

# Optimal Monetary Policy with Downward Nominal Wage Rigidity\*

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## Abstract

At the individual and country level nominal wages have been found to be downwardly rigid, such that they are more likely to increase than decrease. This has strong implications for optimal monetary policy in the standard New-Keynesian model, which typically assumes flexible wages or symmetric nominal wage rigidities. When wages are downwardly rigid and households do not fully internalise the constraint, boom-bust cycles can arise due to agents increasing their wage flexibly and the economy then suffers as wages sluggishly fall and remain elevated even after the shock dissipates. Solving a non-linear model that internalises this constraint endogenously at all periods in time dampens wage increases in a model where agents can flexibly increase their wage. This work adds to the literature by introducing the downward nominal wage rigidity (DNWR) constraint of [Schmitt-Grohé and Uribe \(2016\)](#) into a standard New-Keynesian model and finding the optimal trend inflation when agents fully understand the existence of this DNWR constraint. Furthermore, motivated by the welfare loss generated by using a standard Taylor rule, this paper searches for a new optimal simple rule that can replicate the optimal monetary policy in this framework. Moreover, as with other work on DNWR this paper finds support for ‘greasing the wheels’ - positive trend inflation that helps to deflate real wage increases - at 0.75% to 1% .

## 1 Introduction

In the standard New-Keynesian model wages are assumed to be symmetrically flexible, such that wage increases or decreases are equally effortless to implement. Extensions of this simple model, such as a medium-scale dynamic stochastic general equilibrium model add Calvo wages, which symmetrically dampens wage changes. However, as shown in the data of individuals’ wage changes in [Daly et al. \(2012\)](#) and at the country level for developed and developing countries by [Schmitt-Grohé and Uribe \(2016\)](#), wages are downwardly rigid - we observe increases in nominal wages more often than decreases. Adding in such a constraint into a standard model impacts how the agents in the economy react to shocks, which has further consequences for the optimal monetary policy and the optimal steady state inflation rate compared to a model with flexible wages.

This paper explores monetary policy in the New Keynesian model when the model is affected by downward nominal wage rigidities (DNWR), such that nominal wages can freely rise but are sluggish when adjusting downwards. Including DNWR, instead of Calvo wages or flexible wages, causes an asymmetric response of monetary policy to shocks of the same size but differing signs. Moreover, shocks that temporarily increase the nominal wage, such as a positive demand shock, can create persistent effects since the wage cannot adjust down in a timely manner and therefore causes an increase in unemployment as firms cannot afford to hire the workers but a higher wage incentivises many households to enter the workforce, leading to a boom-bust cycle. This isn’t found when Calvo wages are assumed. Welfare gains can be made on the standard Taylor rule, where the central bank reacts to inflation and the output gap, by using a simple rule that includes

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either unemployment as in Galí (2015) and Galí (2011b) or wage inflation. Furthermore, finding that the optimal monetary policy is asymmetric and allows for price inflation to deflate real wages (both of these found using a related framework of Kim and Ruge-Murcia (2011)). This work contributes to the literature by finding the optimal simple rule in a New-Keynesian model with a simple downward nominal wage rigidity constraint of the form in Schmitt-Grohé and Uribe (2016). Moreover, in the latter part of the paper, optimal trend inflation of around 0.75-1% is found to be welfare improving, thus providing further support for ‘greasing the wheels’ of the economy. In an extension of the model, the downward nominal wage rigidity constraint is endogenised, such that households optimise over it. Once the households understand the existence of this occasionally binding constraint, and that a shock may cause it to bind, it causes them to limit their wage increases even though the household can flexibly raise the wage, this is because once increased the wage is sluggish to fall, a finding made as well by Elsyby (2009) and more recently in a theoretical model by Wolf (2018).

The paper is organised as follows: Section 2 highlights the empirical motivation. Section 3 discusses the papers that have implemented a downward nominal wage constraint and outlines their findings. Section 4 outlines the simple model used, which is a log-linearised New Keynesian model with an exogenous DNWR constraint or DNWR wage setting rule. Section 5 provides the impulse responses from simulating the model and presents the optimal monetary policy and optimal simple rule in this setup. Section 6 solves the non-linear model with a positive trend inflation and endogenous DNWR constraint, allowing the households to be forward looking when setting their wages. Section 7 concludes and the Appendix includes details on the model equations, steady state, computational techniques used and additional tables and figures.

## 2 Empirical Motivation

This paper utilizes the basic New Keynesian model, which includes sticky prices in the standard form of Calvo (1983). The empirical motivation of this type of model has already been covered extensively. Therefore this section aims to empirically motivate the non-standard additions to the New Keynesian model that is used within this paper.

Previous work has been completed concerning wage rigidities, stemming from Erceg et al. (2000), which enter in a symmetric way to the price setting. This setting has also been extended to include unemployment<sup>1</sup>, originating from Galí (1996) and worked upon in Galí (2011a) and Galí et al. (2011). The principle finding of which, “the structural wage equation derived here is shown to account reasonably well for the co-movement of wage inflation and the unemployment rate in the US economy,” Galí (2011a).

However, there exists empirical evidence that wages may not be symmetrically rigid and that for developed countries, as well as some developing countries, wages are downwardly rigid - such that we observe increases in nominal wages more often than decreases, Schmitt-Grohé and Uribe (2016). The parameters of the model included in this paper are based upon U.S. data and hence the evidence for DNWR presented here is U.S. focused. The three main empirical studies that use individual level data are: Gottschalk (2005), Barattieri et al. (2014) and Daly et al. (2012). Gottschalk (2005) applies new methods to data from the Survey of Income and Program Participation and finds that downward flexible wages found in individual level data is due to measurement error and once corrected the data produce findings closer to that found in firm level data where “only 2% to 3% of workers experiencing nominal-wage cuts, which implies substantial rigidity.” In keeping with Gottschalk (2005), Barattieri et al. (2014) finds evidence using the same micro data “that wage changes are significantly right-skewed” therefore seeing an increase in wages is more likely than a decrease. Moreover, they also show that

higher wage stickiness makes it easier for macroeconomic models to match the stylized fact that monetary shocks cause persistent changes in real output and small but relatively persistent changes in prices.(Barattieri et al., 2014)

Lastly Daly et al. (2012) analyses wage growth during the great recession of 2007 using the Current Population Survey and finds that “despite modest economic growth and persistently high unemployment, real wage growth has averaged 1.1% since 2008” and that “ a significant fraction

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<sup>1</sup>Staggered wage contracts and their link to unemployment can be seen even earlier in a rational expectations model of Taylor (1980).

of workers are affected by downward nominal wage rigidities.” One possible reason for observing DNWR during that period is that the low inflation environment meant that real wages were not being eroded by inflation. Furthermore, employers are hesitant to reduce pay as it can reduce morale and prompt resistance [Kahneman et al. \(1986\)](#).

Further empirical support of downward nominal wage rigidities has been found for European countries by the Wage Dynamics Network, a research network consisting of the European Central Bank (ECB) and the National Central Banks (NCBs) of the EU Member States. Using a firm-level survey spanning 15 European countries during the late 2007 and early 2008, [Babecky et al. \(2010\)](#) find evidence of downward nominal wage rigidity (defined in their study as wage freezes) and downward real wage rigidity (defined through wage indexation). Further evidence is provided by research conducted by the Wage Dynamics Network focuses on downward real wage rigidity for specific countries over a longer period of time, including [Lunnemann and Wintr \(2010\)](#) who focuses on Luxembourg between 2001 and 2007, and [Du Caju et al. \(2012\)](#) for Belgium between 1990 and 2002.

However recent work by [Elsby et al. \(2016\)](#), which focuses on the USA and UK labour markets, argue that downward nominal wage rigidity may be less binding than originally thought. Through the use of higher quality data - payroll data instead of self-reported surveys that may be subject to reporting error - and a comparison between male and female workers they find a higher frequency of wage reductions than previous studies. Thus motivating further empirical research into the existence and impact of downward nominal wage rigidities.

### 3 Literature Review

Downward Nominal Wage Rigidities have been studied within economic models previously. The innovation within this paper is to include it in the New Keynesian model with unemployment using a simple constraint, exploring the optimal monetary policy and optimal simple rule. Below provides an outline and briefly discusses the most prominent papers within the literature that include DNWR.

