)\sigma^2+\sigma^4}{\sigma^2}\right\} \\
&= -\frac{\bar{\omega}}{\sigma}\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\frac{(\log\bar{\omega}-0.5\sigma^2)^2}{\sigma^2}\right\}\exp(-\log\bar{\omega}) \\
&= -\frac{1}{\sigma}\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\phi\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right).
\end{aligned}$$

Therefore,

$$\begin{aligned}
\Gamma_\sigma &= G_\sigma - \bar{\omega}F_\sigma \\
&= -\frac{1}{\sigma}\phi\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\left(\frac{\log\bar{\omega}+0.5\sigma^2}{\sigma}\right) + \frac{1}{\sigma}\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\phi\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right) \\
&= -\frac{1}{\sigma}\phi\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\left[\left(\frac{\log\bar{\omega}+0.5\sigma^2}{\sigma}\right) - \left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\right] \\
&= -\phi\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right) < 0.
\end{aligned}$$

Using the above relations it is easy to show that  $\rho^G, \rho^B \geq 1$ , and

$$\begin{aligned}\rho_\omega^B &= \frac{\Gamma_{\omega\omega}(1-\Gamma)(\Gamma_\omega - \mu G_\omega) - \Gamma_\omega(1-\Gamma)(\Gamma_{\omega\omega} - \mu G_{\omega\omega})}{[(1-\Gamma)(\Gamma_\omega - \mu G_\omega) + (\Gamma - \mu G)\Gamma_\omega]^2} \\ &= \frac{\mu(1-\Gamma)}{[(1-\Gamma)(\Gamma_\omega - \mu G_\omega) + (\Gamma - \mu G)\Gamma_\omega]^2}(\Gamma_\omega G_{\omega\omega} - \Gamma_{\omega\omega} G_\omega) > 0.\end{aligned}\quad (\text{B.28})$$

$$\rho_\omega^G = \frac{(1-\xi\Gamma)}{[(1-\xi\Gamma)(\Gamma_\omega - G_\omega) + \xi(\Gamma - G)\Gamma_\omega]^2}(\Gamma_\omega G_{\omega\omega} - \Gamma_{\omega\omega} G_\omega) > 0. \quad (\text{B.29})$$

$$\begin{aligned}\rho_\sigma^B &= \frac{(1-\Gamma)[(\Gamma_\omega - \mu G_\omega)\Gamma_{\omega\sigma} - \Gamma_\omega(\Gamma_{\omega\sigma} - \mu G_{\omega\sigma})] + \Gamma_\omega[\Gamma_\sigma(\Gamma_\omega - \mu G_\omega) - \Gamma_\omega(\Gamma_\sigma - \mu G_\sigma)]}{[(1-\Gamma)(\Gamma_\omega - \mu G_\omega) + (\Gamma - \mu G)\Gamma_\omega]^2} \\ &= \frac{(1-\Gamma)\mu[\Gamma_\omega G_{\omega\sigma} - G_\omega \Gamma_{\omega\sigma}] + \mu\Gamma_\omega[\Gamma_\omega G_\sigma - \Gamma_\sigma G_\omega]}{[(1-\Gamma)(\Gamma_\omega - \mu G_\omega) + (\Gamma - \mu G)\Gamma_\omega]^2} > 0.\end{aligned}\quad (\text{B.30})$$

$$\rho_\sigma^G = \frac{(1-\xi\Gamma)[\Gamma_\omega G_{\omega\sigma} - G_\omega \Gamma_{\omega\sigma}] + \xi\Gamma_\omega[\Gamma_\omega G_\sigma - \Gamma_\sigma G_\omega]}{[(1-\xi\Gamma)(\Gamma_\omega - G_\omega) + \xi(\Gamma - G)\Gamma_\omega]^2} > 0. \quad (\text{B.31})$$

Next, we show that  $\rho_\xi^G < 0$ . Clearly,

$$\rho_\xi^G = -\frac{\Gamma_\omega}{[(1-\xi\Gamma)(\Gamma_\omega - G_\omega) + \xi(\Gamma - G)\Gamma_\omega]^2}(\Gamma G_\omega - G\Gamma_\omega). \quad (\text{B.32})$$

Notice that

$$\begin{aligned}\Gamma G_\omega - G\Gamma_\omega &= [G + \bar{\omega}(1-F)]G_\omega - G(1-F) \\ &= GG_\omega + (1-F)(\bar{\omega}G_\omega - G).\end{aligned}$$

