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The Aggregate Consequences of Tax Evasion

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Abstract

There is a sizable overall tax gap in the U.S., albeit tax non-compliance differs sharply across income types. While only small percentages of wages and salaries are underreported, the estimated misreporting rate of self-employment business income is substantial. This paper studies how tax evasion in the self-employment sector affects aggregate outcomes and welfare. We develop a dynamic general equilibrium model with incomplete markets in which heterogeneous agents choose between being a worker or self-employed. Self-employed agents may hide a share of their business income but face the risk of being detected by the tax authority. Our model replicates important quantitative features of the U.S. economy in terms of income, wealth, self-employment, and tax evasion. Our quantitative findings suggest that tax evasion leads to a larger self-employment sector but it depresses the average size and productivity of self-employed businesses. Tax evasion generates positive aggregate welfare effects because it acts as a subsidy for the self-employed. Workers, however, suffer from substantial welfare losses.

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Keywords: Tax evasion, Self-employment, Wealth inequality, Tax policy.

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1 Introduction

The evasion of individual income taxes is substantial in the United States. The Internal Revenue Service (IRS) estimates that the lost tax revenue due to underreported income is $197 billion in 2001, which is 18 percent of the actual income tax liability (U.S. Department of the Treasury 2009). Tax evasion is concentrated among the self-employed businesses. While only 1 percent of wages and salaries are not reported, this figure rises to 57 percent of self-employed income (Johns and Slemrod 2010). Self-employed businesses constitute an important component of the U.S. economy. They account for 39 percent of the assets and 21 percent of the income in the economy.[1]

What are the aggregate consequences of tax evasion in the self-employment sector in the U.S.? Does evading taxes by small businesses matter for aggregate outcomes, inequality, and welfare? What are the channels through which such effects operate? What are the implications for tax enforcement and tax policy?

To answer these questions we develop a dynamic general equilibrium model with incomplete markets and occupational choice and analyze how imperfect tax enforcement affects aggregate outcomes, the distribution of wealth, and welfare. In our model environment, infinitely lived agents face idiosyncratic and persistent shocks to their labor productivity and their talent of running a business. They pay progressive income taxes and choose between being a worker or running a self-employed business each period. Workers supply inelastically their effective time endowment to corporate firms, which operate with a constant returns to scale technology. These firms use competitively labor and capital and produce a consumption good. Workers cannot evade taxes and make consumption and saving decisions. Self-employed business owners use a decreasing returns to scale technology in capital to produce the consumption good. They may hide a share of their business income but are confronted with the probability of being detected by the tax authorities and punished by paying the evaded taxes and a proportional fine. Self-employed business owners optimally determine the size of their firms by choosing capital, taking into account that detection becomes more likely as their firm grows. In doing so, they face borrowing constraints proportional to the amount of their savings.

We calibrate the model to the U.S. economy at the start of the 2000s. First, we use the Panel Survey of Income Dynamics (PSID) data to estimate the labor productivity process for workers and impose parametric functions for the progressive income taxes paid by workers and self-employed business owners. Second, we set the parameter values related to production, the talent of running a self-employed business, and tax evasion by matching a selected number of data targets via a method-of-moments estimation. In particular, we target the capital-output ratio and the interest rate of the U.S. economy, the share of

[1]These numbers are derived from the Panel Study of Income Dynamics (PSID). For more details on the data work, see Appendix A.

self-employed business owners, their assets and income, and the annual exit rate from self-employment. Importantly, the parameters related to tax evasion are set to match the average misreporting rate of income as well as the cross-sectional misreporting rates conditional on the level of income.

The model replicates the empirical distributions of income and wealth even though they are not explicitly targeted. Another non-targeted dimension, which the model successfully matches, is the size distribution of self-employed businesses. The overall excellent fit of the model with respect to this broad set of empirical facts for the U.S. economy gives us confidence to use the model for a quantitative analysis.

In our quantitative analysis, we study the impact of tax evasion by comparing our benchmark economy with a counterfactual economy in which taxes are perfectly enforced. The optimal decision rules highlight three important channels through which tax evasion may affect aggregate outcomes. *(i)* The *subsidy channel*: tax evasion acts like a subsidy and stimulates asset accumulation, allowing higher investment in business capital. *(ii)* The *selection channel*: The opportunity to evade taxes induces less talented agents to run self-employed businesses. *(iii)* The *detection channel*: self-employed business owners have incentives to keep their businesses small to stay under the radar of the tax authorities and to reduce the chances of being audited.

The quantitative analysis of the stationary equilibrium suggests that tax evasion by small self-employed businesses matters for aggregate outcomes and inequality. The opportunity to evade taxes increases the number of self-employed businesses but reduces the average productivity of the self-employment sector. Moreover, tax evasion increases the share of small businesses, which is crucial for replicating the empirical self-employed firm size distribution. Furthermore, the economy with tax evasion is characterized by higher aggregate savings and larger aggregate output than the counterfactual economy with perfect tax enforcement. The increase in the aggregate capital stock lowers the interest rate and raises the wage generating a lower wealth inequality. A quantitative decomposition of the aggregate effects of tax evasion reveals that the *subsidy channel* as well as the *selection channel* are crucial while the *detection channel* is quantitatively less important.

Next, we study the welfare implications of tax evasion. To this end, we calculate the welfare effects of eliminating tax evasion by adopting a perfect tax enforcement technology. Our analysis suggests that the elimination of tax evasion leads to an aggregate welfare loss of about 4 percent, measured in consumption equivalence units, which is not surprising because tax evasion increases aggregate output in the economy. However, perfect tax enforcement raises tax revenues by around 1.6 percent of GDP, which roughly corresponds to the empirical estimate of the U.S. tax gap of 2 percent of GDP.[2] If these additional tax revenues are redistributed to the households via lump-sum transfers or tax cuts, the aggregate welfare loss of perfect tax enforcement is substantially reduced. Importantly, there is

[2] U.S. Department of the Treasury (2009).

a large degree of heterogeneity: perfect tax enforcement is associated with sizable welfare gains for the workers while self-employed business owners face substantial welfare losses. This pattern is emphasized for poor workers and poor self-employed. If the additional tax revenues are used for tax cuts targeted to the poor self-employed business owners, then perfect tax enforcement generates higher aggregate productivity and an overall aggregate welfare gain of 0.89 percent.

Finally, we study the implications of our analysis for tax enforcement and tax policy. First, we vary the fine that detected evaders have to pay to the tax authorities. It turns out that within a reasonable range of the penalty, tax revenues follow a Laffer curve in the size of the fine. This hump-shaped pattern is generated by two opposing forces. On the one hand, a higher fine allows the government to collect more revenues. On the other hand, a higher fine makes misreporting more risky and reduces the share of self-employed businesses. This, in turn, reduces aggregate output such that the lower tax base decreases tax revenues. Our quantitative findings suggest that a fine of around 65 percent maximizes tax revenues, which is 10 percentage points lower than the existing civil fraud penalty of 75 percent on missing taxes in the U.S.

Second, we vary the average tax burden on workers and self-employed businesses by scaling their respective non-linear tax functions. Our results show that the tax revenues collected from self-employed businesses follow a Laffer curve. While tax hikes increase revenues directly, they induce more and less productive agents to become self-employed and to escape taxation by evading. Lower productivity and higher distortionary taxes decrease output, which, in turn, reduces the tax base and adversely affects the tax revenues coming from self-employed businesses. The tax revenues collected from workers increase strongly if taxes are raised, suggesting that the elasticity of self-employed taxable income is much higher than the elasticity of taxable labor income. Thus, explicitly modeling tax evasion has direct quantitative implications for the assessment of tax policy.

The rest of the paper is organized as follows. The next subsection discusses the related literature. In Section 2, we provide further details on the technology of tax evasion in the U.S. Section 3 presents the model. Section 4 explains the calibration procedure and shows the model fit. In Section 5, we present and discuss how tax evasion affects aggregate outcomes, inequality, and welfare. Section 6 studies the impact of tax enforcement and tax policy on tax revenues. The last section concludes.

1.1 Related Literature

The economic theory of the technology and practices of tax evasion was initiated by the seminal works of Allingham and Sandmo (1972) and Yitzhaki (1974). They present a stylized model of tax evasion by a risk-averse agent who faces the probability of getting caught and penalized by the tax authorities. The theoretical analysis shows that it depends on income and risk aversion how much individuals evade. Andreoni (1992) extends this framework to a two-period model with income uncertainty and borrowing constraints. Other notable extensions of the static theory are presented by Yitzhaki (1974) and Pencavel (1979) who allow for a more general penalization structure and introduce labor supply choice, respectively. For a detailed summary of the literature, see Andreoni et al. (1998), Slemrod and Yitzhaki (2002), and Slemrod (2007). We take this classic modeling approach to tax evasion and incorporate it in a modern heterogeneous agent macroeconomic model of income and wealth inequality.

The macroeconomic literature on the aggregate effects of tax evasion is scarce. Our paper is related Maffezzoli (2011) who looks at the distributional effects of income tax evasion in a heterogeneous agent framework with incomplete markets. His model, similarly to ours, successfully replicates the cross-sectional pattern of tax evasion which increases in true individual income levels. The results point out that moving from a progressive taxation to a proportional tax rate reduces the amount of evaded taxes and raises government revenues. In contrast to his model, our framework explicitly accounts for the role of self-employed businesses in tax evasion. This allows us to quantitatively document the consequences of tax evasion for capital accumulation and aggregate productivity. In a related contribution, Bastidas (2018) studies the impact of a flat tax reform in the presence of tax evasion.

Another related study is Dessy and Pallage (2003). Their two-period heterogeneous agent model features formal and informal sectors of production with taxes financing the provision of productive public infrastructure. While the study outlines the differential role of tax evasion for aggregate productivity and inequality in poor and rich countries, it does not attempt to quantitatively explore the role of tax evasion for aggregate outcomes.

Our work builds on existing quantitative macroeconomic models with heterogeneous agents and incomplete markets in which entrepreneurs face borrowing constraints. The seminal works of Quadrini (2000) and Cagetti and De Nardi (2006) paved the way for generating adequate distributions of wealth in macroeconomic environments due to the savings behavior of entrepreneurs. Kitao (2008), on the other hand, explores the productive and welfare effects of capital taxation in a similar framework and shows that these effects depend on whether entrepreneurial or non-entrepreneurial capital is taxed. We complement these works by introducing the possibility of tax evasion for self-employed businesses and by exploring its role for aggregate economic outcomes and welfare.

Finally, we contribute to the macroeconomic literature of occupational choice and informality featuring two-sector models with formal and informal production. Amaral and Quintin (2006) emphasize the fact that informal sectors in developing countries feature less skilled workers than formal sectors. Antunes and Cavalcanti (2007) argue that the variation in regulation costs and enforcement of financial contracts account for the cross-country differences in informality. In a similar spirit, Kuehn (2014) explains the variation of informality across OECD countries through differences in taxes and government quality. Ordonez (2014), like us, emphasizes the role of imperfect tax enforcement for aggregate output and productivity for the case of Mexico. We extend these models to an environment with richer heterogeneity and a realistic self-employed business sector, which allows us to conduct an elaborate quantitative analysis on the role of tax evasion for the U.S. economy.

2 Tax Evasion in the United States

The Internal Revenue Code contains three primary obligations on taxpayers: *(i)* to file timely returns, *(ii)* to report accurately on those returns, and *(iii)* to pay the required tax voluntarily and on time. Thus, non-compliance takes three forms: *(i)* underreporting (not reporting full liability on a timely-filed return), *(ii)* underpayment (not paying the full amount of tax reported on a timely-filed return), and, *(iii)* non-filing (not filing the required returns on time). Given the scope of this paper, we concentrate our attention to the first component of non-compliance, namely, underreporting.

Individual income tax evasion and its distribution. The underreporting tax gap is defined as the amount of tax liability, which is not reported voluntarily by taxpayers who file tax returns on time. The IRS estimates that in 2001 underreporting of individual income led to a tax gap of $197 billion (U.S. Department of the Treasury 2009). This amounts to around 2 percent of the U.S. GDP in this year.[3]

Only 1 percent of the wages and salaries and 4 percent of taxable interest and dividends are misreported to the IRS. In contrast, 57 percent of self-employment business income is not reported. Johns and Slemrod (2010) analyze the micro data from the NRP in order to assess the distribution of tax non-compliance for the fiscal year of 2001. In their analysis, tax payers are grouped according to percentiles of their true income, i.e., the gross income they should have reported if not evading. According to their calculations, 11 percent of true income is misreported to the IRS. However, the misreporting rate varies with income levels. True income levels in the first decile of income are not misreported at all. Income in all other deciles below the median are misreported at a steady rate of around 5 percent. Around 7-8 percent of income in the four deciles above the median are hidden. Finally, tax evasion is highest in the top decile, where more than 15 percent of income is misreported.

Detecting and punishing tax evasion. The IRS had around 13,000 revenue and tax agents in 2002 whose main responsibility is detecting tax evasion (Dubin 2018). The individual income tax examination coverage, i.e., the audit rate was 1.27 percent in 1997.

[3] The estimate is based on the data collected through the National Research Program (NRP) Individual Income Tax Reporting Compliance Study for the 2001 tax year. The NRP analyzes approximately 46,000 randomly selected individual income tax returns. The estimated underreporting gap excludes unpaid taxes due to purely illegal activities.

In the following years, the audit rate declined and fell below 1 percent.(TIGTA 2002) The aggregate numbers mask, however, a large variation by type and size of reported income, as documented in Slemrod and Gillitzer (2014) and the U.S. Department of Treasure (2011). For individuals the probability of auditing is generally rising in reported income: individuals tax returns with reported income between \$25,000 and \$50,000 had a 0.73 percent probability of being audited. The probability raises to 29.93 percent for tax returns with reported income over \$10 million. Auditing rates depend considerably on the type of income declared. Individuals with business income (which is subject to more evasion) face a higher audit probability than those not reporting business income (2.1 percent and 0.6 percent, respectively). Likewise, for corporations the audit rate dramatically rises with the amount of total assets.[4]

Legally, it is very demanding to prove that a taxpayer knowingly committed a fraudulent act when evading taxes. Therefore, the IRS performs very few criminal investigations and more often pursues civil charges for evasion. Accuracy-related penalties vary between 20-40 percent of the missing taxes, while the civil fraud penalty is fixed at 75 percent (U.S. Department of the Treasury 2016).

3 The Model

The model builds on the seminal contributions of Quadrini (2000) and Cagetti and De Nardi (2006) who introduce entrepreneurs in macroeconomic models of wealth inequality but it differs from them in three key aspects. First, we introduce income tax evasion following the classic papers by Allingham and Sandmo (1972) and Yitzhaki (1974). Second, we allow for non-linear taxes, which describe the existing tax code more accurately. Third, in the light of the empirical facts on tax evasion, we concentrate our attention on the self-employed businesses and not on the general category of entrepreneurs.

