Optimal Monetary Policy, Determinacy and Policy Puzzles at the Effective Lower Bound

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Abstract

Should we expect inflation to be stable at the Effective Lower Bound? According to the workhorse model for monetary policy analysis, the answer should be no as the economy experiences sunspots/multiple equilibria. In this paper, I show that if we take monetary policy seriously, this possibility becomes much less likely. Using a model in which monetary policy is optimal subject to a loose commitment constraint, I show that there is a threshold degree of commitment such that the equilibrium is unique at the ELB. In this case, inflation will be stable at the ELB. Further, if the degree of commitment is high enough then (i) the model does not feature policy puzzles and (ii) converges smoothly to its flexible price limit. These features help to reconcile the New Keynesian model with recent empirical evidence.

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1 Introduction

Should we expect inflation to be stable at the Effective Lower Bound? According to the workhorse model for monetary policy analysis —the New Keynesian model with the Central Bank following a Taylor rule, the economy should be plagued with sunspot driven fluctuations. In this model, sunspot-driven fluctuations are ruled out when the Central Bank is able to react aggressively enough to inflation fluctuations. Since it is not able to do so at the Effective Lower Bound, sunspots can potentially affect inflation/output fluctuations.

After an ELB spell of roughly 7 years in the United States, the verdict is in: inflation/output fluctuations have been remarkably mild (see Cochrane (2018) and Christiano (2018) for a comment). This is illustrated in Figure 1, where I plot both inflation and output growth in recent years alongside the effective Federal Funds Rate. There is no sign of increased volatility during the ELB spell for these two variables; if anything, both look more volatile before than during the ELB spell.

Figure 1: Inflation and Output during the U.S. Great Recession
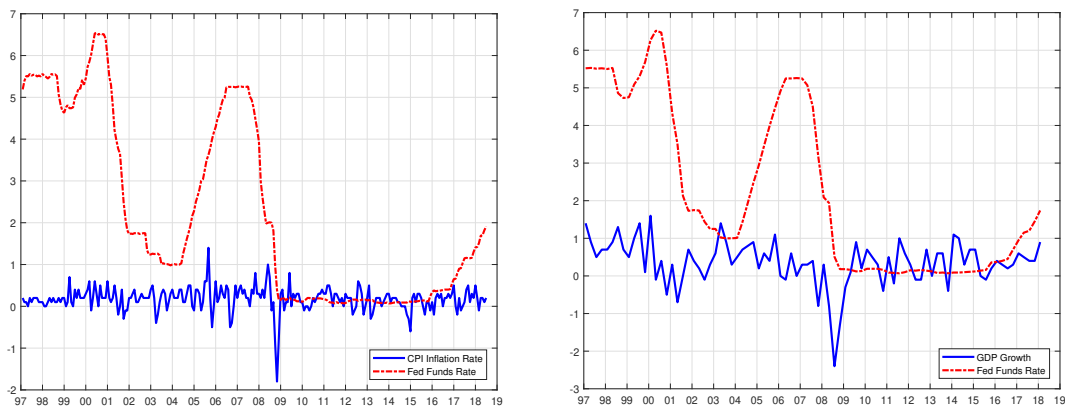
In this paper, I show that taking monetary policy seriously can reconcile these two seemingly contradictory statements. Indeed, it is hard to argue that

the Federal Reserve has only been helplessly following a Taylor rule during the period 2009-2016. There were many forward guidance announcements during this period[1] as well as a large and bold policy of quantitative easing. It therefore does not follow that, because the interest rate was at its ELB then the Central Bank had no traction on the economy.

To give a (potentially) meaningful role for monetary policy at the ELB, I study a New Keynesian model with loose commitment[2]. This model nests both discretion and full commitment as special cases. The main result of this paper is that there exists a threshold degree of commitment over which the Central Bank will be able to implement a unique equilibrium at the ELB. It follows that (i) sunspot driven fluctuations are to be expected under discretion and (ii) they will vanish under full commitment.

If the degree of commitment is high enough then the New Keynesian model behaves well at the ELB. The policy paradoxes that usually emerge under a Taylor rule[3] are no longer there. To single out one of those, the paradox of flexibility implies that as prices become more flexible, the standard New Keynesian model displays explosive behavior at the ELB. In my setup, if the degree of commitment is high enough the model converges smoothly to its flexible price limit. Finally, the required degree of commitment is not implausibly large : if the Central Bank is able to commit for at least 5 quarters then these conclusion hold.

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[1]According to the typology developed in Filardo & Hofmann (2014), the Federal Reserve engaged in *open-ended* forward guidance from December 2008 to July 2011. It then switched to *calendar-based* forward guidance until November 2012. Finally it turned to *threshold-based* forward guidance until the end of 2016.

[2]See Schaumburg & Tambalotti (2007), Bodenstein et al. (2012) and Debortoli & Lakdawala (2016).

[3]See Eggertsson (2010), Christiano et al. (2011), Eggertsson & Krugman (2012) and Eggertsson et al. (2014).

1.1 Related Literature

This paper builds on a vast and rapidly expanding literature. The seminal article about price determinacy in monetary models is Sargent & Wallace (1975), who show that *exogenous* interest rules lead to indeterminacy. In another seminal contribution, Clarida et al. (2000) show that the key is that interest rules[4] should be *endogenous* to ensure determinacy. While this is necessary, the sufficient condition is that such rules should react aggressively enough to inflation/output fluctuations. This has been known as the Taylor Principle. It should be noted that the latter only guarantees that the equilibrium will be unique locally.

How exactly following this kind of rule weeds out multiple equilibria has been a subject of debate. Both Atkeson et al. (2010) and Cochrane (2011) study escape clauses coupled with interest rules to get rid of multiple equilibria and conclude that adhering to the Taylor Principle does not really solve the problem of multiple equilibria. A recent counterpoint can be found in Christiano & Takahashi (2018).

In an influential paper, Benhabib et al. (2001) show that the presence of an ELB generates global multiplicity of equilibria in New Keynesian models. Recent contributions that study the multiplicity of equilibria at the ELB include Aruoba & Schorfheide (2013), Cochrane (2013), Mertens & Ravn (2014), Schmidt & Nakata (2015), Armenter (2016) and Holden (2017). This paper is also related to the literature on optimal monetary policy in the New Keynesian framework. Here, the seminal contribution is Gertler et al. (1999). This paper is mostly related to the follow-up literature that focused on optimal monetary policy at the ELB, which includes Eggertsson & Woodford (2003), Jung et al.