[Kim and Ruge-Murcia \(2011\)](#), which builds on one of their earlier papers <sup>2</sup> utilize a convex cost function for changing prices and wages that can be asymmetric or reduced down to a quadratic cost a la [Rotemberg \(1982\)](#). Moreover, this cost function encompasses the ‘L’ shaped cost function of [Benigno and Ricci \(2011\)](#) which corresponds to the situation where cutting wages is infinitely costly and raising wages is costless. They find that ‘greasing the wheels’, having a low but strictly positive inflation target is welfare improving. Therefore,

for an economy with downwardly rigid wages, the benefits of positive inflation conjectured by [Tobin \(1972\)](#) may overcome [Friedman \(1969\)](#)’s general prescription of negative inflation. ([Kim and Ruge-Murcia, 2011](#))

This inflation target is estimated to be around 1% but will change depending on the model specifications and country estimated to. [Kim and Ruge-Murcia \(2011\)](#)’s paper is similar to this paper in execution and conclusion however this model utilizes fully flexible increases in nominal wages with DNWR and Calvo prices. Moreover, the model is able to analyze the response of employment, unemployment and the labor force to exogenous shocks as well as finding the coefficients for an optimal simple rule.

[Benigno and Ricci \(2011\)](#) introduce DNWR in a DSGE model with flexible prices and find also that ‘greasing the wheels’ and allowing for moderate inflation may help intratemporal and intertemporal relative wage adjustments and that “those experiencing large volatility or lower productivity growth may find it desirable to target a higher inflation rate.” They also link the steepness of the Phillips Curve and wage rigidities and find that the Phillips Curve would steepen if wage rigidities declined. Furthermore, when wage rigidities are present there exists a “non-negligible long-run trade-off between inflation and the output gap,” [Benigno and Ricci \(2011\)](#).

[Schmitt-Grohé and Uribe \(2016\)](#) motivated the manner in which the DNWR constraint is included as the constraint follows the same form utilised in their paper. Differences arise between our papers as [Schmitt-Grohé and Uribe \(2016\)](#) is a real model that focus on developing open economies and how “the combination of a currency peg and free capital mobility creates a negative externality that causes overborrowing during booms and high unemployment during contractions.”

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<sup>2</sup>See [Kim and Ruge-Murcia \(2009\)](#).

The value of the degree of DNWR,  $\gamma$ , is explored by [Schmitt-Grohé and Uribe \(2016\)](#) and is around 1 for the developed and developing countries they analyze. This paper follows [Schmitt-Grohé and Uribe \(2016\)](#) in choosing  $\gamma = 0.99$ , which means that nominal wages can decline up to 4% per year.

[Fahr and Smets \(2010\)](#) combines the convex cost function of [Kim and Ruge-Murcia \(2009\)](#) with regard to prices and wages with the labor market of [Erceg et al. \(2000\)](#). Using a two country model and real wage rigidity, instead of nominal rigidity, allows them to focus on transmission of monetary policy in a monetary union. Their main findings that pertain to this paper are that

Downward nominal wage rigidities lead to a positively skewed response in nominal wage changes and a sizeable positive optimal inflation rate. This effect is stronger the lower price rigidity. The greasing effects of inflation vanish if wages are either indexed (real wage rigidity) or if adjustment costs are symmetric. ([Fahr and Smets, 2010](#))

The range of optimal trend inflation proposed can be vast, with [Gross \(2018\)](#) finding optimal inflation in their model, which is an extension of [Daly and Hobijn \(2014\)](#), to be 5.4%. [Gross \(2018\)](#) uses a [Calvo \(1983\)](#) approach to the DNWR constraint, such that with some probability the household cannot lower their wage.

Moreover as well as adding motivation for positive trend inflation DNWR has also been shown to provide wage restraint. This has been shown in [Wolf \(2018\)](#), which also takes seriously the wage constraint found in [Schmitt-Grohé and Uribe \(2016\)](#) and uses it to assess wage inflation rates in the euro area.

Thus this paper provides further support to grease the wheels of the economy and wage restraint. Moreover, the paper advances the literature by highlighting the differences in responses when the households follow a wage setting rule versus when they are forward looking. Furthermore, I find an optimal simple rule, one which can be followed by a central bank, which is close to the optimal monetary policy.

## 4 Methodology

The model is a fairly simple New Keynesian model but with unemployment as in [Galí et al. \(2011\)](#) and the addition of downward nominal wage rigidities. The model follows closely to a standard New Keynesian model that can be found in [Galí \(2015\)](#) ‘Chapter 6: Sticky Wages and Prices’ as well as ‘Chapter 7: Unemployment and Monetary Policy’. The main difference between these models is that the one used in this paper suffers from has Downward Nominal Wage Rigidities instead of Calvo wages.

### 4.1 Firms

As in the standard New Keynesian mode, a continuum of firms are assumed to exist and are indexed by  $i \in [0, 1]$ . Each firm produces a differentiated good with a technology represented by the production function

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \quad (1)$$

where  $A_t$  is an exogenous technology <sup>3</sup> parameter that is common to all firms,  $Y_t(i)$  denotes the output of good  $i$ , and  $N_t(i)$  is a labour input used by firm  $i$  and can thought of as employment or hours worked. The definition of  $N_t(i)$  is given by

$$N_t(i) = \left( \int_0^1 N_t(i, j)^{1-\frac{1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w-1}}. \quad (2)$$

Here  $N_t(i, j)$  denotes the quantity of type- $j$  labour employed by firm  $i$  in period  $t$ . Moreover, there is a continuum of labour types indexed by  $j \in [0, 1]$ . The parameter  $\epsilon_w$  represents the elasticity of substitution among labour types.

$W_t(j)$  denotes the nominal wage for type  $j$  labour that prevails in period  $t$ , for all  $j \in [0, 1]$ . Wages are set by the household and can be increased flexibly, however wages face downward rigidities such that the nominal wage cannot decrease freely - this is outlined further in [Section 4.3](#). Therefore the cost minization yields a set of demand schedules for each firm  $i$  and labour type

<sup>3</sup>Positive trend growth in technology is neglected here for simplicity, providing motivation for future extensions. Adding in positive growth in technology has important implications for optimal steady state price inflation.

$j$ , given the firm's total employment  $N_t(i)$ , which highlights that hiring of a particular labor type is due to their relative wage and substitutability

$$N_t(i, j) = \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t(i) \quad (3)$$

for all  $i, j \in [0, 1]$ , where

$$W_t \equiv \left( \int_0^1 W_t(j)^{1-\epsilon_w} dj \right)^{\frac{1}{1-\epsilon_w}} \quad (4)$$

is an aggregate wage index. Through substituting equation (4) into equation (3) it can be shown that

$$\int_0^1 W_t(j) N_t(i, j) dj = W_t N_t(i),$$

which is a convenient aggregation result that will subsequently be used.

As well as hiring workers firms set the price of final goods in the economy following Calvo (1983), which is typical in a New Keynesian model. A firm in period  $t$  will choose the price  $P_t^*$  to maximize their current market value of profits, however, a firm may only reset their price with a probability  $1 - \theta$  in any given period. Hence their problem can be shown to be,

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \{ \Lambda_{t,t+k} (1/P_{t+k}) (P_t^* Y_{t+k|t} - C_{t+k}(Y_{t+k|t})) \}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left( \frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} \quad (5)$$

for  $k = 0, 1, 2, \dots$  where  $\Lambda_{t,t+k} \equiv \beta^k U_{c,t+k}/U_{c,t}$  is the stochastic discount factor,  $C(\cdot)$  is the nominal cost function. As shown in Galí (2015) solving this problem and rearranging accordingly, as well as conducting a first-order Taylor-approximation in the neighborhood of the zero inflation steady state, the following equation for price inflation  $\pi_t^p \equiv p_t - p_{t-1}$ <sup>4</sup>

$$\pi_t^p = \beta E_t \{ \pi_{t+1}^p - \lambda_p \hat{\mu}_t^p \} \quad (6)$$

where as in Galí (2015),  $\hat{\mu}_t^p \equiv \mu_t^p - \mu^p$  is the deviation of the average (log) price markup from its flexible price counterpart and  $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$ . Firms wish to raise their prices when the average price markup in the economy today or in the future is below the desired levels of the firms and hence prices rise when firms are able to change their prices and inflation arises from this.

## 4.2 Households and Unemployment

This section follows closely Galí (2015)'s Chapter 7 on unemployment and monetary policy.

