

# Optimal Monetary Policy under Rigid Wages and Decreasing Returns

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This paper studies optimal Ramsey monetary policy in a search and matching model that combines real wage rigidity and decreasing returns to scale in production. Adding decreasing returns to scale significantly reduces the trade-off between employment and inflation stabilization usually associated with real wage rigidity. As firms adjust employment in response to an aggregate productivity shock, the resulting change in the marginal product of labor partly offsets the effect of a rigid real wage on marginal costs. The effect is quantitatively important. Optimal inflation volatility is reduced by a factor of four compared to a model with constant returns.

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# 1. Introduction

This paper studies optimal monetary policy in a New Keynesian (NK) model with search and matching on the labor market, rigid real wages, and decreasing returns to scale in production. Previous research has shown that real wage rigidity and its combination with decreasing returns to scale has important positive and normative policy implications. Michaillat (2012) shows that a search and matching model that features both characteristics gives rise to “rationing unemployment”: some unemployment would prevail in equilibrium even if recruiting were costless.<sup>1</sup> Michaillat (2014) demonstrates that in such a model fiscal policy in the form of public employment programs is more effective during recessions. Furthermore, in a model version with endogenous search effort, Landais et al. (2013) show that optimal unemployment insurance should be more generous in times of a slack labor market. Real wage rigidity, in turn, also has important implications for the conduct of monetary policy. Blanchard and Galí (2010) show that real wage rigidity in a model with search frictions creates a trade-off between inflation and employment stabilization and that optimal monetary policy deviates from strict price stability. Thus, in the presence of rigid real wages, the literature so far lends support to a more active fiscal and monetary policy with a stronger focus on employment stabilization over the business cycle.

However, the consequences of real wage rigidity for optimal monetary policy have not been analyzed in combination with decreasing returns to scale. This paper fills this gap. It studies optimal Ramsey monetary policy in a New Keynesian model with a search and matching labor market that is characterized by both rigid real wages and decreasing returns to scale. Given that the existing literature provides an argument for a more active fiscal and monetary policy, the results may seem surprising at first. When real wage rigidity is coupled with decreasing returns to scale, the policy trade-off between inflation and employment stabilization is reduced significantly. Compared to a model with rigid real wages but constant returns to scale, the optimal inflation volatility is reduced by a factor of four. Optimal inflation volatility still deviates from zero. However, the differences to a policy that completely stabilizes inflation are relatively small.

Optimal monetary policy in this paper is based on the Ramsey approach. The Ramsey approach derives the welfare maximizing policy of a planner – e.g. the government or central bank – that acts in a competitive and potentially distorted economy. It has become a popular tool for analyzing optimal fiscal and monetary policy (see e.g. Lucas and Stokey, 1983; Chari et al., 1991; Khan et al., 2003; Schmitt-Grohé and Uribe, 2004,

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<sup>1</sup>More specifically, “rationing unemployment” refers to the level of unemployment in the counterfactual case of zero recruiting costs.

2007; Yun, 2005). Specifically, the policy maker sets a single instrument – in this case the nominal interest rate – to maximize the sum of households’ discounted utility, taking into account the constraints of the competitive economy. In this model, both price adjustments (through price adjustment costs) and employment adjustments (through hiring expenses) are costly and the Ramsey planner would like to mitigate both with only one instrument at his disposal. Previous research has shown that a simultaneous stabilization of inflation and employment is no longer feasible if, in addition to the price and labor market frictions, wages are not allowed to adjust freely and/or deviate from Hosios’ (1990) rule. As a consequence, the policy planner has to trade off between two competing goals.<sup>2</sup> This paper shows that while not eliminating it, decreasing returns to scale reduce the trade-off significantly.

The intuition behind this result is straightforward. With decreasing returns to scale, an adjustment in labor input changes the marginal product of workers. If real wages are rigid, a negative aggregate productivity shock leads to an increase in the relative wage and hence marginal costs. However, if firms reduce employment at the same time, the resulting increase in the marginal product partly offsets the effect of the rigid real wage. For a given change in employment, marginal costs move by less. This results in a lower inflation volatility in response to aggregate productivity shocks. For a policy maker that aims at mitigating both costly adjustments in employment and prices, stabilizing inflation comes at a much lower cost in terms of employment and vice versa.

The paper is organized as follows. Section 2 provides a discussion of the relevant literature. Section 3 presents the model. The constrained efficient allocation is derived in Section 4 and Section 5 explains the model mechanism in the presence of real wage rigidity and decreasing returns. It discusses the implications for monetary policy comparing the competitive economy to the solution of a constrained social planner. Section 6 defines the Ramsey problem, presents the calibration, and discusses numerical results. Section 7 briefly concludes.

## 2. Relation to the Literature

Both real wage rigidity and decreasing returns to scale have become increasingly popular tools in macroeconomic models of the labor market. Hall (2005) and Shimer (2005) have both suggested that real wage rigidity could be a driver of the large labor market fluctuations observed in U.S. data. Decreasing returns to scale, in turn, are a useful mechanism for modeling firm size dynamics (see e.g. Elsby and Michaels, 2013). In ad-

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<sup>2</sup>For a seminal paper on the role of wage rigidity for optimal monetary policy see Erceg et al. (2000).

dition, decreasing returns to scale feature prominently in models of intra-firm bargaining (see e.g. Stole and Zwiebel, 1996; Cahuc et al., 2008).

There are a number of papers that have studied optimal monetary policy in the context of a search and matching labor market. Thomas (2008) finds that optimal monetary policy based on a quadratic loss function should deviate from strict price stability in the presence of nominal wage rigidity. Strict price stabilization is associated with large welfare losses. Faia (2009), using a Ramsey approach, shows that the same is true if workers bargaining power is inefficiently high. Ravenna and Walsh (2011) develop a linear-quadratic framework for a model with sticky prices and search frictions. In their framework, stochastic fluctuations in the bargaining power of workers generate policy trade-offs. As changes in bargaining power lead to deviations of the wage from its efficient level, they can have a similar function as real wage rigidity. Faia et al. (2014) study Ramsey optimal monetary policy in a labor selection model with labor turnover costs. In their model, efficiency in the labor market cannot be achieved by a standard Nash bargaining process and optimal inflation volatility increases with higher firing costs. Sala et al. (2008) evaluate monetary policy for the U.S. economy in an estimated model with search frictions and staggered nominal wage bargaining. They find that the trade-off between inflation and unemployment is quantitatively important. Sala et al. (2008) also study optimal monetary policy rules based on a loss function that reflects the mandate of the Federal Reserve. They find that the interest rate should respond to the lagged interest rate more than one to one. Finally, as already discussed, Blanchard and Galí (2010) focus on real wage rigidity. They show in a model with search frictions that the policy maker can stabilize both inflation and unemployment if wages are flexible. When real wages are rigid, however, inflation stabilization leads to large and inefficient movements in unemployment. As common in the search and matching literature and in contrast to the present paper, all these models are based on a constant returns to scale production function.<sup>3</sup> The current paper is further related to Erceg et al. (2000) who investigate monetary policy trade-offs in a model with staggered nominal wages. However, they do not consider a search and matching labor market. Finally, Arseneau and Chugh (2008) study optimal fiscal and monetary policy in a model with costly nominal wage adjustments and a search and matching labor market. In their model, in turn, product prices are flexible.

This paper also adds to the growing literature that features the combination of real wage rigidity and decreasing returns to scale in a search and matching labor market

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<sup>3</sup>Sala et al. (2008) study a model with capital where the production function is constant returns with respect to both input factors and a competitive rental market for capital.

framework. When real wages are rigid, a negative aggregate productivity shock results in a higher wage relative to productivity. When production exhibits decreasing returns, firms can adjust labor input to partly offset this effect. More specifically, in response to a negative aggregate productivity shock, firms increase the marginal product of labor by reducing their labor demand. This combination of rigid real wages and decreasing returns has first been analyzed in the context of a search and matching model by Michaillat (2012). He shows that in such an environment only part of unemployment is due to search frictions. Even if recruiting were costless, firms' optimal decision would not lead to full employment if aggregate productivity is relatively low. Michaillat (2012) refers to this non-frictional part of unemployment as "rationing unemployment". The combination of rigid wages and decreasing returns has important policy implications. In a positive sense, Michaillat (2014) shows that fiscal policy in the form of public employment has a larger multiplier in recessions than in booms. In recessions, firms' labor demand does not react as much to changes in aggregate market tightness. Intuitively, employment is low not only because of the relative costs of search – which are affected by market tightness – but because jobs are rationed to bring the marginal product more in line with the real wage. Hence, fiscal policy in the form of an increase of public employment causes less crowding-out in the private labor market. On the normative side, Landais et al. (2013) show that optimal unemployment insurance is countercyclical. They find that in a version of the model with endogenous search effort, unemployment insurance should be more generous than the standard Baily-Chetty formula (Baily, 1978; Chetty, 2006) if labor market tightness is below its optimum. The optimal policy, however, is only time-varying if unemployment insurance affects tightness. Landais et al. (2013) show that this is the case with rigid wages and decreasing returns but not with constant returns.

