

# Business cycle accounting for the German fiscal stimulus program

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## Abstract

We apply the business cycle accounting method proposed by Chari, Kehoe, and McGrattan (2007, *Econometrica*) for the German stimulus programs and monetary policy measures in 2009. Since the fiscal program includes a cash for clunkers program, we extend the model by durable goods. Government consumption and net exports are separate wedges. Wedges correspond to the following variables: *government consumption, durables, investment, labor, net exports and efficiency*. We map the fiscal and monetary measures to the prototype economy and explore the consequences. We estimate the wedges as well as adjustment costs to the stock of durables and market capital with maximum-likelihood methods. Our findings are: the labor market wedge induce a fast recovery, the durable wedge and government spending wedge are counter-cyclical. The cash for clunkers program is more efficient than pure government consumption.

# 1 INTRODUCTION

Fiscal stimuli is, besides monetary policy, the most discussed anti-cyclical measurement in recessions. Quantifying this measurement is a strong exercise, as e.g. [Parker \(2011\)](#) describes. The paper quantifies the German Fiscal Stimulus Program in 2008 and 2009 with the Business Cycle Accounting (BCA) approach of [Chari et al. \(2007\)](#). [Brinca et al. \(2016\)](#) revisit this approach in detail. BCA bases on an estimated benchmark RBC-Model with wedges in nearly every market. The wedges are modeled as taxes. Nevertheless, the wedges reflect all market distortions, such as taxes, nominal and real frictions, or current changes in preferences and expectations. Maximum-likelihood estimation determines the stochastic process behind the wedges. The resulting residuals explain the wedges' states and the wedges' states explain the wedges' influence on the business cycle.

We modify the BCA procedure in three ways. First, we distinguish between growth and business cycle accounting. The wedges include a long and a short-run component. Subaggregates of demand grow with different rates since German reunification. The estimation of the wedges' stochastic process would be non-stationary without growth accounting. Second, we separate government spending and net export. The separation enables a government spending analysis. Third, the model contains a durable consumption good market. The durable market was heavily distorted during the great recession and we attribute these distortions to a cash for clunkers program. In BCA jargon: the distortions are the equivalent result to a durable good subsidy. Changes in Gross Domestic Product (GDP) induced by both the government and the durables wedge are accordingly due to fiscal measurements. This method enables to discover the substitution effect between durables and non-durables as well as the intertemporal substitution of non-durables. The same holds for crowding-out due to higher government consumption. There were also large measures in other markets. Besides these fiscal distortions, other market distortions are probably large as well, e.g. due to financial frictions, but such distortions during a crisis are likely to be negative. Hence, pro-cyclical wedges give no evidence for ineffective measures in the concerning market, but counter-cyclical wedges give evidence for effective measures.

The paper contributes additionally to maximum-likelihood methods and BCA. Likelihood function optimization and identification is often difficult. [Brinca et al. \(2017\)](#) provide strategies for identification and BCA. Unsolved problems concerning likelihood optimization and BCA are reported e.g. by [Gerth and Otsu \(2016\)](#).<sup>1</sup> As described briefly by [Chari et al. \(2007\)](#) and [Brinca et al. \(2016\)](#) and more noticeable in their online code, they exert the steady-state Kalman-filter. We apply a method proposed by [Huber \(2018\)](#), which enables an analytical and unique estimation of the covariance matrix. The result is equivalent to the steady-state Kalman-filter. We follow further [Huber \(2018\)](#) and use this result only as a guess for the Kalman-filter initialized with the unconditional variance. This is the most common initialization for the Kalman-filter in macroeconometrics (see e.g. [DeJong and Dave \(2011\)](#)). This procedure is solid to identify a global maximum and maximums for a broad set of parameters.

We find a positive robust effect for the cash for clunkers program and government con-

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<sup>1</sup>[Gerth and Otsu \(2016\)](#) do not account for growth, which potentially explains the problem.

sumption. Furthermore, the labor wedge induce a fast recovery. Our study suggests that subsidies on durable goods are more effective than pure government consumption shocks and measures in the labor market are potentially effective but delayed in time. BCA in general is only a useful first step in the identification and consequences of distortions. Thus, we aim to motivate further research on durable and labor market subsidies as an instrument for fiscal policy. Recent work investigates the labor market measures with focus on unemployment, e.g. [Balleer et al. \(2016\)](#). [Berger and Vavra \(2015\)](#) investigate durable consumption in recessions and [Baker et al. \(2017\)](#) dynamics with changing price expectations.

The remainder of the paper reads as follows. The next section sketches the German fiscal stimulus programs and the monetary policy of the European Central Bank (ECB). Furthermore, we provide long time series with focus on the crises from 2008 till 2011 for the reunified German economy. Thereafter, we describe the model. We map the single measures of the program to the wedges. In a next step, we present our calibration exercises and the new estimation strategy. We present our results with a robustness section. Afterwards the paper concludes. Our Appendix presents the entire model as well as the source of our data and the corresponding manipulation.

## 2 THE GERMAN CASE

### 2.1 The fiscal stimulus program I and II in detail

The German government drew two fiscal stimulus programs. The first one became effective at the end of 2008 ([Bundesgesetzblatt, 2008](#)) and the second one in the beginning of 2009 ([Bundesgesetzblatt, 2009](#)).

As [Rosenberger \(2013\)](#) describes, the first fiscal stimulus was amounted 32 Billions € plus a loan program of 15 Billions €. The fiscal stimulus consisted of: a year tax exemption for new cars, higher tax deductions by permitting the reducing-balance method and increasing child allowance, lower employment insurance tax, and higher transfers for students and retirees.

The second one amounted to 50 Billion € plus both a loan and guarantee program of 100 Billion € and an increase of the German export credit guarantee program (Hermes cover) of round about 2 Billion €. The fiscal stimulus consisted of investments in public infrastructure and financial support for local and state authority spending, a subsidy of new cars at the amount of 2500 € per car and in total 5 Billion €, subsidies for private innovations as well as lower income taxes and social contributions. Short-time work possibilities and benefits were broadened, further training was supported and the Federal Employment Agency increased the number of job agents.

As calculated by the [OECD \(2009\)](#) and presented in [Table 1](#), the size of the fiscal package was on equal terms by reducing tax and increasing transfers and spending. The fiscal packages amounted to 3.2% of GDP, excluding all measures which do not affect the national budget directly, e.g. the loan and guarantee programs.

**Table 1:** Composition of the fiscal programs in % of GDP (non domestic use)

Tax	Individuals -1.6	Social Contribution -0.7	Business -0.6	Total -1.6
Spending	Transfers to households 0.3	Transfers to business 0.3	Government spending 0.8	Total 1.6

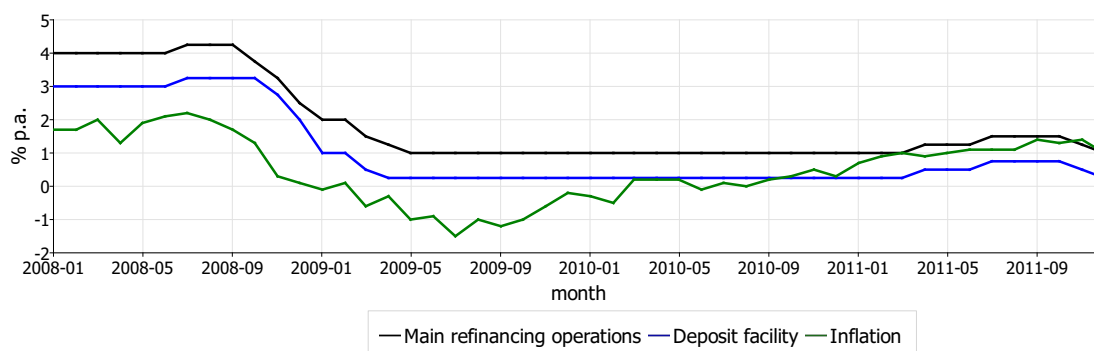
Source: OECD (2009).

In Section 3.1.2 we show how to map individual measures of the stimulus program in the extended prototype economy of Chari et al. (2007).

## 2.2 Monetary policy in the great recession

Monetary policy of the ECB reacts also to the recession. Figure 1 shows the minimum bid rate on main refinancing operations and the interest rate on deposit facilities declined in the aftermath of the declining inflation rate. The first one declined from 4.25% in mid 2008 to 1% by mid 2009. Both interest rates were persistent from there on.