### 4.2.1 Households

There exists a large number of identical households, whereby each household has a continuum of members represented by the unit square and indexed by a pair  $(j, s) \in [0, 1] \times [0, 1]$ . Here  $j \in [0, 1]$  represents the type of labour that the household member specialises in and  $s \in [0, 1]$  is the disutility that each household member faces from working. Disutility from work is given by  $\chi s^\varphi$  if he is employed and zero otherwise, where  $\chi > 0$  and  $\varphi > 0$  are exogenous parameters. Full risk sharing within the household is assumed and therefore given the separability of preferences this implies the same level of consumption for each household member. The household's period utility is given by the integral of its members' utilities and can therefore be written as follows

<sup>4</sup>Lower case letters throughout this short paper denote the (natural) log of the corresponding variable, e.g.,  $\pi \equiv \log \Pi$

$$\begin{aligned}
U(C_t\{\mathcal{N}_t(j)\}; Z_t) &\equiv \left( \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi \int_0^1 \int_0^{\mathcal{N}_t(j)} s^\varphi ds dj \right) Z_t \\
&= \left( \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi \int_0^1 \frac{[\mathcal{N}_t(j)]^{1+\varphi}}{1+\varphi} dj \right) Z_t
\end{aligned}$$

where  $C_t \equiv \left( \int_0^1 C_t(i)^{1-\frac{1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p-1}}$  is a consumption index,  $C_t(i)$  is the quantity consumed of good  $i$ , for  $i \in [0, 1]$ , and  $\mathcal{N}_t(j) \in [0, 1]$  is the fraction of members specialised in type  $j$  labour who are employed in period  $t$ . The preference shifter,  $z_t$ , is assumed to follow an AR(1) process with  $\rho_z = 0.5$  as in Galí (2015) and  $\varepsilon_t^z$  is a white noise process with zero mean and variance  $\sigma_z^2 = 1$  and can be rationalised as a demand shock.

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$

Each household seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t\{\mathcal{N}_t(j)\}; Z_t)$$

subject to a sequence of flow budget constraints given by

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + \int_0^1 W_t(j) N_t(j) dj + D_t. \quad (7)$$

Here  $P_t(i)$  is the price of good  $i$ ,  $W_t(j)$  is the nominal wage for labour type  $j$  and  $B_t$  represents purchases of a nominally riskless one-period bond,  $Q_t$  is the price of that bond and  $D_t$  is a lump-sum component of income, which can be thought of as dividends from ownership of firms.

Optimal demand for each good resulting from utility maximization takes the form:

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon_p} C_t \quad (8)$$

where  $P_t \equiv \left( \int_0^1 P_t(i)^{1-\frac{1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p-1}}$  denotes the price index for final goods. This takes a familiar form and can be shown that  $\int_0^1 P_t(i) C_t(i) di = P_t C_t$ .

The household's intertemporal optimality condition is given by and Euler equation of the form

$$Q_t = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{Z_{t+1}}{Z_t} \right) \left( \frac{P_t}{P_{t+1}} \right) \right\} \quad (9)$$

The wage setting is done by the workers, or a union that represents all workers specialised in it. The contribution of this short paper is to have wages adhere to DNWR as seen in Section 4.3

#### 4.2.2 Unemployment

Unemployment in this model follows that of Galí et al. (2011), hence unemployment arises due to the discrepancy in wages set by the labour union and firm. Taking into account the household members' disutility from working that individual will be willing to work, and therefore be a part of the labour force if and only if

$$\frac{W_t(j)}{P_t} \geq \chi C_t^\sigma s^\varphi.$$

Therefore, the individual will be willing to work if the real wage achieved exceeds the disutility from working given in units of consumption, hence multiplied by the household's marginal utility of consumption.

The marginal supplier of type  $j$  labour, denoted  $L_t(j)$ , is given by

$$\frac{W_t(j)}{P_t} = \chi C_t^\sigma L_t(j)^\varphi \quad (10)$$

Following this we can define the aggregate labour force as  $L_t \equiv \int_0^1 L_t(j) dj$ . Then taking logs and integrating over  $j$  it is possible to derive the following approximate relation:

$$w_t - p_t = \sigma c_t + \varphi l_t + \xi. \quad (11)$$

Equation (11) can be thought of as a participation equation where by first-order approximation around the symmetric steady state  $w_t \simeq \int_0^1 w_t(j) dj$ ,  $l_t \simeq \int_0^1 l_t(j) dj$  and  $\log(\chi) = \xi$ .

Following Galí (2011a); Galí et al. (2011) the unemployment rate  $u_t$  is defined as the log difference between the labour force and employment:

$$u_t \equiv l_t - n_t. \quad (12)$$

Combining the average wage markup  $\mu_t^w \equiv (w_t - p_t) - (\sigma c_t + \varphi n_t + \xi)$  with equation (11) and equation (12), provides us with a linear relation between the wage markup and the unemployment rate

$$\mu_t^w = \varphi u_t. \quad (13)$$

Employment is demand determined with the labour demand given by the inverse production function in logs

$$n_t = \frac{1}{1 - \alpha} (y_t - a_t). \quad (14)$$

Following Galí (2015) the natural rate of unemployment,  $u_t^n$ , is defined as that which would prevail in the absence of nominal wage rigidities. In Galí (2015) the natural rate of unemployment is set to 0.05, consistent with an average unemployment rate of 5%, to take into account frictional unemployment.

$$u^n = \frac{\mu^w}{\varphi}. \quad (15)$$

It is important to note that due to the monopoly the households have over labour that even under flexible wages a wage markup,  $\mu^w$ , will still be positive and hence natural rate of unemployment will also be greater than 0.

### 4.3 Wage Setting and Downward Nominal Wage Rigidity

Households set wages by maximising their utility with respect to their budget constraint as well as the sequence of labour demand schedules given in equation (3). Since the households have market power over wage setting in a flexible wage setting environment, one without any nominal rigidities, they would set wages as in equation (16) as a markup over their marginal rate of substitution. The markup here is given by as  $\mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$ .

$$\frac{W_t^*}{P_t} = \frac{W_t}{P_t} = \mathcal{M}_w C_t^\sigma N_t^\varphi \quad (16)$$

The main innovation of this short paper is the inclusion of Downward Nominal Wage Rigidities that has been inspired by Schmitt-Grohé and Uribe (2016) and empirically motivated in Section 2. With DNWR the occasionally binding constraint is imposed of

$$W_t \geq \gamma W_{t-1} \quad \gamma > 0, \quad (17)$$

where  $\gamma$  defines the degree of downward nominal wage rigidity. Such that when  $\gamma = 0$  there is full wage flexibility and the higher  $\gamma$ , the more downwardly rigid are nominal wages. If  $\gamma \geq 1$  we see absolute downward wage rigidity, found empirically in Schmitt-Grohé and Uribe (2016) for many countries<sup>5</sup>. The parameter  $\gamma$  is chosen to emulate that of Schmitt-Grohé and Uribe (2016), such that  $\gamma = 0.99$  at a quarterly frequency implies that nominal wages can decline up to 4 percent per year. Therefore the wage setting rule divided by the price level for convenience can now be written as:

$$\frac{W_t}{P_t} = \max\left\{ \mathcal{M}_w C_t^\sigma N_t^\varphi, \gamma W_{t-1} \frac{1}{P_t} \right\} \quad (18)$$

<sup>5</sup>See Schmitt-Grohé and Uribe (2016) for an extensive list, as an example it includes countries such as Bulgaria, Ireland, Italy, Spain, Slovenia.

For the first half of this paper the downward nominal wage rigidity constraint is added into the model exogenously. This means that the households are only able to see the constraint once they reach it. This lends itself to a first attempt at understanding the effect of adding in such an occasionally binding constraint and provides a simple modeling environment using Occbin as discussed in [Guerrieri and Iacoviello \(2015\)](#) and explained in [Appendix A](#) for this model.

Later in [section 6](#) households will be able to maximise their utility with respect to their wage while taking into account the downward nominal wage constraint. A model of this type cannot be solved using perturbation technique and therefore I use Smolyak collocation, a projection method, to solve the model. The computational technique is outlined in [Appendix B.2](#).

#### 4.4 Equilibrium and Calibration

Below are the equations that characterize the equilibrium conditions in the New Keynesian Framework developed above. It is important to note that these correspond exactly to that in [Galí \(2015\)](#) except equation (21), which relates to log-linearizing the DNWR condition. Where  $\tilde{y}_t = y_t - y^n$ , the output gap and  $\tilde{\omega}_t$  is the real wage gap.

$$\tilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}^p\} - r_t^n) + E_t\{\tilde{y}_{t+1}\} \quad (19)$$

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \varkappa_p \tilde{y}_t + \lambda_p \tilde{\omega}_t \quad (20)$$

$$w_t \geq \log(\gamma) + w_{t-1} \quad (21)$$

$$\tilde{\omega}_t \equiv \tilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta\omega_t^n \quad (22)$$

$$\text{Taylor rule: } i_t = \rho + \phi_\pi \pi_t^p + \phi_y \tilde{y}_t \quad (23)$$

The calibration used is standard except the downward nominal wage rigidity parameter  $\gamma$ , which is taken from [Schmitt-Grohé and Uribe \(2016\)](#).