Thus, real wage rigidity calls for large deviations from price stability in the conduct of optimal monetary policy. Rigid real wages in combination with decreasing returns to scale have been shown to give rise to countercyclical fiscal multipliers and countercyclical optimal unemployment insurance. This paper will explore how the combination of decreasing returns and real wage rigidity affects the trade-off between inflation and employment stabilization and thus optimal monetary policy.

### **3. The Model**

The model studied here is a search and matching model embedded in a New Keynesian framework that features both real wage rigidity and decreasing returns to scale in

production. It is thus similar to the model presented in Michailat (2014).

### 3.1. Household

There is a big representative household that pools income and consumption of its members, called workers. The household derives utility  $U(c_t)$  from an aggregate consumption good  $c_t$ .<sup>4</sup> It receives real wage income,  $w_t n_t$ , from its employed members  $n_t$  and nominal transfers  $T_t$  from intermediate firms. In addition, the household has access to one period bonds  $B_t$  that pay a gross nominal interest rate  $R_t$  in the next period. The future is discounted with discount factor  $0 < \beta < 1$ .

The household chooses consumption and nominal bond holdings to maximize the stream of discounted lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \quad (1)$$

subject to the following budget constraint:

$$c_t + \frac{B_t}{p_t} = n_t w_t + R_{t-1} \frac{B_{t-1}}{p_t} + \frac{T_t}{p_t}, \quad (2)$$

with  $p_t$  denoting the price of the aggregate consumption good. Utility is given by:

$$U(c_t) = \frac{c_t^{1-\nu}}{1-\nu}. \quad (3)$$

This yields the standard Euler equation:

$$\lambda_t = \beta E_t \left[ \frac{R_t}{1 + \pi_{t+1}} \lambda_{t+1} \right], \quad (4)$$

where

$$\lambda_t = U'(c_t) = c_t^{-\nu} \quad (5)$$

represents the marginal utility of consumption and  $\pi_t = (p_t/p_{t-1} - 1)$  is net inflation.

**Workers** The household consists of a measure one of workers that supply labor inelastically. The labor market is characterized by search and matching frictions, i.e. only a fraction of workers is employed in every period. Unemployed workers search for jobs.

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<sup>4</sup>As common in search and matching models, I assume that there is no disutility from work. This also implies that labor only adjusts at the extensive margin. This assumption seems justified given that the empirical literature (see e.g. Hansen (1985) and more recently Merkl and Wesselbaum (2011)) shows that most of the volatility of aggregate hours is driven by the extensive margin.

Employed workers earn a real wage  $w_t$  and face the risk of losing their job with an exogenous probability  $\sigma$  in every period. Newly unemployed workers can be immediately rehired. The number of searching workers at the beginning of every period is thus

$$u_t^s = 1 - (1 - \sigma)n_{t-1}, \quad (6)$$

with  $n_t$  representing the number of employed workers in period  $t$ . Searching workers are matched to vacant jobs  $v_t$  via a standard Cobb-Douglas matching function

$$m_t = \vartheta v_t^\xi (u_t^s)^{1-\xi}, \quad (7)$$

where  $m_t$  are the number of new matches at the beginning of period  $t$  and  $0 < \xi < 1$ . New matches become productive immediately. The job-finding probability for a worker is thus

$$f(\theta_t) = \vartheta \theta_t^\xi, \quad (8)$$

where  $\theta_t = v_t/u_t^s$  denotes market tightness.

## 3.2. Firms and Production

There are two types of firms. Firms in the retail sector produce the final consumption good from differentiated intermediate goods purchased on the wholesale market. They sell the consumption good on a competitive market to the household. The wholesale sector consists of a continuum of firms on the unit interval. They produce a differentiated intermediate good sold on a monopolistically competitive market using labor as input. They face a downward sloping demand curve, search and matching frictions on the labor market, and quadratic adjustment costs in price setting.

### 3.2.1. Retail Firms

Retail firms purchase the intermediate goods and transform them into the aggregate consumption good using a CES-technology. Specifically, they maximize the following objective function, where  $p_t(i)$  and  $y_t(i)$  denote the price and quantity of intermediate good  $i$  and  $y_t$  is the final consumption good:

$$p_t y_t - \int_0^1 p_t(i) y_t(i) di, \quad (9)$$

subject to

$$y_t = \left[ \int_0^1 y_t(i)^{(\epsilon-1)/\epsilon} di \right]^{\epsilon/(\epsilon-1)}. \quad (10)$$

Optimization yields the following optimal demand for intermediate good  $i$ :

$$y_t(i) = y_t \left( \frac{p_t(i)}{p_t} \right)^{-\epsilon}. \quad (11)$$

The aggregate price level satisfies:

$$p_t = \left( \int_0^1 p_t(i)^{1-\epsilon} di \right)^{1/(1-\epsilon)}. \quad (12)$$

### 3.2.2. Wholesale Firms

Firms in the wholesale sector act under monopolistic competition, they face search and matching frictions on the labor market, and quadratic price adjustment costs as in Rotemberg (1982). In addition, production is characterized by decreasing returns to scale and real wages are rigid.

**Production** Firms in the wholesale sector use labor as their sole input factor in production:

$$y_t(i) = a_t n_t(i)^\alpha, \quad (13)$$

where  $a_t$  is aggregate productivity and  $0 < \alpha < 1$ . The production technology thus features decreasing returns to scale. In contrast to the one-firm one-job model in e.g. Pissarides (2000), firms are large and can have more than one employee. Decreasing returns to scale in production could arise because capital is predetermined or costly to adjust in the short term. Decreasing returns to scale in a labor market context have been used by Stole and Zwiebel (1996) in their model of intra-firm bargaining. Cahuc et al. (2008) have incorporated this approach into the search and matching framework. In Elsby and Michaels (2013), decreasing returns to scale serve to study firm size dynamics within a search and matching model with endogenous job destruction.

**Search Frictions** Firms in the wholesale sector have to post vacancies  $v_t(i)$  at period cost  $\kappa a_t$  in order to make new hires.<sup>5</sup> Vacancy posting costs are measured in terms of the final output good. Given the matching technology, the vacancy-filling rate is:

$$q(\theta_t) = \frac{m_t}{v_t} = \vartheta \theta_t^{\xi-1}, \quad (14)$$

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<sup>5</sup>Recruiting costs are thus increasing in aggregate productivity as e.g. in Pissarides (2000, chapter 1). The main results of this paper are robust to this assumption.



where  $v_t$  denotes the aggregate number of vacancies in the economy. In every period, a fraction  $\sigma$  of existing employment relations is dissolved. Thus, for a desired number of new hires  $h_t(i)$  the firm needs to post  $h_t(i)/q(\theta_t)$  vacancies. The total costs for new hires are thus:

$$\kappa a_t v_t(i) = \frac{\kappa a_t}{q(\theta_t)} h_t(i) = \frac{\kappa a_t}{q(\theta_t)} [n_t(i) - (1 - \sigma)n_{t-1}(i)]. \quad (15)$$

**Prices** Wholesale firms face two sorts of price rigidity: price setting is subject to quadratic adjustment costs and real wages are rigid with respect to aggregate productivity.

Price changes are subject to Rotemberg (1982) adjustment costs:

$$\frac{\Phi}{2} \left( \frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 y_t, \quad (16)$$

i.e. firms pay a quadratic cost measured in the final consumption good whenever they change their price level from one period to the next. As decreasing returns to scale in this model imply that firm level employment is the outcome of a firm's optimal choice, I choose Rotemberg (1982) adjustment costs instead of a Calvo (1983) mechanism to model price stickiness. If firms were allowed to reset prices at different points in time, this would also result in different employment levels across firms. With Rotemberg (1982) adjustment costs, in contrast, firms are confronted with symmetric problems in every period which allows for a better tractability of the model. Note, however, that these two mechanisms are equivalent up to a first-order approximation if trend inflation is zero, which is the case in this setup.<sup>6</sup>

**Wage** The wage follows the simple wage rule that is also used in Blanchard and Galí (2010) and Michaillat (2012, 2014):

$$w_t = \omega a_t^\zeta, \quad (17)$$

with  $0 < \zeta < 1$ . The real wage is thus rigid with respect to aggregate productivity. Hall (2005) shows that in a search and matching labor market even a constant wage could be privately efficient.<sup>7</sup> Michaillat (2012) extends this argument to the case with decreasing returns to scale. However, Brügemann (2014) demonstrates that private efficiency with

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<sup>6</sup>Equivalence does not hold, however, if trend inflation deviates from zero (Ascari and Rossi, 2012).

