Monetary policy implications of state-dependent prices and wages

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Abstract

This paper studies the dynamic general equilibrium effects of monetary shocks in a "control cost" model of state-dependent retail price adjustment and state-dependent wage adjustment. Both suppliers of retail goods and suppliers of labor are monopolistic competitors that face idiosyncratic productivity shocks and nominal rigidities. Stickiness arises because precise choice is costly: decision-makers tolerate errors both in the timing of adjustments, and in the new level at which the price or wage is set, because making these choices with perfect precision would be excessively costly.

The model is calibrated to match the size and frequency of price and wage changes. We find that the impact multiplier of a money growth shock on consumption and labor in our calibrated state-dependent model is similar to that in a Calvo model with the same adjustment frequencies, though the response lasts roughly twice as long under the Calvo mechanism. Wage rigidity accounts for most of the nonneutrality that occurs in a model where both prices and wages are sticky; hence, a model with both rigidities produces substantially larger real effects of monetary shocks than does a model with sticky prices only.

We find that the state-dependence of nominal rigidity strongly decreases the slope of the Phillips curve as trend inflation declines. This result is not driven by downward wage rigidity; adjustment costs are symmetric in our model. Here, instead, price- and wage-setters prefer to adjust less frequently when trend inflation is low, making short-run inflation less reactive to shocks.

Keywords: Sticky prices, sticky wages, state-dependent adjustment, logit equilibrium, near rationality, control costs

JEL Codes: E31, D81, C73

1 Introduction[1]

The nominal rigidity of prices and/or wages is a prominent assumption in monetary macroeconomics today. For reasons of analytical tractability, many studies are based on Calvo's (1983) framework, in which the probability of adjustment is constant. But several influential papers have claimed that if nominal stickiness is derived from rational decision-making, instead of being imposed in an *ad hoc* way, then the real macroeconomic effects monetary policy are negligible (see for example the menu cost models of Caplin and Spulber, 1987, and Golosov and Lucas, 2007). This finding motivates a wave of new research investigating how the conclusions of Calvo-style models and menu cost models hold up in a variety of state-dependent pricing frameworks that are closely calibrated to retail price microdata (*e.g.* Klenow and Kryvtsov, 2008; Gagnon, 2009; Matejka, 2010; Midrigan, 2011; Álvarez, González-Rozada, Neumeyer, and Beraja, 2011; Eichenbaum, Jaimovich, and Rebelo, 2011; Kehoe and Midrigan, 2012; Dotsey, King, and Wolman, 2013; Álvarez, Lippi, and Paciello, 2014; Costain and Nakov, 2011, 2018).

Much of this new literature concludes, to quote Kehoe and Midrigan, that "prices are sticky after all". That is, while money is almost neutral in stripped-down menu cost models like Golosov and Lucas (2007), related frameworks that fit retail microdata better show that price stickiness does matter at the aggregate level, delivering nontrivial real effects of monetary policy.[2] This apparent consensus represents an encouraging improvement in the link between microdata and modern macroeconomics, but it derives from studies where, for computational reasons, price stickiness was the only friction considered. This contrasts with the current generation of empirical DSGE models that rely not only on nominal rigidity of prices and wages, but also on many other frictions, such as consumption habits, investment adjustment costs, and labor matching frictions. Hence, to better assess the quantitative role of nominal rigidity for macroeconomic dynamics, it is still relevant to study models in which multiple frictions interact.

As a modest step forward, this paper analyzes a model with one additional layer of state-dependent adjustment, allowing for wage stickiness as well as price stickiness. A natural point of departure for our analysis is Erceg, Henderson, and Levin's (2000) study of monopolistic retail price setters and monopolistic wage setters, both operating under the Calvo framework. Following Erceg *et al.*, we set up the wage setters' problem so that it closely parallels the price setting problem, but we allow for state dependence in both decisions. More precisely, we compare a framework in which both price and wage setters are constrained by the Calvo friction to a framework in which price and wage setters are both constrained by a state-dependent friction, and in addition we compare these with scenarios in which price setting and/or wage setting approaches perfect flexibility. We emphasize that our goal is to compare different specifications of price stickiness and wage stickiness while abstracting from any other frictions that might affect the labor market (or other markets). While the interaction of nominal rigidities with labor market matching is a major theme of the macro-labor literature, here we quantify the effects of state-dependent prices and wages by themselves, leaving their interaction with matching frictions for future work.