Table 1: Table summarizing parameter values

Symbol	Parameter	Value	Reasoning
$\alpha$	Capital Share	0.25	Standard
$\beta$	Discount Factor	0.99	Standard
$\varphi$	Inverse Frisch elasticity	5	Galí 2015
$\epsilon_p$	Demand elasticity for goods	9	Galí 2015
$\epsilon_w$	Demand elasticity for labour services	4.5	Galí 2015
$\theta_p$	Calvo parameter for price	0.75	Galí 2015
$\phi_\pi$	Taylor weight on inflation	1.5	Galí 2015
$\phi_y$	Taylor weight on output gap	0.125	Galí 2015
$u^n$	Natural rate of unemployment	5%	Galí 2015
$\gamma$	DNWR Parameter	0.99	Schmitt-Grohé and Uribe

## 5 Results

This section houses the main results of the paper for the simple model, whereby the model is solved around a zero inflation steady state and exogenous downward nominal wage rigidity constraint. A more realistic model that is solved around positive trend inflation and allows the household to maximise over the occasionally binding constraint can be seen in [Section 6](#).

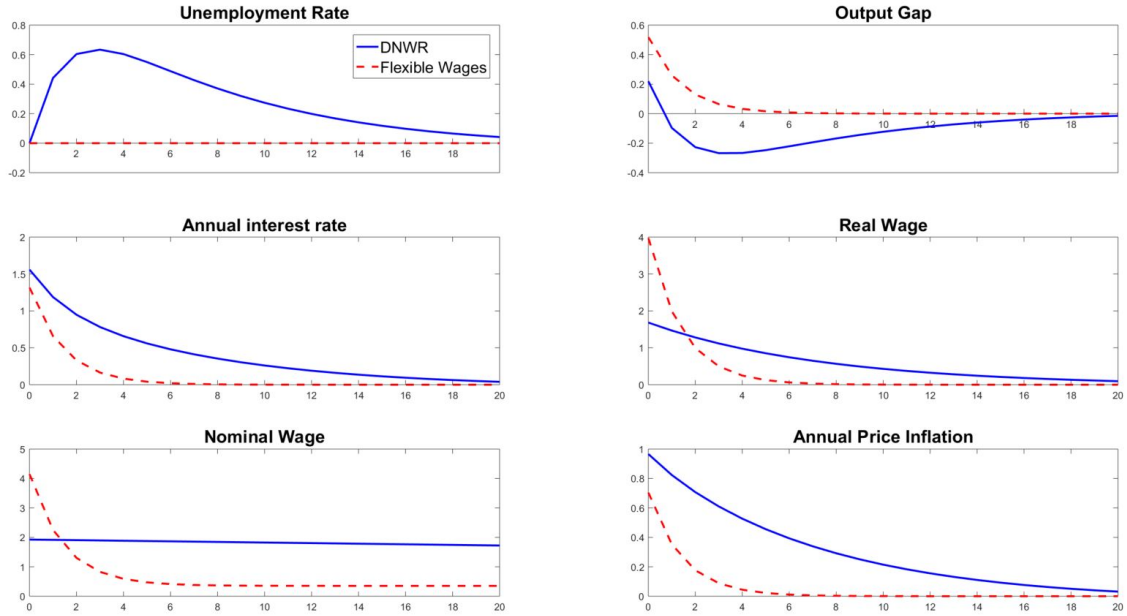
The figures below highlight how a simple New-Keynesian economy is affected by adding in downward nominal wage rigidities. Of significance is the asymmetric response of the interest rate under monetary policy rules, as well as the optimal monetary policy. Furthermore, comparing the welfare loss under different monetary policy rules motivates exploration for an optimal simple rule that attempts to mimic the optimal simple rule.

[Figure 1](#) contrasts the response of a New-Keynesian model with flexible wage versus downward nominal wage rigidities. As shown below under flexible wages the wage is free to adjust such that unemployment stays constant at the steady state. This is in contrast to an unrealistic response of an increase in unemployment rate due to a sustained increase to firms costs through artificially high wages. This strange results subsides when the downward nominal wage rigidity constraint is



endogenised in Section 6. Due to the increase in unemployment we see a negative output gap. The high price inflation caused by the artificially high wages helps to deflate the real wage back to its steady state value.

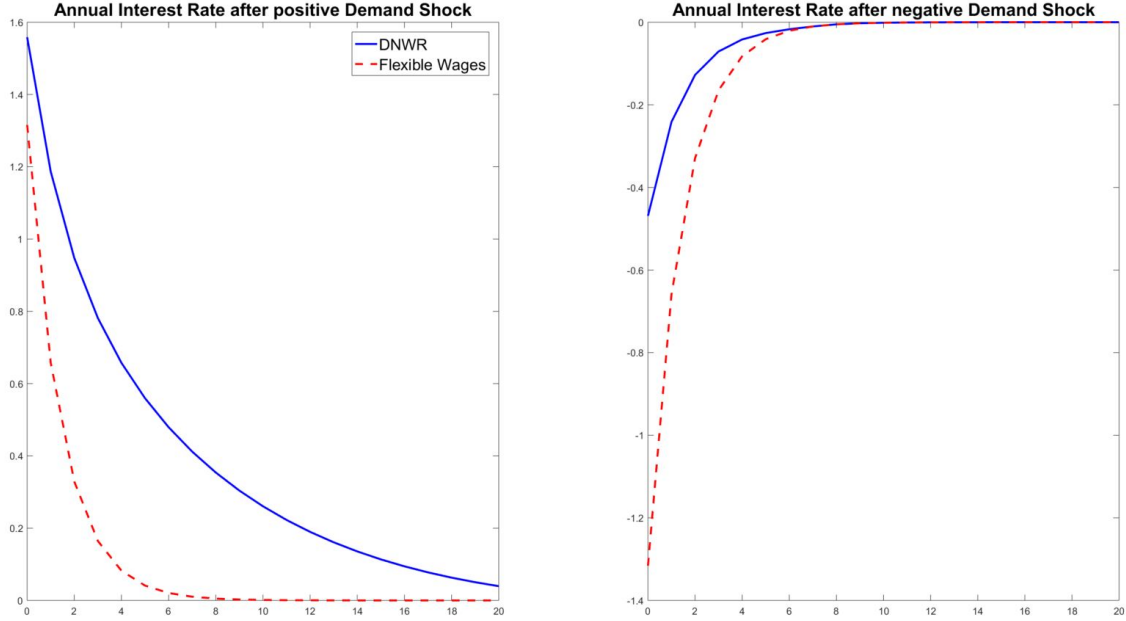
Figure 1: Positive Demand Shock under the Taylor Rule



Impulse response for variables facing a positive demand shock. The demand shock follows an AR(1) with  $\rho_z = 0.5$  and  $\sigma = 1$ . The Taylor rule is  $i_t = 0.001 + 1.5\pi_t^p + 0.125\hat{y}_t + \nu_t$ .

Due to the constraint occasionally binding we see in Figure 2 the asymmetric response of monetary policy when the central bank follows a Taylor rule. Under flexible or Calvo wages the response of the central bank is symmetric, however, when the economy suffers from DNWR the central bank reaction is stronger under a positive demand shock than a negative demand shock. This is because under a positive demand shock nominal wages rise, pushing up inflation and causing the central bank to react strongly, however, since wages cannot fall during a negative demand shock inflation will remain close to its steady state the central bank, which cares most about inflation deviations, will not react as much.

Figure 2: Taylor Rule following a +ve and -ve Demand Shock



Impulse response of the nominal interest rate facing a positive and negative demand shock. The demand shock follows an AR(1) with  $\rho_z = 0.5$  and  $\sigma = 1$ . The Taylor rule is  $i_t = 0.001 + 1.5\pi_t^p + 0.125\hat{y}_t + \nu_t$ .

The optimal monetary policy is a perfect-foresight solution derived from minimising the discounted sum of welfare loss, shown in equation (24), subject to the equilibrium condition in section 4.4. As is standard in the New Keynesian model the optimal monetary policy is able to fully neutralise any effect a demand shock would have on the welfare loss through changes in the interest rate. This is not the case when faced with a technology shock as it affects the natural output and interest rate of the economy.