<sup>7</sup>This is a notable difference to a frictionless labor market, where a rigid real wage would be subject to the Barro (1977) critique.

decreasing returns to scale can only be guaranteed along the equilibrium path, but not off equilibrium. With decreasing returns, private efficiency could always be violated if firms chose a large enough expansion of employment. Two comments are in order: First, in equilibrium, such an allocation would never arise. Second, Brügemann (2014) shows that a way around this problem is to assume a wage that follows equation (17) up to a threshold and afterwards adjusts such that firms' profit function is flat. Brügemann (2014) calls this the full-appropriation wage schedule.<sup>8</sup> For the sake of simplicity, this paper keeps the simple wage of equation (17) as equilibrium allocations would be the same under both schedules.

**Optimization** A wholesale sector firm chooses employment and prices to maximize the following objective function taking into account its demand curve from equation (11) and the household's stochastic discount factor  $\beta^t \frac{\lambda_t}{\lambda_0}$ :

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left\{ \left( \frac{p_t(i)}{p_t} \right)^{1-\epsilon} y_t - w_t n_t(i) - \frac{\Phi}{2} \left( \frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 y_t - \frac{\kappa a_t}{q(\theta_t)} [n_t(i) - (1 - \sigma)n_{t-1}(i)] + \Lambda_t(i) \left[ a_t n_t(i)^\alpha - \left( \frac{p_t(i)}{p_t} \right)^{-\epsilon} y_t \right] \right\}, \quad (18)$$

where  $\Lambda_t(i)$  is the Lagrange multiplier on the production constraint and thus represents real marginal costs. Optimization yields the following first-order conditions for employment,

$$\Lambda_t(i) \alpha n_t(i)^{\alpha-1} = \frac{w_t}{a_t} + \frac{\kappa}{q(\theta_t)} - (1 - \sigma) \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{a_{t+1}}{a_t} \frac{\kappa}{q(\theta_{t+1})} \right], \quad (19)$$

and the price level  $p_t(i)$ ,

$$\frac{p_t(i)}{p_t} = \frac{\epsilon}{\epsilon - 1} \Lambda_t(i) + \frac{\Phi}{\epsilon - 1} \left( \frac{p_t(i)}{p_t} \right)^\epsilon \times \left[ \beta \cdot E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{y_{t+1}}{y_t} \left( \frac{p_{t+1}(i)}{p_t(i)} - 1 \right) \frac{p_{t+1}(i)}{p_t(i)} \right] - \left( \frac{p_t(i)}{p_{t-1}(i)} - 1 \right) \frac{p_t(i)}{p_{t-1}(i)} \right]. \quad (20)$$

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<sup>8</sup>The threshold here is the level of employment that would arise absent any recruiting costs, i.e. the point where profits would start falling.

### 3.3. Equilibrium

In equilibrium, firms are identical such that  $p_t(i) = p_t$ ,  $n_t(i) = n_t$ ,  $y_t(i) = y_t$ , and all markets clear. Aggregate labor demand thus becomes:

$$\Lambda_t \cdot \alpha n_t^{\alpha-1} = \frac{w_t}{a_t} + \frac{\kappa}{q(\theta_t)} - (1 - \sigma)\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{a_{t+1}}{a_t} \frac{\kappa}{q(\theta_{t+1})} \right], \quad (21)$$

with

$$q(\theta_t) = \vartheta \theta_t^{\xi-1}.$$

Note that in contrast to the classical search and matching model with constant returns to scale (see e.g. Pissarides, 2000, chapter 1) there is no free-entry condition for firms. Instead, the number of firms is assumed to be fixed and employment is determined by the optimal choice of labor input.

Equation (20) reduces to the familiar nonlinear Phillips curve:

$$(\pi_t + 1)\pi_t = \frac{1}{\Phi} \cdot [\epsilon \Lambda_t - (\epsilon - 1)] + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{y_{t+1}}{y_t} (\pi_{t+1} + 1)\pi_{t+1} \right]. \quad (22)$$

Aggregate production is given by:

$$y_t = a_t n_t^\alpha. \quad (23)$$

By bond market clearing,  $B_t = R_{t-1}B_{t-1}$ , and assuming that firms' profits are transferred to the household, the aggregate resource constraint becomes:

$$y_t = c_t + \frac{\Phi}{2}\pi_t^2 y_t + \frac{\kappa a_t}{q(\theta_t)} [n_t - (1 - \sigma)n_{t-1}]. \quad (24)$$

The law of motion for employment is given by:

$$n_t = (1 - \sigma) \cdot n_{t-1} + [1 - (1 - \sigma) \cdot n_{t-1}] \cdot f(\theta_t), \quad (25)$$

with

$$f(\theta_t) = \vartheta \theta_t^\xi.$$

Finally, the nominal interest rate  $R_t$  is set by the central bank in accordance with a Ramsey policy as described in Section 6.1. Alternatively, the optimal Ramsey policy is compared to a policy that enforces zero inflation and a policy that follows a classical Taylor rule with a single focus on inflation:

$$R_t = \frac{1}{\beta} \left( \frac{\pi_t + 1}{\bar{\pi} + 1} \right)^{\mu_\pi}, \quad (26)$$

where  $\bar{\pi}$  is the long-run net inflation rate, which is set to zero.

**Definition 1.** *Given initial employment and bond holdings,  $\{n_0, b_0\}$ , a stochastic path for productivity  $\{a_t\}_{t=0}^\infty$ , and a policy for the nominal interest rate  $\{R_t\}_{t=0}^\infty$ , the equilibrium is a collection of processes  $\{c_t, \lambda_t, \pi_t, w_t, \Lambda_t, n_t, \theta_t, y_t\}_{t=0}^\infty$  that satisfy equations (4), (5), (17), (21), (22), (23), (24), and (25).*

## 4. Constrained Efficient Allocation

In order to illustrate potential trade-offs between inflation and employment stabilization, it is useful to first set a benchmark by deriving the constrained efficient allocation of a social planner. A benevolent social planner would like to mitigate both inefficient fluctuations in prices and employment as both are associated with adjustment costs in the model. He is constrained by the search technology in the labor market but is not subject to frictions arising from price rigidity or monopolistic competition.<sup>9</sup> The constrained social planner chooses consumption, employment, and vacancies to maximize the stream of the household's discounted lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \quad (27)$$

subject to the resource constraint and the employment dynamics equation:

$$a_t n_t^\alpha - \kappa a_t v_t - c_t = 0, \quad (28)$$

$$v_t q(\theta_t) + (1 - \sigma)n_{t-1} - n_t = 0, \quad (29)$$

with

$$\theta_t = \frac{v_t}{1 - (1 - \sigma)n_{t-1}}. \quad (30)$$

Let  $\lambda_t$  and  $\tau_t$  be the multipliers on constraints (28) and (29) respectively. Taking first-order conditions and rearranging gives the following optimality conditions (see Appendix A for details):

$$\tau_t = \lambda_t \alpha a_t n_t^{\alpha-1} + \beta E_t \left[ \tau_{t+1} \left( (1 - \sigma) + v_{t+1} q'(\theta_{t+1}) \frac{\partial \theta_{t+1}}{\partial n_t} \right) \right], \quad (31)$$

$$\lambda_t \kappa a_t = \tau_t \left( q(\theta_t) + v_t q'(\theta_t) \frac{\partial \theta_t}{\partial v_t} \right). \quad (32)$$

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<sup>9</sup>Assume that the social planner can always provide a subsidy on production to annihilate the distortions arising from monopolistic competition.