**Figure 1:** Monetary policy and usage of the deposit facility



The ECB applies further tools of monetary policy besides the conventional interest rate policy. Here we give a short overview from the detailed reports of the European Central Bank (2010) and European Central Bank (2011). In October 2008 the ECB switched from a variable-rate to a fixed-rate tender, slacked collateral requirements and enhanced the provision of liquidity. The ECB's Governing Council prolonged several times these measures. The Governing Council decided to purchase bonds issued in the Euro area in May 2009. The Security Markets Program started in June 2009. This program conducts interventions on public and private debt securities markets in the Euro area. The Governing Council decided to switch back and forth between a variable- and a fixed-rate tender in March and May 2010. The Governing Council also decided to intervene on the Euro area public and private debt securities markets again. The Council determined long-term refinance operations to provide liquidity in August and October 2010.

In Section 3.1.2 we show how to map the monetary policy in the extended prototype economy of Chari et al. (2007)

### 2.3 Stylized facts for the German economy

Table 2 presents the average long run share of subaggregates. Private consumption expenditure (PCE) account for 56%, whereby durables accounts for 6% and non-durables for the half of GDP. The share of investment is 21% and of government consumption nearby 19%. Net exports are nearby 4%.

**Table 2:** Long run ratios in % of GDP

Description	$\overline{x_t}/\text{GDP}_t$	$\sigma_x/\text{GDP}$
PCE	56.05	0.0152
non-durables	49.72	0.0129
durables	06.33	0.0050
investment	21.32	0.0188
government consump.	18.87	0.0053
net exports	03.76	0.0287

Figure 2 presents the cyclical behavior of GDP and Figure 3 of subaggregates and hours worked. The time series are the relative deviations from the concerning linear trend. We choose a linear trend filter instead of the commonly used HP-filter to be consistent to our estimation strategy. Flor (2014) presents an overview of HP-filtered second moments of similar data.

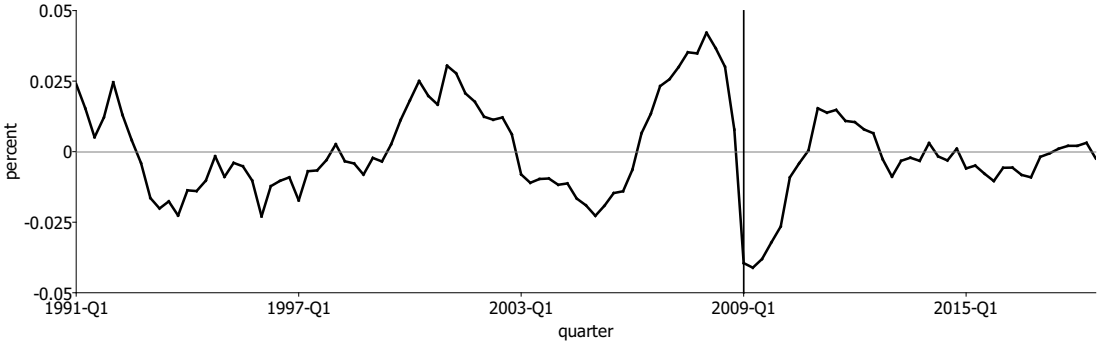
We observe a boom-bust-cycle in GDP around the time of the dot-com bubble. This cycle is followed by a recovery from 2005 till 2008, which ends in a heavy drop. This drop depicts the great recession. The GDP recovers fast and from there on moves along the long run trend.

Investment co-moves with GDP, but with larger fluctuations. Durables show two heavy short boom-bust-cycles. The first one peaks at the end of 2006. The reason was an announcement of a value added tax increase. This is followed by a bust at the beginning of 2007, where the increase took place. We observe the second peak at the time of the German cash for clunkers program, followed by an burst at its end. Government consumption is above the trend in the middle and late 1990's. It decrease at the beginning of the 2000's and increase from 2008 till 2010. Since 2010 it fluctuates around the trend. Non-durable consumption is below its trend after the reunification, is above the trend in the 2000's until the great recession and decrease slightly afterwards. Net exports relative to GDP decreases from 1997 till 2001 and increases sharply afterwards till 2003. From there on untill the crises it moves above the trend. Since the crises it fluctuates around the long run trend. Hours worked decline after the German reunification. Hours worked co-moves with GDP from 2000 onwards.

The vertical line in Figure 2 and 3 focus on the great recession. GDP, hours worked and investment begin to decrease from the end of 2008 until the end of 2009 to 5%, 4% and 12%, respectively. The recovery completes in 2011. Durables increase during the time of the cash for clunkers program by 8% and decrease by 18%-points afterwards. Durables

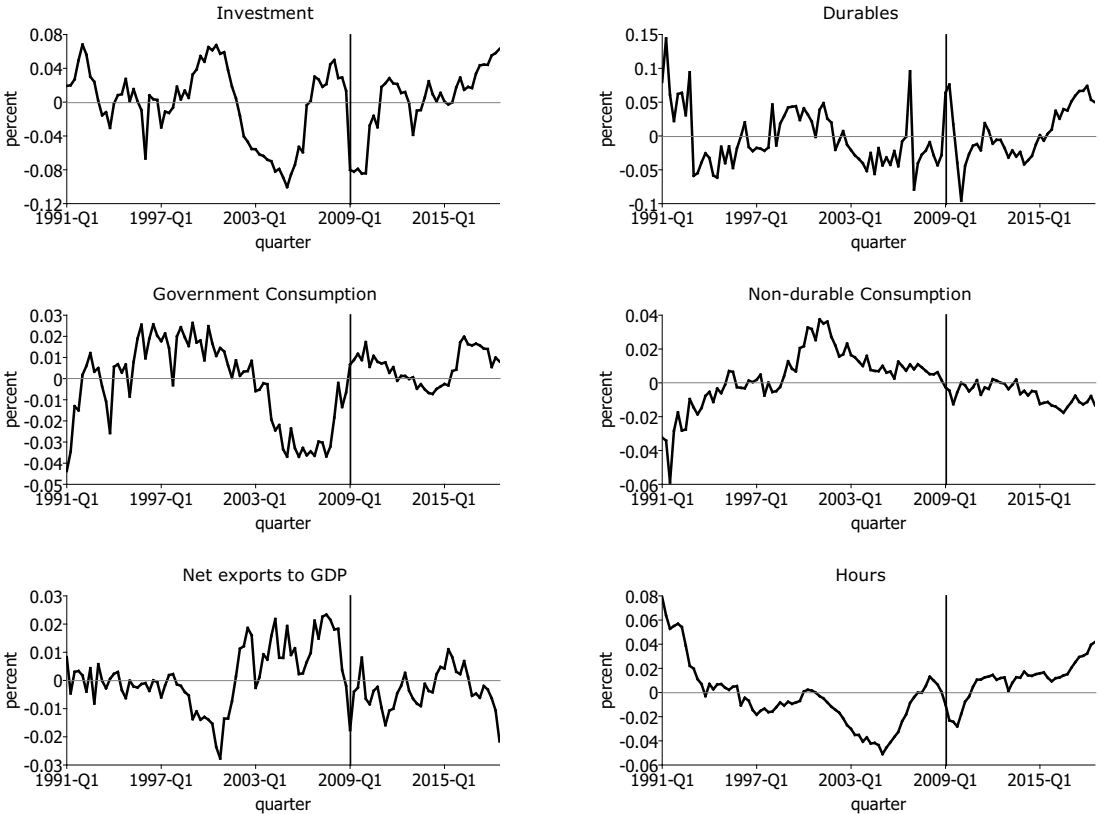
recovers at the end of 2010. Government consumption increase at the beginning of 2009 by 5% and remains till the end of 2011 by 4% above balanced growth. Non-durables are 2% below the trend growth at the end of 2009 and still remains about 1-2% below until the end of 2011.

**Figure 2: Cyclical behavior of GDP**



The data are presented as relative deviations from linear trend. The line indicates 2009q1

**Figure 3: Cyclical behavior of different economic measures**



Despite hours worked, the data are presented as relative deviations from the corresponding linear trend. Hours worked is the relative deviation from the average. The line indicates 2009q1

### 3 METHODS

#### 3.1 The prototype economy

We employ a prototype economy which consists of an infinitely-lived household, a firm facing perfect competition, and a government instance which finances its exogenous expenditure by levying taxes on labor, durables and investment. The model differs from [Chari et al. \(2007\)](#) in the distinction of durable consumption investment and productive capital investment as well as the distinction of net exports and government consumption. The model accounts for capital adjustment costs. [Chang \(2000\)](#) shows that adjustment costs face problems with excess volatility and negative co-movements with two or more different capital stocks. The model is written in per capita terms. Furthermore, wedges are composed of a growth and a business cycle component.