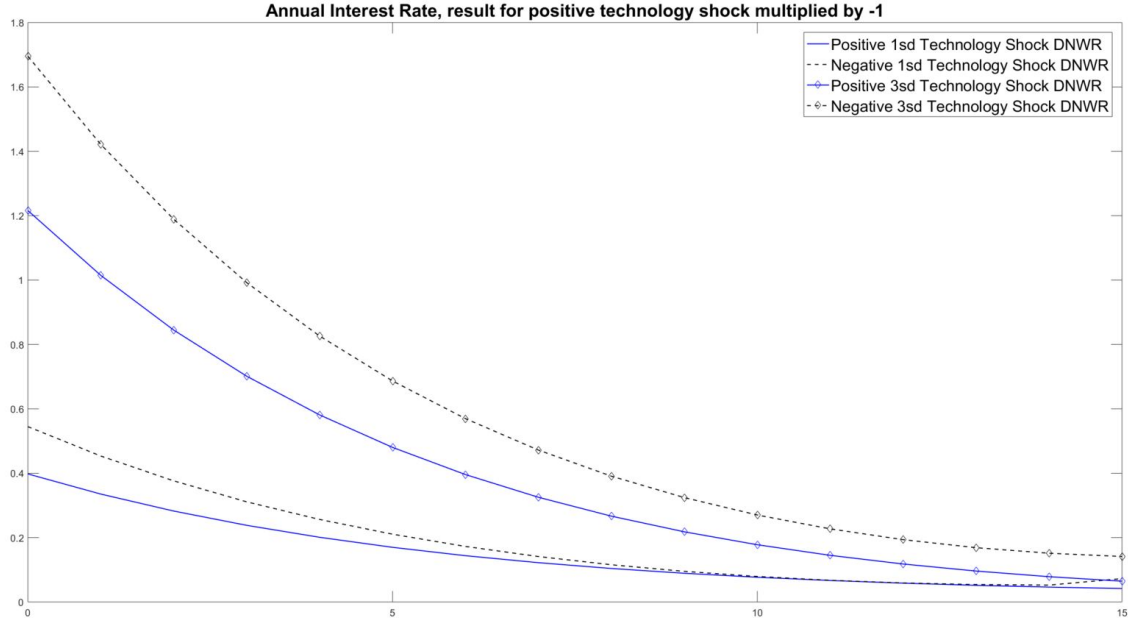
$$\frac{1}{2}E_0 \sum_{t=0}^{\infty} \beta^t \left[ \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) var(\tilde{y}_t) + \frac{\epsilon_p}{\lambda_p} var(\pi_t^p) \right] \quad (24)$$

Therefore the average welfare loss used to compare different monetary policy rules is given by:

$$\mathbb{L} = \frac{1}{2} \left[ \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) var(\tilde{y}_t) + \frac{\epsilon_p}{\lambda_p} var(\pi_t^p) \right] \quad (25)$$

The optimal monetary policy response to a demand shock is symmetric, due to the aforementioned reasons. Therefore figure 3 analyses if the optimal monetary policy is asymmetric following a positive and negative technology shock of one standard deviation and three standard deviations. Figure 3 shows that it is an optimal response of the central bank to act asymmetrically. However, in this economy an amplification effect does not exist with DNWR.

Figure 3: Asymmetric Optimal Monetary Policy: Technology Shock



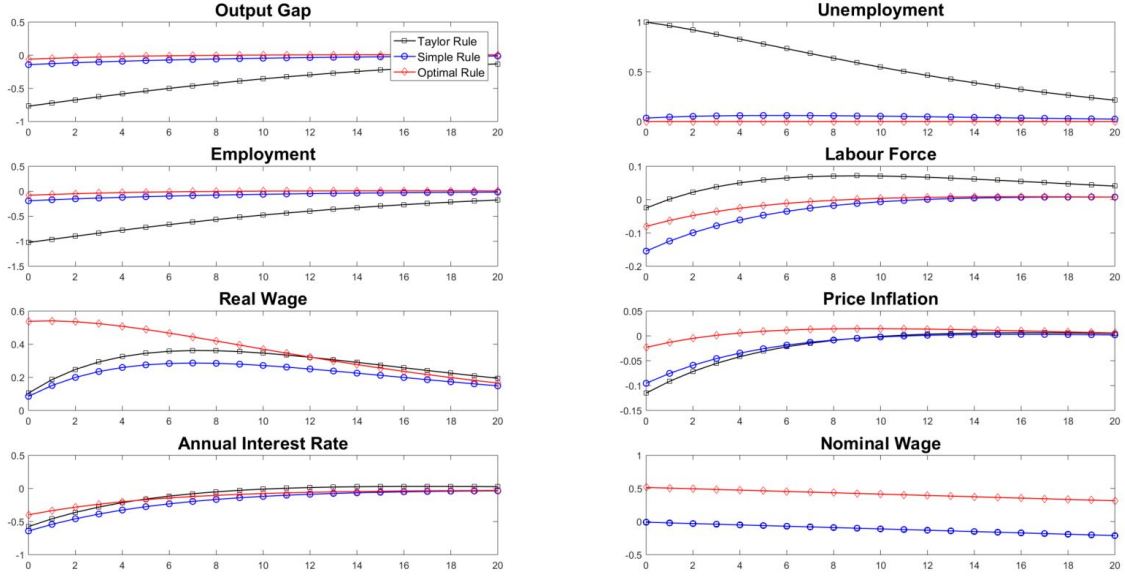
Impulse response of the nominal interest rate under optimal monetary policy facing a positive and negative technology shock. The technology shock follows an AR(1) with  $\rho_a = 0.9$  and  $\sigma = \{1, 3\}$ .

Figure 4 assesses whether current monetary policy rules can match the optimal monetary policy response in this environment. The simple rule is taken from Galí (2015) and performs well under Calvo price and wage rigidities.  $\hat{u}_t$  here is defined as the log difference between the unemployment rate and natural level of unemployment:

$$i_t = 0.01 + 1.5\pi_t^p - 0.5\hat{u}_t. \tag{26}$$

In Figure 4 it can be seen that the simple rule proposed in Galí (2015) performs well in limiting the fall in the output gap due to reacting to changes in the unemployment, however the response of price inflation is closer to the Taylor rule than the optimal monetary policy.

Figure 4: Optimal Policy vs Taylor Rule vs Simple Rule: +ve Techno Shock



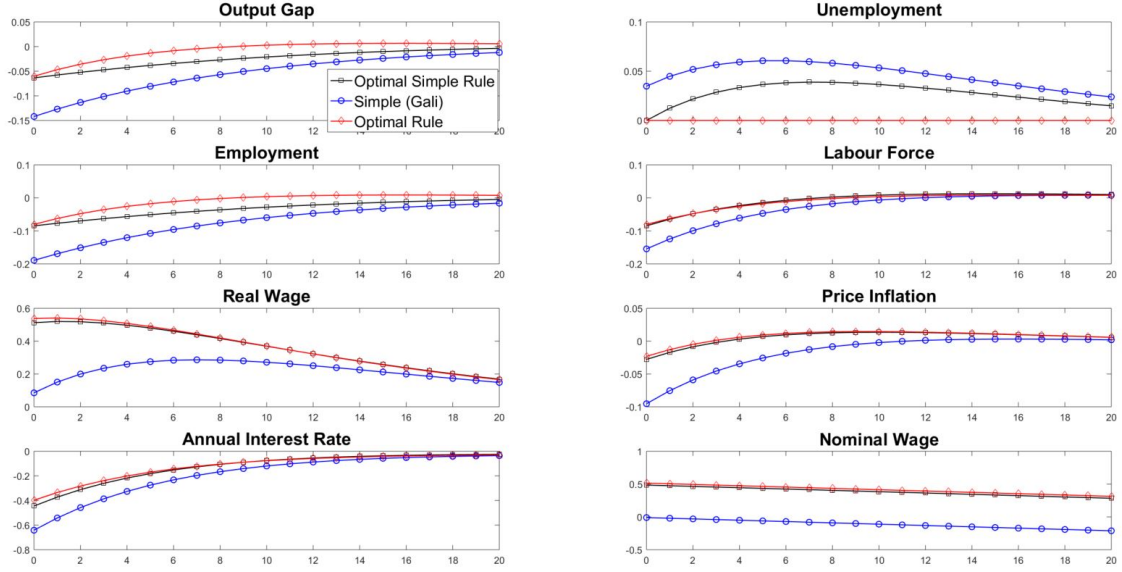
Impulse response for variables facing a positive technology shock. The technology shock follows an AR(1) with  $\rho_a = 0.9$  and  $\sigma = 1$ . The Taylor rule is  $i_t = 0.001 + 1.5\pi_t^p + 0.125\hat{y}_t + \nu_t$  and simple rule is  $i_t = 0.01 + 1.5\pi_t^p - 0.5\hat{u}_t$ .