Equation (31) describes the value of an additional worker. It is given by the current marginal product weighted by the household's marginal utility and the continuation value of a worker. The latter is composed of two parts: the value of a worker in the next period weighted by the retention probability and a term that takes into account that an employed worker reduces the chances of finding new workers in the next period – a congestion effect. Equation (32) equates the costs of posting a vacancy (the left hand side) with the benefits of a vacancy (right hand side). Again, the social planner takes into account that posting an additional vacancy increases market tightness and thus decreases the chance of filling a vacancy. Combining equations (31) and (32) and using the definition of the vacancy-filling rate (14) and market tightness (30) yields the following optimality condition, which equates the benefits of a new hire to its costs:

$$\begin{aligned} \frac{\kappa a_t}{q(\theta_t)} &= \xi \alpha a_t n_t^{\alpha-1} + (1 - \sigma) \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa a_{t+1}}{q(\theta_{t+1})} \right] \\ &\quad - (1 - \xi) (1 - \sigma) \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \kappa a_{t+1} \theta_{t+1} \right]. \end{aligned} \quad (33)$$

Now, let us compare the planner's solution to the optimality condition from the competitive economy (equation (21)).

$$\frac{\kappa a_t}{q(\theta_t)} = \Lambda_t \cdot \alpha a_t n_t^{\alpha-1} + (1 - \sigma) \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa a_{t+1}}{q(\theta_{t+1})} \right] - w_t. \quad (34)$$

Assume that the social planner can subsidize production such that steady state marginal costs  $\Lambda$  are equal to one. It is straightforward to see in this case that a wage norm that achieves the constrained social planner's allocation would have to take the following form:<sup>10</sup>

$$w_t^* = (1 - \xi) \left( \alpha a_t n_t^{\alpha-1} + (1 - \sigma) \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \kappa a_{t+1} \theta_{t+1} \right] \right). \quad (35)$$

The optimal wage moves proportionally with the marginal product of workers, with the factor of proportionality equal to the elasticity of matches with respect to unemployment.<sup>11</sup> It is thus flexible with respect to both aggregate productivity and firm level employment. The second term in equation (35) captures the effect of market tightness

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<sup>10</sup>Again, this is only exact if the planner uses a different tool, i.e. a subsidy on production, to offset the inefficiency arising from monopolistic competition.

<sup>11</sup>This wage looks very similar to the wage under Nash bargaining and Hosios (1990) rule in a standard search and matching model. Note, however, that decreasing returns to scale change the nature of the individual bargaining game as (marginal) workers are not identical anymore. For a theory of intra-firm bargaining see Stole and Zwiebel (1996). For an incorporation of the Stole and Zwiebel (1996) bargaining into an equilibrium search and matching model see e.g. Cahuc and Wasmer (2001), Cahuc et al. (2008), and Elsby and Michaels (2013).

on the wage. Plugging the optimal wage back into the competitive economy's optimality condition (34) yields an expression of marginal costs as a function of labor market variables. Keep in mind that given the Phillips curve relation in equation (22), movements of real marginal costs drive inflation volatility. The equation reads as follows:

$$\Lambda_t = (1 - \xi) + \frac{n_t^{1-\alpha}}{\alpha} \left( \frac{\kappa}{q(\theta_t)} - (1 - \sigma)\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{a_{t+1}}{a_t} \left( \frac{\kappa}{q(\theta_{t+1})} + (1 - \xi)\kappa\theta_{t+1} \right) \right] \right). \quad (36)$$

The first term on the right hand side is a constant. The second term depends only on labor market variables, productivity, and the stochastic discount factor. If the stochastic discount factor moves one to one with changes in productivity,<sup>12</sup> stabilizing real marginal costs, and thus inflation, stabilizes employment as well. Thus, with decreasing returns but a flexible, optimal wage, there is no trade-off between inflation and employment stabilization. The next section explores how the relationship between real marginal costs and employment changes if real wages are rigid.

## 5. The Inflation-Unemployment Trade-off

### 5.1. The Inflation-Unemployment Trade-off under Rigid Wages and Constant Returns

Why does real wage rigidity create trade-offs for monetary policy? Let us first illustrate the case for constant returns to scale. Consider equation (21) with constant returns to scale and the wage norm (17):

$$\Lambda_t = \omega a_t^{\zeta-1} + \left( \frac{\kappa}{q(\theta_t)} - (1 - \sigma)\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{a_{t+1}}{a_t} \frac{\kappa}{q(\theta_{t+1})} \right] \right). \quad (37)$$

Compare this to equation (36). In (36), the first term on the right hand side is a constant. In contrast, the first term on the right hand side of equation (37) will fluctuate in response to aggregate productivity shocks if real wages are rigid, i.e.  $\zeta < 1$ . Keeping marginal costs constant would mean that the second term on the right hand side of equation (37), i.e. the value of a match, would have to move by a lot and thus create large and costly fluctuations in employment.

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<sup>12</sup>This would be the case for log utility. However, even if households are more risk averse, the effect is quantitatively close to zero as shown in Section 6.3. Blanchard and Galí (2010) show that for more general preferences, where households enjoy leisure, inflation and employment can be simultaneously stabilized if income and substitution effects cancel.

## 5.2. The Inflation-Unemployment Trade-off under Rigid Wages and Decreasing Returns

As already explained in the previous section, the simultaneous stabilization of both inflation and employment is no longer feasible once real wages are rigid. However, in contrast to a case with constant returns to scale, decreasing returns to scale substantially reduce the trade-off, although not eliminating it. Consider equation (34) if the wage is given by equation (17), i.e.  $w_t = \omega a_t^\zeta$ , and solve for real marginal costs:

$$\begin{aligned}\Lambda_t &= \frac{w_t}{a_t} \frac{a_t}{(\alpha a_t n_t^{\alpha-1})} + \frac{n_t^{1-\alpha}}{\alpha} \left( \frac{\kappa}{q(\theta_t)} - (1-\sigma)\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{a_{t+1}}{a_t} \frac{\kappa}{q(\theta_{t+1})} \right] \right) \\ &= \frac{\omega}{\alpha} a_t^{\zeta-1} n_t^{1-\alpha} + \frac{n_t^{1-\alpha}}{\alpha} \left( \frac{\kappa}{q(\theta_t)} - (1-\sigma)\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{a_{t+1}}{a_t} \frac{\kappa}{q(\theta_{t+1})} \right] \right).\end{aligned}\quad (38)$$

The wage deviates from the optimal wage in three ways: it is rigid with respect to aggregate productivity, it is invariant to firm-level employment and thus the marginal product of labor, and it does not react to aggregate labor market conditions, i.e. market tightness. As the latter effect – the dependence on market tightness – is quantitatively of minor importance for the trade-off between inflation and employment stabilization, the discussion focuses on the first two characteristics of the wage.<sup>13</sup> The first term on the right hand side of equation (38) is the ratio between the relative real wage  $w_t/a_t$  and the relative marginal product of labor. If the wage adjusts optimally, the term is constant in response to an aggregate productivity shock as demonstrated in equation (36). With real wage rigidity, the relative real wage rises if productivity falls. However, if employment also falls in response, the marginal product of workers rises. This reduces the effect on real marginal costs. As discussed in Section 5.1, with constant returns to scale, stabilizing marginal costs in the presence of rigid real wages means that all the adjustment has to come from the value of the match. Employment has to move by a lot. With decreasing returns to scale, employment also adjusts but by doing so mitigates the effect on real marginal costs. In other words, for a given level of employment volatility, the resulting volatility of real marginal costs and hence inflation is lower. In response to productivity shocks, firms adjust employment to countervail the change of the relative real wage and thus reduce the effect on real marginal costs.

Interestingly, this feedback effect on inflation volatility is lost if the wage can adjust to firm-level employment. Consider a wage rule that is rigid with respect to aggregate

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<sup>13</sup>See Section 6.3 for numerical results.

productivity but flexible with respect to firm-level employment:

$$w_t = \omega a_t^\zeta n_t^{\alpha-1}(i) = \omega a_t^\zeta n_t^{\alpha-1}. \quad (39)$$

This wage is closer to the optimal wage (35). In addition, Brügemann (2014) shows that this wage satisfies private efficiency for any allocation on and off equilibrium. The optimal labor demand equation now becomes

$$\Lambda_t = \frac{\omega}{\alpha} a_t^{\zeta-1} + \frac{n_t^{1-\alpha}}{\alpha} \left( \frac{\kappa}{q(\theta_t)} - (1-\sigma)\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{a_{t+1}}{a_t} \frac{\kappa}{q(\theta_{t+1})} \right] \right), \quad (40)$$

which is similar to the case with constant returns to scale (numerical results are displayed in Appendix C). If the wage adjusts proportionally to firm-level employment, firms cannot use employment to reduce marginal costs in response to a negative productivity shock. Thus, a wage rigidity with respect to one variable – firm-level employment – helps firms to partly offset the wage rigidity with respect to another variable – aggregate productivity.

The next section shows that the reduction in optimal inflation volatility is quantitatively important using the Ramsey approach for optimal monetary policy.