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[4]See also Kerr & King (1996), Bernanke & Woodford (1997) and Bullard & Mitra (2007).

(2005), Sugo & Teranishi (2005) and Adam & Billi (2006). Recent contributions include Schmidt (2013), Hasui et al. (2016), Schmidt & Nakata (2015) and Bilbiie (2016).

The literature on optimal monetary policy at the ELB has been focused on two polar cases : discretion and full commitment. Outside the ELB, some contributions have focused on the spectrum between those two polar cases : see Schaumburg & Tambalotti (2007), Kara (2007) and Debortoli & Lakdawala (2016). An exception that studies a framework with loose commitment at the ELB is Bodenstein et al. (2012).

Finally, there is a recent (mostly empirical) literature trying to gauge how much of a constraint on monetary policy the ELB actually was. The main result from this literature is that the ELB was not a very tight constraint on monetary policy. Swanson & Williams (2014) show that Treasury yields of relatively long maturities were still sensitive to economic news during the ELB period. Wu & Xia (2016) compute a shadow interest rate and show that, even though the effective Federal Funds Rate was at the ELB, monetary policy still had traction on the economy. Finally, Davide Debortoli & Gambetti (2018) study a time-varying structural VAR and find no change of transmission mechanisms at the ELB.

To the best of my knowledge, none of those papers study how the determinacy properties of the New Keynesian model at the ELB depend on the degree of Monetary Policy commitment. Both Schmidt & Nakata (2015) and Armenter (2016) show that the New Keynesian model has multiple *Markov* equilibria at the ELB under discretionary policy. Since they both focus on Markov equilibria, they rule out sunspots by construction. The findings reported in this paper are thus complementary as I show that under discretion, the model features infinitely many equilibria. While Bodenstein et al. (2012) do study a model of

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loose commitment at the ELB, they look for minimum state variable solutions. Therefore, they cannot analyze equilibrium determinacy by construction.

2 Model

I start from a baseline, two-equations New Keynesian model as in Schmidt (2013) :

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t \tag{1}$$

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \sigma(R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - R_t^*), \tag{2}$$

where $\tilde{y}_t = y_t - y_t^*$ is the deviation of actual output from potential and R_t^* is the natural rate of interest. The slope of the Phillips curve is

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\eta\theta)}(\sigma^{-1} + \eta),$$

where α is the standard Calvo probability. The natural output/rate of interest are given by

$$y_t^* = \Theta a_t + \Gamma(g_t + \xi_t) \tag{3}$$

$$R_t^* = \sigma^{-1}\left(\mathbb{E}_t y_{t+1}^* - y_t^* + g_t - \mathbb{E}_t g_{t+1}\right), \tag{4}$$

where a_t is a technology shock, g_t a government spending shock and ξ_t a preference shock. The constant parameters are defined as

$$\Theta = \frac{\eta+1}{\eta+\sigma^{-1}} \quad \& \quad \Gamma = \frac{\sigma^{-1}}{\eta+\sigma^{-1}},$$

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where σ is the elasticity of intertemporal substitution and η is the (inverse of the) Frisch elasticity of labor supply.

3   Indeterminacy and Policy Puzzles at the ELB with a Taylor Rule

Suppose that the Central Bank follows a Taylor rule given by

$$R_t = \max \left\{ \phi_\pi \pi_t - \log(\beta), 0 \right\}. \tag{5}$$

It is well known that in this case, there is a unique equilibrium if and only if $\phi_\pi > 1$. It can be also shown that if there is a large recessionary shock $\tilde{\xi}_t$, the Zero Lower Bound will be a binding constraint. In this case, the equilibrium is given (up to a constant) by the standard equilibrium in which I set $\phi_\pi = 0$. Therefore, there are multiple equilibria at the ELB. This comes from the fact that the Central Bank looses its ability to shape (expected) inflation.

To derive analytically the multiplier effects of government spending and technology shocks at the ELB, assume that all shocks have a two state Markovian structure. Once a shock occurs, it persists next period with a probability given by p. This allows us to write $\mathbb{E}_t z_{t+1} = p \cdot z_t$ for each variable z_t. Given this, setting $R_t = 0$ and using equations (1)-(2) I get

$$\tilde{y}_t = \frac{1 - \beta p}{(1 - \beta p)(1 - p) - \sigma p \kappa} \sigma (R_t^* - \log(\beta)). \tag{6}$$

To ensure that the economy ends up in a liquidity trap due to fundamentals and not expectations (see Mertens & Ravn (2014)), I set the parameters such that the numerator of the right-hand side is positive. If the decline in ξ_t is large enough, then this ensures that the economy experiences a negative output gap at the ELB.

Now I want to compute the effect of government spending and technology shocks at the ELB. Let us begin with the technology shock. From equations (4) and (6), I get

$$
\begin{aligned}
\frac{\partial \tilde{y}_t}{\partial a_t} &= \frac{1-\beta p}{(1-\beta p)(1-p)-\sigma p\kappa}\sigma\frac{\partial R_t^*}{\partial a_t} \\
&= \frac{1-\beta p}{(1-\beta p)(1-p)-\sigma p\kappa}\sigma\frac{\Theta(p-1)}{\sigma} \\
&= \frac{1-\beta p}{(1-\beta p)(1-p)-\sigma p\kappa}\Theta(p-1) \\
&= -\frac{(1-\beta p)(1-p)}{(1-\beta p)(1-p)-\sigma p\kappa}\Theta.
\end{aligned}
$$

By definition of the output gap and using equation (3), the effect on actual output at the ELB will be

$$
\begin{aligned}
\frac{\partial y_t}{\partial a_t} &= \frac{\partial \tilde{y}_t}{\partial a_t} + \frac{\partial y_t^*}{\partial a_t} \\
&= -\frac{(1-\beta p)(1-p)}{(1-\beta p)(1-p)-\sigma p\kappa}\Theta + \Theta \\
&= -\Theta\frac{\sigma p\kappa}{(1-\beta p)(1-p)-\sigma p\kappa} < 0,
\end{aligned}
$$

so that a positive technology shock decreases actual output. Furthermore, it can be seen that in the limit of flexible prices (ie when $\kappa \to \infty$), I naturally get

$$
\frac{\partial y_t}{\partial a_t} = \frac{\partial y_t^*}{\partial a_t}.
$$

But the left-hand side does not converge smoothly to the right-hand side. This point has been made recently by Cochrane (2013). It can be easily seen that has κ increases, there will be a value for which the denominator of $\partial y_t / \partial a_t$ goes to zero and the multiplier explodes. Formally,

$$\frac{\partial \tilde{y}_t}{\partial a_t} \to \infty \quad as \quad \kappa \to \frac{(1 - \beta p)(1 - p)}{p\sigma}$$