The outcome of the simple rule in Figure 4 and welfare losses presented in table 2 and table 5 in Appendix C.1 motivated a search for a new optimal simple rule. The coefficients on the new optimal simple rule are found by simulating the economy over demand and technology shocks and optimising these values to produce the lowest welfare loss possible. The optimal simple rule displayed in equation (27) is similar to the simple rule provided in Galí (2015), however, reacts stronger to both inflation and the unemployment gap - difference in unemployment and its natural rate. Moreover, this new optimal simple rule assigns a higher weight on deviations in unemployment compared to inflation than the alternative simple rule in equation (26). Here  $\rho = -\log(\beta) = 0.01$  and therefore the OSR can be given as:

$$\text{Optimal Simple Rule (OSR)} : i_t = 0.01 + 4\pi_t^p - 2\hat{u}_t. \quad (27)$$

Figure 5 presents the impulse response from a positive technology shock comparing the optimal monetary policy, optimal simple rule and the simple rule. The optimal simple rule closely follows the optimal monetary policy and is able to approximately replicate the optimal simple rule. In comparison to Figure 4, which shows the outcome under a Taylor rule, unemployment deviations have been significantly dampened. Paradoxically the optimal monetary policy and optimal simple rule allow for increases in nominal wage from a positive technology shock even though wage decreases are sluggish. Allowing a higher wage allows for price inflation to fall less and the labour force decrease to be muted. Under the optimal simple rule the output gap also falls less compared to the alternative simple rule, due to employment decreasing less.

Figure 5: Optimal Monetary Policy, OSR, Simple Rule: +ve Techno Shock



Impulse response for variables facing a positive technology shock. The technology shock follows an AR(1) with  $\rho_a = 0.9$  and  $\sigma = 1$ . The Taylor rule is  $i_t = 0.001 + 1.5\pi_t^p + 0.125\hat{y}_t + \nu_t$ , simple rule is  $i_t = 0.01 + 1.5\pi_t^p - 0.5\hat{u}_t$  and optimal simple rule is  $i_t = 0.01 + 4\pi_t^p - 2\hat{u}_t$

Table 2 displays the welfare loss, using equation (25), from positive technology and demand shocks. The negative shock counterpart to this table can be found in Appendix C.1. Strict targeting rules keep price inflation and wage inflation, respectively, at their steady state values and adjust the interest rate accordingly. The optimal rule provides a lower bound on the welfare loss in the table. From the table it is evident that the optimal simple rule (OSR) performs well with both positive technology and demand shocks. This is in contrast to the Taylor rule, which provides a relatively high welfare loss in comparison to the other rules in Table 2.

Table 2: Evaluation of MP rules following **positive** Technology and Demand Shocks

	Optimal	Strict Targeting		Other rules		
		Price	Wage	Taylor	Simple	OSR
Technology shocks						
$\sigma(\pi^p)$	0.008	0	0.025	0.08	0.042	0.010
$\sigma(\pi^w)$	0.096	0.215	0	0.157	0.044	0.110
$\sigma(\hat{y})$	0.017	0.489	0.033	0.113	0.032	0.019
$\mathbb{L}$	0.016	0.956	0.130	1.468	0.372	0.023
Demand Shocks						
$\sigma(\pi^p)$	0	0	0	0.067	0.132	0.039
$\sigma(\pi^w)$	0	0	0	0.353	0.663	0.201
$\sigma(\hat{y})$	0	0	0	0.106	0.083	0.025
$\mathbb{L}$	0	0	0	0.994	3.7018	0.320

## 6 Extensions

This section houses part of the extensions of the model presented in the main text above. The main differences are i) the model is not log-linearised around a zero percent steady-state inflation rate and ii) the model endogenises the occasionally binding constraint, which allows the households to maximise their utility taking into account that wages are downwardly rigid. This moves the model to a more appealing setting, enables the exploration of the optimal trend inflation rate and provides more sensible results to exogenous shocks<sup>6</sup>.

<sup>6</sup>The main illustration of this is the increase in unemployment from a positive demand shock seen in Fig 1

Endogenising the downward nominal wage rigidity constraint means that the household's maximisation problem needs to be revisited. Since the only optimisation problem impacted is the wage maximisation this is the focus on the equations below. The variables in parenthesis  $\lambda_t$  and  $\Omega_t$  correspond to the lagrange multipliers, or shadow cost of the constraints. The wage setter (household or labour union for a worker of type  $j$ ) seeks to maximise their utility flow subject to labour demand, the budget constraint and the downward nominal wage rigidity.

$$\max_{W_t(j)} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t(j)^{1+\eta}}{1+\eta} \right] \right\} Z_t$$

subject to:

$$N_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t$$

$$(\lambda_t) \quad P_t C_t + E_t[Q_t D_{t+1}] \leq D_t + W_t(j) N_t(j) - T_t$$

$$(\Omega_t) \quad W_t(j) \leq \gamma W_{t-1}(j)$$

The solution to this problem, combining the previous first order conditions found in Section 4, can be seen below. It is convenient when simulating the model to represent this condition in terms of nominal wage inflation  $\Pi_t^w = W_t/W_{t-1}$  and the real wage rather than solely nominal wages and nominal wage changes.

$$\Pi_t^w \Omega_t = (\epsilon_w - 1) \frac{W_t}{P_t} Z_t C_t^{-\sigma} N_t - \epsilon_w N_t^{1+\eta} Z_t - \beta E_t[\Omega_{t+1} \Pi_{t+1}^w]$$

$$\text{Complementary slackness: } \Omega_t (\Pi_t^w - \gamma) = 0$$

Non-negativity constraints can be shown to be:

$$\Omega_t \geq 0$$

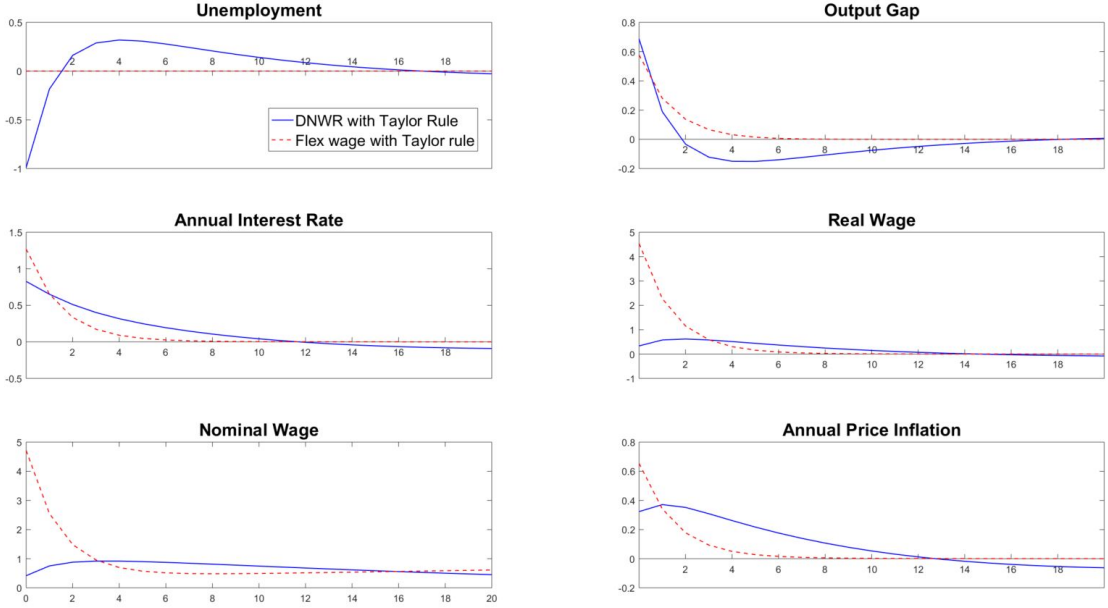
$$\Pi_t^w - \gamma \geq 0$$

Therefore when the DNWR constraint does not bind the associated lagrange multiplier will equal zero,  $\Omega_t = 0$ , and we are back to the flexible wage schedule where the real wage is a markup over the marginal rate of substitution between consumption and labour.

## 6.1 Extension Results

Figure (6) displays the impulse response from a one standard deviation positive demand shock, comparing the response of an economy with downward nominal wage rigidities and flexible wages under a Taylor rule with  $\phi_\pi = 1.5$ . For this figure no positive trend inflation is assumed, which allows for a direct comparison to Figure (1). Figure (6) can be used as a comparison to Figure (1) which had exogenous DNWR and a zero inflation steady state. Endogenising the occasionally binding constraint means that the households now only increase their nominal wage by 0.4% instead of 2% since they understand that they are downwardly rigid. Therefore the existence of the constraint causes wage increases to be muted, creating an endogenous rigidity when increasing wages. Figure (6) also shows a more sensible response of unemployment to a positive demand shock, with it decreasing. However, even though households understand that the DNWR shock exists they still find themselves constrained as the wage is sluggish to fall.