## 6. Optimal Monetary Policy

### 6.1. The Ramsey Policy

Optimal monetary policy is determined by the Ramsey approach. The policy maker chooses the interest rate such as to maximize the sum of the household's discounted utility, taking into account the constraints of the competitive economy. Given the utility function, matching function, production function, and the wage rule, the policy maker is subject to the following constraints:

$$c_t^{-\nu} = \beta E_t \left[ \frac{R_t}{1 + \pi_{t+1}} c_{t+1}^{-\nu} \right], \quad (41)$$

$$\Lambda_t \cdot \alpha n_t^{\alpha-1} = \omega a_t^{\zeta-1} + \frac{\kappa}{\vartheta \theta_t^{\xi-1}} - (1-\sigma)\beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\nu} \frac{a_{t+1}}{a_t} \frac{\kappa}{\vartheta \theta_{t+1}^{\xi-1}} \right], \quad (42)$$

$$\begin{aligned} (\pi_t + 1)\pi_t &= \frac{1}{\Phi} \cdot [\epsilon \Lambda_t - (\epsilon - 1)] \\ &+ \beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\nu} \frac{a_{t+1}}{a_t} \left( \frac{n_{t+1}}{n_t} \right)^\alpha (\pi_{t+1} + 1)\pi_{t+1} \right], \end{aligned} \quad (43)$$



$$a_t n_t^\alpha = c_t + \frac{\Phi}{2} \pi_t^2 a_t n_t^\alpha + \frac{\kappa a_t}{\vartheta \theta_t^{\xi-1}} [n_t - (1 - \sigma) n_{t-1}], \quad (44)$$

$$n_t = (1 - \sigma) \cdot n_{t-1} + [1 - (1 - \sigma) \cdot n_{t-1}] \vartheta \theta_t^\xi. \quad (45)$$

Formally, the Ramsey problem is defined as follows:

**Definition 2.** Let  $\{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}\}_{t=0}^\infty$  be the Lagrange multipliers on constraints (41), (42), (43), (44), and (45). For a given path of productivity  $\{a_t\}_{t=0}^\infty$ , a first-best constrained allocation is a plan for the control variables  $\Xi_t^n = \{c_t, n_t, \theta_t, \pi_t, \Lambda_t, R_t\}_{t=0}^\infty$  and the co-state variables  $\Lambda_t^n = \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}\}_{t=0}^\infty$  that solves the following maximization problem:

$$\text{Min}_{\{\Lambda_t^n\}_{t=0}^\infty} \text{Max}_{\{\Xi_t^n\}_{t=0}^\infty} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t) \right\}, \quad (46)$$

subject to (41), (42), (43), (44), and (45).

As constraints (41), (42), and (43) are forward looking, the private agents' decisions are influenced by their expectation about future policy. However, their current choices are also based on past hiring decisions, influenced in turn by past expectations about the evolution of monetary policy. The Ramsey policy thus suffers from a time-inconsistency problem (see e.g. Kydland and Prescott, 1977; Prescott, 1977; Calvo, 1978). Fortunately, as suggested by Kydland and Prescott (1980), this problem can be transformed into a recursive problem by enhancing the state space with additional co-state variables, i.e. the multipliers on the forward looking constraints. Intuitively, these co-state variables represent the costs of the planner of sticking to earlier policy commitments. At time zero, the values of the co-state variables are set to their steady state values which is consistent with a timeless perspective. In other words, it is assumed that the economy has already been evolving around that steady state for some time.

## 6.2. Calibration

The model is calibrated on a quarterly frequency to the U.S. economy. The parameter values are displayed in Table 1.

The household's discount factor is 0.99, consistent with a 4% annual interest rate and the coefficient of relative risk aversion is  $\nu = 2$ . The parameter determining the elasticity of substitution between different goods is  $\epsilon = 11$ , which corresponds to a 10% markup in steady state.

In line with the data, the quarterly separation rate is set to  $\sigma = 0.1$ . I target a steady

state unemployment rate of 6%.<sup>14</sup> Given the law of motion for employment, this yields a quarterly job-finding rate of 0.6105. The steady state value of market tightness is normalized to 1. Given the matching function, this pins down the value of matching efficiency at  $\vartheta = 0.6105$ . I set the weight on vacancies in the matching function to  $\xi = 0.3$ , which is in line with the survey of matching function estimations by Petrongolo and Pissarides (2001).<sup>15</sup> The coefficient on labor in the production function is set to  $\alpha = 0.66$ , a value commonly taken for the labor share.

Haefke et al. (2013) estimate an elasticity of the real (hourly) wage with respect to aggregate productivity for new workers of 0.79. For earnings per person the value is 0.83. I therefore set the elasticity of the real wage with respect to productivity in the model to  $\zeta = 0.8$ . Following Michaillat (2014), who derives the price adjustment costs from microeconomic evidence presented in Zbaracki et al. (2004), I set  $\Phi = 61$ . This is an intermediate value between Krause and Lubik (2007), who use a value of 40, and Faia et al. (2014), who use a value of 116.5 for a quarterly calibration.

Note that because of condition (42), choosing recruiting costs simultaneously determines the steady state real wage. Silva and Toledo (2009), using evidence provided by Dolfin (2006), calculate that recruiting costs per hire amount to 4.3% of the quarterly wage of a newly hired. However, these costs only include the manhours spent by the company. Assuming that firms also incur direct financial costs (costs of posting advertisements, travel costs etc.), I set total vacancy posting costs to 10% of the steady state real wage. This simultaneously pins down  $\omega$  and hence the steady state real wage at 0.0602.<sup>16</sup> In Appendix B, I provide robustness checks for different values of recruiting costs. When I compare the baseline to the case with constant returns to scale, I keep the labor market steady state and the relative magnitudes of recruiting costs vis-a-vis the wage fixed.<sup>17</sup>

Finally, aggregate productivity follows an AR(1) process. I estimate the process using data for quarterly total factor productivity (TFP) from 1964 to 2013 from the database constructed by Fernald (2012).<sup>18</sup> This yields an autocorrelation coefficient of 0.96 and

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<sup>14</sup>Unemployment refers to  $u_t = 1 - n_t$ , i.e. not the number of searching workers, which, given immediate rehiring, is higher.

<sup>15</sup>Michaillat (2014) uses the same value.

<sup>16</sup>Using the definition of Michaillat (2012), 1% of total unemployment is rationed and 5% is frictional in steady state. Rationing unemployment refers to the counterfactual case in which recruiting costs are zero.

<sup>17</sup>This means, of course, that the absolute size of recruiting costs as well as the wage could change. I also offer an alternative specification in the Appendix, where instead of assuming constant returns to scale, the wage responds to firm-level employment. As this offsets the influence of decreasing returns on profits, the effect is similar to the case with constant returns. The advantage of this specification, however, is that the steady state production value is the same under both scenarios.

<sup>18</sup>Data at: [www.frbsf.org/economic-research/files/quarterly\\_tfp.xls](http://www.frbsf.org/economic-research/files/quarterly_tfp.xls).

a standard deviation for the shock process of 0.00965.

I simulate the model 1000 times using first-order approximations.<sup>19</sup> In the Ramsey problem, all co-state variables are set to their steady state value in period zero consistent with a timeless perspective. Each time, I simulate 1200 periods and discard the first 1000 data points. In line with the data for productivity, the length of the resulting time series corresponds to a time span of 50 years. The simulated data is filtered using a Hodrick-Prescott filter with smoothing parameter 1600.

Parameter	Description	Value
$\beta$	Discount factor	0.99
$\nu$	Risk aversion coefficient	2
$\epsilon$	Elasticity of substitution parameter	11
$\sigma$	Separation rate	0.1
$\xi$	Elasticity in matching function	0.3
$\Phi$	Price adjustment costs	61
$\vartheta$	Matching efficiency	0.6105
$\alpha$	Labor elasticity	0.66
$\zeta$	Real wage elasticity	0.8
$\kappa$	Recruiting costs	$0.1 \cdot \omega$
$\omega$	Steady state real wage	0.602
$\rho$	AR-coef. productivity	0.96
$sd(a)$	SD productivity	0.00965

**Table 1:** Parameters (baseline)

### 6.3. Results

Before analyzing the interaction between decreasing returns to scale and real wage rigidity, let us first establish a benchmark by looking at both characteristics individually.