##### 3.1.1 Model

The per period utility of the representative household is parameterized as follows

$$u(C_t, D_t, N_t) = \begin{cases} \phi \ln(C_t) + (1 - \phi) \ln(K_{Dt}) + \psi \ln(1 - N_t) & \text{for } \eta = 1, \\ \frac{(C_t^\phi \cdot K_{Dt}^{1-\phi} \cdot (1-N_t)^\psi)^{1-\eta} - 1}{1-\eta} & \text{for } \eta \neq 1, \end{cases} \quad (1)$$

where  $C_t$  denotes consumption of non-durable goods and  $N_t$  is the household's labor supply. The stock of durable consumption goods  $K_{Dt}$  accumulates according to

$$\gamma_n K_{Dt+1} = (1 - \delta_D) K_{Dt} + D_t - \Theta_{Dt} \left( \frac{D_t}{K_{Dt}} \right) \cdot K_{Dt}. \quad (2)$$

$\gamma_n$  denotes the population growth factor and  $D_t$  are investments in durable consumption goods. We assume  $\Theta_{Dt}$  follows

$$\Theta_{Dt} \left( \frac{D_t}{K_{Dt}} \right) = \frac{a_D}{2} \left( \frac{D_t}{K_{Dt}} - b_D \right)^2 \quad (3)$$

without costs on the balanced growth path. The household maximizes its expected life-time-utility

$$U_t = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \gamma_n)^s u(C_{t+s}, K_{Dt+s}, N_{t+s}) \quad (4)$$

subject to the budget constraint

$$C_t + (1 + \tau_{It}) P_{It} I_t + (1 + \tau_{Dt}) P_{Dt} D_t \leq R_t K_{It} + (1 - \tau_{Nt}) W_t N_t + T_t - P_{Et} E_t, \quad (5)$$

where  $K_{It}$  denotes the productive capital stock (capital stock hereafter),  $I_t$  investment in capital,  $T_t$  lump-sum transfers,  $E_t$  net exports,  $R_t$  the rental rate on capital, and  $W_t$  the

real wage. The tax rates or more general the wedges' driver  $\tau_{N_t}$ ,  $\tau_{I_t}$  and  $\tau_{D_t}$ , belong to the labor, investment and durable market, respectively.  $P_{E_t}$ ,  $P_{I_t}$  and  $P_{D_t}$  are the prices for net exports, investment and durable goods. The consumption good is the numeraire. The capital stock follows the law-of-motion

$$\gamma_n K_{I_{t+1}} = (1 - \delta_I)K_{I_t} + I_t - \Theta_{I_t} \left( \frac{I_t}{K_{I_t}} \right) \cdot K_{I_t}. \quad (6)$$

We assume  $\Theta_{I_t}$  follows

$$\Theta_{I_t} \left( \frac{I_t}{K_{I_t}} \right) = \frac{a_I}{2} \left( \frac{I_t}{K_{I_t}} - b_I \right)^2 \quad (7)$$

again without costs on the balanced growth path.

The representative firm produces its output good  $Y_t$  with the Cobb-Douglas technology

$$Y_t = K_{I_t}^\alpha (\gamma_z^t Z_t N_t)^{1-\alpha} \quad (8)$$

and faces perfect competition. The parameter  $\gamma_z$  denotes growth factor of labor augmenting technical progress and  $Z_t$  is the efficiency wedge.

The expenditures  $G_t$  are exogenous and the government chooses  $T_t$  that its budget constraint

$$P_{G_t} G_t + T_t \leq \tau_{N_t} W_t N_t + \tau_{I_t} P_{I_t} I_t + \tau_{D_t} P_{D_t} D_t \quad (9)$$

always binds. Thereby, the resource constraint of the economy is

$$Y_t = C_t + P_{I_t} I_t + P_{D_t} D_t + P_{G_t} G_t + P_{E_t} E_t. \quad (10)$$

**Growth component:** As already mentioned the population grows with  $\gamma_n$  and technical progress with  $\gamma_z$ . Furthermore, the wedges evolve differently. The relative prices reflect this. In the long-run  $P_{x_t} \in \{P_{I_t}, P_{D_t}, P_{G_t}, P_{E_t}\}$  evolves with  $P_{x_t} = g_{P_x} P_{x_{t-1}}$ . The ensuing trend growth rates of different variables are described in Table 3.



**Table 3:** Growth rates

$X_t$	$g_x$
$N_t$	1
$Y_t, C_t, W_t, T_t$	$g_y$
$I_t, K_{It}$	$g_I$
$D_t, K_{Dt}$	$g_D$
$G_t$	$g_G$
$E_t$	$g_E$
$R_t$	$g_y/g_I$
$P_{Xt}$	$g_y/g_x$
$\gamma_z$	$g_y^{\frac{1}{1-\alpha}} g_K^{\frac{\alpha}{\alpha-1}}$

The growing variables

$$X_t \in \{Y_t, C_t, I_t, D_t, G_t, E_t, K_{It}, K_{Dt}, T_t, W_t, P_{Xt}\}.$$

are scaled by  $x_t = \frac{X_t}{g_x^t}$  and thus are stationary variables.

**Business cycle component:** The fluctuation in the model is driven by the VAR(1)-process

$$\underbrace{\begin{bmatrix} \ln(s_{At+1}) \\ s_{Nt+1} \\ s_{It+1} \\ s_{Dt+1} \\ s_{Et+1} \\ \ln(s_{Gt+1}) \end{bmatrix}}_{:=s_{t+1}} = \Pi \underbrace{\begin{bmatrix} \ln(s_{At}) \\ s_{Nt} \\ s_{It} \\ s_{Dt} \\ s_{Et} \\ \ln(s_{Gt}) \end{bmatrix}}_{:=s_t} + \underbrace{\begin{bmatrix} \epsilon_{At+1} \\ \epsilon_{Nt+1} \\ \epsilon_{It+1} \\ \epsilon_{Dt+1} \\ \epsilon_{Et+1} \\ \epsilon_{Gt+1} \end{bmatrix}}_{:=\epsilon_{t+1}}, \quad \epsilon_t \sim \mathcal{N}(0, \Sigma). \quad (11)$$

The stochastic process affects the wedges as follows

$$Z_t = A^* \cdot s_{At}, \quad (12)$$

$$\tau_{Nt} = \tau_N^* + s_{Nt}, \quad (13)$$

$$\tau_{It} = \tau_I^* + s_{It}, \quad (14)$$

$$\tau_{Dt} = \tau_D^* + s_{Dt}, \quad (15)$$

$$e_t = e^* + s_{Et}, \quad (16)$$

$$g_t = g^* \cdot s_{Gt}, \quad (17)$$

where  $A^*$ ,  $\tau_N^*$ ,  $\tau_I^*$ ,  $\tau_D^*$ ,  $e^*$  and  $g^*$  are the corresponding steady-state component of the different distortions. We follow [Chari et al. \(2007\)](#) and define six wedges as follows: The efficiency wedge  $Z_t$ , the labor wedge  $1 - \tau_{Nt}$ , the investment wedge  $\frac{1}{1+\tau_{It}}$ , the durables wedge

$\frac{1}{1+\tau_{Dt}}$ , the net export wedge  $e_t$  and the government spending wedge  $g_t$ . Since the cyclical component includes the steady state component, detrended prices  $p_{Et}$ ,  $p_{Gt}$ ,  $p_{It}$ ,  $p_{Dt}$  are normed to one.

To derive the models decision rules, we use a linear perturbation method. In detail we apply the method of undetermined coefficients as described by Uhlig (1999) and Christiano (2002) to solve the log-linearized model. The solved model then can be written as

$$\mathbf{y}_t = \mathbf{L}_x^y \cdot \mathbf{x}_t + \mathbf{L}_s^y \cdot \mathbf{s}_t, \quad (18)$$

$$\mathbf{x}_{t+1} = \mathbf{L}_x^x \cdot \mathbf{x}_t + \mathbf{L}_s^x \cdot \mathbf{s}_t, \quad (19)$$

where the matrices  $\mathbf{L}_x^x$  and  $\mathbf{L}_x^y$  characterize the deterministic part of the model's solution, while  $\mathbf{L}_s^x$  and  $\mathbf{L}_s^y$  describe the stochastic part.  $\mathbf{y}_t = [\hat{y}_t \ \hat{N}_t \ \hat{i}_t \ \hat{i}_{dt} \ \hat{g}_t \ \hat{e}_t/\hat{y}_t]^T$  is the vector of observed variables and  $\mathbf{x}_t = [\hat{k}_t \ \hat{d}_t]^T$  is the vector of endogenous states.<sup>2</sup>

### 3.1.2 Mapping the stimulus programs and further distortions to wedges

Chari et al. (2007), Brinca et al. (2016) and various other authors map well specified models into the presented prototype economy. First, we show how to map the stimulus programs to the prototype economy. Since the wedges' drivers are modeled as taxes, this is simple for the most measures. Secondly, we reflect monetary policy.