The first term is clearly positive. We show that the second term is also positive by studying the function  $G_\omega$ :

$$G_\omega = \frac{1}{\sigma\tilde{\omega}^G}\phi\left(\frac{\log\tilde{\omega}^G - 0.5\sigma^2}{\sigma}\right) = \frac{1}{\sigma}\phi\left(\frac{\log\tilde{\omega}^G + 0.5\sigma^2}{\sigma}\right).$$

This means that  $\lim_{\tilde{\omega}^G \rightarrow 0} G_\omega(\tilde{\omega}^G) = 0$ . Furthermore,  $G_{\omega\omega} > 0$  for  $\omega \in [0, \tilde{\omega}^G]$ . These mean that

$$\tilde{\omega}^G G_\omega(\tilde{\omega}^G) > \int_0^{\tilde{\omega}^G} G_\omega d\omega = G(\tilde{\omega}^G) - \lim_{\tilde{\omega}^G \rightarrow 0} G(\tilde{\omega}^G) = G(\tilde{\omega}^G).$$

Therefore,  $\tilde{\omega}^G G_\omega > G$ , so  $\Gamma G_\omega - G\Gamma_\omega > 0$ , which means that  $\rho_\xi^G < 0$ .

Finally, since  $\lim_{\bar{\omega} \rightarrow 0} G_\omega(\bar{\omega}) = 0$  and  $\lim_{\bar{\omega} \rightarrow 0} G(\bar{\omega}) = 0$ , we substitute these results into  $\rho^G, \rho^B$  to get  $\lim_{\bar{\omega} \rightarrow 0} \rho^G(\bar{\omega}) = \lim_{\bar{\omega} \rightarrow 0} \rho^B(\bar{\omega}) = 1$ .

#### Proof of proposition 4

Consider  $\rho_\omega^B, \rho_\omega^G$  in (B.28) and (B.29). We evaluate these functions at a given  $\bar{\omega} > 0$ . Clearly, the numerator of  $\rho_\omega^B$  is smaller than the numerator of  $\rho_\omega^G$ . Furthermore, the denominator of

$\rho_\omega^B$  is larger than the denominator of  $\rho_\omega^G$ . To see this, notice that

$$\begin{aligned}
& [(1 - \Gamma)(\Gamma_\omega - \mu G_\omega) + (\Gamma - \mu G)\Gamma_\omega] - [(1 - \xi\Gamma)(\Gamma_\omega - G_\omega) + \xi(\Gamma - G)\Gamma_\omega] \\
&= G_\omega[1 - \mu + \Gamma(\mu - \xi)] + (\xi - \mu)G\Gamma_\omega \\
&> G_\omega[\Gamma(1 - \mu) + \Gamma(\mu - \xi)] + (\xi - \mu)G\Gamma_\omega \\
&> G_\omega\Gamma(1 - \xi) \\
&> 0,
\end{aligned}$$

where the second last inequality follows from the fact that  $\xi > 1 - (1 - \mu)\Omega^B/\Omega^G > \mu$ . Therefore,  $\frac{\partial \rho^G(\bar{\omega}_t, \xi_t; \sigma_{t-1})}{\partial \bar{\omega}_t} > \frac{\partial \rho^B(\bar{\omega}_t; \sigma_{t-1})}{\partial \bar{\omega}_t}$ .

### Proof of proposition 5

We consider the two participation constraints:

$$1 - \frac{1}{\phi_{t-1}^B} = \frac{R_t^K}{R_{t-1}}[\Gamma(\bar{\omega}_t^B) - \mu G(\bar{\omega}_t^B)], \quad (\text{B.33})$$

$$1 - \frac{1}{\phi_{t-1}^G} = \frac{R_t^K}{R_{t-1}}\xi_t[\Gamma(\tilde{\omega}_t^G) - G(\tilde{\omega}_t^G)]. \quad (\text{B.34})$$

When  $\bar{\omega}_t = \bar{\omega}_t^B = \tilde{\omega}_t^G$ ,

$$1 - \frac{1}{\phi_{t-1}^B} = \frac{R_t^K}{R_{t-1}}[\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)] > \frac{R_t^K}{R_{t-1}}\xi_t[\Gamma(\bar{\omega}_t) - G(\bar{\omega}_t)] = 1 - \frac{1}{\phi_{t-1}^G}. \quad (\text{B.35})$$

Therefore,  $\phi_{t-1}^B > \phi_{t-1}^G$ .