Table 2 shows the results of optimal monetary policy when production is characterized by decreasing returns to scale but real wages are flexible. The first line of Table 2 displays the optimal volatility of each variable relative to productivity if the wage follows equation (17) but with  $\zeta$  set to one, i.e. the wage moves one to one with aggregate productivity. As expected, the optimal inflation volatility is nearly zero. The same applies to employment. The Ramsey planner is very close to achieving a simultaneous stabilization of both inflation and employment. The small deviation from price stability

<sup>19</sup>As the state space is not restricted, it is possible that in some of the 1000 simulations the private efficiency condition is violated at some point in time. If this happens, I discard the run. This never happens in the Ramsey problem.

arises because the wage is not fully optimal as it does not respond to labor market conditions. The remaining volatility, however, is quantitatively small. The second row of Table 2 confirms that once an optimal wage rule as in equation (35) is in place,<sup>20</sup> optimal monetary policy completely stabilizes inflation and employment.<sup>21</sup>

Flexible Wage & Decreasing Returns									
Scenario	$a$	$w$	$\pi$	$\theta$	$n$	$\Lambda$	$c$	$y$	$u$
Simple wage	1	1.00	0.01	0.15	0.01	0.00	1.00	1.00	0.10
Optimal wage	1	1.01	0.00	0.02	0.00	0.00	1.00	1.00	0.04

**Table 2:** Optimal volatility of model variables with flexible wage and decreasing returns. Simple wage refers to equation (17) with  $\zeta = 1$ . Optimal wage refers to equation (35). Standard deviations are relative to the standard deviation of productivity. All data is in log-deviations from a Hodrick-Prescott filter with smoothing parameter 1600. Results are means over 1000 simulations. Inflation is annualized.

Next, let us turn to the opposite case with a rigid real wage but constant returns to scale, i.e.  $\alpha = 1$ , the case considered in Blanchard and Galí (2010). Results are displayed in the first row of Table 3. In line with the literature and theoretical considerations in Section 5.1, real wage rigidity provokes a large deviation from price stability. Optimal policy implies that inflation should fluctuate nearly as much as productivity. Optimal employment volatility is about 20% of the standard deviation of productivity. A simultaneous stabilization of both employment and inflation is no longer feasible in an environment with real wage rigidity and the Ramsey planner strikes a balance between the two goals. A monetary policy that only focuses on inflation is therefore associated with welfare losses. Indeed, the second row in Table 3 shows that with a zero-inflation policy in place the volatility of all labor market variables more than doubles.<sup>22</sup> Consumption and output volatility increase sizably by around 20% and 25% respectively.

Finally, the results for optimal monetary policy in a model with both real wage rigidity and decreasing returns are displayed in Table 4. The differences to the case with constant returns are striking. While optimal employment volatility is comparable to the case with constant returns to scale, optimal inflation volatility is reduced by a factor of four. While complete stabilization is not feasible, decreasing returns to scale reduce the effect of real wage rigidity on inflation volatility. As firms adjust employment in response to productivity shocks, the resulting change in the marginal product of labor mitigates

<sup>20</sup>For the optimal wage, parameter values correspond to the baseline.

<sup>21</sup>There is still some residual volatility in unemployment and market tightness. This is due to the fact that the intertemporal elasticity of substitution deviates from one in the calibration, i.e. the household are more risk averse. With log utility, the volatility of all labor market variables is zero.

<sup>22</sup>Again, the simulation is based on a first-order approximation for comparability. Using a second-order approximation does not change results visibly.

Rigid Wage & Constant Returns									
Scenario	$a$	$w$	$\pi$	$\theta$	$n$	$\Lambda$	$c$	$y$	$u$
Ramsey	1	0.80	0.91	6.63	0.22	0.14	1.14	1.21	3.74
0-Inflation	1	0.80	0.00	14.32	0.53	0.00	1.36	1.51	7.74

**Table 3:** Volatility of model variables with real wage rigidity and constant returns for Ramsey policy and 0-Inflation policy. Standard deviations are relative to the standard deviation of productivity. All data is in log-deviations from a Hodrick-Prescott filter with smoothing parameter 1600. Results are means over 1000 simulations. Inflation is annualized.

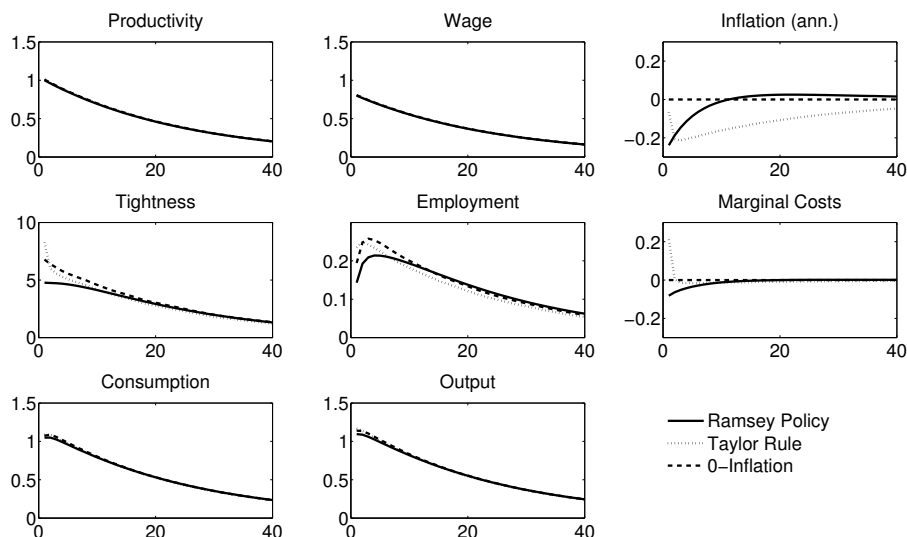
the effect of a changing relative wage. The Ramsey planner still strikes a balance between employment and inflation stabilization. But reducing inefficient fluctuations in employment comes at a much lower cost in terms of inflation volatility. The differences to the case with constant returns to scale are most visible if one compares the Ramsey policy to a zero-inflation policy. While the volatility of labor market variables is higher compared to the optimal policy, the increase is relatively modest. Consumption volatility is only slightly higher under zero-inflation policy compared to the Ramsey policy. The same applies when comparing the Ramsey policy to a simple Taylor rule in which the nominal interest rate only responds to inflation. This rule is meant to represent the policy of a central bank that has inflation stabilization as its single mandate. With the coefficient on inflation set to  $\mu_\pi = 2.5$ , i.e. the policy maker responds strongly to inflation, the Taylor rule matches the optimal inflation volatility quite well.

Rigid Wage & Decreasing Returns									
Scenario	$a$	$w$	$\pi$	$\theta$	$n$	$\Lambda$	$c$	$y$	$u$
Ramsey	1	0.80	0.23	5.28	0.20	0.07	1.09	1.13	3.31
0-Inflation	1	0.80	0.00	6.43	0.26	0.00	1.12	1.17	4.12
Taylor Rule	1	0.80	0.21	7.13	0.26	0.17	1.12	1.17	4.25

**Table 4:** Volatility of model variables with rigid real wage and decreasing returns for Ramsey policy, 0-Inflation policy, and Taylor rule. Standard deviations are relative to the standard deviation of productivity. All data is in log-deviations from a Hodrick-Prescott filter with smoothing parameter 1600. Results are means over 1000 simulations. Inflation is annualized.

Figure 1 shows the model's impulse responses to a 1% productivity shock for the three different policies. In response to a positive productivity shock the Ramsey policy shows a more sluggish response in employment compared to the other policies. The employment level builds up more gradually and market tightness does not jump as much. This should allow for a more efficient distribution of recruiting costs over time. With the optimal Ramsey policy, inflation and marginal costs fall on impact but then show a relatively

steep increase. With the Taylor rule, marginal costs increase on impact as employment increases more sharply. Therefore, inflation peaks with some delay. Overall, however, the effects on output and consumption are extremely small.



**Figure 1:** Impulse responses to an aggregate productivity shock (in percent deviation from steady state).

## 7. Conclusion

This paper investigates optimal Ramsey monetary policy in a New Keynesian model with a search and matching labor market. It shows that optimal inflation volatility is significantly reduced in a model that combines both real wage rigidity and decreasing returns to scale compared to a model with real wage rigidity only. In response to an aggregate productivity shock, firms adjust labor which changes the marginal product of labor. This partly offsets the detrimental effect of a rigid wage on real marginal costs. For a given employment volatility, optimal inflation volatility is about four times smaller compared to an economy with constant returns to scale. While optimal policy still deviates from full price stability, the differences to a policy that focuses on inflation only are significantly smaller. These results are surprising given earlier findings that lend support to a more active fiscal and monetary policy in the presence of real wage rigidity.