Our model economy includes households, firms, and the government. Households are infinitely lived. Each time period corresponds to one year. Each period, households receive a pair of idiosyncratic realizations of their working ability and their ability of running a business. Based on these realizations and their stock of savings, they decide whether to form a self-employed business or to supply their work to a labor market. As in Aiyagari (1994), asset markets are incomplete, i.e., households cannot insure against shocks to working or business ability. In addition, there is another source of market imperfection: borrowing of self-employed businesses is subject to collateral constraints.

[4] We report more data on the relationship between auditing and income levels/size of business activity in Appendix A.5.

3.1 Preferences and Endowments

Households maximize the expected sum of discounted utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $\beta \in (0,1)$ is the time discount factor and c_t is consumption in period t. The utility function u is defined as $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, where $\sigma > 0$ denotes the coefficient of relative risk aversion. For simplicity, we assume that labor is inelastically supplied. Each household is endowed with a working ability $\varepsilon \in E$ and a business ability $\theta \in \Theta$, where ε and θ are drawn from a finite-state Markov process with transition probability $F(\varepsilon', \theta' | \varepsilon, \theta)$.

3.2 Technology

The economy consists of two sectors of production. The single consumption good is produced either in small self-employed businesses each of which is run by a household or in a large corporate sector with a representative firm. Actors in both sectors are price takers.

Self-employed businesses combine their business ability θ and capital k according to a production function,

$$f(k) = \theta k^v,$$

where $0 < v < 1$. The production function exhibits decreasing returns to scale capturing the *span of control* idea introduced by Lucas (1978): self-employed business skills gradually deteriorate as the size of the firm increases. The self-employed can save at a risk-free rate r and use their wealth to finance capital used in the project. In addition to using their own assets, they are allowed to borrow from a financial intermediary at a rate r.[5] This borrowing is limited up to a constant share of the assets, which self-employed businesses can pledge as collateral, $k \leq \lambda a$, where $\lambda \geq 1$. The two polar cases of $\lambda = 1$ and $\lambda = \infty$ capture the two extremes of financial autarky and perfect credit markets, respectively.

The corporate firm operates according to a constant returns to scale technology,

$$F(K_C, N_C) = K_C^{\alpha} N_C^{1-\alpha},$$

where $0 < \alpha < 1$. The corporate firm rents capital from households at rate r and labor services from workers paying a wage w. Capital in both sectors depreciates at a rate $\delta \in (0,1)$. Profit maximization in the corporate sector implies that input prices are set according to their marginal productivity,

$$r = \alpha K_C^{\alpha-1} N_C^{1-\alpha} - \delta \tag{1}$$

[5] The financial intermediary collects deposits from households and lends the proceeds to the corporate firm or the self-employed businesses. The ability of the self-employed is not observed by the intermediary, and, therefore, borrowing contracts cannot be conditioned on it.

and

$$w = (1-\alpha) K_C^{\alpha} N_C^{-\alpha}. \quad (2)$$

3.3 Government and Taxation

The government raises tax revenues to finance wasteful public spending G. Both workers and self-employed are subject to a non-linear personal income tax $T^i(\cdot)$ meant to approximate the actual tax code for the U.S. by capturing not only the statutory tax rates but also deductions, exemptions, and tax credits. We allow the tax schedules to be different for workers and self-employed. In particular, following Gouveia and Strauss (1999), we assume that each agent of type $i = \{W, E\}$, where W stands for worker and E stands for self-employed, has to pay tax liabilities given by the following tax function:[6]

$$T^i(y) = a_0^i \left[y - \left(y^{-a_1^i} + a_2^i \right)^{-1/a_1^i} \right]. \quad (3)$$

Note that for $a_1 > 0$ we have a progressive tax system since the average tax rate,

$$\frac{T^i(y)}{y} = a_0^i \left[1 - \left(1 + a_2^i y^{a_1^i} \right)^{-1/a_1^i} \right],$$

is increasing with income y.[7]

The crucial element of our modeling exercise is the introduction of imperfect tax enforcement. Whereas workers cannot evade taxes, self-employed agents may hide a share $\phi \in [0,1]$ of their business income.[8] The government, knowing that the self-employed evade taxes, can monitor through audits and perfectly verify the individual tax returns. Let $p(k)$, with $p'(k) > 0$, be the probability that a self-employed tax return is subject to monitoring. If the self-employed agent is audited and underreporting is detected, a fine $s > 1$ proportional to the amount of the underreported taxes is issued.[9] In essence, the self-employed needs to pay back the hidden taxes and an additional proportional penalty. For simplicity, we assume that the auditing efforts of the tax authorities are costless.

Our key assumption here is that the probability of being audited depends positively on the size of the business, $p'(k) > 0$, capturing the idea that larger firms are more visible

[6] Guner et al. (2014) show that this tax function is very flexible and provides a good approximation of the effective U.S. tax schedule. This functional form has been used extensively in the quantitative macroeconomic and public finance literature. Notable examples are Conesa and Krueger (2006), Kitao (2008) and Cagetti and De Nardi (2009).

[7] In addition, the degree of tax progressivity is increasing with a_1. If $a_1 \to 0$, then $T^i(y) \to a_0 y$, i.e. taxes are proportional.

[8] We assume that interest income generated by savings cannot be underreported, for both workers and self-employed.

[9] In the seminal work of Allingham and Sandmo (1972), the fine paid upon detection of tax fraud is proportional to the evaded income. However, the administrative penalty for evading taxes in the U.S. is proportional to the amount of unpaid taxes.

to the tax authorities. Our modeling choice is in line with the empirical evidence: Slemrod and Gillitzer (2014) document that in the U.S. the probability of auditing generally rises with income levels.[10] Furthermore, Lewis (2005) and Ordonez (2014) report that government agencies target larger establishments when it comes to audits.

3.4 Household Problem

Timing of events. The sequence of events in this economy unfolds as follows. At the beginning of each period, the idiosyncratic shocks ε and θ for working ability and business ability are realized. After observing these shocks, and conditional on the value of assets a inherited from the previous period, an individual chooses whether to be a worker or a self-employed for the current period. Workers make optimal decisions regarding consumption and savings and pay income taxes to the government. On the other hand, self-employed decide how much to invest (i.e., they choose k, taking the collateral constraint into account) and how much to evade (i.e., they choose ϕ). After business decisions are taken, detection and auditing by the government takes place. After observing if they are detected or not, self-employed agents make consumption and savings decisions. Note that the optimal consumption and saving choice of the self-employed is contingent on detection.

The optimization problem of an agent can be recursively formulated, with the individual states being the assets level a and the current abilities ε and θ. Let V^W and V^E denote the values of being a worker or a self-employed, respectively. The beginning-of-the-period value function is given by:

$$V(a,\varepsilon,\theta) = \max\left\{V^W(a,\varepsilon,\theta), V^E(a,\varepsilon,\theta)\right\}. \tag{4}$$

Let $o(a,\varepsilon,\theta)$ denote the occupational choice associated with problem (4):

$$o(a,\varepsilon,\theta) = \begin{cases} 1, & \text{if } V^E(a,\varepsilon,\theta) \geq V^W(a,\varepsilon,\theta) \\ 0, & \text{otherwise.} \end{cases}$$

Workers. The worker's problem can be written as:

$$V^W(a,\varepsilon,\theta) = \max_{c,a'}\left\{u(c) + \beta E\left[V(a',\varepsilon',\theta')\,|\varepsilon,\theta\right]\right\} \tag{5}$$

subject to

$$y_W = w\varepsilon + ra, \tag{6}$$

$$c + a' \leq y_W + a - T^W(y_W), \tag{7}$$

$$a' \geq 0. \tag{8}$$

[10] See Appendix A.5 for more evidence on this.

$T^W(\cdot)$ is the non-linear tax schedule defined in Section 3.3. Each worker supplies her total amount of time to the corporate sector as employed labor, earning a wage w for each productivity unit ε. Equation (6) represents the worker's taxable income, which consists of labor income $w\varepsilon$ and income from financial assets ra. Equation (7) states that all available resources net of taxes are split between consumption and savings. The last constraint summarizes the assumption that workers cannot borrow.[11] Crucially, employed workers cannot misreport their true income to the tax authority.

Self-employed. The decisions of a self-employed agent over production and tax evasion amount to choosing the operational capital k and the share of business income ϕ, which is not reported to the tax authorities. In doing so, the agent takes into account the probability of an audit by the tax authorities, which is conditional on the amount of capital invested in the business. The beginning-of-the-period value function is given by

$$V^E(a,\varepsilon,\theta) = \max_{k,\phi\in[0,1]} \left\{p(k)\,V_d^E(a,\varepsilon,\theta,k,\phi) + (1-p(k))\,V_n^E(a,\varepsilon,\theta,k,\phi)\right\} \tag{9}$$

subject to

$$0 \leq k \leq \lambda a, \tag{10}$$

where (10) is the collateral constraint. The value function for the case of detection is given by

$$V_d^E(a,\varepsilon,\theta,k,\phi) = \max_{c,a'} \left\{u(c) + \beta E\left[V(a',\varepsilon',\theta')\,|\varepsilon,\theta\right]\right\} \tag{11}$$

subject to

$$\pi = \theta k^v - (\delta + r)k, \tag{12}$$

$$y_E = \pi + ra, \tag{13}$$

$$c + a' \leq y_E + a - T^E\left((1-\phi)\,\pi + ra\right) - s\left[T^E(\pi + ra) - T^E\left((1-\phi)\,\pi + ra\right)\right], \tag{14}$$

$$a' \geq 0. \tag{15}$$

Equations (12) and (13) define respectively the profits from business activity and taxable income, which includes both capital profits π and financial income from savings ra. The budget constraint is given by equation (14) and states that available resources, net of taxes and fines, are allocated between consumption and savings. Since the self-employed is audited, she has to pay a fine $s > 1$ proportional to the amount of the underreported taxes (14). Notice that self-employed agents may hide a fraction ϕ of their business income π but they report truthfully their interest income ra.

The value function for the case of non-detection is defined as

$$V_n^E(a,\varepsilon,\theta,k,\phi) = \max_{c,a'} \left\{u(c) + \beta E\left[V(a',\varepsilon',\theta')\,|\varepsilon,\theta\right]\right\} \tag{16}$$

[11] More generally, equation (8) can be replaced by $a' \geq -\underline{a}$ where $\underline{a} \geq 0$ is an *ad hoc* borrowing limit.

subject to

$$
\begin{aligned}
\pi &= \theta k^{v} - (\delta + r)k, \\
y_E &= \pi + ra, \\
c + a' &\leq y_E + a - T^E\left((1-\phi)\pi + ra\right), \\
a' &\geq 0.
\end{aligned}
\tag{17}
$$

The optimization problem in (16) is very similar to (11) with the only difference coming from the flow budget constraint, which now does not show any tax evasion penalties.

For future reference, let us summarize the policy functions associated with the above problems. After solving the worker's maximization problem (5), we get the policy function for asset holdings a' if the agent is a worker, $g^W(a,\varepsilon,\theta)$. The solutions to the maximization problems of the self-employed (11) and (16) imply the policy function for asset holdings if the agent is self-employed and detected, $g^E_d(a,\varepsilon,\theta)$, and if the agent is self-employed and not detected, $g^E_n(a,\varepsilon,\theta)$. $k(a,\varepsilon,\theta)$ refers to the policy function for business capital and $\phi(a,\varepsilon,\theta)\in[0,1]$ is the policy function for tax evasion.

3.5 Equilibrium

The stationary equilibrium in the economy is characterized by a stationary distribution of agents over assets and ability realizations when the optimal behavior of agents and firms is taken into account. First, define the functions

$$
\mathbf{1}^W(a',a,\varepsilon,\theta) = \begin{cases} 1, & \text{if } g^W(a,\epsilon,\theta) = a' \\ 0, & \text{otherwise} \end{cases}
$$

$$
\mathbf{1}^E_d(a',a,\varepsilon,\theta) = \begin{cases} 1, & \text{if } g^E_d(a,\varepsilon,\theta) = a' \\ 0, & \text{otherwise} \end{cases}
$$

$$
\mathbf{1}^E_n(a',a,\varepsilon,\theta) = \begin{cases} 1, & \text{if } g^E_n(a,\varepsilon,\theta) = a' \\ 0, & \text{otherwise.} \end{cases}
$$

These functions take the value of one if the current realizations of the state variables $\{a,\varepsilon,\theta\}$ are associated with a future realization of the asset position a' according to the policy functions for workers and self-employed. Second, redefine the probability of detection as a function of the state variables using the policy function for business capital, $p_k(a,\varepsilon,\theta)\equiv p(k(a,\varepsilon,\theta))$. Then, the stationary distribution μ is defined as

$$
\begin{aligned}
\mu(a',\epsilon',\theta') = \int [\,&(1-o(a,\varepsilon,\theta))\,\mathbf{1}^W(a',a,\varepsilon,\theta) \\
&+ o(a,\varepsilon,\theta)\,p_k(a,\varepsilon,\theta)\,\mathbf{1}^E_d(a',a,\varepsilon,\theta) \\
&+ o(a,\varepsilon,\theta)\,(1-p_k(a,\varepsilon,\theta))\,\mathbf{1}^E_n(a',a,\varepsilon,\theta)]F(\varepsilon',\theta'|\varepsilon,\theta)d\mu(a,\epsilon,\theta)\,.
\end{aligned}
\tag{18}
$$

The first row of equation (18) counts those agents who decide to be workers and reach future states $\{a', \epsilon', \theta'\}$ given that they are at current states $\{a, \varepsilon, \theta\}$. The second and third lines represent the flow of self-employed who transit between current states $\{a, \varepsilon, \theta\}$ and future states $\{a', \epsilon', \theta'\}$ depending on whether they are detected or not by the tax authorities.

In a competitive stationary equilibrium workers, self-employed businesses, and the corporate firm solve their problems, all markets clear, and the distribution over the state variables that govern the behavior of households is stationary over time. Let the vector $x = (a, \varepsilon, \theta)$ contain the state variables, which summarize all the information necessary to solve the household problems in the economy. Specifically, a stationary competitive equilibrium consists of value functions $V(x)$, $V^W(x)$, and $V^E(x)$, policy functions for the household $o\left(x\right)$, $g^W\left(x\right)$, $g_d^E\left(x\right)$, $g_n^E\left(x\right)$, $k\left(x\right)$, and $\phi\left(x\right)$, input prices r and w, government spending G, income tax functions $T^W\left(\cdot\right)$ and $T^E\left(\cdot\right)$, and a probability distribution $\mu\left(x\right)$ such that:

1. Given prices $\{r, w\}$ and tax functions $\left\{T^W\left(\cdot\right), T^E\left(\cdot\right)\right\}$, the value functions $\left\{V(x), V^W(x), V^E(x)\right\}$ and the policy functions $\left\{g^W\left(x\right), g_d^E\left(x\right), g_{nd}^E\left(x\right), k\left(x\right), \phi\left(x\right)\right\}$ solve problems (4), (5), (9), (11) and (16).