#### Mapping the stimulus programs:

*Government Wedge:* We assign total government spending to the government spending wedge. These are mainly investments in infrastructure and financial support for local and state authority spending. Hence, the stimulus programs increased the government wedge directly.

*Durable Wedge:* The two measures concerning new cars affect the durable wedge. For a given producer price, both measures reduce the absolute tax or the relative price of durables from the households perspective. Hence, they increase the durable wedge.

*Investment Wedge:* The first part of the stimulus programs which affects the investment wedge are subsidies for investments in innovations. The second one are increased tax deductions by allowing for a reducing-balance method. For given producer prices, absolute taxes or the relative price of investment decrease and thus the investment wedge increases.

Chari et al. (2007) show how to map financial frictions in terms of a financial accelerator with default and Brinca et al. (2016) show how to map financial frictions in terms of collateral constraints into a prototype economy with an investment wedge. The loan and guarantee programs lower financial frictions, particularly they slack the bank's collateral

<sup>2</sup>Where  $\hat{x}_t = \ln(x_t) - \ln(x)$  is approximately the relative derivation of a variable  $x_t$  from its steady-state  $x$ .

constraints. Following this, the loan and guarantee programs also raise the investment wedge.

*Labor Wedge:* On the one hand, the stimulus programs lower income tax and social contribution, which increases the labor wedge. On the other hand, possibilities of short-time work were broadened and employers' contributions for employees in short-time work were taken-over by the Federal Employment Agency up to 100%, which subsidizes and incentivizes short-time work.

Brinca et al. (2016) show the link between a prototype economy with efficiency and labor wedges and an economy with search and matching frictions. The mentioned labor market actions, e.g. broadened short-time work, reduce such frictions. Hence, the labor market wedge increases. The effects should be delayed in time due to lower hiring frictions in the aftermath of the crises.

*Efficiency Wedge:* Due to the labor market actions in the previous paragraph, also the efficiency wedge increases e.g. due to a better matching. The effects of short-time work and further training on efficiency should also be delayed in time.

As shown by Chari et al. (2007), input-financing frictions are associated with efficiency wedges. This friction appears when firms must borrow for an input good and some firms are financially more constrained than others. Those firms have to pay higher interest rates. The loan and guarantee programs lower financial constraints and increase thus the efficiency wedge.

*Net exports:* The increase in Hermes-cover advances the conditions for exports. Nevertheless, the effects are probably small.

### **Mapping monetary policy:**

*Government Wedge:* Purchasing bonds lowers the bonds' interest rates. This lowers the costs of debt-financed government spending, which potentially increases indirectly the government wedge.

*Durable Wedge:* Since refinancing is cheaper, for a given real rate of return, investment increases. Hence, monetary policy changes the intertemporal decision of a household, which is reflected in a higher durable wedge. Furthermore, provision of liquidity also changes the intertemporal decisions of liquidity constrained households, which is also reflected in a higher durable wedge.

*Investment Wedge:* Both mentioned effects of the durable wedge have the same effect on the investment wedge. The provision of liquidity and cheaper refinancing lowers frictions in the investment market.

As already mentioned, [Brinca et al. \(2016\)](#) show how to map an economy with a collateral constrained bank into a the prototype economy with an investment wedge. Lower collateral constraints lower frictions in the investment market. Thus, the slacked collateral requirements by the ECB increase the investment wedge.

*Efficiency Wedge:* As mentioned above, [Chari et al. \(2007\)](#) input-financing frictions are associated with efficiency wedges. The friction appears when firms must borrow for input goods and some firms are financially more constrained than others. Those firms have to pay higher interest rates. The Security Markets Program can lower these frictions and increase the efficiency.

### 3.1.3 Calibration

We estimate the elasticity,  $\eta_I = \frac{I}{K_I} \Phi_I''$ , of the price of capital with respect to the investment-capital ratio as well as the elasticity,  $\eta_D = \frac{D}{K_D} \Phi_D''$ , of the price of the stock of durables with respect to the new durables-stock of durables ratio in addition to the parameters that characterize the stochastic process  $\mathbf{s}_t$ . The remaining parameters of the model are calibrated as follows:

The capital elasticity  $\alpha$  is set to 0.34, which is the capital share for German data from 1991 to 2012 calculated by [Flor \(2014\)](#). The discount parameter  $\beta = 0.994$  for the German economy is taken from [Heer and Maussner \(2009\)](#) in line with [Flor \(2014\)](#). We pin down the annual rate of capital depreciation at the average ratio of gross fixed capital formation and the net stock of fixed assets. So that the average quarterly capital depreciation rate arises from

$$\delta_I = 1 - (1 - \delta_{I,annual})^{\frac{1}{4}}. \quad (20)$$

In the same manner the rate of durables depreciation  $\delta_D$  is computed. The growth factors of population and labor augmenting technical progress are the linear trends of the logged population and GDP series. More problematic is the choice of  $\psi$ ,  $\phi$  and  $\eta$ , which characterize the household's preferences. For  $\psi$  and  $\eta$  we follow the baseline calibration from [Chari et al. \(2007\)](#) and fix  $\psi$  at 2.24 and  $\eta$  at 1. We calibrate the preference weight of durables  $\phi$  from the steady state marginal rate of substitution between consumption and durables. We do not estimate the steady-state values of the different wedges. Instead, we compute them from the model's static equilibrium equations. We fix the steady-state values of output, government consumption, investment in capital and investment in durables to their average shares of output, which are reported in [Table 2](#). The steady-state labor supply  $N$  is 0.122, which equals the average share of hours worked on the available time budget of a household.<sup>3</sup> Our calibration exercises are summarized in [Table 4](#).

<sup>3</sup>Here we follow [Heer and Maussner \(2009\)](#), who assume that the household's maximal amount of time to work is  $1440 = (16 \cdot 90)$  hours per quarter.

**Table 4:** Calibration of the model

Parameter	Description	Value
$\alpha$	Capital share	0.34
$\beta$	Discount factor	0.994
$\delta_K$	Rate of capital depreciation	0.017
$\delta_D$	Rate of durable depreciation	0.045
$\psi$	Preference weight of labor	2.24
$\phi$	Preference weight of consumption	0.879
$\eta$	Risk aversion	1

### 3.2 The business cycle accounting procedure

The BCA procedure is divided into three separate steps. In the first step we use a maximum-likelihood-approach to estimate the matrices  $\Pi$  and  $\Sigma$  that characterize the stochastic process  $\mathbf{s}_t$  as well as the elasticities  $\eta_K$  and  $\eta_D$  that define the level of adjustment costs. Details on the estimation procedure and on data manipulation follow in subsection 3.2.1 and subsection 3.2.2.

After all parameters are pinned down either by calibration or estimation, in a second step we use a state smoothing algorithm as described in [Durbin and Koopman \(2012\)](#) to estimate time series for the data generating process  $\mathbf{s}_t$ .

In the last step of the accounting procedure, we feed the wedges separately back into the model, while others are set constant, as proposed by [Chari et al. \(2007\)](#).<sup>4</sup> This step quantifies the contribution of each wedge to the total fluctuation in the observed data.

#### 3.2.1 Maximum-likelihood estimation (MLE)

For the parameter estimation we transform the solved model into a linear state-space model

$$\mathbf{y}_t = \mathbf{H} \cdot \mathbf{z}_t, \quad (21)$$

$$\mathbf{z}_{t+1} = \mathbf{F} \cdot \mathbf{z}_t + \mathbf{v}_{t+1}, \quad (22)$$

with

$$\mathbf{z}_t = \begin{bmatrix} \mathbf{s}_t \\ \mathbf{x}_t \end{bmatrix}, \quad \mathbf{v}_t = \begin{bmatrix} \epsilon_t \\ 0 \end{bmatrix}, \quad \mathbf{v}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}),$$

$$\mathbf{F} = \begin{bmatrix} \Pi & \mathbf{0} \\ \mathbf{L}_s^x & \mathbf{L}_x^x \end{bmatrix}, \quad \mathbf{H} = [\mathbf{L}_s^y \quad \mathbf{L}_x^y], \quad \mathbf{Q} = \begin{bmatrix} \Sigma & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

<sup>4</sup>See the technical appendix by [Chari et al. \(2007\)](#) for more details.