Our model of state-dependent adjustment is an extension of the "control cost" model of price stickiness proposed by Costain and Nakov (2018), henceforth CN18. Control costs are a modeling device from game theory intended to capture the idea that the costs of precise decision-making sometimes lead

[1]Thanks to Isaac Baley, Jordi Galí, Erwan Gautier, Alok Johri, Julián Messina, Michael Reiter, Ernesto Villanueva, and seminar participants at CEF (2016 and 2017), EEA-ESEM (2016 and 2017), DYNARE 2016, T2M 2017, the Catalan Economic Society 2017, the 2017 Inflation Targeting Seminar of the Banco Central do Brasil, the 2017 ESCB Monetary Economics Cluster Workshop, the 2018 Workshop on Theoretical and Experimental Macroeconomics, and at De Nederlandsche Bank (2017) and Danmarks Nationalbanken (2018) for helpful comments. Views expressed here are those of the authors and do not necessarily coincide with those of the Bank of Spain, the Eurosystem, the ECB, or the CEPR.

[2]The reason for nonneutrality is that the microdata seem to favor specifications in which the "selection effect" is weaker than Golosov and Lucas (2007) found.

players to make some mistakes.[3] Under the control cost framework, a decision is regarded as a random variable defined over a set of feasible alternatives, and the decision-maker is assumed to face a cost function that increases with the precision of that random variable. Placing probability one on the optimal alternative is a very precise decision, so the decision-maker may instead economize on the costs of choice by tolerating some randomness (some errors) in the alternative chosen. CN18 models nominal rigidity by applying this framework *both* to the prices firms choose, and to firms' control of the *timing* of their adjustments. In equilibrium, managers of retail firms economize on the time devoted to decision-making by tolerating some low-cost errors in the prices they set, and some low-cost errors in the timing of their price adjustments.

There are a number of reasons why it seems interesting to extend the CN18 framework to other frictions, beyond price stickiness. First, it describes adjustment costs in a sparsely parameterized way; the benchmark scenario in CN18 simultaneously fits many "puzzling" features of retail price setting by calibrating only two free parameters in the decision cost function. Second, these costs have an appealing interpretation: the costs of price adjustment are interpreted as time devoted by management to decision-making. These may plausibly be larger than the menu-type fixed costs associated with the physical act of changing the price, and may be compared, at least roughly, to case studies on time use by management. Third, the model is no harder to solve numerically than comparable menu cost models, but it is far more tractable than "rational inattention" models in the tradition of Sims (2003). Fourth, the mathematical structure of the model— resetting a control variable at intermittent points of time— seems applicable to many decisions other than price adjustment, potentially allowing us to describe many margins of a general equilibrium model in a mutually consistent and mutually comparable way. Finally, since the calibration strategy in the recent state-dependent pricing literature involves matching many moments of the distribution of individual price adjustments, it stretches credulity to abstract from errors. When matching (for example) the standard deviation of observed price adjustments, inferences about the standard deviation of the underlying shocks may differ greatly depending on whether or not we insist that every single price adjustment represents a precisely optimal action.

1.1 Related literature

Time-dependent price and wage rigidities frequently interact in contemporary DSGE models, such as Blanchard and Galí (2007) and Galí, Smets, and Wouters (2012). One of the key papers that first examined the interplay of these two rigidities, under the Calvo mechanism, was Erceg, Henderson, and Levin (2000), which identified a tradeoff between stabilization of output, price inflation, and wage inflation. Huang and Liu (2002) studied the relative importance of price and wage rigidity in a time-dependent model, concluding that wage rigidity matters more for monetary non-neutrality; Christiano, Eichenbaum and Evans (2005) concur. We revisit this question in a state-dependent model.

The literature that contrasts state-dependent pricing models to micro- and macrodata is extensive, as we discussed above; surveys include Klenow and Malin (2010) and Nakamura and Steinsson (2013). We know of only one previous study of state-dependent prices and wages in a DSGE model (Takahashi, 2017). Takahashi's paper differs from ours in that it analyzes a stochastic menu cost model (following Dotsey *et al.*, 1999) rather than a control cost model. But more importantly, it abstracts from idiosyncratic shocks, so it cannot be closely assessed relative to patterns in microdata on price and wage changes. Annual data relevant for analyzing the distribution of wage adjustments include those of the International Wage Flexibility Project (Dickens *et al.* 2007), which we will use here. Barattieri, Basu, and Gottschalk

[3] See Stahl (1990), Mattsson and Weibull (2002), or van Damme (1991), Ch. 4.

(2014) analyze quarterly wage adjustments in SIPP data. Very recently, wage change data with higher frequency and higher coverage have also become available (Grigsby, Hurst, and Yildirmaz, 2018).

Since our framework abstracts from any frictions in labor mobility, it is not directly related to the search and matching literature. However, it can shed light on macro-labor issues such as the slope of the Phillips curve and the cyclicality of real wages and markups. Akerlof, Dickens, and Perry (1996), Fahr and Smets (2008), Benigno and Ricci (2011), and Lindé and Trabandt (2018) have argued that downward nominal wage rigidity makes the Phillips curve flatter when inflation is low. We will show that the same result is obtained without downward rigidity, if the adjustment hazard varies with inflation.