## References

- ARSENEAU, D. M. AND S. K. CHUGH (2008): “Optimal Fiscal and Monetary Policy with Costly Wage Bargaining,” *Journal of Monetary Economics*, 55, 1401–1414.
- ASCARI, G. AND L. ROSSI (2012): “Trend Inflation and Firms Price-Setting: Rotemberg Versus Calvo,” *The Economic Journal*, 122, 1115–1141.
- BAILY, M. N. (1978): “Some Aspects of Optimal Unemployment Insurance,” *Journal of Public Economics*, 10, 379–402.
- BARRO, R. J. (1977): “Long-term Contracting, Sticky Prices, and Monetary Policy,” *Journal of Monetary Economics*, 3, 305–316.
- BLANCHARD, O. J. AND J. GALÍ (2010): “Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment,” *American Economic Journal: Macroeconomics*, 2, 1–30.
- BRÜGEMANN, B. (2014): “Privately Efficient Wage Rigidity Under Diminishing Returns,” VU University Amsterdam and Tinbergen Institute, mimeo.
- CAHUC, P., F. MARQUE, AND E. WASMER (2008): “A Theory of Wages and Labor Demand with Intra-Firm Bargaining and Matching Frictions,” *International Economic Review*, 49, 943–972.
- CAHUC, P. AND E. WASMER (2001): “Does Intrafirm Bargaining Matter In The Large Firm’s Matching Model?” *Macroeconomic Dynamics*, 5, 742–747.
- CALVO, G. A. (1978): “On the Time Consistency of Optimal Policy in a Monetary Economy,” *Econometrica*, 46, 1411–28.
- (1983): “Staggered Prices in a Utility-maximizing Framework,” *Journal of Monetary Economics*, 12, 383–398.
- CHARI, V. V., L. J. CHRISTIANO, AND P. J. KEHOE (1991): “Optimal Fiscal and Monetary Policy: Some Recent Results,” *Journal of Money, Credit and Banking*, 23, 519–39.
- CHETTY, R. (2006): “A General Formula for the Optimal Level of Social Insurance,” *Journal of Public Economics*, 90, 1879 – 1901.
- DOLFIN, S. (2006): “An Examination of Firms’ Employment Costs,” *Applied Economics*, 38, 861–878.
- ELSBY, M. W. L. AND R. MICHAELS (2013): “Marginal Jobs, Heterogeneous Firms, and Unemployment Flows,” *American Economic Journal: Macroeconomics*, 5, 1–48.
- ERCEG, C. J., D. W. HENDERSON, AND A. T. LEVIN (2000): “Optimal Monetary Policy with Staggered Wage and Price Contracts,” *Journal of Monetary Economics*, 46, 281–313.

- FAIA, E. (2009): “Ramsey Monetary Policy with Labor Market Frictions,” *Journal of Monetary Economics*, 56, 570–581.
- FAIA, E., W. LECHTHALER, AND C. MERKL (2014): “Labor Selection, Turnover Costs, and Optimal Monetary Policy,” *Journal of Money, Credit and Banking*, 46, 115–144.
- FERNALD, J. G. (2012): “A Quarterly, Utilization-adjusted Series on Total Factor Productivity,” Working Paper Series 2012-19, Federal Reserve Bank of San Francisco.
- HAEFKE, C., M. SONNTAG, AND T. VAN RENS (2013): “Wage Rigidity and Job Creation,” *Journal of Monetary Economics*, 60, 887 – 899.
- HALL, R. E. (2005): “Employment Fluctuations with Equilibrium Wage Stickiness,” *American Economic Review*, 95, 50–65.
- HANSEN, G. D. (1985): “Indivisible Labor and the Business Cycle,” *Journal of Monetary Economics*, 16, 309–327.
- HOSIOS, A. (1990): “On the Efficiency of Matching and Related Models of Search and Unemployment,” *Review of Economic Studies*, 57, 279–298.
- KHAN, A., R. G. KING, AND A. L. WOLMAN (2003): “Optimal Monetary Policy,” *The Review of Economic Studies*, 70, 825–860.
- KRAUSE, M. U. AND T. A. LUBIK (2007): “The (Ir)relevance of Real Wage Rigidity in the New Keynesian Model with Search Frictions,” *Journal of Monetary Economics*, 54, 706–727.
- KYDLAND, F. E. AND E. C. PRESCOTT (1977): “Rules Rather Than Discretion: The Inconsistency of Optimal Plans,” *Journal of Political Economy*, 85, 473–91.
- (1980): “Dynamic Optimal Taxation, Rational Expectations and Optimal Control,” *Journal of Economic Dynamics and Control*, 2, 79–91.
- LANDAIS, C., P. MICHAILLAT, AND E. SAEZ (2013): “Optimal Unemployment Insurance over the Business Cycle,” Discussion Papers 1303, Centre for Macroeconomics (CFM).
- LUCAS, R. J. AND N. L. STOKEY (1983): “Optimal Fiscal and Monetary Policy in an Economy without Capital,” *Journal of Monetary Economics*, 12, 55–93.
- MERKL, C. AND D. WESSELBAUM (2011): “Extensive versus Intensive Margin in Germany and the United States: Any Differences?” *Applied Economics Letters*, 18, 805–808.
- MICHAILLAT, P. (2012): “Do Matching Frictions Explain Unemployment? Not in Bad Times,” *American Economic Review*, 102, 1721–50.
- (2014): “A Theory of Countercyclical Government Multiplier,” *American Economic Journal: Macroeconomics*, 6, 190–217.



- PETRONGOLO, B. AND C. A. PISSARIDES (2001): “Looking into the Black Box: A Survey of the Matching Function,” *Journal of Economic Literature*, 39, 390–431.
- PISSARIDES, C. A. (2000): *Equilibrium Unemployment Theory*, The MIT Press, 2 ed.
- PRESCOTT, E. C. (1977): “Should Control Theory Be Used for Economic Stabilization?” *Carnegie-Rochester Conference Series on Public Policy*, 7, 13–38.
- RAVENNA, F. AND C. E. WALSH (2011): “Welfare-Based Optimal Monetary Policy with Unemployment and Sticky Prices: A Linear-Quadratic Framework,” *American Economic Journal: Macroeconomics*, 3, 130–62.
- ROTEMBERG, J. J. (1982): “Sticky Prices in the United States,” *Journal of Political Economy*, 90, 1187–1211.
- SALA, L., U. SÖDERSTRÖM, AND A. TRIGARI (2008): “Monetary Policy under Uncertainty in an Estimated Model with Labor Market Frictions,” *Journal of Monetary Economics*, 55, 983–1006.
- SCHMITT-GROHÉ, S. AND M. URIBE (2004): “Optimal Fiscal and Monetary Policy under Sticky Prices,” *Journal of Economic Theory*, 114, 198–230.
- (2007): “Optimal Simple and Implementable Monetary and Fiscal Rules,” *Journal of Monetary Economics*, 54, 1702–1725.
- SHIMER, R. (2005): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review*, 95, 25–49.
- SILVA, J. I. AND M. TOLEDO (2009): “Labor Turnover Costs And The Cyclical Behavior Of Vacancies And Unemployment,” *Macroeconomic Dynamics*, 13, 76–96.
- STOLE, L. A. AND J. ZWIEBEL (1996): “Intra-Firm Bargaining under Non-Binding Contracts,” *The Review of Economic Studies*, 63, 375–410.
- THOMAS, C. (2008): “Search and Matching Frictions and Optimal Monetary Policy,” *Journal of Monetary Economics*, 55, 936–956.
- YUN, T. (2005): “Optimal Monetary Policy with Relative Price Distortions,” *American Economic Review*, 95, 89–109.
- ZBARACKI, M. J., M. RITSON, D. LEVY, S. DUTTA, AND M. BERGEN (2004): “Managerial and Customer Costs of Price Adjustment: Direct Evidence from Industrial Markets,” *The Review of Economics and Statistics*, 86, 514–533.