To evaluate the likelihood function of this linear state-space model most of the literature uses the Kalman-Filter initialized at the unconditional mean and the unconditional variance of the state vector  $\mathbf{z}_t$  (see e.g. [DeJong and Dave \(2011\)](#)). However, since the Kalman-Filter is asymptotically time-invariant<sup>5</sup> for the state-space model described by (21) and (22), the mean squared error (MSE)  $\mathbf{p}_{t|t}$  of the point estimate for  $\mathbf{z}_t$  given on all information available at time  $t$  converges to a matrix  $\mathbf{p}$  as  $t$  goes to infinity.<sup>6</sup> Exploiting this property [Chari et al. \(2007\)](#) use the steady-state MSE  $\mathbf{p}$  instead of the unconditional variance to initialize their steady-state Kalman-Filter. As pointed out by [Huber \(2018\)](#) it can be shown that for a standard Dynamic Stochastic General Equilibrium (DSGE) model, as the one described in Section 3.1.1, the steady-state MSE  $\mathbf{p}$  is equal zero. To see this imagine  $\mathbf{z}_0$  is uncertain. Since we observe the investments in capital and durables it is straightforward that the uncertainty concerning both stocks disappears as  $t$  goes to infinity. Assuming  $\mathbf{L}_s^y$  is non-singular<sup>7</sup> from equation (18) follows that

$$\mathbf{s}_t = [\mathbf{L}_s^y]^{-1} (\mathbf{y}_t - \mathbf{L}_x^y \cdot \mathbf{x}_t). \quad (23)$$

Thus, as we get more information on the endogenous states  $\mathbf{x}_t$ , the uncertainty over the exogenous states  $\mathbf{s}_t$  decreases, too. Using a steady-state Kalman-Filter is therefore equivalent to the assumption that at time  $t = 0$  there is no uncertainty over the state vector  $\mathbf{z}_t$ .<sup>8</sup> The big advantage of the steady-state Kalman-Filter is that it provides an analytical Maximum-Likelihood-Estimator for  $\Sigma$  since we can observe the residuals  $\epsilon_t$  independently of  $\Sigma$ . Hence the Maximum-Likelihood-Estimator of  $\Sigma$  for a given  $\mathbf{F}$  is

$$\hat{\Sigma} = \frac{1}{N} \sum_{t=1}^N [(\mathbf{s}_t - \mathbf{F} \cdot \mathbf{s}_{t-1}) \cdot (\mathbf{s}_t - \mathbf{F} \cdot \mathbf{s}_{t-1})^T]. \quad (24)$$

The estimates of a textbook Kalman-Filter initialized at the unconditional variance are in general more accurate, since the initial states are usually unknown. In addition it can be shown that the estimates of the steady-state Kalman-Filter converges to the textbook Kalman-filter. Therefore, we follow [Huber \(2018\)](#) and use the estimates of the steady-state Kalman-Filter as the initial guess for a second estimation, where we initialize the Kalman-Filter at the unconditional variance.

### 3.2.2 Data manipulation

GDP, investment, durables, government expenditures, and net exports are the observables. A regression with the logarithm of the first four observables as dependent variable and time as independent variable provides the necessary components. The coefficient approximates

<sup>5</sup>In this context we assume that all eigenvalues of  $\mathbf{F}$  lie strictly inside the unit circle.

<sup>6</sup>For a formal proof see for example [Hamilton \(1994\)](#).

<sup>7</sup>We are sure this is also a necessary condition for the steady-state and all steady-state converging Kalman-filter initializations, such as the normally used initialization with unconditional mean and unconditional variance. We will show this in future work.

<sup>8</sup>[Huber \(2018\)](#) presents a detailed and more general version, Monte Carlo studies and further applications of this approach.

the grow rate for growth accounting and the residuals the relative deviation from balanced growth. The second one is used for business cycle accounting.

Negative values for net exports prevent logarithmization. A regression with net exports relative to GDP as dependent variable and time as independent variable provides an auxiliary variable. The coefficient is the excess growth rate of net exports compared to GDP growth. The residuals are the deviation from the long-run net exports to GDP rate, which is computable in the model.

## 4 RESULTS

### 4.1 Growth accounting

Table 5 presents the growth rates of the observables. The GDP trend growth rate is 1.32% p.a. The amount of durables and investment goods grows slower than GDP, while net exports growth faster. Government consumption grows similar to GDP.

**Table 5:** Growth accounting

Parameter	Description	Value
$\ln(\gamma_n^4)$	yearly growth rate of population	0.03%
$\ln(g_y^4)$	yearly growth rate of GDP	1.32%
$\ln(g_I^4)$	yearly growth rate of investment	0.93%
$\ln(g_D^4)$	yearly growth rate of durables	0.35%
$\ln(g_G^4)$	yearly growth rate of gov. cons.	1.40%
$\ln(g_E^4)$	yearly growth rate of net exports	1.65%

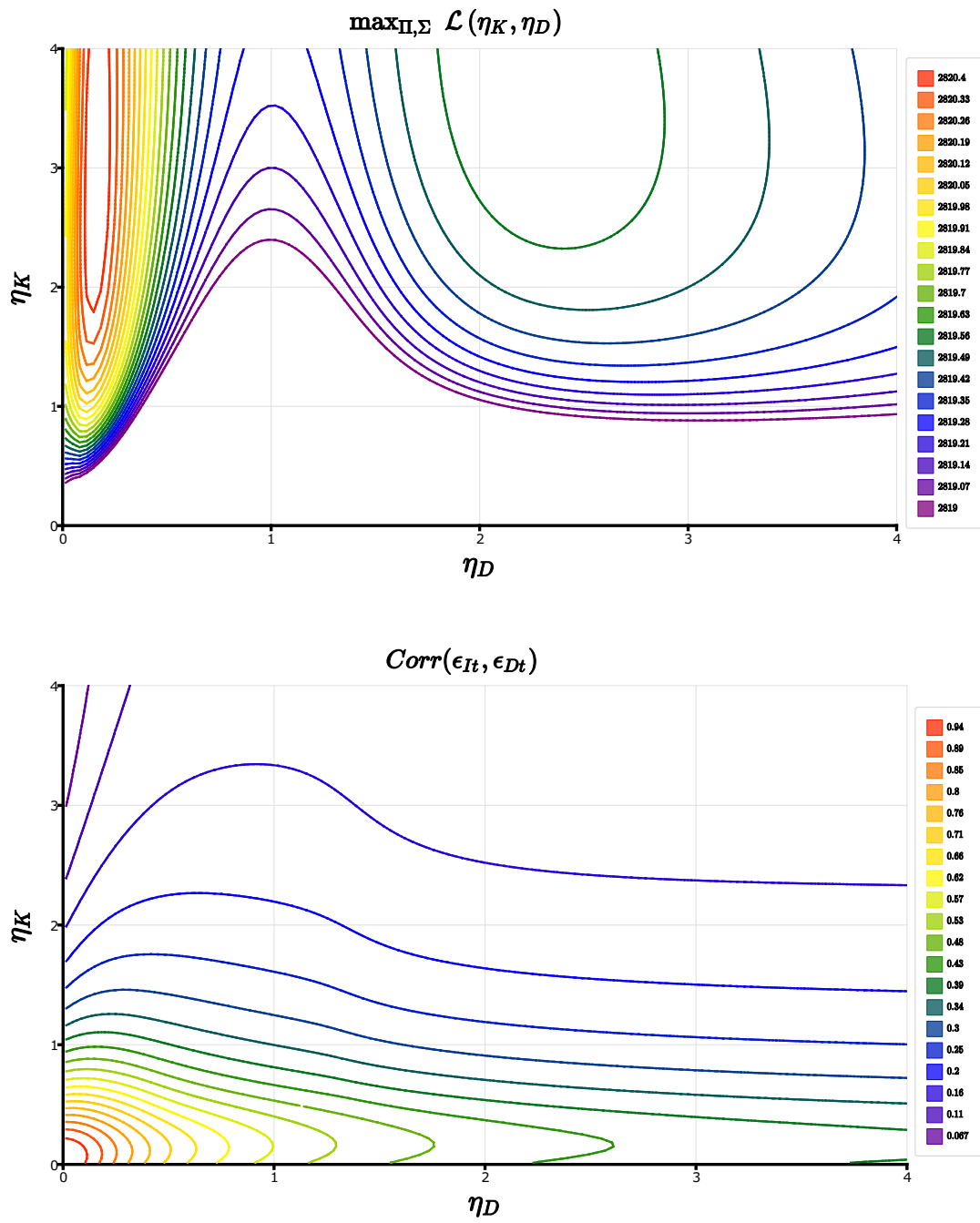
The differences in long-run component of the durable and the investment wedge may be due to investment-specific technological change as Greenwood et al. (1997) describe. in't Veld et al. (2014) investigate potential causes for the increase in the German net exports since the launch of the Euro, which reflects in  $P_{Et}$ . They find the most important factors are: A higher German savings rate, positive supply shocks especially due to labor market reforms as well as a higher demand for German goods of non Euro area members.