This comes from the fact that an increase in technology generates a decrease in inflation. As the nominal interest rate is stuck at zero, this causes the real interest rate to increase and households to reduce their consumption. As prices become more flexible, the increase in real interest rate is magnified and becomes unbounded. Note that this comes from the fact that the Central Bank is essentially helpless in this situation, which is most likely not the case in reality. Wu & Xia (2016) derive a shadow interest rate for the Federal Reserve which shows that the Central Bank still had traction on effective lending/borrowing costs at the Zero Lower Bound. Relatedly, Chen et al. (2017) shows that a Central Bank that is optimizing every period outperforms a model in which it follows a Taylor rule.

Moving on to the government spending shock, in a similar manner I get

$$\frac{\partial \tilde{y}_t}{\partial g_t} = \frac{1 - \beta p}{(1 - \beta p)(1 - p) - \sigma p\kappa} \sigma \frac{\partial R_t^*}{\partial g_t}$$

$$= \frac{1 - \beta p}{(1 - \beta p)(1 - p) - \sigma p\kappa} (1 - \Gamma)(1 - p).$$

It follows that

$$\frac{\partial y_t}{\partial g_t} = \frac{\partial \tilde{y}_t}{\partial g_t} + \frac{\partial y_t^*}{\partial g_t}$$

$$= \frac{1 - \beta p}{(1 - \beta p)(1 - p) - \sigma p\kappa} (1 - \Gamma)(1 - p) + \Gamma.$$

The puzzle aspect of government spending increases at the ELB is that they tend to give multipliers that are higher than 1. To see this, I can show that

$$\frac{\partial y_t}{\partial g_t} - 1 = (1 - \Gamma)\frac{\sigma p \kappa}{(1 - \beta p)(1 - p) - \sigma p \kappa} > 0.$$

This comes from the fact that, by definition $\Gamma < 1$. As before, when prices are flexible the multiplier effect will converge to its flexible part counterpart but the multiplier effect under sticky prices will become infinite along the way. This is because higher government spending generates higher inflation and a lower real interest rate at the ELB, which is just the opposite of what happens after a positive technology shock.

There has been attempts to make New Keynesian models more in line with data by changing key aspects of the aggregate supply/demand equations or the structure of information. In this paper, I show that by making monetary policy more in line with how it is carried out in reality can make these puzzles disappear.

4 Model with Imperfect Commitment

Instead of assuming that the Central Bank follows a Taylor-type rule, I now assume that it will set the (path of) the nominal interest rate optimally. Following Schaumburg & Tambalotti (2007), I assume that the Central Bank only has access to a loose commitment policy. The length of central bankers' tenure depends on a sequence of i.i.d Bernoulli signals $\{\eta_t\}_{t \geq 0}$ with $\mathbb{E}\eta_t = \delta$, a new central banker takes office at the beginning of period t. Otherwise, the incumbent stays on. All agents observe the realization of η_t. Following Kara (2007), I also assume that private agents expect the central bank to reformulate policy

with probability μ (what he calls "imperfect credibility of intentions"). This means that, from the point of view of private agents the expected duration of a tenure is $\delta + \mu$. That is, private agents always expect a tenure that is shorter than the actual one. This can be motivated by the fact that they know that commitment is dynamically inefficient in this setup as policymakers would like to re-optimize instead of upholding past promises.

The central bank is aware of that. Each policymaker can credibly commit to a state-contingent plan for the entire duration of her tenure, but cannot constrain the actions of her successors.

Given the simple structure of the New Keynesian model, it can be shown (see Woodford (2011)) that a second order approximation of the utility function gives a loss function

$$L_t \equiv \pi_t^2 + \frac{\kappa}{\theta}\tilde{y}_t^2 + \lambda_R(R_t - R)^2,$$

where θ is the elasticity of substitution across goods and λ_R is a positive parameter[5]. The Central Bank then seeks to maximize the following objective:

$$\mathbb{E}_0\Bigg\{ \sum_{j=0}^{\infty} \delta(1-\delta)^{j-1}\bigg[\delta^j\beta V(R_j^*) + \frac{1}{2}\sum_{t=0}^{j-1}\beta^t L_t$$

$$+ \phi_{1,t}\big[\pi_t - \kappa\tilde{y}_t - \beta(1-(\delta+\mu))\pi_{t+1}\big]$$

$$+ \phi_{2,t}\big[\tilde{y}_t + \sigma(R_t - (1-(\delta+\mu))\pi_{t+1} - R_t^*) - (1-(\delta+\mu))\tilde{y}_{t+1}\big]$$

$$+ \phi_{3,t}\big[R_t - 0\big]\bigg]\Bigg\} + \text{t.i.p},$$

where t.i.p stands for terms independent of policy. In particular, expected in-

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[5]See Woodford (2011), Chapter 6.

flation can be written as

$$\mathbb{E}_t \pi_{t+1} = (1 - (\delta + \mu))\pi_{t+1} + (\delta + \mu)\pi_{t+1}^{\text{reop}},$$

where the second term on the right hand side is expected inflation in case the Central Bank re-optimizes. Since I will be focusing on what happens *within* a given commitment regime, this term can be considered independent of policy. Following Kara (2007), the objective can be rewritten as

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} ((1-\delta)\beta)^t \left[\delta\beta V(R_{t+1}^*) + \frac{1}{2}L_t \right. \right.$$
$$+ \phi_{1,t} \left[\pi_t - \kappa\tilde{y}_t - \beta(1 - (\delta + \mu))\pi_{t+1} \right]$$
$$+ \phi_{2,t} \left[\tilde{y}_t + \sigma(R_t + \log(\beta) - (1 - (\delta + \mu))\pi_{t+1} - R_t^*) - (1 - (\delta + \mu))\tilde{y}_{t+1} \right]$$
$$\left. \left. + \phi_{3,t} \left[R_t - 0 \right] \right] \right\} + \text{t.i.p.}$$