2. Prices $\{r, w\}$ satisfy the optimization conditions of corporate firms, (1) and (2).

3. The government budget constraint is satisfied:

$$G = \int [(1 - o\left(x\right))T^W\left(y_W\left(x\right)\right) + o\left(x\right)T^E\left(\left(1 - \phi\left(x\right)\right)\pi\left(x\right) + ra\right) + \tag{19}$$
$$o\left(x\right)sp\left(k\left(x\right)\right)\left[T^E\left(\pi\left(x\right) + ra\right) - T^E\left(\left(1 - \phi\left(x\right)\right)\pi\left(x\right) + ra\right)\right]]d\mu\left(x\right).$$

4. The capital and labor markets clear. Capital demand (by corporate sector and by self-employed businesses) is equal to capital supply:

$$K_C + \int o\left(x\right)k\left(x\right)d\mu\left(x\right) = \int a d\mu\left(x\right).$$

   Labor demand is equal to labor supply:

$$N_C = \int (1 - o(x))\varepsilon d\mu\left(x\right).$$

   By Walras' law the goods market clearing condition holds in equilibrium as well. Total output can be defined as the sum of aggregate production in the self-employed sector and in the corporate sector:

$$Y = \int o\left(x\right)\theta k\left(x\right)^{\nu} d\mu\left(x\right) + \left(K_C\right)^{\alpha}\left(N_C\right)^{1-\alpha}.$$

5. The distribution $\mu\left(x\right)$ is stationary as implied by (18).

We describe the algorithm for the numerical solution of the stationary equilibrium in Appendix B.

4 Fitting the Model to the Data

We choose the parameters in our model in order to replicate important quantitative features of the U.S. economy. In particular, the focus is on matching *(i)* the share of self-employed households and their income and assets, and, *(ii)* the overall misreporting rate and the misreporting rates across quintiles of income.

We use the PSID for the years 1990-2003 to estimate the data moments related to *(i)*. In addition, we use wealth supplements, which are available for the years 1994, 1999, 2001 and 2003. We consider all households with a male household head of age 25-65 who has worked at least 260 hours during the year. We follow Heathcote et al. (2017) and drop observations if *(a)* there is no information on the age for either the household head or his spouse, *(b)* either the head or the spouse has positive labor income but zero annual hours, and *(c)* either the head or spouse has an hourly wage which is less than half of the corresponding federal minimum wage in that year. The data targets related to misreporting are taken from Johns and Slemrod (2010). For more details on our data work, we refer the reader to Appendix A.

The rest of this section is organized as follows. First, we present parameters that are fixed outside the model. Then, we discuss internally calibrated parameters, which are set so that the model matches the selected data targets. Finally, we report the model fit along several dimensions of the targeted and non-targeted data.

4.1 Externally Calibrated Parameters

Personal income tax. As explained in Section 3.3, we specify the income tax functions separately for workers and self-employed, using the functional form of Gouveia and Strauss (1999),

$$T^i(y) = a_0^i \left[y - \left(y^{-a_1^i} + a_2^i \right)^{-1/a_1^i} \right], \tag{20}$$

where $i = \{W, E\}$. The parameters $\{a_0^i, a_1^i, a_2^i\}$ are taken from Cagetti and De Nardi (2009) who estimate this functional form on federal taxes levied on the household pre-government income.[12]

Working ability process. We estimate a stochastic process for working ability following two steps, as it is standard in the literature (see, e.g., Guvenen (2009) and Heathcote et al. (2017)). First, we regress labor earnings on observable household characteristics such as education and experience in order to obtain a measure of labor income residuals ε_t.

[12]The parameters a_2^W and a_2^E are re-scaled in order to balance the government budget in equilibrium. We re-scale both a_2^W and a_2^E by a constant factor χ, $a_2'^i = a_2^i \chi^{a_1^i}$ for $i = \{W, E\}$. As explained in Section 4.2, the parameter χ is set to match a ratio of total income taxes to GDP of 15.2% as in Maffezzoli (2011).

Second, we model the residuals as a first order auto-regressive process:

$$\log \varepsilon_{t+1} = \rho_\varepsilon \log \varepsilon_t + \eta_{\varepsilon,t+1}, \tag{21}$$

where $\eta_{\varepsilon,t+1} \sim N(0, \sigma_\varepsilon^2)$. We estimate this process for workers and obtain a persistence parameter $\rho_\varepsilon = 0.89$, whereas the dispersion parameter is $\sigma_\varepsilon = 0.21$. We approximate the stochastic process in (21) with a discrete Markov chain following the procedure described in Tauchen and Hussey (1991). More details can be found in Appendix A.2.

Further parameters. We fix the coefficient of relative risk aversion σ to 2 which is standard in the macroeconomic literature. The parameter α represents the corporate capital share and is set to 0.38, which is the average corporate capital share for the period 1990-2007 (Karabarbounis and Neiman 2014). The choice for the parameter in the collateral constraint (10) is more delicate. When the borrowing constraint is binding, $\lambda = \frac{k}{a}$. Therefore, λ controls the maximum amount of leverage in the self-employed sector. Since we cannot observe the share of business capital financed with external sources in our data, we set λ to 1.2 as in Diaz-Gimenez et al. (1992). We perform robustness analysis with respect to this parameter; the results are available upon request. All externally set parameter values are reported in Table 1.

Table 1: Externally Calibrated Parameters

Parameter	Description	Value	Source/Target
σ	Elasticity of substitution	2	Standard value
α	Corp. capital share	0.38	Karabarbounis and Neiman (2014)
λ	Leverage ratio	1.2	Diaz-Gimenez et al. (1992)
Working ability			
ρ_ε	Persistence	0.89	Micro data - PSID
σ_ε	Standard deviation	0.21	Micro data - PSID
Tax functions			
a_0^W	Workers	0.32	Cagetti and De Nardi (2009)
a_1^W	Workers	0.76	Cagetti and De Nardi (2009)
a_2^W	Workers	0.22	Cagetti and De Nardi (2009)
a_0^E	Self-employed	0.26	Cagetti and De Nardi (2009)
a_1^E	Self-employed	1.40	Cagetti and De Nardi (2009)
a_2^E	Self-employed	0.44	Cagetti and De Nardi (2009)

4.2 Internally Calibrated Parameters

Business ability is assumed to follow a first order auto-regressive process:

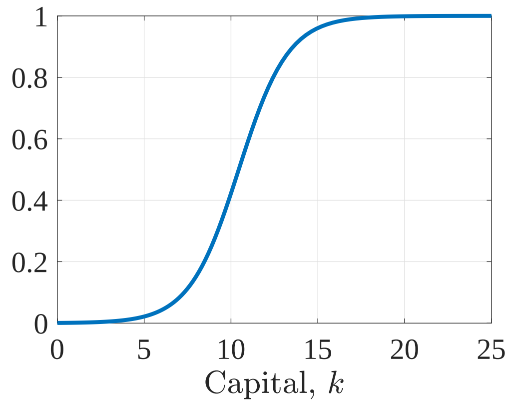
$$\log \theta_{t+1} = \mu_\theta + \rho_\theta \log \theta_t + v_{\theta,t+1}, \tag{22}$$

where $v_{\theta,t} \sim N(0, \sigma_\theta^2)$. The probability of tax fraud detection is a logistic function of business capital. In particular, we assume that

$$p(k) = \frac{1}{1 + p_1 \exp(-p_2 k)}, \tag{23}$$

with $p_1 > 0$ and $p_2 > 0$.[13] As argued before, we assume that the probability of being audited increases with the size of a business unit, following the empirical evidence reported in Slemrod and Gillitzer (2014), Lewis (2005), and Ordonez (2014). Figure 1 shows the function $p(k)$ evaluated at the estimated parameters.

Figure 1: Probability of Auditing



We need to assign values to the following parameters: the household discount factor β, capital depreciation δ, the span of control for self-employed businesses v, the three parameters for the business ability process (ρ_θ, σ_θ, μ_θ), the fine on tax evasion s, and the two parameters for the auditing probability, $p_1 > 0$ and $p_2 > 0$. Additionally, we need to pin down the scaling factor χ for the parameters controlling government revenue level in the tax functions (20).

A number of data targets are considered, which are sensitive to variations in the parameters. It is well-understood that all the model parameters affect all the targets but we can nonetheless outline which data moment is most informative about a certain parameter. The interest rate and the capital-output ratio identify the discount factor β and the capital depreciation δ. The parameter v controls the share of income of self-employed agents. The persistence ρ_θ in the stochastic process for the business ability is identified mainly by the annual exit rate from self-employment: a higher persistence of the ability process implies that self-employed change their occupation less frequently. The standard deviation σ_θ crucially affects the strength of the precautionary saving motive by self-employed

[13]We choose the logistic function for its flexibility. The parameter p_1 affects the vertical intercept of the function, $p(0) = 1/(1 + p_1)$. The parameter p_2 affects the inflexion point of the function. A higher p_2 shifts the inflexion to the left. We have experimented with other functional forms, namely, *(i)* a constant, and, *(ii)* an increasing and concave function. Results are available upon request.

agents, and thus, the share of assets owned by them. The parameter μ_θ, which relates to the unconditional mean of (22), determines the share of self-employed agents in the population.

When setting the penalty for tax evasion s, we target the overall taxable income misreporting rate in the U.S. economy. The parameters p_1 and p_2 of the probability function $p(k)$ are set to match the relationship between tax evasion and income. More precisely, we target the taxable income misreporting rate over quintiles of true household income, which are reported by Johns and Slemrod (2010). Finally, we need to determine the value of the scaling factor χ which adjusts the parameters a_2^W and a_2^E of the tax functions, so that that income tax revenue is an appropriate fraction of GDP.

To summarize, we set the 10 parameters $\Theta = \{\beta, \delta, v, \rho_\theta, \sigma_\theta, \mu_\theta, s, p_1, p_2, \chi\}$ by matching the following data targets:

1. *Share of self-employed, shares of total income and assets in possession of self-employed and their annual exit rate. These targets are derived from the PSID. [4 targets].*

2. *Capital-output ratio (NIPA) and an interest rate of 4% [2 targets].*

3. *Overall taxable income misreporting rate and taxable income misreporting rates in each quintile of income (Johns and Slemrod 2010) [6 targets].*[14]

4. *Tax revenue to GDP of 15.2% (Maffezzoli 2011) [1 target].*

In doing so, we use an overidentified method of moments approach. We minimize the squared difference between the 13 model moments and their counterparts in the U.S. data. We compute the difference of the model moments $\hat{m}_i(\Theta)$ from the data moments m_i as $d_i(\Theta) = m_i - \hat{m}_i(\Theta)$. Let $D(\Theta) = (d_1(\Theta), ..., d_{13}(\Theta))$ be the vector of differences between the model moments and the data moments. Then, the minimization problem is given by

$$\hat{\Theta} = \min_{\Theta} D(\Theta)' \mathcal{W} D(\Theta),$$

where $\mathcal{W}$ is a diagonal weighting matrix. The recovered values for the internally set parameters are presented in Table 2. It is worth mentioning that the recovered value of the tax evasion fine $s = 1.75$ is equivalent to the existing penalty for civil fraud of 75% (U.S. Department of the Treasury (2016)).

[14]We use the data provided by Johns and Slemrod (2010), Table 2, for the targets related to tax evasion. Note that the overall misreporting rate is an independent target from the quintile misreporting rates. In essence, the overall misreporting rate is the share of misreported overall true taxable income. Therefore, by themselves, the income quintile misreporting rates are not sufficient to compute the overall misreporting rate. What is needed for such a computation are the misreporting rates in each quintile and the share of true taxable income out of total taxable income in each quintile.

Table 2: Internally Calibrated Parameters

Parameter	Description	Value	Target
Preferences			
β	Discount factor	0.935	4% interest rate
Production			
δ	Capital depreciation	0.11	Capital-output ratio
v	Span of control	0.62	Share of income, self-employed
Self-employed ability			
ρ_θ	Persistence	0.935	Exit rate, self-employed
σ_θ	Standard deviation	0.77	Share of assets, self-employed
μ_θ	Unconditional mean	-1.29	Share, self-employed
Tax evasion detection			
s	Fine	1.75	Misreporting rate
p_1	Parameter of $p(k)$	1,500	Tax evasion by income (quintiles)
p_2	Parameter of $p(k)$	0.7	Tax evasion by income (quintiles)
Tax functions rescale			
χ	Rescaling parameter	1.4	Tax revenues as share of GDP

4.3 Model Fit

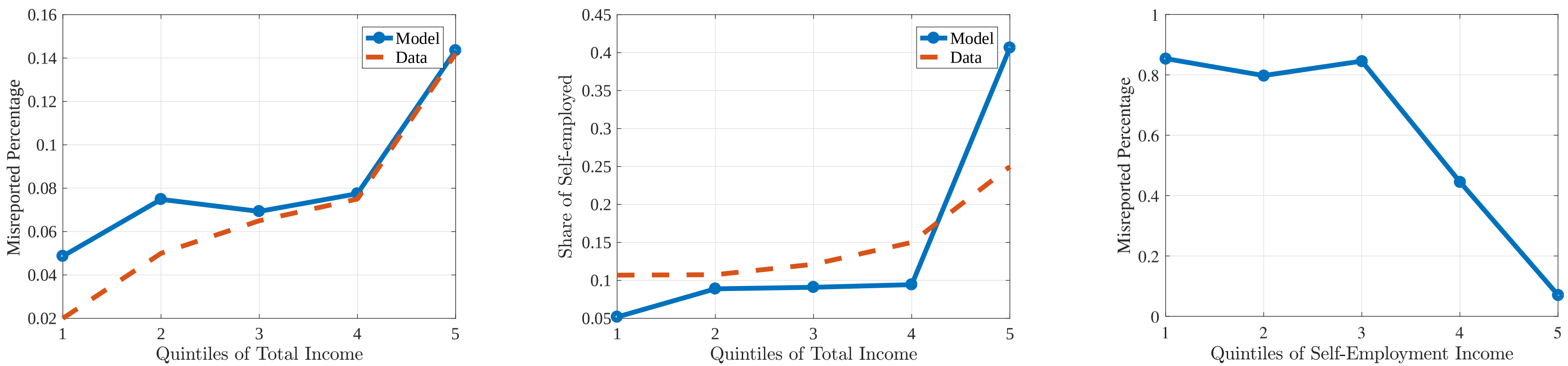
In this section, we compare the outcomes generated by the model with the corresponding statistics for the U.S. economy, both targeted and non-targeted. A good fit of the model along dimensions that are not explicitly targeted in the parameterization process reinforces our confidence in the validity of our approach when it comes to the counterfactual analysis.

Table 3 shows the model fit in terms of the first set of targeted moments of the U.S. data. The interest rate and the capital-output ratio are matched very closely. The model generates all basic targets on the share of self-employed, their income, assets, and exit rate as in the data. The average misreporting rate for taxable income matches the empirical value. Finally, tax revenues from income taxation are matched as part of the budget balancing condition for the government.

The other targeted moments relate to the patterns of misreporting of taxable income by quintiles of true household income. Figure 2 reports the data facts and the model outcomes of misreporting by income level. The model is able to match the increasing pattern of tax evasion with total income (Figure 2a). For lower deciles of true household income the share of workers is higher and workers do not evade. For higher deciles of total income there are more self-employed who can potentially evade (Figure 2b). The overall effect is, however, non-trivial because richer self-employed agents tend to evade less due to the probability of auditing, which rises in the size of the business as shown in Figure 2c.