### 4.2 Estimation

As already mentioned, the maximum-likelihood estimation includes  $\Pi$ ,  $\Sigma$ ,  $\eta_d$  and  $\eta_k$ . The upper panel of Figure 4 illustrates the likelihood function with respect to  $\eta_d$  and  $\eta_k$ , while  $\Pi$ ,  $\Sigma$  are the argument maximum of the function for given  $\eta_k$  and  $\eta_d$ . Two local maximums are identified. The global is at  $\eta_{d,max_{\Pi,\Sigma}} = 0.19$  and  $\eta_{k,max_{\Pi,\Sigma}} = 3.00$ .

Table 6 presents the estimates for the autoregressive matrix  $\Pi$  as well as second moments of the innovations  $\epsilon_i$ . All wedges are highly autoregressive. The investment wedge depends heavily on the other wedges with one lag. The innovations of the investment wedge have the highest volatility and are negatively correlated with the efficiency wedge.

Figure 4: Maximum-Likelihood-Estimation





**Table 6:** Estimation of exogenous shock process

Autoregressive Matrix						
$\Pi$	$S_A$	$S_N$	$S_I$	$S_D$	$S_E$	$S_G$
$S_A$	0.90	0.411	0.00	0.07	-0.21	-0.16
$S_N$	0.01	0.83	0.01	-0.02	-0.12	-0.01
$S_I$	0.70	-1.71	0.96	-0.52	1.44	1.07
$S_D$	0.27	-0.05	-0.00	0.66	0.16	-0.01
$S_E$	0.06	-0.03	0.01	-0.05	0.62	-0.12
$S_G$	-0.05	0.17	-0.01	-0.05	-0.22	0.80

Correlation and standard errors						
$Corr(\epsilon_i, \epsilon_j)$	$\epsilon_A$	$\epsilon_N$	$\epsilon_I$	$\epsilon_D$	$\epsilon_E$	$100StD(\epsilon_i)$
$\epsilon_A$						0.94
$\epsilon_N$	0.03					0.34
$\epsilon_I$	-0.49	-0.06				7.12
$\epsilon_D$	0.27	-0.83	0.13			1.44
$\epsilon_E$	0.31	0.70	-0.02	-0.36		0.59
$\epsilon_G$	-0.10	0.13	-0.19	-0.16	-0.13	0.80

The innovations of the durable wedge correlates strongly negative with the innovations of the labor wedge. The net export wedge's innovation correlates with the labor wedge.

The lower panel of Figure 4 illustrates the innovations of durables and investments are perfectly correlated in the absence of adjustment costs. Fehrlé (2019) investigates different investment goods, vector-autoregressive processes and adjustment costs in detail. Here, it is sufficient to recognize a separation of investments and durables is ineffectual in the absence of adjustment costs.

#### 4.3 Business cycle account for the great recession and the German fiscal stimulus, 2008:1-2011:2

The graphical analysis of our BCA exercise is reported in Figures 5 and 6. In Figure 5 we confront the observations of GDP and in Figure 6 of subaggregates of GDP and hours with the model's prediction for using only one wedge.

The crisis was mainly driven by the efficiency wedge. The investment wedge and net export also causes a part of the crises. The three wedges together induce the decrease in GDP. The labor wedge accounts for a fraction of the crisis in 2009q2-q4. Besides the wedge mitigates the crises and introduce the recovery. Durables and government consumption wedge mitigated the crisis at all.

The investment wedge accounts for the decline in investment. The efficiency wedge influences durables strongly negatively. The durable wedge of its own increases durables by nearly 50%. The efficiency wedge alone accounts for the decline in non-durable consumption mostly. The durable wedge and government consumption influence non-durable consumption only slightly. The decline in net exports and the investment wedge introduce the decline in hours worked. The labor market wedge accounts for the decline in 2009q2-q4. Besides the wedge works counter-cyclical. The other wedges work contrary.

Figure 5: BCA - GDP

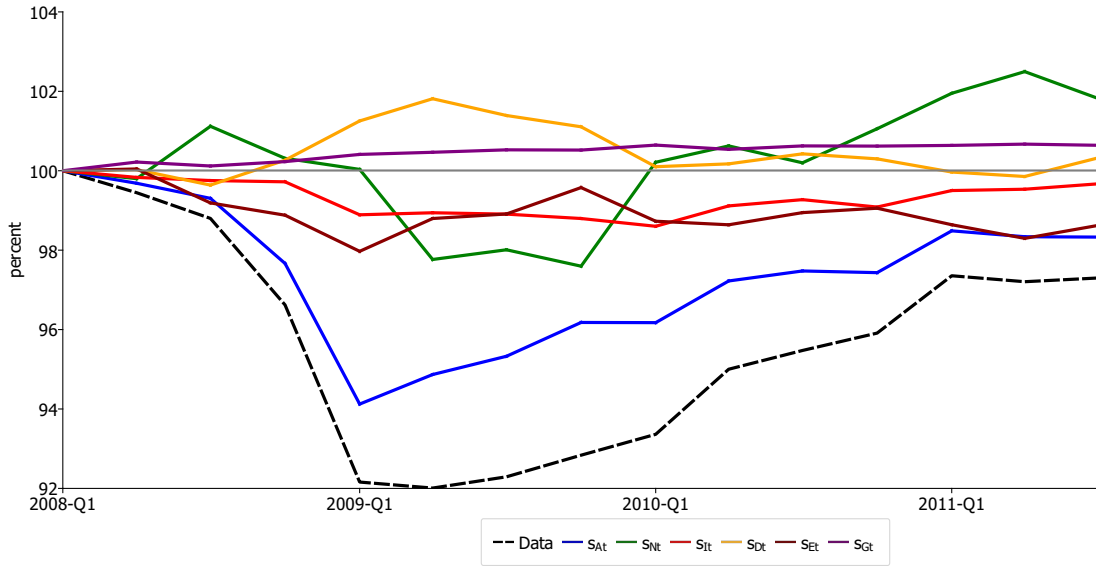
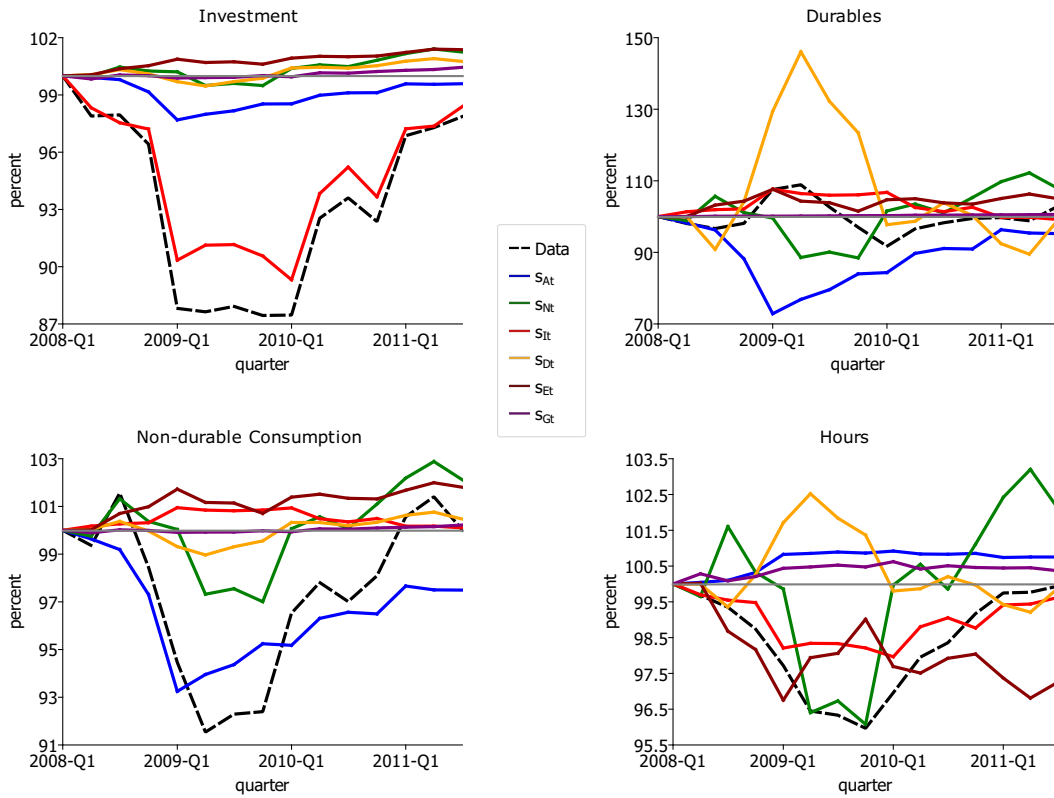


Figure 6: BCA - Subaggregates



Pertaining to GDP and hours, we find that the stimulus program in the durable subsidies and government consumption had a positive effect during the crisis.