For simplicity, I assume that $\mu = \delta \cdot (1 - \delta)$ and define $\gamma \equiv 1 - \delta$. With these assumptions, the first order conditions are given by

$$0 = \pi_t + \phi_{1,t} - \gamma\phi_{1,t-1} - \frac{\gamma\sigma}{\beta}\phi_{2,t-1} \tag{7}$$

$$0 = \frac{\kappa}{\theta}\tilde{y}_t - \kappa\phi_{1,t} + \phi_{2,t} - \frac{\gamma}{\beta}\phi_{2,t-1} \tag{8}$$

$$0 = \lambda_R(R_t - R) + \sigma\phi_{2,t} + \phi_{3,t} \tag{9}$$

$$0 = R_t\phi_{3,t}, \; R_t \geq 0, \; \phi_{3,t} \geq 0. \tag{10}$$

The last one is the slackness condition. When the economy is out of the Zero Lower Bound, $R_t > 0$ and thus $\phi_{3,t} = 0$. When the economy is at the Zero Lower Bound, $R_t = 0$ and $\phi_{3,t} > 0$.

Again, this system describes the dynamics of the economy within a given regime. Alternatively, I could assume away imperfect credibility of intentions

($\mu = 0$). In this case, the FOCs will give a system that is identical to the one with full commitment. Again, this is conditional on the regime staying the same. I could then average the behavior of the economy across realizations of the signal η_t; because $\mathbb{E}\eta_t = \delta$, past promises will not be binding $100 \cdot \delta\%$ of the time. To the contrary, they will be binding $100 \cdot (1-\delta)\%$ of the time. Therefore, the present system of equations can be derived under the assumption that I care about the average dynamics of the economy after countless realizations of the signal. With this in mind, I now study the properties of this system.

4.1 Determinacy

If variations in R_t^* are too small for the ELB to become a binding constraint, it is trivial to show that the Central Bank can implement a unique equilibrium by targeting the natural rate of interest. If the ELB becomes a binding constraint however, then the degree of commitment potentially matters. Even though the Central Bank cannot move the interest rate at time t, it can make announcements and act as a coordination device for inflation expectation. Note that since I focus on what happens within a ELB regime, what happens after the trap is irrelevant. What matters for determinacy here is what the Central Bank is able to do *during* the ELB. I turn to this issue now.

To approximate the ELB situation, I consider the case where λ_R ∞. This means that it is infinitely costly for the Central Bank to move the nominal interest rate. To see whether the model features multiple equilibria at the ELB, I re-write it in matrix form. Since $R_t = 0$, then $\phi_{3,t} > 0$ and equation (9) just expresses $\phi_{3,t}$ as a function of $\phi_{2,t}$. Therefore, I only need equations (7)-(8) along the private sector equilibrium conditions to characterize potential equilibria. Let us define $\mathbf{Z}_t = [\pi_t \ \tilde{y}_t \ \phi_{1,t-1} \ \phi_{2,t-1}]'$. Then combining the FOCs

from the private sector and the policymaker, I get:

$$\mathbf{A} \cdot \mathbb{E}_t \mathbf{Z}_{t+1} = \mathbf{B} \cdot \mathbf{Z}_t + \mathbf{S}_t + \mathbf{C},$$

where $\mathbf{S}_t$ is a matrix that regroups linear combinations of the shocks and the matrices $\mathbf{A}$ and $\mathbf{B}$ are given by:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \sigma & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\kappa & 1 \end{bmatrix} \quad \& \quad \mathbf{B} = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & \gamma & \frac{\gamma\sigma}{\beta} \\ 0 & -\frac{\kappa}{\theta} & 0 & \frac{\gamma}{\beta} \end{bmatrix}.$$

Given this, I have two jump variables (π_t and $\tilde{y}_t$) and two backward-looking ones ($\phi_{1,t-1}$ and $\phi_{2,t-1}$). So the system features a unique equilibrium if and only if the matrix

$$D \equiv \mathbf{A}^{-1}\mathbf{B},$$

has two eigenvalues with modulus strictly higher than 1 and two below. Since I want to study both (i) the effect of imperfect commitment and (ii) how the sticky price model relates to its flexible price limit, I will study the determinacy properties of the model as a function of both γ and α. To do so, I need first to characterize the four eigenvalues. As a first step, the following Lemma will be useful.

Lemma 1 *The eigenvalues of matrix D are the roots of*

$$P_1(\lambda) \cdot P_2(\lambda) = 0,$$

where P_1 and P_2 are both second order polynomials.

The proof of this Lemma is in the Appendix. Now Analysing each polynomial

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separately, I can show the following Proposition:

Proposition 1 *Let λ_j^i be the i-th eigenvalue of polynomial j. Then it can be shown that*

$$|\lambda_1^1| > 1 \quad , \quad |\lambda_1^2| < 1 \quad and \quad |\lambda_2^2| < 1.$$

The proof of this Proposition is in the Appendix. As it stands there are two eigenvalues inside the unit circle and one outside. Since the model features two forward-looking variables, two eigenvalues outside the unit circle are needed to ensure determinacy. For the first three eigenvalues already described, their position with respect to the unit circle does not depend on the degree of commitment. This is not the case for the last eigenvalue, for which the following proposition applies.

Proposition 2 *Let λ_2^1 be the first eigenvalue of polynomial 2. Then it can be shown that*

$$|\lambda_2^1| > 1 \quad \Leftrightarrow \quad \gamma > \gamma^*.$$

Furthermore, $\gamma^* \in (0,1)$.

So there exists a minimal degree of commitment from the Central Bank that will ensure that there is a unique equilibrium at the ELB. The result in **Proposition 2** can be used to study the two polar cases that have been extensively studied in the literature, namely the case of discretion ($\gamma = 0$) and full commitment ($\gamma = 1$).

Corollary .1 *It follows from **Proposition 2** that the two polar cases of discretion and full commitment can be characterized as follows:*

1. *The discretion case with $\gamma = 0$ will necessarily feature multiple equilibrium/sunspots at the ELB*

2. *The full commitment case with $\gamma = 1$ will necessarily feature a unique equilibrium/no sunspots at the ELB.*

The first result echoes findings reported in a recent paper by Armenter (2016), who shows that a Central Banker acting under discretion will fail to implement its target inflation at the Zero Lower Bound. The present result is different in that Armenter (2016) focuses of Markov equilibria and thus rules out sunspot-driven equilibria. In my setup, since the Central Banker optimizes on a quarter to quarter basis, it has no ability to shape the private sector's expectations about future policy. As a consequence, these expectations remain deflationary and the economy is vulnerable to sunspot-induced fluctuations. This finding thus complements the ones reported in Armenter (2016) : discretionary monetary policy at the ELB is highly unlikely to feature a unique equilibrium.