We report selected moments of the distribution of wealth and income in Table 4. Even

Figure 2: Self-Employment and Tax Evasion by Income



(a) Tax Evasion by Total Income (b) Share of Self-Employed by Income (c) Tax Evasion by Self-Employed Income

Notes: U.S. data on the percentage misreporting rate by true income groups are taken from Johns and Slemrod (2010), Table 2.

Table 3: Basic Model Targets

Moments	Data	Model
Interest rate (%)	4.00	3.97
Capital-output ratio	2.65	2.62
Share of self-employed (%)	14.70	14.65
Share of assets, self-employed (%)	39.11	42.72
Share of income, self-employed (%)	21.04	23.76
Exit rate, self-employed (%)	15.73	15.90
Misreporting rate (%)	11.00	10.33
Tax revenues/GDP (%)	15.20	14.96

Notes: The table shows the model targets and the corresponding data targets based on U.S. PSID data for the years 1990-2003. The misreporting rate is taken from Johns and Slemrod (2010).

though we do not target the Gini coefficient, the mean-to-median ratio, and the other measures of wealth and income concentration, the model fits the data very closely in all these dimensions. The model replicates reasonably well both the bottom and the top of the wealth and income distributions, even though it slightly undershoots the concentration of wealth in the top 1 percent.[15] Figure 3a shows the average of self-employed net wealth for different quintiles of wealth, while Figure 3b reports the model fit in terms of business capital distribution.[16] The fit of the model in terms of quintiles of net wealth and firm size is quite good. In particular, the model is able to reproduce the fact that around 70 percent of firms are concentrated in the first bin of the size distribution (with capital of less than $522,000).

[15]It is well known that a standard Bewley model falls short of replicating the high degree of wealth concentration observed in the U.S. data. Modelling entrepreneurship as in the present framework improves significantly the ability of the model in fitting the data along this dimension.

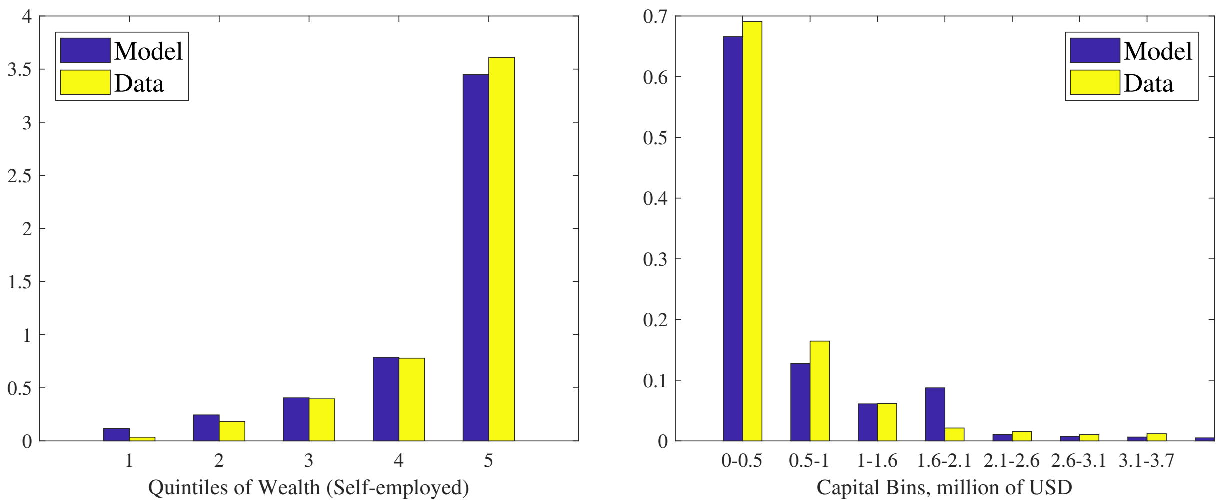
[16]Self-employed firm size and net wealth are strictly related due to the collateral constraint $k \leq \lambda a$.

Table 4: Wealth and Income Distribution

	Gini	Mean/Median	Bottom 40	Top 20	Top 10	Top 1
Wealth						
Model	73.5	2.90	3.26	76.38	63.32	21.53
U.S. Data	71.1	3.10	2.71	75.64	60.56	26.53
Income						
Model	36.6	1.34	19.84	45.03	31.71	10.69
U.S. Data	35.2	1.23	19.32	42.77	28.27	7.60

Notes: U.S. data are from the PSID, 1993-2003. Data on wealth are from the PSID wealth supplements for the period 1994 and 1999-2003.

Figure 3: Distribution of Self-employed Businesses



(a) Average Net Worth

(b) Size of Self-employed Businesses

Notes: U.S. data on net worth are from the PSID, 1993-2003. Panel (a) shows average net worth for each quintile, normalized by average net worth in the population. In panel (b), firm size is measured in terms of capital.

5 The Aggregate Effects of Tax Evasion

To highlight the aggregate effects of tax evasion, we provide a comparison between our benchmark economy and a counterfactual economy in which taxes are perfectly enforced. Think of this economy as an economy in which the penalty on misreporting is so high that tax evasion does not occur. In a first step, to understand the mechanisms, we study how tax evasion affects the optimal decision rules. In a second step, we analyze the impact of tax evasion on aggregate outcomes. Finally, we discuss the welfare implications of tax evasion.

5.1 Understanding the Mechanisms

In this section, we analyze the economic mechanisms of tax evasion and discuss the policy functions displayed in Figure 4.

In our two-sector model with incomplete credit markets, the agents' occupational choice, depicted in Figure 4a, depends both on business ability θ and wealth a (given average working ability). For a given level of business ability θ, agents become self-employed as long as they hold sufficient wealth. Poor talented agents who receive a high realization of business ability are credit-constrained so that they are not able to generate sufficient business income. There exists a wealth threshold $a^*(\varepsilon, \theta)$, (weakly) decreasing with business ability θ, such that agents with $a < a^*(\varepsilon, \theta)$ become workers and those with $a \geq a^*(\varepsilon, \theta)$ become self-employed.

The solid line in Figure 4a represents the wealth threshold for running a self-employed business as a function of business ability θ (given average working ability) in the benchmark economy, while the dashed line refers to the same threshold rule in the counterfactual economy with perfect tax enforcement. Tax evasion distorts the occupational choice at the margin because it makes self-employment more attractive. The opportunity to evade taxes raises the share of self-employed in the economy because a group of relatively less able agents (those between the solid and the dotted line in Figure 4a) find it profitable to run self-employed businesses. This suggests that on average the business ability of a self-employed agent in the economy with tax evasion is lower than in the counterfactual economy with perfect tax enforcement. This mechanism, through which tax evasion affects occupational choice and therefore the aggregates in the economy, is dubbed the *selection channel.*

In Figure 4b, we show the policy function for savings of the self-employed as a function of asset holdings (given average working and business ability). The blue solid line refers to the benchmark economy while the dashed red line refers to the counterfactual economy in which taxes are perfectly enforced. Tax evasion reduces the tax burden of self-employed agents and acts as a subsidy that facilitates higher savings. We refer to this as the *subsidy channel.*

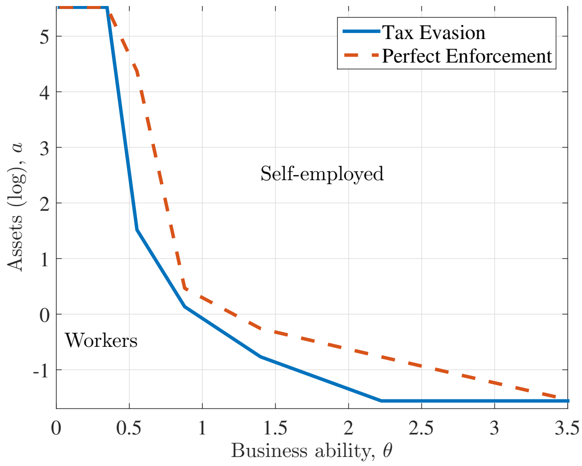
Figure 4c shows the optimal decision rule for business capital for the self-employed as a function of asset holdings. The productive abilities ε and θ are fixed at their average values. The blue solid line shows the decision rule for capital in the benchmark economy and the dashed red line refers to the counterfactual economy in which tax evasion is absent. For high level of assets the collateral constraint $k \leq \lambda a$ is not binding such that the optimal choice for capital is independent of the asset level. For lower values of assets, instead, the financing constraint binds and the self-employed are not able to run their projects at the optimal scale: in such a case their optimal capital choice does depend on wealth.

Interestingly, tax evasion creates a distortion in capital accumulation at low and medium values of assets. Indeed, the presence of a kink in $k(a, \varepsilon, \theta)$ (blue solid line) shows that wealth-constrained self-employed agents choose a sub-optimal level of capital in order to avoid a sharp increase in the probability of being audited. The intuition goes as follows. For low values of capital, Figure 1 shows that $p(k)$ is flat and small so that there is no distortion on capital accumulation: the optimal choice for $k(a, \varepsilon, \theta)$ is increasing. Then, as k approaches the inflexion point in $p(k)$, agents keep $k(a, \varepsilon, \theta)$ flat to avoid a sharp increase in the probability of detection. For larger k, however, they stop evading ($\phi = 0$) and can thus freely increase their choice of capital, until the first best is reached. Under perfect tax enforcement, the kink in $k(a, \varepsilon, \theta)$ disappears (red dotted line). We refer to this effect of tax evasion as the *detection channel*.

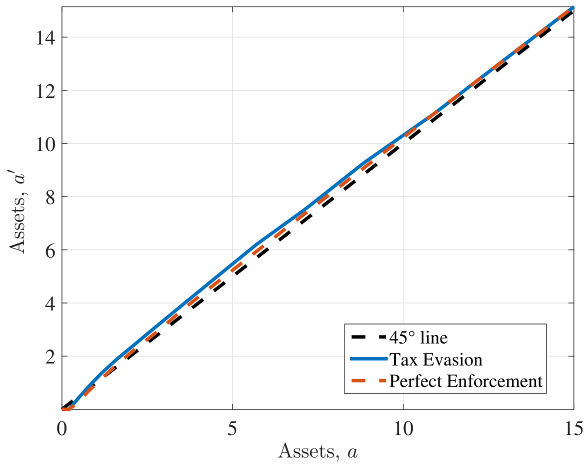
Figure 4c also shows that in the presence of tax evasion wealthy self-employed businesses with non-binding collateral constraints utilize more capital in production than in the counterfactual economy with perfect tax enforcement. This finding can be explained by a general equilibrium effect. Due to the subsidy effect of tax evasion, self-employed agents accumulate more capital in the aggregate, which decreases the equilibrium interest rate. In turn, the lower interest rate raises the first best level of capital.

Figure 4d displays the misreporting rate of a self-employed agent as a function of assets for different values of her business ability (given average working ability). Clearly, less talented agents misreport higher shares of their income as they are financially more constrained. Moreover, because of their small business size they face a lower probability of getting detected by the tax authorities inducing them to evade more.

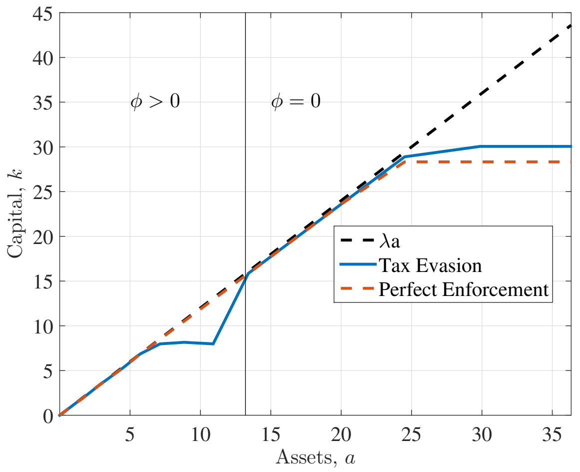
Figure 4: Policy Functions



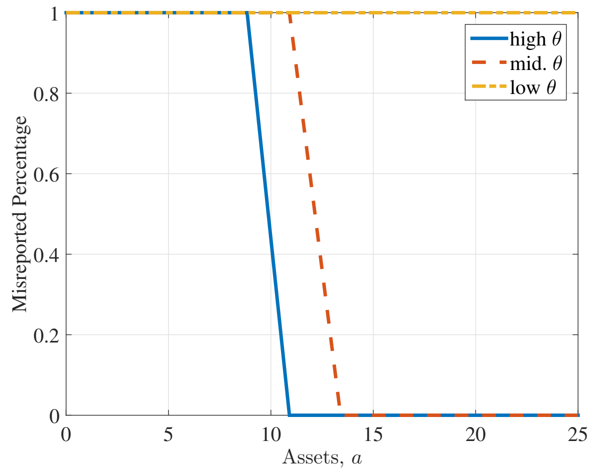
(a) Occupational Choice



(b) Savings of Self-Employed



(c) Business Capital



(d) Misreporting Rate

Notes: In panel (a), the solid (dotted) line demarcates the occupational choice $o(a, \varepsilon, \theta)$ in the benchmark economy with tax evasion (in the counterfactual economy with perfect tax enforcement). Agents with low wealth and/or low business ability become workers (southwest of the demarcation lines). Panel (b) reports the policy functions for next-period assets of self-employed agents $a' = g^E(a, \varepsilon, \theta)$ in the benchmark economy with tax evasion and in the counterfactual economy with perfect tax enforcement. Panel (c) shows the policy function for business capital $k(a, \varepsilon, \theta)$ in the benchmark economy with tax evasion and in the counterfactual economy with perfect tax enforcement. The vertical line demarcates the regions in which self-employed evade ($\phi > 0$) from regions in which they do not ($\phi = 0$). Panel (d) displays the misreporting rate $\phi(a, \varepsilon, \theta)$ in our benchmark economy with tax evasion for different business abilities θ. In panels (a) and (d), working ability is fixed to the average value. In panels (b) and (c), working ability ε and business ability θ are fixed to their average values.

5.2 Tax Evasion and Aggregate Outcomes

Table 5 presents selected aggregate statistics for the benchmark economy and the counterfactual economy in which taxes are perfectly enforced.

In our benchmark economy the share of self-employed agents is about 4 percentage points larger than in the economy with perfect tax enforcement. At the same time, the average business ability $E(\theta|E)$ is lower highlighting the *selection channel*: the opportunity to evade taxes induces less talented agents to run self-employed businesses.

There are several opposing forces affecting capital in the self-employment sector. On the one hand, the *subsidy channel* stimulates asset accumulation and allows higher investment in business capital. On the other hand, the *detection channel* provides incentives to keep self-employed businesses small to stay under the radar of the tax authorities and to reduce the chances of being audited. In addition, the *selection channel* lowers the average productive capacity of the self-employed businesses, and thus, their average size. Our quantitative findings suggest that the business capital decision of a self-employed business owner is critically affected by the *detection* and the *selection channels*: in the economy with tax evasion the mean value of business capital of a self-employed agent, $E(k|E)$, is lower than in the counterfactual economy with perfect tax enforcement.