Furthermore,

$$\frac{\partial \left(1 - \frac{1}{\phi_{t-1}^B}\right)}{\partial \bar{\omega}_t} = \frac{R_t^K}{R_{t-1}}[\Gamma_\omega(\bar{\omega}_t) - \mu G_\omega(\bar{\omega}_t)] > \frac{R_t^K}{R_{t-1}}\xi_t[\Gamma_\omega(\bar{\omega}_t) - G_\omega(\bar{\omega}_t)] = \frac{\partial \left(1 - \frac{1}{\phi_{t-1}^G}\right)}{\partial \bar{\omega}_t}$$

Therefore,

$$\frac{\partial \phi_{t-1}^B}{\partial \bar{\omega}_t} > \left(\frac{\phi_{t-1}^B}{\phi_{t-1}^G}\right)^2 \frac{\partial \phi_{t-1}^G}{\partial \bar{\omega}_t} > \frac{\partial \phi_{t-1}^G}{\partial \bar{\omega}_t}. \quad (\text{B.36})$$

Finally we prove that the two participation constraints are upward-sloping. We consider the function  $\Psi(\bar{\omega}) \equiv \Gamma(\bar{\omega}) - G(\bar{\omega})$  and show that  $\Psi_\omega > 0$  for a relevant range of  $\omega$ . To see

this we write:

$$\begin{aligned}
G_\omega(\bar{\omega}) &= \bar{\omega}f(\bar{\omega}) = \bar{\omega}h(\bar{\omega})(1 - F(\bar{\omega})) > 0, \\
\Gamma_\omega(\bar{\omega}) &= G_\omega(\bar{\omega}) + (1 - F(\bar{\omega})) - \bar{\omega}f(\bar{\omega}) = 1 - F(\bar{\omega}) > 0, \\
\Psi_\omega(\bar{\omega}) &= \Gamma_\omega(\bar{\omega}) - G_\omega(\bar{\omega}) = (1 - F(\bar{\omega}))(1 - \bar{\omega}h(\bar{\omega})),
\end{aligned}$$

where  $h(\bar{\omega}) = f(\bar{\omega})/(1 - F(\bar{\omega}))$  is the hazard rate. For the log-normal distribution,  $\bar{\omega}h(\bar{\omega}) = 0$  when  $\bar{\omega} = 0$ ,  $\lim_{\bar{\omega} \rightarrow \infty} \bar{\omega}h(\bar{\omega}) = \infty$ , and  $\bar{\omega}h(\bar{\omega})$  is increasing in  $\bar{\omega}$ . Hence, there exists an  $\bar{\omega}^*$  such that  $\Psi_\omega(\bar{\omega}) > 0$  for  $\bar{\omega} < \bar{\omega}^*$  and  $\Psi_\omega(\bar{\omega}) < 0$  for  $\bar{\omega} > \bar{\omega}^*$ . For any  $\bar{\omega}_1$  such that  $\bar{\omega}_1 > \bar{\omega}^*$ , there exist a  $\bar{\omega}_2$  such that  $\bar{\omega}_2 < \bar{\omega}^* < \bar{\omega}_1$  and  $\Psi(\bar{\omega}_2) = \Psi(\bar{\omega}_1)$ . Since the smaller  $\bar{\omega}_2$  implies a smaller bankruptcy rate for the borrower than  $\bar{\omega}_1$  while keeping the lenders' share of profit unchanged, any  $\bar{\omega}_1 > \bar{\omega}^*$  will never be chosen. Hence,  $\bar{\omega}$  has an interior solution and in the optimal contract  $\Psi_\omega(\bar{\omega}) > 0$ . This means that:

$$\frac{\partial PC^B \left( \bar{\omega}_t, \frac{R_t^K}{R_{t-1}} \right)}{\partial \bar{\omega}_t} > \frac{\partial PC^G \left( \bar{\omega}_t, \xi_t, \frac{R_t^K}{R_{t-1}} \right)}{\partial \bar{\omega}_t} > 0. \quad (\text{B.37})$$

### Proof of proposition 6

We know from Propositions 2 and 3 that  $\lim_{\bar{\omega}_t \rightarrow 0} \rho^B(\bar{\omega}_t) = \lim_{\bar{\omega}_t \rightarrow 0} \rho^G(\bar{\omega}_t, \xi_t) = 1$ , and  $\rho^B, \rho^G$  are increasing in  $\bar{\omega}_t$ . Moreover, Proposition 4 show that  $\rho_\omega^G(\bar{\omega}_t, \xi_t) > \rho_\omega^B(\bar{\omega}_t)$ . These mean that, for any external finance premium such that  $E_t(R_{t+1}^K)/R_t = E_t \rho^G(\tilde{\omega}_{t+1}^G, \xi_{t+1}) = E_t \rho^B(\tilde{\omega}_{t+1}^B)$ , we must have  $\tilde{\omega}_{t+1}^G < \tilde{\omega}_{t+1}^B$ .