In the case of full commitment, the Central Banker is allowed to make credible announcements about future policy and so can shape expected inflation. It follows that, even though the Central Banker has no ability to move its interest rate *now*, it still has some traction on economic activity. This property has been noted in the numerical experiments in Roulleau-Pasdeloup (2018). This result also formalizes the idea that a Central Bank is not powerless at the ELB (see Wu & Xia (2016), Swanson (2018))

The question is now : How does this depends on the primitives of the model? For example, will more flexible prices make it more difficult for the Central Bank to ensure a unique equilibrium at the ELB? I answer these questions in the following proposition.

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Proposition 3 *The threshold level of commitment to ensure a unique equilibrium at the ELB has the following properties:*

1. $\dfrac{\partial \gamma^*}{\partial \sigma} < 0$

2. $\dfrac{\partial \gamma^*}{\partial \alpha} > 0$

3. $sign\left(\dfrac{\partial \gamma^*}{\partial \eta}\right) = sign\left(\sigma - \theta\right)$

Part 1 says that the higher the elasticity of intertemporal substitution, the lower the threshold degree of commitment needed for a unique equilibrium at the ELB. A higher elasticity of substitution means that variations in expected inflation —and thus, variations in the real interest rate at the ELB —have more traction on the actual output gap. Thus, the more willing households are to move resources across time, the easier it is for the Central Bank to engineer a unique equilibrium at the ELB.

Part 2 says that as prices becomes more sticky (remember that the average duration of a given price reset is $1/(1 - \alpha)$), the threshold degree of commitment needed for a unique equilibrium at the ELB is now higher. Stickier prices means that the Central Bank will find it more difficult to move inflation around as desired at the ELB. As a result, it will be more difficult to deliver a unique equilibrium.

Finally, part 3 says that the effect of varying the (inverse of the) Frisch elasticity has ambiguous effect on the threshold degree of commitment. If $\sigma < \theta$ (which is true under most usual calibrations of these two parameters), then a lower Frisch elasticity will decrease the threshold degree of commitment. Decreasing the Frisch elasticity relative to a baseline calibration will lead household members to ask for a higher real wage to work more hours. This means that a given variation in output gap will translate into a larger variation in

real marginal costs and inflation. To put it differently, as long as $\sigma < \theta$, a lower Frisch elasticity will increase the slope of the Phillips Curve. In turn, this higher slope will make it easier for the Central Bank to engineer the desired fluctuations in (expected) inflation, and thus to implement a unique equilibrium.

5 Policy Puzzles with Loose Commitment

5.1 A Special Case : Discretion

Whenever $\gamma > 0$, past promises matter to some extent and thus the dynamic system has two endogenous state variables. In this subsection, I focus on the polar case of discretion for which past promises do not matter. This implies that I can solve the model analytically and study whether it displays the same kind of puzzles as the one in which the Central Bank follows a Taylor rule. Given the results in section 4.1, the multiplier effects are derived for the minimum state variable solution. Since the Central Bank is not able to commit on any promises, the question is whether the allocation after a natural rate shock at the ELB is different from the Taylor rule specification. The answer is in Proposition 4:

Proposition 4 *Assume full discretion so that the Central Banker cannot commit beyond the current quarter. The multiplier effect on output gap of a variation in the natural rate of interest is:*

$$\frac{\partial \tilde{y}_t}{\partial R_t^*} = \frac{1 - \beta p}{(1 - \beta p)(1 - p) - \sigma p \kappa} \sigma,$$

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which is exactly the same as in the setup in which the Central Bank follows a Taylor rule.

The proof is in the Appendix. Since the Central Bank is not able to commit, then there is nothing that it can do above and beyond what a Taylor rule would prescribe. As such, the dynamics at the ELB are identical whether the Central Bank follows a Taylor rule or acts under discretion. This is different from what would happen in normal times however. Away from the ELB, the optimal discretionary policy will call for a path of interest rate that will be different[6] from the one prescribed by a Taylor rule.

It follows that under discretion, the multiplier effect of variations in the natural interest rate will explode under some paramter configurations and thus will not converge to their flexible price limit smoothly.

5.2   The General Case

To illustrate further how the degree of commitment matters, I plot the impact response to a 1% government spending shock as a function of the probability to keep the same price α after a preference shock sends the economy at the ELB. For the rest of the parameters, the calibration follows **??** and is described in the Appendix.

First, in Figure 2 I consider the case in which γ takes the value estimated in Debortoli & Lakdawala (2016), which is 0.81. I want to know whether the multiplier effect converges to its flexible price counterpart smoothly, so I plot the difference between the actual multiplier effect under sticky prices and Γ,

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[6]Unless the Taylor rule is optimal in the sense that it is derived such as to replicate the equilibrium under discretion as in see Giannoni & Woodford (2003).

Figure 2: Fiscal Multiplier as a Function of Price Flexibility ($\gamma = 0.81$)
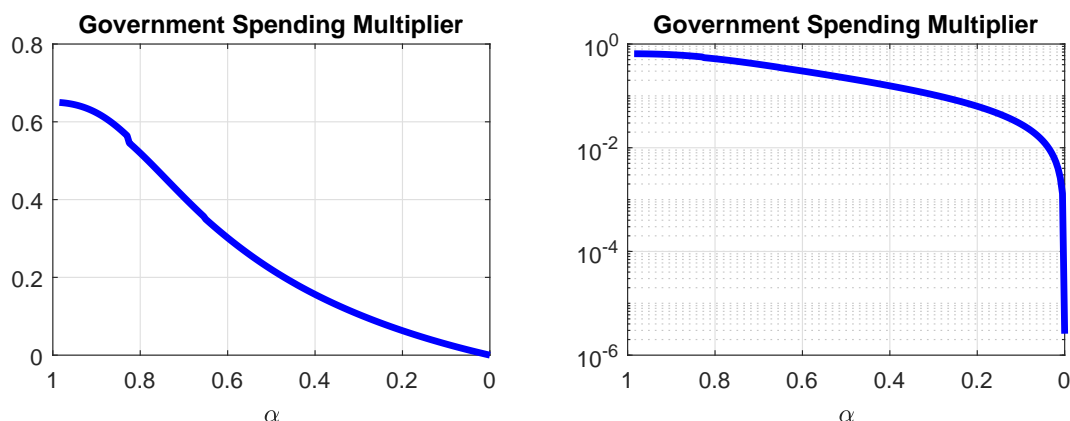

the multiplier under flexible prices. From Figure 2, one can see that the multiplier does converge smoothly to its flexible price counterpart as prices become more flexible. I also plot the difference in multipliers on a log scale to highlight that it does actually converge to zero.