In the aggregate, however, business capital K^E in the self-employment sector increases when tax evasion is allowed due to the higher share of self-employed businesses in the economy. As a consequence, tax evasion raises the aggregate output of the self-employed sector. The impact of tax evasion on the firm size distribution is shown in Figure 5. Due to the *detection channel* of tax evasion there are more firms in the smallest bin (from \$0 to \$522,000). Note that our benchmark economy with tax evasion provides a better description of the empirical firm size distribution than our counterfactual economy in which taxes are perfectly enforced.

Since the opportunity to evade taxes increases the share of self-employed, fewer households become workers and aggregate labor N^C in the corporate sector decreases. The increase in labor productivity is reflected in a higher real wage w. The benchmark economy with tax evasion is characterized by a larger aggregate capital stock than the counterfactual economy such that the interest rate r decreases. Both the higher wage and the lower interest rate contribute to a lower wealth inequality measured by the Gini coefficient of the household wealth distribution. In addition, wealth is less concentrated because tax evasion allows poor self-employed agents to save more.

In our quantitative application of the theoretical model to the U.S. economy, tax evasion reduces tax revenues by 1.6 percentage points of GDP. This figure is close to the empirical estimate of the U.S. tax gap of 2 percent of GDP (U.S. Department of the Treasury 2009).

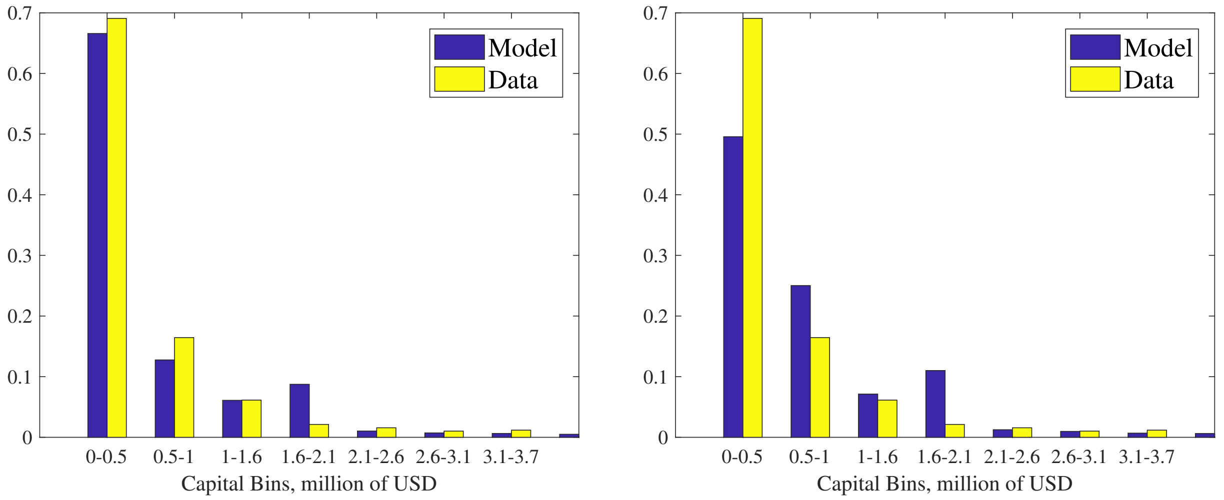
In our discussion so far, we highlighted the three channels through which tax evasion affects the aggregate outcomes in the economy. In the following, we seek to evaluate the quantitative importance of the three channels. Table 6 summarizes the main findings of

Table 5: Aggregate Effects of Tax Evasion

	Tax Evasion (Benchmark)	Perfect Tax Enforcement	Change (%)
Sector of self-employment			
Share of self-employed (%)	14.65	10.51	+4.14
$E(\theta\|E)$	0.93	1.02	-10.14
$E(k\|E)$	12.86	14.65	-13.90
K^E	1.88	1.54	+18.30
Y^E	0.68	0.56	+17.90
Corporate sector			
K^C	3.84	3.82	+0.53
N^C	0.85	0.89	-4.34
Y^C	1.51	1.54	-2.46
Prices			
r (%)	3.97	4.34	-0.37
w	1.10	1.08	+1.48
Tax revenues			
T/Y (%)	14.96	16.61	-1.65
Wealth inequality			
Gini	73.50	75.24	-1.74

Notes: The table compares macroeconomic aggregates of the benchmark economy with tax evasion with those of the counterfactual economy with perfect tax enforcement. The last column reports the percentage changes in the macroeconomic aggregates when moving from the counterfactual economy with perfect tax enforcement to the benchmark economy with tax evasion. $E(\theta/E)$ and $E(k|E)$ denote the mean value of business ability and business capital of a self-employed agent. K^E and Y^E refers to aggregate capital and aggregate output in the self-employment sector. K^C, N^C, and Y^C denote capital, labor, and output in the corporate sector, respectively. Tax revenues T/Y are given as percentage share of total output. w denotes the wage rate, while r is the interest rate in percent.

Figure 5: Tax Evasion and the Distribution of Self-Employed Businesses



(a) Tax Evasion (Benchmark)

(b) Perfect Tax Enforcement

Notes: U.S. data on the size of self-employed businesses are from the PSID, 1993-2003. Panel (a) and (b) provide a comparison of the data with the benchmark model with tax evasion and the counterfactual economy with perfect tax enforcement.

our decomposition exercise. We consider the following aggregate outcomes: the share of self-employed business owners, the average capital of self-employed businesses $E(k|E)$, the aggregate capital in the self-employed sector K^E, the aggregate capital in the corporate sector K^C, and the income misreporting rate.

As a starting point, in column (1) of Table 6, we consider the counterfactual economy in which taxes are perfectly enforced. Then, we move to a tax evasion economy in a partial equilibrium fashion, i.e., we keep the wage and the interest rate at the values of the perfect tax enforcement economy. In this way, we can document the changes in the aggregate economic outcomes solely due to the presence of tax evasion abstracting from general equilibrium effects. We report our findings in column (5). To facilitate a comparison, in column (6), we list the outcomes of the benchmark economy with tax evasion in general equilibrium. Comparing columns (1), (5), and (6) reveals that when the wage and the interest rate are not allowed to adjust, tax evasion has a larger positive effect on the self-employment rate and aggregate capital in both sectors. The opportunity to evade taxes induces more but less talented agents to become self-employed, which increases the misreporting rate and reduces the mean value of business capital of a self-employed agent. In general equilibrium, the wage increases while the interest rate decreases, such that the impact of tax evasion on aggregate outcomes is mitigated.

In the next step, we decompose the partial equilibrium effects to deduce the strength of the *subsidy*, *selection* and *detection channels*. To this end, we run a series of additional counterfactual experiments. Let $\tilde{o}(x)$ and $\tilde{k}(x)$ denote the policy functions for the occupational choice and for business capital in the economy with perfect tax enforcement, respectively. To isolate the effect of the *subsidy channel* of tax evasion, in column (2), we impose exogenously the policy functions $o(x) = \tilde{o}(x)$ and $k(x) = \tilde{k}(x)$ in the partial equilibrium economy with tax evasion. Thus, we allow for tax evasion but the decisions on occupational choice and business capital are fixed such that the *selection* and *detection channels* are shut down. Now tax evasion affects the outcomes of the economy only through the savings behavior of agents. Next, in column (3), we fix only the occupational choice $o(x) = \tilde{o}(x)$ in the partial equilibrium economy with tax evasion and shut down the *selection channel*. Finally, in column (4), we fix only the choice of business capital $k(x) = \tilde{k}(x)$ to eliminate the *detection channel*.

Let us first focus on the quantitative importance of the *subsidy channel*. Our findings in column (2) show that in the absence of the *detection* and *selection channels* the opportunity to evade taxes increases the average business capital of a self-employed agent by 7.1 percent (from 14.65 to 15.69). Misreporting income allows self-employed business owners to pay less taxes and to accumulate more savings and, in turn, to invest more in their business capital. Column (3) shows that the *detection channel* has a mitigating impact on average business capital because the probability of being audited by the tax authorities induces self-employed businesses to stay small. However, quantitatively this *detection channel* is less important than the *subsidy channel*.

A comparison of column (4) with column (2) reveals that the *selection channel* is of great quantitative importance. In the economy in which both the *subsidy* and *selection channels* are in place, average capital of self-employed businesses is reduced by 16 percent compared to the economy in which only the *subsidy channel* is present ($E(k|E)$ drops from 15.69 to 13.23). At the same time, the share of self-employment raises by around 3.7 percentage points (from 11.22 to 14.93), which is close to the total increase in self-employment due to tax evasion shown in column (5). The rise in the number of self-employed implies a substantial increase in aggregate capital in the self-employment sector: K^E goes up by 12.5 percent from column (2) to column (4). Moreover, the selection of more but less talented self-employed agents generates a lot of the observed tax noncompliance as evident by the high misreporting rate reported in column (4).

Table 6: Decomposition of Aggregate Effects

		Tax Evasion Economies				
	Perfect Tax Enforcement	Counterfactual Experiments Partial Equilibrium				Benchmark General Equilibrium
	(1)	(2)	(3)	(4)	(5)	(6)
Setup						
Fixed prices from (1)	-	r, w	r, w	r, w	r, w	-
Fixed decisions from (1)	-	$o(x), k(x)$	$o(x)$	$k(x)$	-	-
Endogenous decisions	*all*	$a'(x), \phi(x)$	$a'(x), k(x), \phi(x)$	$o(x), a'(x), \phi(x)$	*all*	*all*
Operational channels	-	*Subsidy*	*Subsidy +Detection*	*Subsidy +Selection*	*Subsidy+Detection +Selection*	*Subsidy+Detection +Selection+Prices*
Outcomes						
Share of self-employed (%)	10.51	11.22	11.29	14.93	15.09	14.65
$E(k\|E)$	14.65	15.69	15.04	13.23	12.64	12.86
K^E	1.54	1.76	1.70	1.98	1.91	1.88
K^C	3.82	4.23	4.31	4.16	4.27	3.84
Misreporting rate (%)	0	7.77	8.63	9.83	10.74	10.33

Notes: The table provides macroeconomic outcomes for a series of counterfactual experiments in which prices and policy functions are fixed at the perfect tax enforcement economy. $E(k|E)$ denotes the mean business capital of a self-employed agent. K^E and K^C denote aggregate capital in the self-employment sector and corporate sector, respectively.

5.3 Tax Evasion and Welfare

In this section, we evaluate how tax evasion affects average welfare. The welfare effects of eliminating tax evasion are in terms of consumption equivalent variations, i.e., we calculate the consumption gain or loss of moving from the benchmark economy with tax evasion to an economy in which taxes are perfectly enforced.[17] Thereby, we compare the stationary equilibria and abstract from transitional dynamics. Table 7 summarizes the results.

Table 7: Tax Evasion and Aggregate Welfare

	Tax Evasion Benchmark	Perfect Tax Enforcement			
		No Redis-tribution	Redistribution		
			Lump-Sum All	Tax Cut All	Tax Cut Self-Employed
	(1)	(2)	(3)	(4)	(5)
Share of self-employed (%)	14.65	10.51	10.45	10.80	13.92
Y	2.18	2.10	2.10	2.13	2.21
r (%)	3.97	4.34	4.40	4.23	3.81
w	1.10	1.08	1.08	1.09	1.11
CEV (%)	N.A.	-4.09	-1.72	-1.25	-0.60

Notes: The table summarizes selected statistics for the benchmark economy with tax evasion and four counterfactual economies in which taxes are perfectly enforced. CEV (consumption equivalent units) shows the percentage change in consumption needed to make a household indifferent between being born in the benchmark economy (column (1)) and being born in any of the four counterfactual economies with perfect tax enforcement (columns (2) to (5)). Column (2) is the counterfactual economy without fiscal neutrality in which additional tax revenues are not redistributed. In column (3) the government balances the budget with lump-sum transfers to all households. In column (4) the government balances the budget by implementing tax cuts for all households while in column (5) the tax cut is implemented for self-employed agents only. Y refers to total aggregate output. r is the interest rate in percent while w refers to the wage rate.

Since the elimination of tax evasion increases tax revenues, we distinguish several fiscal policy scenarios. In a first step, in column (2) of Table 7, we assume that the additional tax revenues are not redistributed to the agents in the economy. In a second step, we consider fiscal scenarios under fiscal neutrality, i.e., we assume that tax policies are adjusted such that the same level of tax revenues is achieved as in the benchmark economy. Specifically, in column (3), we report the welfare results if the additional tax revenues are redistributed via *lump-sum transfers* to all households. In column (4), the additional tax revenues are redistributed by *tax cuts* for all households. To this end, the tax level is decreased by re-scaling proportionately down the terms (a_2^W, a_2^E) in the non-linear tax functions (3). Finally, in column (5), fiscal neutrality is imposed by a *tax cut for the self-employed* only.

[17] Appendix B.5 provides technical details.

Eliminating tax evasion without imposing fiscal neutrality has a negative effect on welfare (column (2)). This is not surprising since aggregate capital and output fall when taxes are perfectly enforceable. If the additional taxes are rebated via lump-sum transfers to all households, the welfare loss from perfect tax enforcement is much smaller, dropping from 4 percent to 1.7 percent (column (3)). If redistribution is accomplished by slashing the level of non-linear taxes (column (4)), the welfare loss is even smaller than in the previous case. If the tax cut is implemented for self-employed agents only (column (5)), welfare is roughly in line with the benchmark economy, but aggregate output is 1.2 percent higher. The reason is that the additional tax revenues allow the government to reduce the distortionary effect of taxation, which increases aggregate production. This positive effect is enhanced if the tax cut is targeted to the self-employed sector.

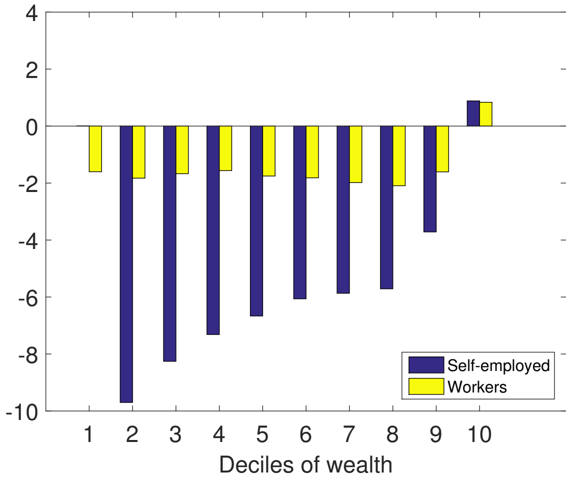
To deepen our understanding of the welfare results, we decompose them to study whether workers and self-employed agents are affected differently by tax evasion. Figure 6 shows the average welfare changes for workers and self-employed agents along the wealth distribution for the four fiscal policy scenarios. In order to compute these values, first, we identify the decile of the overall wealth distribution that each household type (in terms of asset holdings and abilities) belongs to in the benchmark economy. Second, we identify the occupational choice made by different household types in the benchmark economy. Finally, we compute the average welfare effects in each counterfactual scenario for households occupying each of the original benchmark economy deciles, while distinguishing between workers and self-employed according to their benchmark choice. However, the distribution over household types used in the welfare effects computation comes from the counterfactual economy. Defining tight household categorization according to the abilities and choices made in the benchmark economy allows us to derive the heterogeneous welfare effects of moving to the new economy for the same agents while taking into account that the weights of different household types in the new economy's distribution change.[18]

Figure 6a displays the welfare results if the additional tax revenues are not redistributed back. The elimination of tax evasion leads to welfare losses for both self-employed agents and workers, except for those in the last decile of wealth. Self-employed agents incur larger losses since perfect tax enforcement has a direct negative impact on them, whereas workers are hurt indirectly from the future option value of becoming self-employed. In addition, the wage is reduced as seen in Table 7.