The model predicts an approximately 2.5 % bigger decline in GDP and an approximately 3% bigger decline in hours without changes in those wedges. In matters of non-durable consumption and investment the effect of the stimulus program is negative. Nevertheless, during the crises the stimulus of durables and government consumption increases GDP and is not completely substituted by lower investments and non-durable consumption. Due to intertemporal substitution durables decline after the cash for clunkers program. The durable wedge does not influence GDP negatively in this time period. The labor market wedge mitigate the crises at the beginning and the end of the crises. The model predicts an increase over 2% in GDP and over 3% in hours worked at the end of the crisis.

The measurement  $\omega_i$  quantifies the contribution of each wedge to GDP via:

$$\omega_i = \frac{\sum_t (\hat{y}_t^{GDP} - \hat{y}_t^i)}{\sum_j \sum_t (\hat{y}_t^{GDP} - \hat{y}_t^j)} \text{ with } i, j \in \{s_A, s_N, s_I, s_D, s_G, s_E\}, t \in [2008Q1, \dots, 2011Q2], \quad (25)$$

where  $\hat{y}_t^{GDP}$  is the GDP when all wedges are non-changing and  $\hat{y}_t^i$  is the model outcome of wedge  $i$  alone. Thus, the contribution of all wedges together sums to 1, while the sign of  $\omega_i$  points out if wedge  $i$  has mitigated (−) or amplified (+) the crisis.

The efficiency wedge accounts for 62% of the decline in GDP during this period, net exports for 26%, the investment wedge 19%, and the labor market accounts for 3%. Government consumption accounts for -5% and the durable wedge for -4%. Since the effect of the durable subsidies and government consumption were similar but expenditures for subsidies were notably smaller, the cash for clunkers program seems more efficient than the increased government consumption.

#### 4.4 Robustness

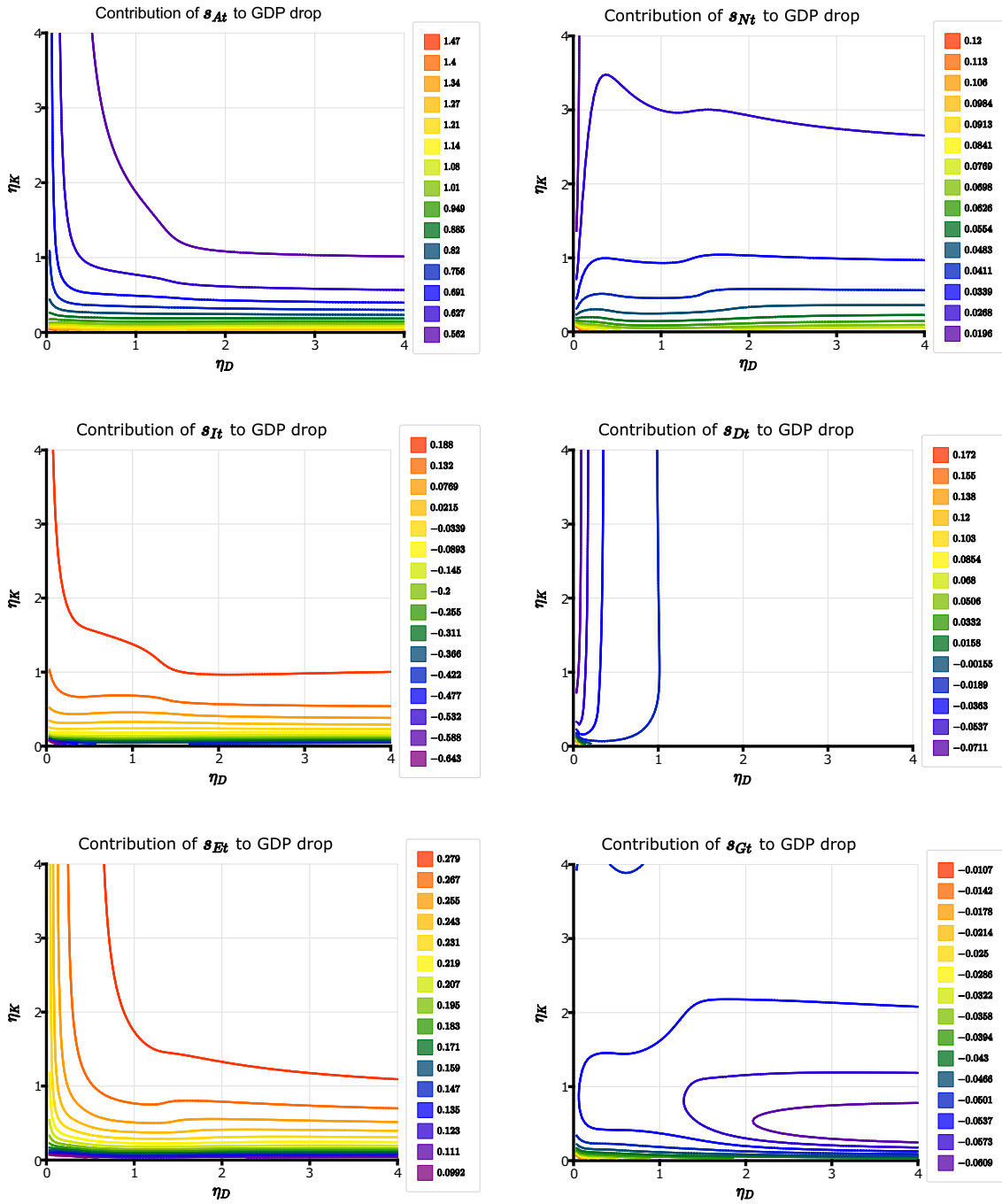
*Robustness in Parameters:* The results depend potentially on the values of adjustment costs  $\eta_k, \eta_d$  and on the intertemporal elasticity of substitution  $\eta$ .<sup>9</sup>

Figure 7 illustrates the contribution of the concerning wedges for different amounts of adjustment costs. The efficiency wedge contributes mostly to the decline in GDP, followed by net export for the whole set of adjustment costs. The labor wedge contributes robustly to the crises, it even induces the recovery. Government consumption mitigates the crisis for the whole set of parameters. The durables wedge mitigates the crises for most of the parameters. The contribution is pro-cyclical in the absence of adjustment costs. As mentioned above in the absence of adjustment costs, a separation of the durable and investment wedge is ineffectual. The investment wedge's contribution to the crises is negative for  $\eta_k < 0.5$  where the likelihood is small (see Figure 4) and positive otherwise.

Subsidies in durables change the intertemporal rate of substitution. Hence, a robustness check to the elasticity of the substitution rate is relevant. Figure 8 presents the contribution to the decline in GDP over  $\eta_{\max_{\Pi, \Sigma, \eta_k, \eta_d}} \in \{0.9, \dots, 2.1\}$ . The contribution of the labor, in-

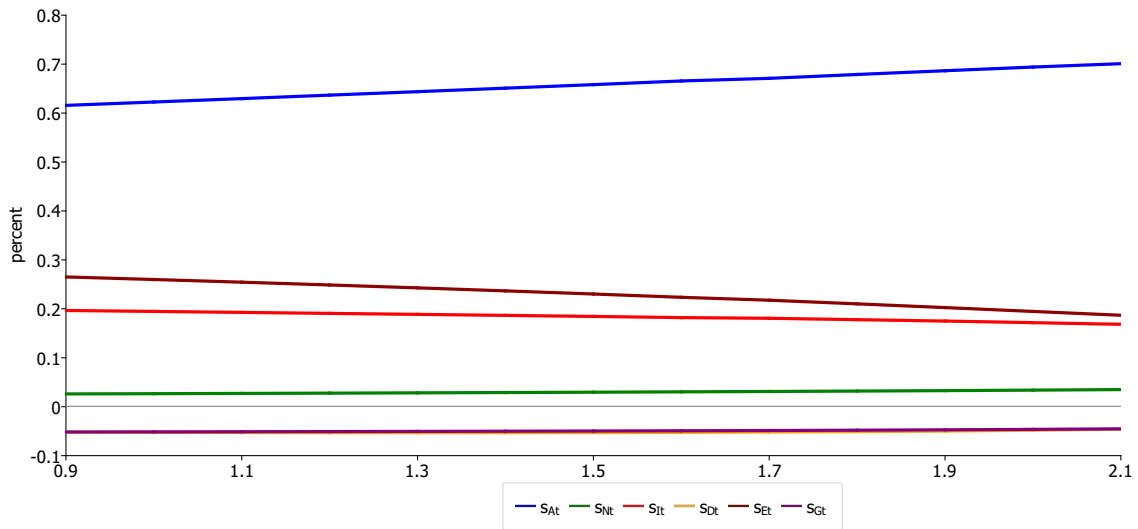
<sup>9</sup>We also checked for  $\psi$ . Changes were not identifiable.