Now I turn to the case of a technology shock. Has I have shown before, the puzzle here is that in a standard New-Keynesian model an *increase* in productivity will be contractionary. I first begin with the case $\gamma = 0.81$ and plot the results in Figure 3. In this case, one can see that the impact of a temporary increase in technology is always positive and converges smoothly to its flexible price counterpart Θ. Given the baseline calibration, I have $\Theta \simeq 2.33$.

6   Conclusion

Figure 3: Technology Multiplier as a Function of Price Flexibility ($\gamma = 0.81$)
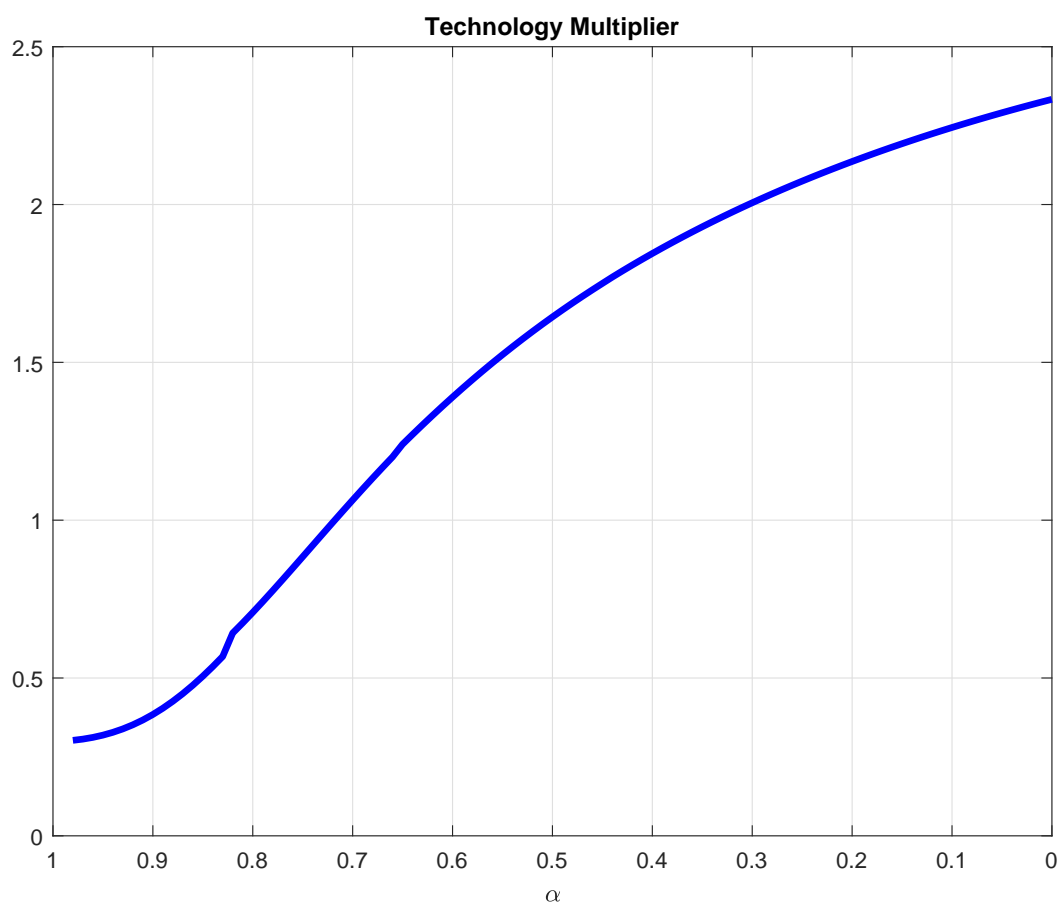
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7 Proofs

7.1 Proof of Lemma 1

Let the eigenvalues of D be λ. I know that $|D - \lambda I| = 0$, where I is the 4x4 identity matrix. Using minors and cofactors, I get:

$$0 = \begin{vmatrix} \frac{1}{\beta} - \lambda & -\frac{\kappa}{\beta} & 0 & 0 \\ -\frac{\sigma}{\beta} & \frac{\sigma\kappa}{\beta} + 1 - \lambda & 0 & 0 \\ -1 & 0 & \gamma - \lambda & \frac{\gamma\sigma}{\beta} \\ -\kappa & -\frac{\kappa}{\theta} & \gamma\kappa & \frac{\gamma\sigma\kappa+\gamma}{\beta} - \lambda \end{vmatrix}$$

$$\Leftrightarrow \quad 0 = (\frac{1}{\beta} - \lambda) \begin{vmatrix} \frac{\sigma\kappa+\beta}{\beta} - \lambda & 0 & 0 \\ 0 & \gamma - \lambda & \frac{\gamma\sigma}{\beta} \\ -\frac{\kappa}{\theta} & \gamma\kappa & \frac{\gamma\sigma\kappa+\gamma}{\beta} - \lambda \end{vmatrix} + \frac{\kappa}{\beta} \begin{vmatrix} -\frac{\sigma}{\beta} & 0 & 0 \\ -1 & \gamma - \lambda & \frac{\gamma\sigma}{\beta} \\ -\kappa & \gamma\kappa & \frac{\gamma\sigma\kappa+\gamma}{\beta} - \lambda \end{vmatrix}$$