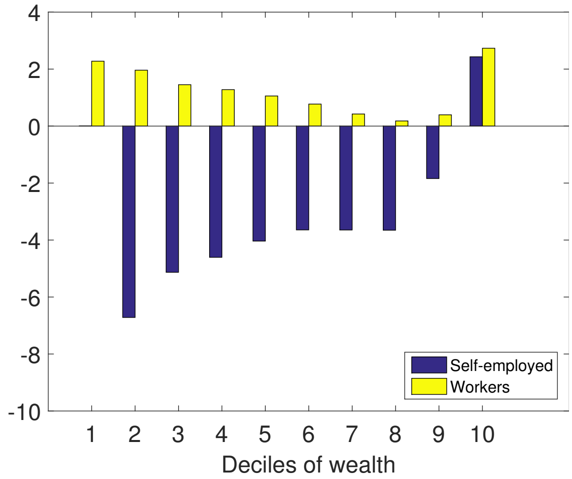
The welfare losses of self-employed business owners decrease in the relative wealth position. Recall that poor self-employed business owners have a higher misreporting rate (Figure 2c). Therefore, they lose more if taxes are perfectly enforced. Workers and self-employed in the top decile of wealth encounter welfare gains from the elimination of tax

[18] Our approach to calculating heterogeneous welfare effects in the presence of occupational choice resembles the welfare analysis of tax policies in Brüggemann (2017). Further details are described in Appendix B.5.

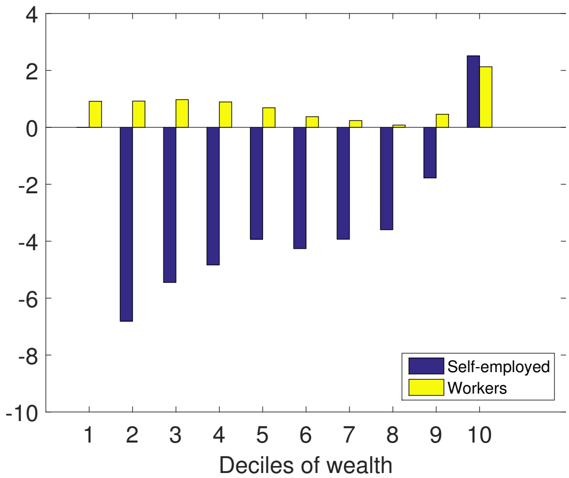
Figure 6: Tax Evasion and Welfare Across Wealth and Occupation



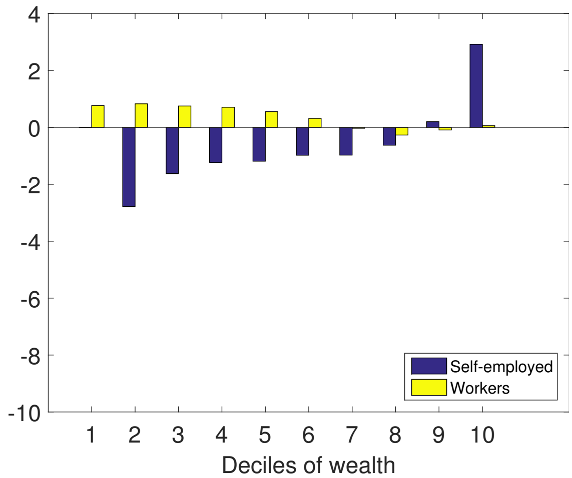
(a) No redistribution



(b) Lump-sum redistribution



(c) Decrease taxes for all



(d) Decrease taxes for self-employed

Notes: This figure shows the percentage change in consumption needed to make a household (worker or self-employed) indifferent between being born in the benchmark economy and being born in any of the four counterfactual economies in which taxes are perfectly enforced. A formal definition of the certainty equivalents among different groups is given in Appendix B.5. Panel (a) is the counterfactual economy without fiscal neutrality in which additional tax revenues are not redistributed. In panel (b), the government balances the budget with lump-sum transfers to all households. In panel (c), the government balances the budget by implementing tax cuts for all households while in panel (d) the tax cut is implemented for self-employed agents only. Note that there are no self-employed agents in the first decile of the wealth distribution.

evasion because they benefit from higher interest rates (Table 7).

The overall welfare effect of eliminating tax evasion is negative if the additional tax revenues are redistributed via lump-sum transfers to all households (Table 7). However, Figure 6b reveals that the welfare changes are heterogeneous along the dimensions of occupation and wealth. Importantly, workers *gain* while the majority of self-employed business owners *lose* from the elimination of tax evasion accompanied by lump-sum transfers. Note that poor workers have larger welfare gains because they benefit more from lump-sum transfers. In contrast, if taxes are rebated via tax cuts for all, the welfare gains of workers vary less across deciles of wealth (see Figure 6c).

Figure 6d displays the average welfare changes for workers and self-employed business owners if the tax cut is implemented for the self-employed only. Clearly, the welfare losses of self-employed agents are much smaller than in the previous scenarios. Although there is no tax cut for the workers, they still have sizable welfare gains because they benefit from a higher wage rate.

To sum up, our welfare results point out that under fiscal neutrality moving to an economy where taxes are perfectly enforced makes workers better off while self-employed, and in particular the poor ones, lose. This observation motivates the last fiscal policy scenario presented in this section: we impose fiscal neutrality by cutting taxes, but only for a sub-group of the self-employed agents who might be particularly affected by perfect enforcement.

In Table 8 we target self-employed agents with asset holdings below a given threshold measured as percentile of the benchmark wealth distribution. Our results reveal that targeting self-employed agents with assets holdings below the 60th percentile leads to an average welfare gain of 0.89 percent for *all* households. Interestingly, when tax cuts are implemented only for the poorest self-employed with wealth levels below the 50th percentile, eliminating tax evasion generates an overall welfare loss. The intuition behind this finding is that these poor self-employed business owners are likely to have a lower business ability as well. Therefore, subsidizing them via tax reductions has adverse welfare effects.

Table 8: Tax Evasion and Aggregate Welfare: Tax Cuts for Self-Employed

	Threshold as Percentage of the Wealth Distribution								
	p10	p20	p30	p40	p50	p60	p70	p80	p90
Share of self-employed (%)	15.25	14.71	14.52	14.45	14.54	14.83	14.38	14.41	14.15
Y	2.14	2.14	2.14	2.16	2.17	2.22	2.20	2.23	2.22
r (%)	4.13	4.14	4.14	4.12	4.05	3.92	3.96	3.85	3.84
w	1.09	1.09	1.09	1.09	1.09	1.10	1.10	1.10	1.10
CEV (%)	-1.33	-1.09	-1.03	-0.82	-0.3	0.89	0.38	0.78	0.25

Notes: The table shows selected statistics for counterfactual economies in which taxes are perfectly enforced. CEV (consumption equivalent units) shows the percentage change in consumption needed to make a household indifferent between being born in the benchmark economy with tax evasion (column (1) of Table 7) and being born in a counterfactual economy with perfect tax enforcement in which the government balances the budget by reducing tax rates only for self-employed agents with asset holdings below a given threshold (10th, 20th,...,90th percentiles of the self-employed wealth distribution). Y refers to total aggregate output. r is the interest rate in percent while w refers to the wage rate.

6 Tax Evasion and Laffer Curves

In this section, we study the impact of tax evasion on tax revenues and perform two types of exercises. First, we focus on the fine that detected evaders have to pay to the tax authorities. We consider the penalty as a policy variable and study the impact of tax enforcement on tax revenues and aggregate outcomes. Second, we analyze how shifts in the tax scheme affect tax revenues and aggregate outcomes in the presence of tax evasion.

6.1 The Fine on Tax Evasion

According to the OECD (2011), in developed countries the penalties for tax evasion vary between 10 and 30 percent for minor offenses and between 40 and 100 percent for frauds. To study the impact of the size of the penalty on aggregate outcomes and tax revenues, we use our benchmark model and vary the fine s within a range between 20 and 100 percent of the tax evaded. In Figure 7, we display how the share of self-employed business owners, the average productivity in the self-employment sector, aggregate capital and output as well as tax revenues change in response to the size of the penalty.

Figure 7c shows that the share of self-employed businesses decreases as the level of the fine rises. This goes hand in hand with an increase in the average productivity of the self-employment sector. The reason is intuitive: if tax evasion is punished with a higher fine, agents with a lower business ability leave the sector of self-employment because misreporting becomes too risky for them. The smaller size of the self-employment sector, however, decreases aggregate capital and output in the economy, as shown in Figure 7d.

Figure 7b suggests that total tax revenues follow a hump-shaped pattern in the fine

s. This behavior resembles a Laffer curve and is driven by the tax revenues collected from the self-employed business owners as the tax revenues collected from the workers hardly change. The hump-shape is generated by two opposing forces. First, an increase in the penalty for tax evasion incentivize self-employed business owners to report business income more truthfully: the income gap shrinks as shown in Figure 7a and the government can collect higher revenues. Second, an increase in s decreases the share of self-employed businesses as well as aggregate output and capital (see Figures 7c and 7d). The drop in aggregate output reduces the tax base and lowers tax revenues. Within the considered range, we find that a fine of around 65 percent of the tax evaded maximizes tax revenues. This revenue-maximizing fine is 10 percentage points lower than the existing civil fraud penalty of 75 percent on evaded taxes in the U.S.

Note that for penalties larger than 200 percent tax revenues start to increase as the fine s is raised. Very large penalties eliminate tax evasion and the outcomes correspond to the ones of our counterfactual economy in which taxes are perfectly enforced. However, we argue that penalties of this size are politically not feasible.

6.2 The Tax Scheme

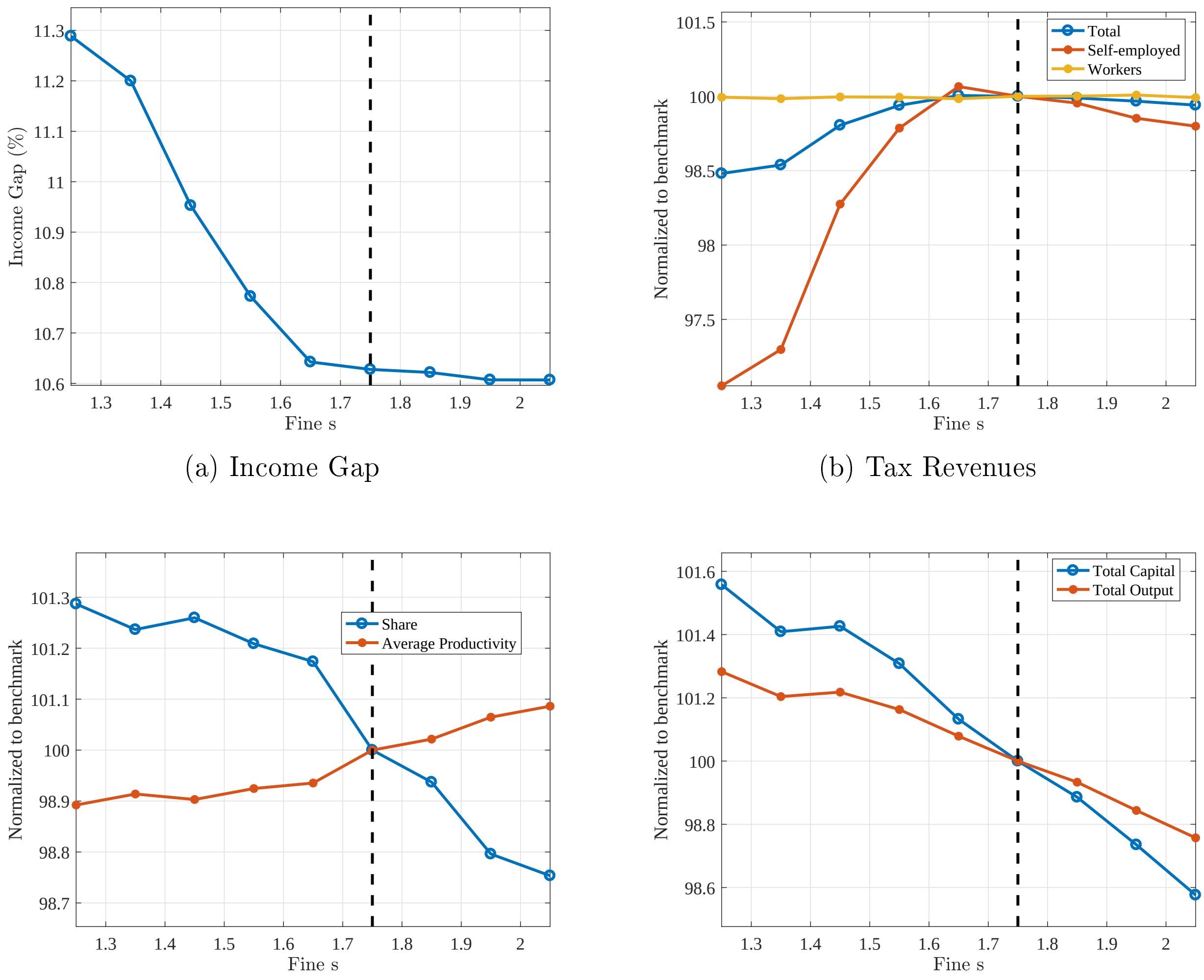
In this section, we analyze the impact of the tax scheme on aggregate outcomes and tax revenues in the presence of tax evasion. In particular, we are interested in how shifts in the tax scheme affect the size of the self-employment sector and the misreporting rate. To shift the tax scheme, we increase the tax parameters a_0^W and a_0^E proportionally, see the tax function (20), such that both workers and self-employed business owners face higher taxes. We display our findings in Figure 8.

Figure 8b displays total tax revenues and tax revenues collected from workers. Both types of tax revenues increase as the tax rates for workers and self-employed are shifted proportionally. Interestingly, the tax revenues collected from self-employed businesses follow a hump-shaped pattern. Furthermore, the tax revenues coming from workers increase much stronger than the tax revenues from business owners suggesting that the elasticity of taxable income for self-employed businesses is high.