Figure 6: Taylor Rule vs Optimal Monetary Policy - Positive Demand Shock



Wage restraint, the phenomenon displayed in Figure (6), has also been found empirically in [Elsby \(2009\)](#) and motivated by a stylised model of workers resistant to nominal wage cuts. Instead in this paper workers understand that wages are downwardly rigid and therefore limit their demand for higher wages as unemployment will arise when the DNWR constraint binds. This mechanism is due to the equilibrium wage being artificially high if the DNWR constraint binds, which causes labour supply to remain high, the wage to not adjust downwards and therefore the firm cannot afford to hire all available workers and unemployment arises.

## 6.2 Optimal Trend Inflation and Taylor Rule

Unlike in Section 5 I now introduce welfare as the present discounted value of the flow utility of a representative agent, which will be used to assess the optimal steady state inflation rate and Taylor rule coefficients under DNWR. The previous measurement of optimality, which used a second order approximation around a zero inflation steady state, cannot be used to assess positive trend inflation in that form. Moreover, this measure should be able to handle the highly non-linear nature of the occasionally binding constraint and therefore provide a more accurate measure of welfare. The optimal inflation and Taylor rule coefficients will be disciplined by choosing the values which maximise the present discounted value of the flow utility of a representative agent seen in equation (28).

$$V_t = U_t(C_t, N_t) + \beta E_t V_{t+1} \quad (28)$$

In contrast, the Ramsey Planner, which provides the optimal solution to this model, would maximise the households welfare taking into account the first order conditions from our non-linear model seen in Appendix A. For now I focus on a standard Taylor rule that focuses on deviations in inflation and output from their steady state levels, outlined below.

$$\frac{R_t}{R} = \left( \frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left( \frac{Y_t}{Y} \right)^{\phi_y} \quad (29)$$

Using a grid search method<sup>7</sup> over  $\{\Pi, \phi_\pi, \phi_y\}$  the economy is simulated for 300,000 periods of shocks<sup>8</sup> and the mean value of the households welfare is calculated  $V_t$  and transformed into its consumption equivalent amount in comparison to a zero trend inflation steady state and standard

<sup>7</sup>Future work which will provide a robustness check will use a non-linear solver such as the Newton-Raphson Method to determine the optimal  $\{\Pi, \phi_\pi, \phi_y\}$ . Moreover, technological growth will need to be added to the model as well as this plays an important role in finding the optimal level of trend inflation.

<sup>8</sup>Technology and Demand shocks are simulated separately and the welfare values from the simulation are then compared.

Taylor rule coefficients  $\{\phi_\pi = 1.5, \phi_y = 0.25\}$ . The finding is suggestive of ‘greasing the wheels’ and is within a sensible range of what others have found. In this model the optimal trend inflation rate is 0.75% to 1% with  $\phi_\pi \in [3.5, 4]$  and  $\phi_y = 0.25$ . The higher trend inflation helps to deflate the Downward Nominal Wage Rigidity with the higher-than-typical reaction to inflation likely being needed to assure determinacy of the model. Other papers, such as [Kim and Ruge-Murcia \(2009\)](#) who use asymmetric wage adjustment costs, find that the optimal trend inflation is 0.35%. However, using asymmetric wage adjustment costs and heterogeneous agents [Fagan and Messina \(2009\)](#) find a much larger range of optimal trend inflation, 0% to 5% depending on calibration used. The tables below summarise the optimal calibration exercise:

#### Welfare Analysis: Optimal calibration of Taylor Rule

Table 3: Demand shocks

$\Pi$	$\phi_\pi$	$\phi_y$	Consumption Equivalence
0.75%	3.5	0.25	0.58%
1.00%	3.5	0.25	0.56%
0.5%	3.5	0.25	0.54%

Table 4: Technology shocks

$\Pi$	$\phi_\pi$	$\phi_y$	Consumption Equivalence
0.75%	3.5	0.25	0.60%
1.00%	4	0.25	0.59%
1.00%	3.5	0.25	0.59%

Consumption equivalence is calculated from differences in the mean of discounted household flow utility ( $V_t = U_t(C_t, N_t) + \beta E_t V_{t+1}$ ) after 300,000 periods of uniformly distributed shocks. The welfare compared is from a Taylor rule with  $\{\Pi = 0\%, \phi_\pi = 1.5, \phi_y = 0.25\}$ .

Further work will be completed to extend the model, mimicking the work done above to find an optimal simple rule in this set-up - providing a further contribution to the literature.

## 7 Conclusion

In conclusion adding an occasionally binding constraint into the New-Keynesian model such as a downward nominal wage rigidity seen throughout this paper can distort the standard results of a New-Keynesian model with flexible or Calvo wages. This work has found that a DNWR constraint can cause boom-bust cycles from a positive demand shock if the agents within the model are not affected by the constraint until they receive a shock that will cause them to reach the constraint. The optimal monetary policy in this setup is asymmetric and there are gains in welfare to be made over the Taylor rule by finding a new optimal simple rule - one that reacts stronger to changes in unemployment. Taking the constraint seriously and embedding it into the households problem and solving the non-linear model with positive trend inflation leads to further support for ‘greasing the wheels’, allowing positive inflation in the steady state to deflate real wage changes, which leads to welfare gains. Once the constraint is fully internalised, such that the wage setters understand its existence even during periods that they are unconstrained, wage increases become dampened even though they are flexible upwards, a finding also shown in [Elsby \(2009\)](#) and [Wolf \(2018\)](#). The main contribution of this work comes from embedding the DNWR constraint from [Schmitt-Grohé and Uribe \(2016\)](#) into a New-Keynesian model, finding wage restraint and a new optimal simple rule whilst providing more support to positive trend inflation.

In general this work outlines the importance of understanding the distribution of wage changes in the data, how this may change across countries and how it affects the optimal monetary policy.



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## A Derivation

Appendix A houses the equilibrium equations for the full non-linear New Keynesian model with Downward Nominal Wage Rigidities. In the latter part of this section the steady state of this model is outlined. Since the nominal wage is not constrained in the steady-state the lagrange multiplier associated with the downward nominal wage rigidity constraint is zero, such that  $\Omega_t = 0$ , and the steady state equations are similar to those found in most medium-scale New-Keynesian models.

### A.1 Full model

$$Q_t = \frac{\beta \left( \frac{C_{t+1}}{C_t} \right)^{(-\sigma)} \frac{Z_{t+1}}{Z_t}}{\Pi_{t+1}} \quad (30)$$

$$R^n_t = \frac{1}{Q_t} \quad (31)$$

$$Y_t = A_t \left( \frac{N_t}{S_t} \right)^{1-\alpha} \quad (32)$$

$$R^n_t = \Pi_{t+1} R^r_t \quad (33)$$

$$R^n_t = \frac{\Pi_{SS}}{\beta} \left( \frac{\Pi_t}{\Pi_{SS}} \right)^{\phi_\pi} \left( \frac{Y_t}{\bar{Y}} \right)^{\phi_y} e^{\mu_t} \quad (34)$$

$$C_t = Y_t \quad (35)$$

$$\log(A_t) = \rho_a \log(A_{t-1}) + \varepsilon_{at} \quad (36)$$

$$\log(Z_t) = \rho_z \log(Z_{t-1}) - \varepsilon_{zt} \quad (37)$$

$$MC_t = \frac{\frac{W}{P}_t}{S_t \frac{Y_t(1-\alpha)}{N_t}} \quad (38)$$

$$1 = \theta \Pi_t^{\epsilon-1} + (1-\theta) \Pi_t^*{}^{1-\epsilon} \quad (39)$$

$$S_t = (1-\theta) \Pi_t^*{}^{\frac{(-\epsilon)}{1-\alpha}} + \theta \Pi_t^{\frac{-\epsilon}{1-\alpha}} S_{t-1} \quad (40)$$

$$\Pi_t^*{}^{1+\epsilon \frac{\alpha}{1-\alpha}} = \frac{\epsilon \frac{x_{1t}}{x_{2t}}}{\epsilon - 1} \quad (41)$$

$$x_{1t} = MC_t Y_t Z_t C_t^{(-\sigma)} + \beta \theta \Pi_{t+1}^{\epsilon + \frac{\alpha \epsilon}{1-\alpha}} x_{1t+1} \quad (42)$$

$$x_{2t} = Y_t Z_t C_t^{(-\sigma)} + \beta \theta \Pi_{t+1}^{\epsilon-1} x_{2t+1} \quad (43)$$

$$\Pi_t^w \Omega_t = (\epsilon_w - 1) \frac{W_t}{P_t} C_t^{-\sigma} N_t - \epsilon_w N_t^{1+\eta} - \beta \Omega_{t+1} \Pi_{t+1}^w \quad (44)$$

$$\frac{W_t}{P_t} = \frac{\Pi_t^w W_{t-1}}{\Pi_t P_{t-1}} \quad (45)$$

$$\mu_t = \rho_a \mu_{t-1} + \epsilon_{\mu t} \quad (46)$$

$$V_t = Z_t \left( \log(C_t) - \frac{N_t^{1+\varphi}}{1+\varphi} \right) + \beta V_{t+1} \quad (47)$$

$$\Omega_t(\Pi_t^w - \gamma) = 0 \quad (48)$$

$$\Pi_t^w - \gamma \geq 0 \quad (49)$$

$$\Omega_t \geq 0 \quad (50)$$

## A.2 Steady State

The steady state is not affected by the DNWR constraint.