Using minors and cofactors again, I get:

$$0 = \left(\frac{1}{\beta} - \lambda\right) \left(\frac{\sigma\kappa + \beta}{\beta} - \lambda\right) \left[(\gamma - \lambda)\left(\frac{\gamma\sigma\kappa + \gamma}{\beta} - \lambda\right) - \frac{\gamma^2\kappa\sigma}{\beta}\right] -$$

$$\frac{\sigma\kappa}{\beta^2} \left[(\gamma - \lambda)\left(\frac{\gamma\sigma\kappa + \gamma}{\beta} - \lambda\right) - \frac{\gamma^2\kappa\sigma}{\beta}\right]$$

$$\Leftrightarrow 0 = \left[(\gamma - \lambda)\left(\frac{\gamma\sigma\kappa + \gamma}{\beta} - \lambda\right) - \frac{\gamma^2\kappa\sigma}{\beta}\right] \left[\frac{\sigma\kappa + \beta}{\beta^2} - \frac{\lambda}{\beta} - \frac{\sigma\kappa + \beta}{\beta}\lambda + \lambda^2 - \frac{\sigma\kappa}{\beta^2}\right]$$

$$\Leftrightarrow 0 = \left[\frac{(\gamma - \lambda)(\gamma\sigma\kappa + \gamma - \beta\lambda) - \gamma^2\kappa\sigma}{\beta}\right] \left[\frac{1 - \lambda - \sigma\kappa\lambda - \beta\lambda + \beta\lambda^2}{\beta}\right]$$

$$\Leftrightarrow 0 = \left[\beta\lambda^2 + (-\gamma\beta - \gamma\sigma\kappa - \gamma)\lambda + \gamma^2\right] \left[\beta\lambda^2 + (-\sigma\kappa - \beta - 1)\lambda + 1\right]$$

$$\Leftrightarrow 0 = \left[\beta\lambda^2 - \lambda\psi + 1\right] \left[\beta\lambda^2 - \gamma\lambda\psi + \gamma^2\right]$$

$$\Leftrightarrow 0 \equiv P_1(\lambda) \cdot P_2(\lambda)$$

where I have defined $\psi = 1 + \beta + \sigma\kappa$.

7.2 Proof of Proposition 1

Let us begin with polynomial 1. The discriminant of this polynomial is given by

$$\Delta_1 = \psi^2 - 4\beta.$$

The polynomial P_1 will have two real roots if $\Delta_1(\gamma) > 0$. In the special case that $\gamma = 0$, then there is a unique root that is trivially given by $\lambda = 0$. Excluding this special case, the discriminant will be strictly positive if

$$\psi^2 - 4\beta > 0$$
$$\Leftrightarrow \quad 1 + \beta^2 + 2\beta + (\sigma\kappa)^2 + 2(1+\beta)\sigma\kappa - 4\beta > 0$$
$$\Leftrightarrow \quad 1 + \beta^2 + (\sigma\kappa)^2 + 2(1+\beta)\sigma\kappa > 2\beta$$
$$\Leftrightarrow \quad \beta^2 + 1 - 2\beta + (\sigma\kappa)^2 + 2(1+\beta)\sigma\kappa > 0$$
$$\Leftrightarrow \quad (\beta - 1)^2 + (\sigma\kappa)^2 + 2(1+\beta)\sigma\kappa > 0,$$

which is always the case. There are thus two real roots given by

$$\lambda_1^1 = \frac{\psi + \sqrt{\Delta_1}}{2\beta} \quad \& \quad \lambda_1^2 = \frac{\psi - \sqrt{\Delta_1}}{2\beta}.$$

I know study whether the lie inside/outside the unit circle.

7.2.1 Sign of $|\lambda_1^1| - 1$

First, note λ_1^1 is strictly positive, so it is sufficient to show that $\lambda_1^1 > 1$. Since $1 + \beta > 2\beta$, then

$$\lambda_1^1 = \frac{1 + \beta + \sigma\kappa + \sqrt{\Delta_1}}{2\beta} > \frac{2\beta + \sqrt{\Delta_1}}{2\beta} > 1.$$

7.2.2 Sign of $|\lambda_1^2| - 1$

At first sight, the sign of λ_1^2 is *a priori* ambiguous. However, since $\beta > 0$, I can write

$$\psi^2 + 4\beta > \psi^2$$
$$\Leftrightarrow \quad \psi^2 > \Delta_1 > 0$$
$$\Leftrightarrow \quad \psi - \sqrt{\Delta_1} > 0.$$

So it follows that $|\lambda_1^2| > 1$ if, and only if $\lambda_1^2 > 1$. This will be the case if

$$\psi - 2\beta > \sqrt{\psi^2 - 4\beta}$$
$$\Leftrightarrow \quad (\psi - 2\beta)^2 > \psi^2 - 4\beta$$
$$\Leftrightarrow \quad \psi^2 + 4\beta^2 - 4\beta\psi > \psi^2 - 4\beta$$
$$\Leftrightarrow \quad 4\beta(\beta + 1 - \psi) > 0$$
$$\Leftrightarrow \quad \beta + 1 - \psi > 0$$
$$\Leftrightarrow \quad -\sigma\kappa > 0,$$

which is not possible, so in the end $|\lambda_1^2| < 1$.

Let us now turn to the second polynomial. The discriminant of this polynomial is given by

$$\Delta_2 = (\psi\gamma)^2 - 4\beta\gamma^2.$$

Depending on the value of γ, there are two cases to consider for the discriminant. If $\gamma = 0$, then $\Delta_2 = 0$ and the polynomial has just one root given by $\lambda = 0$. So in this case the eigenvalue has a modulus strictly lower than 1. In

the general case where $\gamma > 0$, then the discriminant is positive if and only if

$$\psi^2 - 4\beta > 0,$$

which has already been shown to be strictly positive. So when $\gamma > 0$, there are two real eigenvalues given by

$$\lambda_2^1 = \frac{\gamma\psi + \sqrt{\Delta_2}}{2\beta} \quad \& \quad \lambda_2^2 = \frac{\gamma\psi - \sqrt{\Delta_2}}{2\beta}.$$

Let us begin with the second root. First, note that it can be rewritten as

$$\lambda_2^2 = \gamma \frac{\psi - \sqrt{\Delta_1}}{2\beta}$$
$$= \gamma \cdot \lambda_1^2.$$

Since $\gamma \in (0,1)$ and $|\lambda_1^2| < 1$, then it follows that $|\lambda_2^2| < 1$.