To understand the pattern of tax revenues, we display the income gap in Figure 8a, the share and productivity of self-employed businesses in Figure 8c, and total aggregate capital and output in Figure 8d. The Laffer-type behavior of tax revenues collected from self-employed business owners is driven by opposing forces. On the one hand, tax hikes increase tax revenues directly. On the other hand, higher taxes induce more agents to become self-employed such that the average business ability in the self-employment sector decreases. Lower productivity and higher distortionary taxes decrease aggregate capital and output in the economy. This in turn, reduces the tax base and adversely affects the tax revenues collected from self-employed businesses. Note that those agents who remain being workers cannot escape the higher taxation by working less because labor supply is fixed in this economy. Therefore, tax revenues collected from workers increase if the tax scheme is shifted upwards.

We perform the same experiment for our counterfactual economy in which taxes are perfectly enforceable. We find that as the tax rate increases, the share of self-employed businesses decreases in the economy with perfect tax enforcement while the opposite is true in the economy with tax evasion.[19] The reason is quite intuitive: with tax evasion, self-employed agents can protect themselves against increases in taxes by evading more.

Figure 7: The Impact of the Fine



(a) Income Gap

(b) Tax Revenues

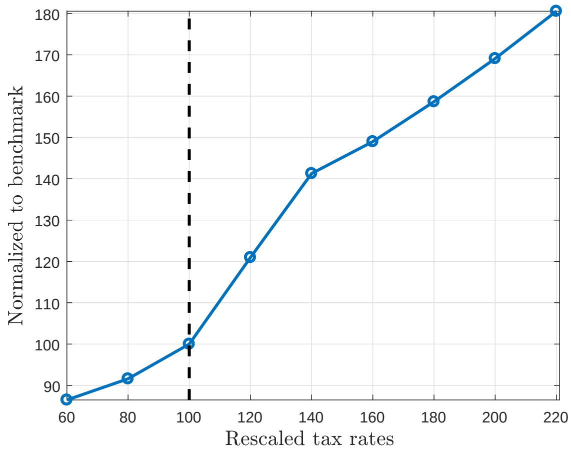
(c) Self-Employed: Share and Productivity

(d) Capital and Output (total)

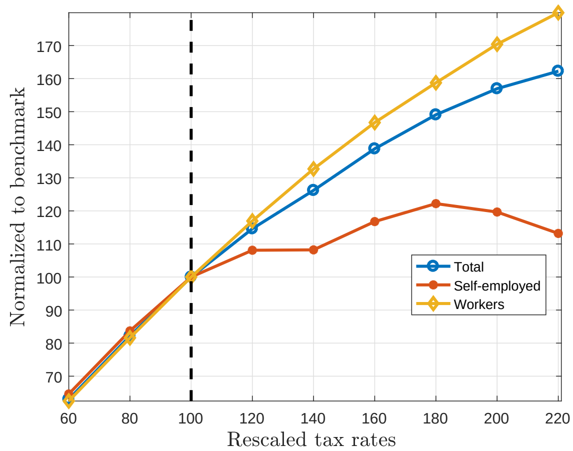
Notes: This figure varies the fine on tax evasion s within a range between 1.2 and 2.2 and report selected variables. The black dotted line indicates the benchmark economy with $s = 1.75$. The percentage income gap shown in panel (a) refers to the share of underreported income for the whole population. The other variables are normalized to the benchmark economy.

[19] To save space we omit the figures, which are available upon request.

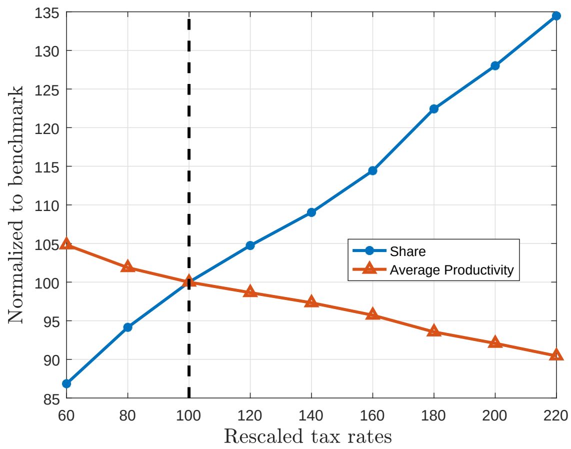
Figure 8: The Impact of the Tax Scheme



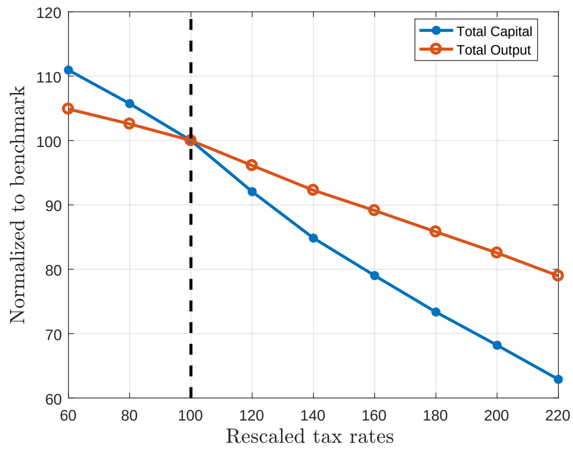
(a) Income Gap



(b) Tax Revenues



(c) Self-Employed: Share and Productivity



(d) Capital and Output (total)

Notes: This figure shifts the tax scheme and reports selected variables. The black dotted line indicates the benchmark economy with tax rates $a_0^W = 0.32$ and $a_0^E = 0.26$. a_0^W and a_0^E are rescaled proportionally. The percentage income gap shown in panel (a) refers to the share of underreported income for the whole population. The other variables are normalized to the benchmark economy.

7 Conclusions

The evasion of individual income taxes in the U.S. is substantial and concentrated among the self-employed businesses. To study the aggregate consequences of tax evasion we develop a dynamic general equilibrium model with incomplete markets and occupational choice in which self-employed business owners may hide a share of their business income but face the risk of being detected by the tax authorities. The model replicates important quantitative features of the U.S. economy in terms of the distribution of income and wealth, self-employment, and tax evasion.

We show that tax evasion in the self-employment sector has a non-trivial quantitative impact on aggregate outcomes and welfare. We quantify three important channels through which tax evasion affects the overall economy. The *subsidy channel* emphasizes that tax evasion acts like a subsidy and stimulates asset accumulation. The *selection channel* highlights that the opportunity to evade taxes induces less talented agents to run self-employed businesses and depresses the average productivity in the self-employment sector. The *detection channel* is important to replicate the empirical firm size distribution because self-employed business owners have incentives to keep their businesses small to stay under radar of the tax authorities.

Tax evasion generates positive welfare effects in the aggregate because it subsidizes self-employed business owners. However, welfare is affected heterogeneously along the dimensions of occupation and wealth. While tax evasion reduces the welfare of poor workers, it is particularly beneficial for poor self-employed agents. In our setup, implementing a perfect tax enforcement technology and using the additional revenues for a targeted tax reduction for poor self-employed agents leads to an increase in aggregate welfare and to a more productive economy.

Our analysis has important implications for tax enforcement and tax policy: macroeconomic models which abstract from tax evasion might deliver biased policy recommendations.

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A Appendix: Data

A.1 Data Description

In this section, we provide details regarding data, sample selection, and definitions used in this paper.

For our calibration, we estimate the moments from the Panel Study of Income Dynamics (PSID). We use a sample from 1990-2003 to estimate most of the relevant moments related to income. For wealth targets we link the main dataset to the *Wealth Supplement File* for the years 1994, 1999, 2001 and 2003.[20] The questions in the survey refer to the previous calendar year.

Sample selection. We create our sample including variables related to the characteristics of the households and occupation and merge it with Sample A of Heathcote et al. (2010), which contains information on household tax liabilities. This allows us to estimate tax functions for self-employed and workers separately. Heathcote et al. (2010) apply basic data cleaning by dropping records if: *(a)* there is no information on age for either the head or spouse, *(b)* either the head or spouse has positive labor income but zero annual hours, or *(c)* either the head or spouse has an hourly wage less than half of the corresponding federal minimum wage in that year. In addition, we select all households where the head of the household is male, has age in the 25-65 range and has worked at least 260 hours during the year.

Definition of self-employed. Traditionally, the entrepreneurial literature distinguishes between two definitions of entrepreneurs, see e.g. Quadrini (2000). According to the first definition, entrepreneurs are families that own a business or have a financial interest in some business enterprise. This definition is based on the PSID variable "Whether Business" which is based on the following interview question: "Did you (Head) or anyone else in the family own a business at any time during the previous year or have a financial interest in any business enterprise?". If the answer is positive this household is recorded as an entrepreneur and if negative this household is thought of as 'a worker'. According to the second definition, entrepreneurs are families in which the head is self-employed in his or her main job and the interview question is: "In your main job, are you (head) self-employed or do you work for someone else". Unlike the previous survey question, which allows only a binary answer (yes/no), this one specifies the occupation of the head and allows to identify a household directly as: a self-employed, an employee, both a self-employed and an employee, or an unemployed.

In our study, we opt for the second definition. First, this definition is more consistent with the data on tax evasion since underreported self-employed business income refers to

[20] Although wealth data are also available for 2005 and 2007, we do not extend the analysis to these years since other variables needed are missing.

Table 9: Summary Statistics for Alternative Definitions

Variable	Self-employed	Business Owners
Share of entrepreneurs (%)	14.70	20.11
Share of income, entrepreneurs (%)	21.04	27.98
Share of assets, entrepreneurs (%)	39.11	46.15
Ratio of median assets (E/W)	4.02	3.65
Exit rate, entrepreneurs (%)	15.73	24.43
Number of observations	22647	22704

Notes: Summary statistics are computed based on PSID for the years 1990-2003.

those who are self-employed (see Johns and Slemrod (2010)). Second, the answer to the first question can be positive if the household has " a financial interest in any business enterprise" and it would not reflect the occupation of the household, which we have in mind in the model. Moreover, the second survey question gives more information on the occupation of the head of the household and allows to clearly distinguish between self-employed and workers.

Based on the second survey question, we define an entrepreneurial household as a household where the head is self-employed, a 'worker' household where the head is an employee or 'both a self-employed and an employee'[21]. We drop those who answered they are 'unemployed' from the sample. As the result, there are 14.70 percent of self-employed households in our sample. Some important summary statistics for the alternative definitions of entrepreneurs are presented in Table 9.

A.2 Estimating Labor Income Process

In our income process estimation, we follow closely the procedure described by Heathcote et al. (2010). Since our model unit is a household, we focus on household labor income. We concentrate on the residual dispersion for logarithm labor household income residuals obtained from a standard Mincerian regression:

$$\text{lninc}_{i,t} = \alpha_0 + \beta_0 \text{educ}_{i,t} + \beta_1 \text{potexp}_{i,t} + \beta_2 \text{potexp}_{i,t}^2 + \varepsilon_{it}, \tag{24}$$

where i is a household index and t is time. The variable `lninc` is the logarithm of household labor income, `educ` refers to years of education and `potexp` represents years of potential experience.

Potential experience is calculated as the difference between the age and years of education less 6, i.e. potexp = age − educ − 6, where 6 is the typical age for entering the

[21] There are 0.7 percent of such households, hence either dropping those households or including them to either of the group does not change the main moments.

elementary school. Hence, someone who is 40 years old, with 16 years of education can potentially have 18 years of working experience.

We assume that the error term follows a first order Markov process of the form:

$$\log \varepsilon_{t+1} = \rho_\varepsilon \log \varepsilon_t + \eta_{\varepsilon,t+1}, \tag{25}$$

where $\eta_{\varepsilon,t+1} \sim N(0, \sigma_\varepsilon^2)$. We estimate this process for workers and obtain a persistence parameter $\rho_\varepsilon = 0.89$, whereas the dispersion parameter is $\sigma_\varepsilon = 0.21$.

A.3 Estimating Entry and Exit Rates

The exit rates are calculated as follows. First, we sort individuals by their identification number and consider two consecutive years. Then, we calculate how many individuals remained workers from one year to another and divide by the initial number of workers. This gives us the share of people who stayed workers. In the same way, we calculate the share of those who stayed self-employed. Exit rates are calculated as one minus the share of those who stayed a worker/self-employed. Finally, we calculate a weighted sum of year-by-year exit rates to get an average number we use for calibration. We get that on average, per year, around 15.73 percent of those who were self-employed exited self-employment. This number is comparable with the number reported by Quadrini (2000) 13.6 percent. Table 10 shows year-by-year exit rates for workers and self-employed.

Table 10: Exit Rates

Year	% Stayed Workers	Number of Workers	Exit Rate Workers (%)	% Stayed Self-employed	Number of Self-employed	Exit Rate Self-employed (%)
1990	96.82	1,572	3.18	88.85	278	11.15
1991	96.32	1,522	3.68	89.93	278	10.07
1992	96.07	1,424	3.93	79.57	235	20.43
1994	96.84	1,709	3.16	85.00	260	15.00
1995	97.96	1,715	2.04	82.14	252	17.86
1996	97.45	1,728	2.55	84.65	241	15.35
1997	96.33	1,609	3.67	83.11	219	16.89
1999	96.07	1,704	3.93	82.45	245	17.55
2001	95.93	1,844	4.07	80.80	224	19.20
Exit Rate						15.73

Notes: Summary statistics are computed based on PSID for the years 1990-2003. The year 1993 is excluded due to no information on the occupation.

A.4 Estimating Income and Wealth Inequality

In this section, we report additional statistics which we do not include in the main text. We base our estimates of the asset distribution on the PSID variable 'Wealth', which includes:

Table 11: Wealth Summary Statistics

	Gini	Mean/Median	Bottom 40	Top 20	Top 10	Top 1
All	71.1	3.10	2.71	75.64	60.56	26.53
Self-employed	68.4	2.61	4.35	72.17	57.23	21.13
Workers	67.0	2.61	3.17	71.86	55.21	21.14

a) 'market' value of farm/business net of debt, *(b)* money in checking and savings accounts, money market funds certificates or deposit, government savings bonds or treasury bills, *(c)* other real estate than your main home, *(d)* shares of stock in publicly held corporations, mutual funds, etc., *(e)* value of home equity. In the data, we recode negative asset positions with zeros to zeros to stay consistent with the model. Some important summary statistics on the asset distribution in the data are summarized in Table 11 .

A.5 Data on Auditing

Table 12 reports auditing rates by type and size of reported income, relative to the fiscal year 2011. On average only 1.11 percent of individual tax returns are audited but this percentage changes quite substantially across income levels. Generally the probability of being audited by the Internal Revenue Service is rising with income, increasing from less than 1 percent to almost 30 percent for tax returns above $10 million. Individuals who include business income in their returns are significantly more likely to be audited. Small corporations, with less than $10 million in total assets, are audited with only 1 percent probability, whereas larger corporations, with more $10 million in total assets, have an audit rate at 17.6 percent.