Then

$$\phi_t^G = PC^G \left( \tilde{\omega}_{t+1}^G, \xi_{t+1}, \frac{R_{t+1}^K}{R_t} \right) < PC^B \left( \tilde{\omega}_{t+1}^G, \frac{R_{t+1}^K}{R_t} \right) < PC^B \left( \tilde{\omega}_{t+1}^B, \frac{R_{t+1}^K}{R_t} \right) = \phi_t^B, \quad (\text{B.38})$$

where the first inequality is proved in Proposition 5, and the second inequality makes use of the fact that  $PC^B$  is increasing in  $\bar{\omega}$  and that  $\tilde{\omega}_{t+1}^G < \tilde{\omega}_{t+1}^B$ .

## Appendix C. Details of calibration

We discuss our calibration strategy of the benchmark model. We first use the following equations for the secured debt contracts:

$$\lambda^B = (1 - \kappa)\phi^B\beta\Omega^B R^K [1 - G(\bar{\omega}^B) - \bar{\omega}^B(1 - F(\bar{\omega}^B))] \quad (\text{C.1})$$

$$1 - \frac{1}{\phi^B} = \beta R^K \{ \bar{\omega}^B [1 - F(\bar{\omega}^B)] + (1 - \mu)G(\bar{\omega}^B) \} \quad (\text{C.2})$$

$$\lambda^B = \frac{(1 - \kappa)\Omega^B [1 - F(\bar{\omega}^B)]}{[1 - F(\bar{\omega}^B) - \mu\bar{\omega}^B f(\bar{\omega}^B)]} \quad (\text{C.3})$$

$$\Omega^B = \theta\lambda^B + 1 - \theta \quad (\text{C.4})$$

We use the steady-state conditions for the unsecured debt contracts:

$$\lambda^G = \phi^G\beta\Omega^G R^K \{1 - \xi[G(\tilde{\omega}^G) + \tilde{\omega}^G(1 - F(\tilde{\omega}^G))]\} \quad (\text{C.5})$$

$$1 - \frac{1}{\phi^G} = \beta R^K \xi \tilde{\omega}^G [1 - F(\tilde{\omega}^G)] \quad (\text{C.6})$$

$$\lambda^G = \frac{\Omega^G \xi (1 - F(\tilde{\omega}^G))}{\xi [1 - F(\tilde{\omega}^G) - \tilde{\omega}^G f(\tilde{\omega}^G)]} \quad (\text{C.7})$$

$$\xi = 1 - \frac{\zeta(1 - \mu)(\theta\lambda^B + 1 - \theta)}{\Omega^G} \quad (\text{C.8})$$

$$\Omega^G = \theta\lambda^G + 1 - \theta \quad (\text{C.9})$$

Furthermore, the steady-state ratio of secured and unsecured debt is given by:

$$\frac{B^G}{B^B} = \frac{K^G - N^G}{K^B - (1 - \kappa)N^B} = \frac{\frac{K^G}{N^G} - 1}{\frac{K^B}{N^B} \frac{N^B}{N^G} - (1 - \kappa)\frac{N^B}{N^G}} = \frac{N^G}{N^B} \times \frac{\phi^G - 1}{(\phi^B - 1)(1 - \kappa)} \quad (\text{C.10})$$

where the evolution of net worth of  $B$  firms in the steady state gives the following relation:

$$\frac{N^G}{N^B} = \frac{1 - \tau - (1 - \kappa)\theta\{1 - G(\bar{\omega}^B) - \bar{\omega}^B[1 - F(\bar{\omega}^B)]\}R^K\phi^B}{\zeta(1 - \mu)G(\tilde{\omega}^G)\theta R^K\phi^G}.$$

The evolution of net worth of  $G$  firms in the steady state gives the following relation:

$$\tau = 1 - \theta R^K \phi^G \{1 - G(\tilde{\omega}^G) - \xi \tilde{\omega}^G [1 - F(\tilde{\omega}^G)]\} \quad (\text{C.11})$$

The four steady-state conditions pin down  $R^K/R, \phi^B, \phi^G, B^G/B^B$ . The above eleven equations solve for the remaining steady-state values of  $\{\bar{\omega}^B, \tilde{\omega}^G, \lambda^B, \lambda^G, \Omega^B, \Omega^G, \xi\}$  and the parameters  $\{\sigma, \kappa, \zeta, \tau\}$ .