7.3 Proof of Proposition 2

Given the definition of λ_2^1, it is strictly positive. Further, it follows that

$$\lambda_2^1 > 1 \quad \Leftrightarrow \quad \gamma > \frac{2\beta}{\psi + \sqrt{\Delta_1}} \equiv \gamma^*.$$

Next, it still needs to be proven that $\gamma^* \in (0, 1)$. It is straightforward to see that $\gamma^* > 0$. Now, given the definition of ψ

$$\psi = 1 + \beta + \sigma\kappa > 2\beta$$
$$\Leftrightarrow \quad \psi + \sqrt{\Delta_1} > 2\beta$$
$$\Leftrightarrow \quad (\psi + \sqrt{\Delta_1})^{-1} < (2\beta)^{-1}$$
$$\Leftrightarrow \quad \gamma^* < 1.$$

7.4   Proof of Proposition 3

Given the definition of γ^* and the fact that $\psi = 1 + \beta + \sigma\kappa$, it is straightforward to see that γ^* is a decreasing function of both σ and α. How γ^* varies with respect to η is less straightforward. Since γ^* is a strictly decreasing function of ψ, it is sufficient to study how the latter varies with respect to η. Formally, given the definition of κ I get:

$$\frac{\partial\psi}{\partial\eta} = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left[\frac{\sigma(1+\eta\theta) - \theta(1+\sigma\eta)}{(1+\eta\theta)^2}\right]$$
$$= \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left[\frac{\sigma - \theta}{(1+\eta\theta)^2}\right].$$

From this, it follows that since

$$\frac{\partial\gamma^*}{\partial\eta} = \frac{\partial\gamma^*}{\partial\psi}\frac{\partial\psi}{\partial\eta} \quad \& \quad \frac{\partial\gamma^*}{\partial\psi} > 0,$$

I have the third result in Proposition 3.

7.5 Proof of Proposition 4

Under discretion ($\gamma = 0$), when I combine the FOCs from the private sector and the policymaker, I get:

$$\mathbf{F} \cdot \mathbb{E}_t \mathbf{X}_{t+1} = \mathbf{G} \cdot \mathbf{X}_t + \mathbf{S}_t + \tilde{\mathbf{C}}, \tag{11}$$

where $\mathbf{X}_t = \begin{bmatrix} \pi_t & \tilde{y}_t & \phi_{2,t-1} \end{bmatrix}'$ and the matrices $\mathbf{F}$, $\mathbf{G}$ and $\mathbf{S}_t$ are given by:

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 \\ \sigma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad, \quad \mathbf{G} = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} & 0 \\ 0 & 1 & 0 \\ -\kappa & -\frac{\kappa}{\theta} & 0 \end{bmatrix} \quad \& \quad \mathbf{S}_t = \begin{bmatrix} 0 \\ -\sigma \\ 0 \end{bmatrix}$$

Ignoring constant terms, this system can be rewritten as

$$\mathbb{E}_t \mathbf{X}_{t+1} = \mathbf{F}^{-1} \mathbf{G} \cdot \mathbf{X}_t + \mathbf{F}^{-1} \mathbf{S}_t$$
$$= \mathbf{F}^{-1} \mathbf{G} \cdot \mathbf{X}_t + \mathbf{S}_t$$
$$= \mathbf{F}^{-1} \mathbf{G} \cdot \mathbf{X}_t + \mathbf{S} \cdot R_t^*$$

Given the Markov structure of R_t^*, I use the method of undetermined coefficients. Specifically, I guess that $\mathbf{X}_t = \Omega R_t^*$. Solving for Ω, I get

$$\mathbb{E}_t \mathbf{X}_{t+1} = \mathbf{F}^{-1} \mathbf{G} \cdot \mathbf{X}_t + \Psi \mathbf{S}_t$$
$$\Leftrightarrow \quad p\Omega \mathbf{S}_t = \mathbf{F}^{-1} \mathbf{G} \cdot \Omega \mathbf{S}_t + \Psi \mathbf{S}_t$$

Comparing coefficients, I get:

$$p\Omega = \mathbf{F}^{-1}\mathbf{G} \cdot \Omega + \Psi$$

$$\Leftrightarrow \quad \Psi = (pI - \mathbf{F}^{-1}\mathbf{G})\Omega,$$

$$\Leftrightarrow \quad \Omega = (pI - \mathbf{F}^{-1}\mathbf{G})^{-1}\Psi,$$

where I is the $3x3$ identity matrix. Given the expression for the matrices, I get:

$$
(pI - F^{-1}G) = \begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ -\sigma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} & 0 \\ 0 & 1 & 0 \\ -\kappa & -\frac{\kappa}{\theta} & 0 \end{bmatrix}
$$

$$
= \begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix} - \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} & 0 \\ -\frac{\sigma}{\beta} & \frac{\sigma\kappa}{\beta}+1 & 0 \\ -\kappa & -\frac{\kappa}{\theta} & 0 \end{bmatrix}
$$

$$
= \begin{bmatrix} p - \frac{1}{\beta} & \frac{\kappa}{\beta} & 0 \\ \frac{\sigma}{\beta} & p - \frac{\sigma\kappa}{\beta} - 1 & 0 \\ \kappa & \frac{\kappa}{\theta} & p \end{bmatrix}
$$

Taking the inverse, I get:

$$
(pI - F^{-1}G)^{-1} = \begin{bmatrix} \frac{\beta-\beta p+\kappa\sigma}{p+\beta p-\beta p^2+\kappa p\sigma-1} & \frac{\kappa}{p+\beta p-\beta p^2+\kappa p\sigma-1} & 0 \\ \frac{\sigma}{p+\beta p-\beta p^2+\kappa p\sigma-1} & -\frac{\beta p-1}{p+\beta p-\beta p^2+\kappa p\sigma-1} & 0 \\ -\frac{\kappa(\sigma+\beta\theta-\beta p\theta+\kappa\sigma\theta)}{\theta(\beta p^2-p-\beta p^3+p^2+\kappa p^2\sigma)} & -\frac{\kappa(\kappa\theta-\beta p+1)}{\theta(\beta p^2-p-\beta p^3+p^2+\kappa p^2\sigma)} & \frac{1}{p} \end{bmatrix}
$$

The impact of a variation in R_t^* on the output gap is then given by

$$\frac{\partial \tilde{y}_t}{\partial R_t^*} = \frac{\beta p - 1}{p + \beta p - \beta p^2 + \kappa p \sigma - 1} \sigma$$

$$= \frac{1 - \beta p}{(1 - \beta p)(1 - p) - \sigma p \kappa} \sigma.$$