The picture varies quite a lot also when it comes to fine and punishment. The IRS imposes a relatively low penalty of 20 percent on underpayment that lacks economic substance, whereas it penalizes at a higher 75 percent underpayment due to fraud. The U.S. tax code follows on this respect most of existing legislatures around developed countries. According to OECD (2011), penalties for minor offenses are set around 10-30 percent of evaded tax liability, while more serious offenses are penalized at 40-100 percent of the tax evaded.

Table 12: Auditing rates by type and size of tax return (U.S. economy, fiscal year 2011)

Type of return	Percent covered
Individual Income Tax	1.11
No adjusted gross income	3.42
$[1, 25000]$	1.22
$[25000, 50000]$	0.73
$[50000, 75000]$	0.83
$[75000, 100000]$	0.82
$[100000, 200000]$	1.00
$[200000, 500000]$	2.66
$[500000, 1\text{m}]$	5.38
$[1\text{m}, 5\text{m}]$	11.80
$[5\text{m}, 10\text{m}]$	20.75
$> 10\text{m}$	29.93
Corporate income tax	1.5
Small firms (<$10m in assets)	1.0
Large firms (>$10m in assets)	17.6

Source: Slemrod and Gillitzer (2014) and U.S. Department of Treasure (2011).

B Appendix: Model

B.1 Solution Algorithm

We summarize the main steps to compute the stationary equilibrium formally defined in Section 3.5.

1. Make a guess for the interest rate r^0.

2. Compute the capital-labor ratio in the corporate sector $k_C = \frac{K_C}{N_C}$, which satisfies the following:

$$r^0 = \alpha A \left(k_C\right)^{\alpha-1} - \delta$$

3. Compute wage w as follows:

$$w^0 = \left(1 - \alpha\right) A \left(k_C\right)^{\alpha}$$

4. Given r^0 and w^0, solve the individual optimization problem described in Section (3.4) and get the relevant policy functions. Given the high non-linearity of the problem we use value function iteration with piecewise linear interpolation on a discrete grid.

5. Compute the invariant distribution μ using the policy functions and the exogenous Markov chains for the shocks ε, θ, as we described in Section (3.5), equation (18).

6. Using the distribution μ and the policy functions, compute the aggregate conditions and get new values for K_C, N_C. In particular, do the following:

$$K_C = \int_x a d\mu(x) - \int_{\{x:o(x)=SE\}} k(x) d\mu(x)$$

$$N_C = \int_{\{x:o(x)=W\}} \varepsilon d\mu(x)$$

7. Excess demand function can be defined as:

$$ED(r^0) = k_C - \frac{K_C}{N_C}$$

Notice that finding a root of $ED(r)$ is equivalent to finding a fixed point of the capital-labor ratio in the corporate sector.

8. If $ED(r^0) = 0$, stop and exit the loop over r. Otherwise, if $ED(r^0) > 0$ set a new guess $r^1 > r^0$. If instead $ED(r^0) < 0$ set $r^1 < r^0$. Go back to step 2. In practice we noticed that it is faster to update the interest rate using a root-finding method based upon a combination of bisection, secant, and inverse quadratic interpolation

9. Stop if either $ED(r)$ or $|r^0 - r^1|$are sufficiently close to zero.

In some of the counterfactual experiments we simulate the effects of a revenue neutral tax reform (e.g., elimination of tax evasion with redistribution). We do the following: we record the absolute value of government spending G arising in the baseline calibration and then we rescale the parameters a_2^W, a_2^E in the non-linear tax function so that actual revenues are equal to G. For updating the tax parameters we follow a standard bisection algorithm.

B.2 Computation of Stationary Distribution: Details

Keep in mind that if we compute the policy function for assets using interpolation, an additional complication arises. Almost surely, the optimal choice does not happen to be one of the grid points (even if we use a finer grid for the distribution). Suppose $a^* \equiv g(a, \epsilon, \theta)$ falls between grid a_J and a_{J+1}, where $J = 1, 2, .., N_a$. If this is the case, we can proceed in two different ways. The simpler one is to force the agents to choose the closest grid point. Another way is that we let the agents draw a lottery and the proportion

$$\frac{a_{J+1} - a^*}{a_{J+1} - a_J}$$

are forced to choose a_J and the rest are forced to choose a_{J+1}. This method is described in greater detail in Heer and Maussner (2008) (see algorithm 7.2.3 pp. 351-352).

B.3 Entry and Exit Rates in the Model: Details

Calculating the exit rate of entrepreneurs requires to track down each entrepreneur who changes occupation status from one period to the next. Hence, we need to define the following transition operator:

$$\begin{aligned} T\left(a,\varepsilon,\theta,a',\varepsilon',\theta'\right) &= \left(1-o\left(a,\varepsilon,\theta\right)\right)\mathbf{1}^{W}\left(a',a,\varepsilon,\theta\right)\Pr\left(\epsilon',\theta'|\epsilon,\theta\right) \\ &+o\left(a,\varepsilon,\theta\right)p_{k}\left(a,\varepsilon,\theta\right)\mathbf{1}_{d}^{SE}\left(a',a,\varepsilon,\theta\right)\Pr\left(\epsilon',\theta'|\epsilon,\theta\right) \\ &+o\left(a,\varepsilon,\theta\right)\left(1-p_{k}\left(a,\varepsilon,\theta\right)\right)\mathbf{1}_{nd}^{SE}\left(a',a,\varepsilon,\theta\right)\Pr\left(\epsilon',\theta'|\epsilon,\theta\right) \end{aligned}$$

which expresses the probability of moving from state (a, ε, θ) today to state a', ε', θ tomorrow. The number of exiting entrepreneurs is then given by:

$$\mathbf{e}_{SE} = \int_A \int_E \int_\Theta \int_A \int_E \int_\Theta o\left(a,\varepsilon,\theta\right)\left(1-o\left(a',\varepsilon',\theta'\right)\right)T\left(a,\varepsilon,\theta,a',\varepsilon',\theta'\right)d\mu\left(a,\varepsilon,\theta\right)$$

where $o \in \{0, 1\}$ is the policy function for the occupational choice ($0 =$ worker, $1 =$ entrepreneur). If we define the state vector as $x = (a, \varepsilon, \theta)$, then the above equation can be written more compactly as

$$\mathbf{e}_{SE} = \int_x \int_{x'} o(x)(1-o(x'))T(x,x')d\mu(x)$$

Recall that the number of entrepreneurs out of all households in the economy is given by

$$\mathbf{e}_{SE} = \int_A \int_E \int_\Theta o(a,\varepsilon,\theta)d\mu(x).$$

Therefore the *exit rate from entrepreneurship*, i.e., the share of entrepreneurs who become workers, can be computed as:

$$\mathbf{E}_{SE} = \frac{\mathbf{e}_{SE}}{\mathbf{s}_{SE}}.$$

Likewise, the number of workers who become entrepreneurs, is:

$$\mathbf{e}_{W} = \int_x \int_{x'} o\left(x'\right)\left(1-o\left(x\right)\right)T\left(x,x'\right)d\mu\left(x\right)$$

and we can compute the *exit rate from working* as

$$\mathbf{E}_{W} = \frac{\mathbf{e}_{W}}{\mathbf{s}_{W}}$$

where $\mathbf{s}_W$ is the number of workers out of the population. Please notice that in a stationary distribution $\mathbf{e}_{SE} = \mathbf{e}_W$ but of course $\mathbf{E}_{SE}$ and $\mathbf{E}_W$ will in general differ. We can summarize the transitions between the different occupational status with the help of the following 2×2 transition matrix:

$t\backslash t+1$	W	SE
W	$1-\mathbf{E}_W$	$\mathbf{E}_W$
SE	$\mathbf{E}_{SE}$	$1-\mathbf{E}_{SE}$

B.4 Tax and Income Gap in the Model: Details

Let $x = (a, \varepsilon, \theta)$ be the state vector. Furthermore, we define the following objects:

$$\texttt{unpaid}(x) \equiv T^E(\pi(x) + ra) - T^E((1 - \phi(x))\pi(x) + ra),$$

$$\texttt{true\_tax}(x) \equiv T^E(\pi(x) + ra),$$

where $\phi \in [0, 1]$ is the fraction of hidden income and π is business income.[22]

Given these definitions, we compute the aggregate tax gap in the model economy as follows:

$$TG = \frac{\int_x \texttt{unpaid}(x)\mu^E(dx)}{\int_x \texttt{true\_tax}(x)\mu^E(dx)}. \tag{26}$$

The data counterpart of this is 57% (see Johns and Slemrod (2010), Table 4).

We compute the tax gap for each true income decile (based on overall income) as follows. We define the tax gap in income decile $i = 1, .., 10$ as the ratio between total unpaid taxes and total due taxes for all individuals whose income belongs to decile i :

$$TG_i = \frac{\int_{\{x:y(x)\in I_i\}} \texttt{unpaid}(x)\mu^E(dx)}{\int_{\{x:y(x)\in I_i\}} \texttt{true\_tax}(x)\mu^E(dx)}. \tag{27}$$

It can be verified that the if we take the weighted sum of (27) we get back the aggregate tax gap in (26):

$$\sum_{i=1}^{10} TG_i \cdot \left(\frac{\int_{\{x:y(x)\in I_i\}} \texttt{true\_tax}(x)\mu^E(dx)}{\int_x \texttt{true\_tax}(x)\mu^E(x)} \right) = TG.$$

The above equation clarifies that the aggregate tax gap is *not* the simple average of the tax gap for each deciles: deciles that account for a higher tax liability receive more weight in the aggregation.

Johns and Slemrod (2010) report also another measure of tax non-compliance: they define the net misreporting percentage (NMP) as the sum of underreported income divided by the absolute value of the corresponding true income. The results are reported in Johns and Slemrod (2010), Table 2, pag.404. The first column shows the NMP by income, whereas the second columns shows the NMP by taxes. The NMPs in each column are computed for each decile of true income. Of course, income misreporting percentages are equal to tax misreporting percentages under a proportional tax scheme,[23] but with

[22] Taxable income for entrepreneurs is defined as $y_E = \pi + ra$.

[23] If true income is y, undeclared income is $e \in [0, y]$, then unpaid taxes are $T(y) - T(y - e)$ and the tax gap is

$$\frac{T(y) - T(y - e)}{T(y)}$$

Clearly, if the tax schedule T is linear the tax gap above is equal to $\frac{e}{y}$ which is the *income gap*, i.e. unreported income divided by true income.

income tax progressivity they may differ substantially, and they do in Slemrod's data. In particular, whereas NMP by income rises with income, NMP by taxes *declines* with income, albeit non-monotonically.

To compute the NMP by income we follow a similar procedure as in (26). First, we compute the aggregate NMP as follows:

$$\begin{aligned} IG &= \frac{\text{Total undeclared income}}{\text{Total true income}} = \frac{\int_x \left[\pi(x) + ra - (1-\phi(x))\,\pi(x) - ra\right] \mu^E(dx)}{\int_x \left[\pi(x) + ra\right] \mu^E(dx)} = \\ &= \frac{\int_x \phi(x)\pi(x)\,\mu^E(dx)}{\int_x \left[\pi(x) + ra\right] \mu^E(dx)}. \end{aligned}$$

Please observe that this delivers a number between 0 and something less than 1 due to asset income. Second, we compute the disaggregated NMP by income deciles and/or quintiles, following the above approach.

B.5 Welfare Analysis: Details

Let us define the state space as $x = (a, \epsilon, \theta)$. The distribution in the benchmark economy is given by $\mu^B(a, \epsilon, \theta)$, while in the counterfactual economy is $\mu^C(a, \epsilon, \theta)$. The corresponding decision rules for the occupational choice are $o^B(a, \varepsilon, \theta)$ and $o^C(a, \varepsilon, \theta)$. Then, the welfare criterion in the benchmark economy is given by

$$EV^B = \int [o^B(a,\varepsilon,\theta)\, V^{B,E}(a,\varepsilon,\theta) + (1 - o^B(a,\varepsilon,\theta)) V^{B,W}(a,\varepsilon,\theta)] d\mu^B(a,\epsilon,\theta),$$

whereas welfare criterion in a given counterfactual economy C is:

$$EV^C = \int [o^C(a,\varepsilon,\theta)\, V^{C,E}(a,\varepsilon,\theta) + (1 - o^C(a,\varepsilon,\theta)) V^{C,W}(a,\varepsilon,\theta)] d\mu^C(a,\epsilon,\theta).$$

Then, to compare welfare changes between benchmark and counterfactual, we compute the consumption equivalent variation (CEV) needed to make a household indifferent between the two economies. Specifically, we calculate:

$$\text{CEV} = (EV^C / EV^B)^{1/(1-\sigma)} - 1,$$

where EV^C and EV^B are defined above. We can further compute the same number based on the decile of household net wealth.[24] We can derive a function which places each household in a particular decile of household net wealth in the benchmark economy. This function, say $D(o^B(a,\varepsilon,\theta), a, \varepsilon, \theta, \mu^B)$, depends on the asset position, the abilities ε and

[24] Note that it is difficult to consider gross income because of the presence of tax evasion and its punishment.

θ, the occupational choice, and the distribution over the state variables. This function takes discrete values from 1 to 10. Then, the household type is defined as (i, j), where

$$i = D(o^B(a, \varepsilon, \theta), a, \varepsilon, \theta, \mu^B)$$

and

$$j = o^B(a, \varepsilon, \theta).$$

We can summarize the types in a series of indicator functions,

$$\mathbf{1}_{i,j}(a, \varepsilon, \theta) = \begin{cases} 1 & \text{if } D(o^B(a, \varepsilon, \theta), a, \varepsilon, \theta, \mu^B) = i \text{ and } o^B(a, \varepsilon, \theta) = j \\ 0 & \text{otherwise.} \end{cases}.$$

Then, we compute 20 numbers,

$$EV_{i,j}^B = \frac{\int \mathbf{1}_{i,j}(a, \varepsilon, \theta)[o^B(a, \varepsilon, \theta)V^{B,E}(a, \varepsilon, \theta) + (1 - o^B(a, \varepsilon, \theta))V^{B,W}(a, \varepsilon, \theta)]d\mu^B(a, \epsilon, \theta)}{\int \mathbf{1}_{i,j}(a, \varepsilon, \theta)d\mu^B(a, \epsilon, \theta)}.$$

Then, we can go to the counterfactual economy. We keep the definition of the household type and compute the new welfare numbers using the new distribution, μ^C,

$$EV_{i,j}^C = \frac{\int \mathbf{1}_{i,j}(a, \varepsilon, \theta)[o^C(a, \varepsilon, \theta)V^{C,E}(a, \varepsilon, \theta) + (1 - o^C(a, \varepsilon, \theta))V^{C,W}(a, \varepsilon, \theta)]d\mu^C(a, \epsilon, \theta)}{\int \mathbf{1}_{i,j}(a, \varepsilon, \theta)d\mu^C(a, \epsilon, \theta)}.$$

Finally, in Figure 6 in the main text we plot

$$\text{CEV}_{i,j} = (EV_{i,j}^C / EV_{i,j}^B)^{1/(1-\sigma)} - 1,$$

where $j = 0, 1$ refers to the occupation (worker or self-employed) and $i = 1, \ldots, 10$ to decile of wealth.

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