$$A = 1$$

$$\mu = 0$$

$$Z = 1$$

$$\Pi^* = \left[ \frac{1 - \theta \Pi^{\epsilon-1}}{1 - \theta} \right]^{\frac{1}{1-\epsilon}}$$

$$S = \frac{(1 - \theta) \Pi^{\frac{-\epsilon}{1-\alpha}}}{1 - \theta \Pi^{\frac{\epsilon}{1-\alpha}}}$$

$$MC = \frac{\epsilon - 1}{\epsilon} \Pi^{*\frac{1+\alpha\epsilon}{1-\alpha}} \frac{1 - \beta \theta \Pi^{\epsilon + \frac{\alpha\epsilon}{1-\alpha}}}{1 - \beta \theta \Pi^{\epsilon-1}}$$

$$Q = \frac{\beta}{\Pi}$$

$$R = \frac{1}{Q}$$

$$r = \frac{R}{\Pi}$$

$$N = \left[ MC(1 - \alpha) \frac{\epsilon_w - 1}{\epsilon_w} S^{\sigma - \alpha\sigma + \alpha} \right]^{\frac{1}{(1-\sigma)\alpha + \varphi + \sigma}}$$

$$C = A \left( \frac{N}{S} \right)^{1-\alpha}$$

$$Y = C$$

$$\frac{W}{P} = w = \frac{MC \cdot S \cdot Y(1 - \alpha)}{N}$$

$$x_1 = \frac{C^{-\sigma} Y \cdot MC}{1 - \beta \theta \Pi^{\frac{\epsilon + \alpha\epsilon}{1-\alpha}}}$$

$$x_2 = \frac{C^{-\sigma} Y}{1 - \beta \theta \Pi^{\epsilon-1}}$$

$$\Omega = 0$$

## B Computational technique

Two computation techniques have been used in this project. Firstly, Occbin by [Guerrieri and Iacoviello \(2015\)](#) is used as a first attempt to analyse the effect of DNWR on a standard New-Keynesian Model. Latterly, Smolyak Projection Method by [Smolyak \(1963\)](#) is used to provide more realistic analysis as it allows the agents to understand that the DNWR constraint exists. Below I outline both of these techniques used.

### B.1 Occbin

Most of the model simulations, impulse response functions and welfare losses were calculated using Dynare<sup>9</sup>, an extension to Matlab used for DSGE models. Dynare cannot typically be used when there is an occasionally binding constraint such as the DNWR, however, with help of the Occbin toolbox seen in [Guerrieri and Iacoviello \(2015\)](#) it is possible. Occbin uses first order perturbation but allows the solution to be highly non-linear. One disadvantage is that all agents within the model have no prior knowledge of the existence of the occasionally binding constraint, and therefore also does not capture precautionary behaviour.

At the start of the period the model is at the steady state and then when the households' wish to lower the nominal wage after the monetary policy shock, the model switches to that of the binding constraint and the wage reduction is forced to be sluggish. Appendix B.2 highlights a projection method, which provides a global solution, used to endogenize this occasionally binding constraint and will form the

### B.2 Smolyak Approximation

Section 6 displays the non-linear model with positive trend inflation and an occasionally binding constraint that the households maximise over. The model is solved using the Smolyak collocation method laid out in [Malin et al. \(2011\)](#) and implemented for a New-Keynesian model with a Zero-Lower-Bound constraint in [Fernández-Villaverde et al. \(2015\)](#). My solution technique closely follows the exercise provided by [Fernández-Villaverde et al. \(2015\)](#). Smolyak collocation allows for more state variables than other common projection methods as the number of terms of the approximating polynomial and grid points do not grow exponentially and therefore do not suffer as much as other techniques from the curse of dimensionality. One prominent example is [Fernández-Villaverde and Levintal \(2016\)](#), which uses 12 state variables and still retains accuracy and speed of computation.

Smolyak's algorithm introduced in [Smolyak \(1963\)](#) is a numerical technique using a sparse grid to efficiently solve multi-dimensional hypercubes. The technique ordered and selected the solution to a tensor-product rule importance of finding the quality of approximation to the problem. Smolyak's algorithm was then adapted by [Krueger and Kubler \(2004\)](#) to be used in an economic setting.

Following the steps found in the technical appendix of [Fernández-Villaverde et al. \(2015\)](#) I start by defining a state vector:

$$\mathbb{S}_t = (S_{t-1}, A_t, P_{t-1}, Z_t, W_{t-1})$$

With the exogeneous states in logs:

$$\hat{\mathbb{S}}_t = (S_{t-1}, \log(A_t), \log(Z_t), w_{t-1})$$

The equilibrium functions  $f = (f^1, f^2, f^3, f^4, f^5)$  characterize the dynamics of the model:

$$\log(C_t) = f^1(\hat{\mathbb{S}}_t)$$

$$\log(\Pi_t) = f^2(\hat{\mathbb{S}}_t)$$

$$\log(x_{1t}) = f^3(\hat{\mathbb{S}}_t)$$

$$\log(\Pi_t^w) = f^4(\hat{\mathbb{S}}_t)$$

$$\Omega_t = f^5(\hat{\mathbb{S}}_t)$$

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<sup>9</sup>The dynare files used were adapted from those created by Dr Johannes Pfeifer to replicate [Galí \(2015\)](#) and provided freely for use, as of which I am extremely grateful. A recent release of an unstable dynare version is required to be downloaded to run these files.

To define the hypercube (grid points) we then choose bounds on the state variables around their steady state levels. The bounds for the exogenous state variables are determined by their unconditional standard deviation.

Then to solve for  $f$  I use a time-iteration procedure:

- Guess on :  $\{\Pi_t, \Pi_t^w, \Omega_t\}$
- Update state to obtain:  $= \{S_t, \log(A_{t+1}), \log(Z_{t+1}), w_t\}$
- Using the state today and weights from a monomial rule calculate expectations of time  $t + 1$  variables in the model.
- Check whether initial guess was correct by using the euler equation, real wage equation and complementary slackness for the occasionally binding constraint - iterate over guess if not correct.
- With the time  $t$  equilibrium found at each of the collocation points, check if they differ from the  $t + 1$  values. If they are similar up to a tolerance level then stop.

## C Additional figures or tables

### C.1 Welfare loss for negative shock

With an occasionally binding constraint the response of a central bank following an interest rate rule or an optimal monetary policy can be asymmetric. Therefore it is important to look at welfare loss for different interest rate rules under positive and negative shocks separately. Hence, table 5 displays the welfare loss from negative shocks to provide a comparison with table 2 found in the main body of the paper.

As with a positive demand shock the optimal policy is able to change the interest rate such that no welfare is lost from the shock. The optimal simple rule in this scenario also does well, for negative technology and demand shocks. Strict price targeting performs well under demand shocks however this regime performs poorly under technology shocks.

Table 5: Evaluation of MP rules following **negative** Technology and Demand Shocks

	Optimal	Strict Targeting		Other rules		
		Price	Wage	Taylor	Simple	OSR
Technology shocks						
$\sigma(\pi^p)$	0.020	0	0.014	0.063	0.031	0.013
$\sigma(\pi^w)$	0.022	0.006	0	0.089	0.031	0.011
$\sigma(\tilde{y})$	0.033	0.582	0.020	0.040	0.020	0.101
$\mathbb{L}$	0.090	3.294	0.133	1.469	0.367	0.090
Demand Shocks						
$\sigma(\pi^p)$	0	0	0	0.003	0.002	0.002
$\sigma(\pi^w)$	0	0	0	0.005	0.006	0.006
$\sigma(\tilde{y})$	0	0	0	0.157	0.078	0.027
$\mathbb{L}$	0	0	0	0.101	0.026	0.004