Figure 7: Adjustment costs specific wedge contribution

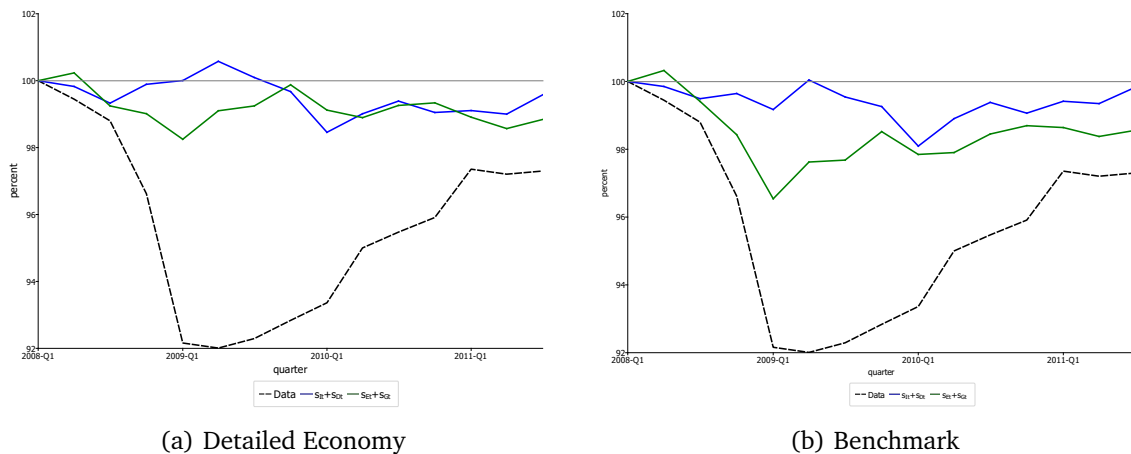


vestment, and durable wedge as well as government consumption is nearly constant. The contribution of net exports decline with a higher elasticity, nevertheless they contribute the second most. The contribution of the efficiency wedge increases.

**Figure 8:** Elasticity of intertemporal substitution specific wedge contribution



**Figure 9:** Robustness CKM Wedges



*Robustness regarding the benchmark model:* The effects of the investment and durables wedge together as well as the government consumption and net exports together maps our economy into the benchmark BCA economy. The left panel of Figure 9 illustrates these effects. The right panel plots the investment and government spending wedge from the BCA in line with CKM ( $\eta_{k,max_{II,\Sigma}} = 0.86$ ). The results are similar, except in the more detailed economy the investment wedge acts slightly counter-cyclical during the cash for clunkers program. Thus, the detailed subaggregates are not counterfactual compared to the benchmark BCA analysis.

## 5 CONCLUSION

In this paper we use the BCA analysis to investigate the impact of the German stimulus program during the great recession from 2008 to 2011. We provide a detailed mapping of the stimulus program into an extended prototype economy with six wedges. Wedges correspond to the following variables: *government consumption, durables, investment, labor, net exports and efficiency*.

In our BCA analysis we find that the great recession was mainly driven by the efficiency, net exports and investment wedge. On the contrary, the durables and the government spending wedge acted counter-cyclical. We argue that these wedges might collect the positive influence of the German stimulus. The labor market wedge was pro-cyclical in 2000q2-q3, besides it mitigate the crises. We check the robustness of our results to different choices of parameters that determine the elasticity of intertemporal substitution as well as capital and durable adjustment costs. We find that our results are robust for all wedges besides the investment wedge. Due to higher expenditures for government consumption and a similar impact compared to the cash for clunkers program, subsidies for durable goods impacts the business cycle more effectively. The efficiency of durable goods subsidies as well as the counter-cyclical labor wedge motivate further research.

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# APPENDIX

(not for publication)

Business cycle accounting for the German fiscal stimulus program

## A MODEL

The following equations determines the model with stationary variables:

$$y_t = k_t^\alpha (A_t N_t)^{1-\alpha} \quad (26)$$

$$r_t = \alpha \frac{y_t}{k_t} \quad (27)$$

$$w_t = (1-\alpha) \frac{y_t}{N_t} \quad (28)$$

$$\lambda_t = \phi c_t^{\phi(1-\eta)-1} d_t^{(1-\phi)(1-\eta)} (1-N_t)^{\psi(1-\eta)} \quad (29)$$

$$(1-\tau_{N_t}) = \frac{\psi}{\phi} \frac{c_t}{(1-N_t)w_t} \quad (30)$$

$$y_t = c_t + i_t + i_{Dt} + g_t + e_t \quad (31)$$

$$\mu_{Kt} = \lambda_t \frac{1 + \tau_{It}}{1 - \Theta'_{Kt}} \quad (32)$$

$$\mu_{Dt} = \lambda_t \frac{1 + \tau_{IDt}}{1 - \Theta'_{Dt}} \quad (33)$$

$$g_K \cdot \gamma_n k_{t+1} = (1 - \delta_K) k_t + i_t - \Theta_{Kt} \cdot k_t, \quad (34)$$

$$g_D \cdot \gamma_n d_{t+1} = (1 - \delta_D) d_t + i_{Dt} - \Theta_{Dt} \cdot d_t, \quad (35)$$

$$\mu_{Kt} = \beta g_{M_K}^{-1} \mathbb{E}_t \left[ \mu_{Kt+1} \left( 1 - \delta_K - \Theta_{Kt+1} + \frac{i_{t+1}}{k_{t+1}} \Theta'_{Kt+1} \right) + \lambda_{t+1} r_{t+1} \right] \quad (36)$$

$$\mu_{Dt} = \beta g_{M_D}^{-1} \mathbb{E}_t \left[ \mu_{Dt+1} \left( 1 - \delta_D - \Theta_{Dt+1} + \frac{i_{Dt+1}}{d_{t+1}} \Theta'_{Dt+1} \right) + \lambda_{t+1} \frac{1-\phi}{\phi} \frac{c_{t+1}}{d_{t+1}} \right] \quad (37)$$

## B DATA

### B.1 Raw Data

The data is taken from the Fachserie 18: National accounts, domestic product from the German Federal Statistical Office.

**Pop:** Total Population 1991:I-2018:I

Source: 2.1.7 Population and labour force participation 1; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

**Hours:** Hours worked by persons in employment 1991:I-2018:I

Source: 2.1.8 Persons in employment, employees and hours worked (domestic concept) 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

**GDP:** 1991:I-2018:I

Nominal source: 2.3.1 Use of gross domestic product at current prices 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

Real source: 2.3.2 Use of gross domestic product, price-adjusted 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

**PCE:** Private Consumption Expenditures of households 1991:I-2018:I

Nominal source: 2.3.3 Final consumption expenditure at current prices 3; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

Real source: 2.3.4 Final consumption expenditure at , price-adjusted; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

**Govern. Cons.:** government final consumption expenditure (domestic use) 1991:I-2018:I

Nominal source: 2.3.3 Final consumption expenditure at current prices 3; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

Real source: 2.3.4 Final consumption expenditure at , price-adjusted; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

**Investment** gross fixed capital formation 1991:I-2018:I

Nominal source: 2.3.1 gross fixed capital formation at current prices 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

Real source: 2.3.2 gross fixed capital formation, price-adjusted 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

**Net Exports:** Nominal source: 2.3.1 Balance of exports and imports at current prices 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

Real source: 2.3.2 Balance of exports and imports, price-adjusted 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

**Durables:** Langlebige Güter (Durable Goods) 1991:I-2018:I

Nominal source: 2.14 Konsumausgaben der privaten Haushalte im Inland nach Dauerhaftigkeit der Güter, Saison- und kalenderbereinigt in jeweiligen Preisen 4; Private Konsumausgaben und Verfügbares Einkommen - 1. Vierteljahr 2018

Real source: 2.14 Konsumausgaben der privaten Haushalte im Inland nach Dauerhaftigkeit der Güter, Saison- und kalenderbereinigt - preisbereinigt 4; Private Konsumausgaben und Verfügbares Einkommen - 1. Vierteljahr 2018

(only in German available: Domestic consumer spending on durable goods, seasonally and calendar adjusted 4; Private consumption expenditure and disposable income - 1st quarter of 2018)