## References

- Azariadis, C., Kaas, L., Wen, Y., 2016. Self-fulfilling credit cycles. *The Review of Economic Studies* 83, 1364–1405.
- Bernanke, B., Gertler, M., Gilchrist, S., 1999. The financial accelerator in a quantitative business cycle framework, in: Taylor, J.B., Woodford, M. (Eds.), *Handbook of Macroeconomics*. North Holland. volume 1, pp. 1341–1393.
- Besanko, D., Kanatas, G., 1993. Credit market equilibrium with bank monitoring and moral hazard. *The review of financial studies* 6, 213–232.
- Bolton, P., Freixas, X., 2000. Equity, bonds, and bank debt: Capital structure and financial market equilibrium under asymmetric information. *Journal of Political Economy* 108, 324–351.
- Boot, A.W., Thakor, A.V., 1997. Financial system architecture. *The Review of Financial Studies* 10, 693–733.
- Carlstrom, C.T., Fuerst, T.S., 1997. Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis. *American Economic Review* 87, 893–910.
- Chemmanur, T.J., Fulghieri, P., 1994. Reputation, renegotiation, and the choice between bank loans and publicly traded debt. *Review of Financial Studies* 7, 475–506.
- Christiano, L.J., Motto, R., Rostagno, M., 2014. Risk shocks. *The American Economic Review* 104, 27–65.
- Crouzet, N., 2017. Aggregate implications of corporate debt choices. *Review of Economics Studies* .
- Cui, W., Kaas, L., 2017. Default cycles. Working Paper.
- Davydenko, S.A., Strebulaev, I.A., Zhao, X., 2012. A market-based study of the cost of default. *The Review of Financial Studies* 25, 2959–2999.
- De Fiore, F., Uhlig, H., 2011. Bank finance versus bond finance. *Journal of Money, Credit and Banking* 43, 1399–1421.
- De Fiore, F., Uhlig, H., 2015. Corporate debt structure and the financial crisis. *Journal of Money, credit and Banking* 47, 1571–1598.
- Denis, D.J., Mihov, V.T., 2003. The choice among bank debt, non-bank private debt, and public debt: evidence from new corporate borrowings. *Journal of financial Economics* 70, 3–28.
- Diamond, D.W., 1991. Monitoring and reputation: The choice between bank loans and directly placed debt. *Journal of political Economy* 99, 689–721.
- Gertler, M., Karadi, P., 2011. A model of unconventional monetary policy. *Journal of Monetary Economics* 58, 17–34.
- Gilchrist, S., Zakrajsek, E., 2012. Credit spreads and business cycle fluctuations. *American Economic Review* 102, 1692–1720.
- Gu, C., Mattesini, F., Monnet, C., Wright, R., 2013. Endogenous credit cycles. *Journal of Political Economy* 121, 940–965.
- Halling, M., Yu, J., Zechner, J., 2016. Leverage dynamics over the business cycle. *Journal of Financial Economics* 122, 21 – 41.
- Jermann, U., Quadrini, V., 2012. Macroeconomic effects of financial shocks. *The American economic review* 102, 238–271.
- Kiyotaki, N., Moore, J., 1997. Credit cycles. *Journal of Political Economy* 105, 211–248.
- Meh, C.A., Moran, K., 2010. The role of bank capital in the propagation of shocks. *Journal of Economic Dynamics and Control* 34, 555 – 576.
- Mora, N., 2012. What determines creditor recovery rates? *Economic Review*. Federal Reserve Bank of Kansas City 97, 79–109.

- Nuno, G., Thomas, C., 2017. Bank leverage cycles. *American Economic Journal: Macroeconomics* 9, 32–72.
- Rannenberg, A., 2016. Bank leverage cycles and the external finance premium. *Journal of Money, Credit and Banking* 48, 1569–1612.
- Rauh, J.D., Sufi, A., 2010. Capital structure and debt structure. *The Review of Financial Studies* 23, 4242–4280.