## A. Derivation of Constrained Efficient Allocation

The constrained social planner chooses consumption, employment, and vacancies to maximize the stream of the household's discounted lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \quad (47)$$

subject to the following constraints:

$$a_t n_t^\alpha - \kappa a_t v_t - c_t = 0, \quad (48)$$

$$v_t q(\theta_t) + (1 - \sigma)n_{t-1} - n_t = 0, \quad (49)$$

with

$$\theta_t = \frac{v_t}{1 - (1 - \sigma)n_{t-1}}. \quad (50)$$

Let  $\lambda_t$  and  $\tau_t$  be the multipliers on constraints (48) and (49) respectively. The three first-order conditions with respect to consumption, employment, and vacancies are:

$$U'(c_t) = \lambda_t, \quad (51)$$

$$\lambda_t \alpha a_t n_t^{\alpha-1} - \tau_t + \beta E_t \tau_{t+1} (1 - \sigma) + \beta E_t \tau_{t+1} v_{t+1} q'(\theta_{t+1}) \frac{\partial \theta_{t+1}}{\partial n_t} = 0, \quad (52)$$

and

$$-\lambda_t \kappa a_t + \tau_t q(\theta_t) + \tau_t v_t q'(\theta_t) \frac{\partial \theta_t}{\partial v_t} = 0. \quad (53)$$

Rearranging yields:

$$\tau_t = \lambda_t \alpha a_t n_t^{\alpha-1} + \beta E_t \left[ \tau_{t+1} \left( (1 - \sigma) + v_{t+1} q'(\theta_{t+1}) \frac{\partial \theta_{t+1}}{\partial n_t} \right) \right], \quad (54)$$

$$\tau_t = \frac{\lambda_t \kappa a_t}{q(\theta_t) + v_t q'(\theta_t) \frac{\partial \theta_t}{\partial v_t}}. \quad (55)$$

Plugging (55) into (54) gives:

$$\begin{aligned} \frac{\lambda_t \kappa a_t}{q(\theta_t) + v_t q'(\theta_t) \frac{\partial \theta_t}{\partial v_t}} &= \lambda_t \alpha a_t n_t^{\alpha-1} \\ &+ \beta E_t \left[ \frac{\lambda_{t+1} \kappa a_{t+1}}{q(\theta_{t+1}) + v_{t+1} q'(\theta_{t+1}) \frac{\partial \theta_{t+1}}{\partial v_{t+1}}} \left( (1 - \sigma) + v_{t+1} q'(\theta_{t+1}) \frac{\partial \theta_{t+1}}{\partial n_t} \right) \right]. \end{aligned} \quad (56)$$

Following the definition of market tightness (equation (50)), the partial derivatives with respect to vacancies and employment are:

$$\frac{\partial \theta_t}{\partial v_t} = \frac{1}{1 - (1 - \sigma)n_{t-1}}, \quad (57)$$

and

$$\frac{\partial \theta_t}{\partial n_t} = \frac{(1 - \sigma)v_t}{(1 - (1 - \sigma)n_{t-1})^2} = \frac{(1 - \sigma)\theta_t}{1 - (1 - \sigma)n_{t-1}}. \quad (58)$$

Plugging these into equation (56) yields:

$$\begin{aligned} \frac{\lambda_t \kappa a_t}{q(\theta_t) + q'(\theta_t)\theta_t} &= \lambda_t \alpha a_t n_t^{\alpha-1} \\ &+ \beta E_t \left[ \frac{\lambda_{t+1} \kappa a_{t+1}}{q(\theta_{t+1}) + q'(\theta_{t+1})\theta_{t+1}} \left( (1 - \sigma) + (1 - \sigma)q'(\theta_{t+1})\theta_{t+1}^2 \right) \right]. \end{aligned} \quad (59)$$

The vacancy-filling rate is:

$$q(\theta_t) = \vartheta \theta_t^{\xi-1}. \quad (60)$$

It follows:

$$q'(\theta_t)\theta_t = (\xi - 1)\vartheta \theta_t^{\xi-2}\theta_t = (\xi - 1)q(\theta_t). \quad (61)$$

Using these in (59) yields:

$$\begin{aligned} \frac{\lambda_t \kappa a_t}{\xi q(\theta_t)} &= \lambda_t \alpha a_t n_t^{\alpha-1} \\ &+ \beta(1 - \sigma)E_t \left[ \frac{\lambda_{t+1} \kappa a_{t+1}}{\xi q(\theta_{t+1})} \left( (1 - (1 - \xi)q(\theta_{t+1})\theta_{t+1}) \right) \right]. \end{aligned} \quad (62)$$

This expression can be further simplified:

$$\begin{aligned} \frac{\kappa a_t}{q(\theta_t)} &= \xi \alpha a_t n_t^{\alpha-1} + (1 - \sigma)\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa a_{t+1}}{q(\theta_{t+1})} \right] \\ &- (1 - \xi)(1 - \sigma)\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \kappa a_{t+1} \theta_{t+1} \right]. \end{aligned} \quad (63)$$

The optimality condition from the competitive economy is given by:

$$\frac{\kappa a_t}{q(\theta_t)} = \Lambda_t \cdot \alpha a_t n_t^{\alpha-1} + (1 - \sigma)\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa a_{t+1}}{q(\theta_{t+1})} \right] - w_t. \quad (64)$$

Under the assumption that the social planner can subsidize production in the competitive economy such that real marginal costs are equal to one, the two equations (63) and (64) are equal if the wage takes on the following form:

$$w_t = w_t^* = (1 - \xi) \left( \alpha a_t n_t^{\alpha-1} + (1 - \sigma)\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \kappa a_{t+1} \theta_{t+1} \right] \right). \quad (65)$$

## B. Different Recruiting Costs

The results could depend on the choice of recruiting costs. For a given steady state employment rate, recruiting costs and the steady state real wage cannot be set independently. The choice of recruiting costs thus determines the steady state real wage which is of course relevant from an efficiency point of view. I compare two opposite cases to the benchmark: In the first case, recruiting costs are relatively high and the real wage is relatively low compared to the baseline. In the second case, recruiting costs are relatively low and hence the real wage relatively high. Recruiting costs amount to 12% and 7% of the real wage respectively.<sup>23</sup> Results are displayed in Table B.1. Both optimal inflation and employment volatility are larger the higher the steady state real wage is compared to productivity. The intuition is straightforward: in the presence of real wage rigidity, the elasticity of employment with respect to productivity is increasing in the relative wage.<sup>24</sup> In the Ramsey problem this translates into both higher employment volatility and higher inflation volatility. The effect, however, is relatively small, such that it is safe to conclude that the main results are not driven by an arbitrary choice of recruiting costs.

Rigid Wage & Decreasing Returns									
Scenario	$a$	$w$	$\pi$	$\theta$	$n$	$\Lambda$	$c$	$y$	$u$
High costs	1	0.80	0.21	4.92	0.19	0.07	1.08	1.12	3.10
Baseline	1	0.80	0.23	5.28	0.20	0.07	1.09	1.13	3.31
Low costs	1	0.80	0.25	6.99	0.26	0.09	1.13	1.16	4.23

**Table B.1:** Ramsey policy with different levels of recruiting costs: Standard deviations relative to the standard deviation of productivity. All data in log-deviations from a Hodrick-Prescott filter with smoothing parameter 1600. Results are means over 1000 simulations. Inflation is annualized.

## C. Wage with Firm-Level Employment

Another way to compare the constant returns and the decreasing returns scenario in the presence of rigid wages is to look at the model with decreasing returns to scale, but to have the wage respond to firm-level employment as in equation (39).

$$w_t = \omega a_t^\zeta n_t^{\alpha-1}(i) = \omega a_t^\zeta n_t^{\alpha-1}. \quad (39)$$

The advantage of this approach is that not only the steady state labor market variables but also the production value are equal under both scenarios. This establishes a better

<sup>23</sup>Recruiting costs and the steady state real wage are  $\kappa = 0.0715$  and  $\omega = 0.6$  in the first and  $\kappa = 0.0424$  and  $\omega = 0.6052$  in the second case. For a given steady state unemployment rate, these also determine which share of unemployment is rationed or frictional in the sense of Michaillat (2012). In the first case, there is no rationing unemployment in steady state, i.e. all unemployment is frictional. In the second case, 2.5% of unemployment is due to rationing.

<sup>24</sup>For a nice discussion of this effect see e.g. Brügemann (2014).

comparability between both cases. The results are displayed in Table C.2. As expected, once the wage reacts to firm-level employment, most of the reduction in optimal inflation volatility is lost. If the wage is rigid with respect to aggregate productivity but flexible with respect to firm-level employment, firms do not benefit from a change in the marginal product of labor and therefore do not use it as an adjustment mechanism in response to aggregate shocks.

Scenario	$a$	$w$	$\pi$	$\theta$	$n$	$\Lambda$	$c$	$y$	$u$
Wage eq. (39)	1	0.80	0.71	9.07	0.27	0.12	1.12	1.17	4.67

**Table C.2:** Ramsey policy with real wage rigidity and decreasing returns if the wage responds to firm-level employment. Standard deviations relative to the standard deviation of productivity. All data in log-deviations from a Hodrick-Prescott filter with smoothing parameter 1600. Results are means over 1000 simulations. Inflation is annualized.