The cyclicality of the real wage has long been controversial (Huang, Liu, and Phaneuf, 2004; McCallum and Smets, 2006; Smets and Wouters, 2007). Christiano, Eichenbaum, and Trabandt (2016) report a small and insignificant procyclical response of the real wage to monetary shocks. Shimer (2007) argues that the "labor wedge", defined as the marginal product of labor minus workers' marginal rate of substitution, is strongly countercyclical. Equivalently, Galí, Gertler, and López-Salido (2007) define an "efficiency gap" (marginal rate of substitution minus marginal product of labor) which they show is strongly procyclical. They further argue that the wedge (the negative of the gap) decomposes into two terms: a highly countercyclical markup of wages over the marginal rate of substitution, and a moderately countercyclical markup of prices over firms' marginal costs. The latter property is controversial: Nekarda and Ramey (2013) show that a wide variety of estimation procedures reject countercyclical markups of prices over firms' marginal costs. Thus, they reject the central tranmission mechanism of the simplest New Keynesian models, in which prices, but not wages, are rigid. Nonetheless, this leaves open the possibility that monetary nonneutrality may derive primarily from wage rigidity.

2 Model

We embed the near-rational nominal adjustment model of Costain and Nakov (2018) in a discrete-time New Keynesian general equilibrium framework that combines elements of Erceg, Henderson, and Levin (2000) and of Golosov and Lucas (2007). There is a continuum of retail firms and a continuum of workers; retail goods markets and labor markets are both monopolistically competitive. Each firm is the unique seller of a differentiated retail good, and resets its nominal price intermittently. Each worker is the unique seller of a differentiated type of labor, and resets its nominal wage intermittently. Price and wage adjustments are driven by idiosyncratic as well as aggregate shocks. Workers belong to a representative household; the budget constraint is defined at the household level. In addition, there is also a monetary authority that sets an exogenous growth process for the nominal money supply.

2.1 Household

The worker's period utility function is $u(C_t) - X(H_t) + v\left(M_t/P_t\right)$, where C_t is consumption, H_t is total time devoted to working *or* decision-making, and M_t/P_t is real money balances. The functions u and v are assumed increasing and concave. We assume the increasing, convex disutility function $X(H) = \frac{\chi}{1+\zeta}H^{1+\zeta}$. We will focus initially on the linear case $\zeta = 0$, implying $X(H) = \chi H$, which is easier to solve, but we will soon see that the nonlinear specification $\zeta > 0$ is necessary to match wage adjustment data. Utility is discounted by factor $\beta \equiv \beta_I \beta_D$ per period, where β_I represents the effect of pure impatience, and β_D reflects the possibility of death (each individual worker dies and is replaced by a new individual with probability $1 - \beta_D$ per period). Consumption is a CES aggregate of differentiated products C_{jt}, with elasticity of substitution ϵ:

$$C_t = \left\{ \int_0^1 C_{jt}^{\frac{\epsilon-1}{\epsilon}} dj \right\}^{\frac{\epsilon}{\epsilon-1}} . \tag{1}$$

The representative household consists of a continuum of workers, and aggregates their resources. Its period budget constraint, in nominal terms, is

$$\int_0^1 P_{jt} C_{jt} dj + M_t + R_t^{-1} B_t = \int_0^1 W_{it} H_{it} di + M_{t-1} + B_{t-1} + T_t^M + T_t^D. \tag{2}$$

Here $\int_0^1 P_{jt} C_{jt} dj$ is total nominal consumption, and $\int_0^1 W_{it} H_{it} di$ is total labor compensation received from supplying the differentiated labor varieties H_{it}. B_t represents nominal bond holdings, with interest rate $R_t - 1$; T_t^M is a lump sum transfer from the central bank, and T_t^D is a dividend payment from the firms.

Households choose $\{C_{jt}, B_t, M_t\}_{t=0}^{\infty}$ to maximize expected discounted utility, subject to the budget constraint (2).[4] The workers in each household set nominal wages intermittently, as we will discuss in Sec. 2.3, and they supply labor to fulfill the demand that arises given the nominal wages they have set. Optimal consumption across the differentiated goods implies

$$C_{jt} = (P_{jt}/P_t)^{-\epsilon} C_t, \tag{3}$$

so nominal spending can be written as $P_t C_t = \int_0^1 P_{jt} C_{jt} dj$ under the price index

$$P_t \equiv \left\{ \int_0^1 P_{jt}{}^{1-\epsilon} dj \right\}^{\frac{1}{1-\epsilon}} . \tag{4}$$

The first-order conditions for total consumption and for money use are:

$$R_t^{-1} = \beta E_t \left(\frac{P_t u'(C_{t+1})}{P_{t+1} u'(C_t)} \right), \tag{5}$$

$$1 - \frac{v'(M_t/P_t)}{u'(C_t)} = \beta E_t \left(\frac{P_t u'(C_{t+1})}{P_{t+1} u'(C_t)} \right). \tag{6}$$

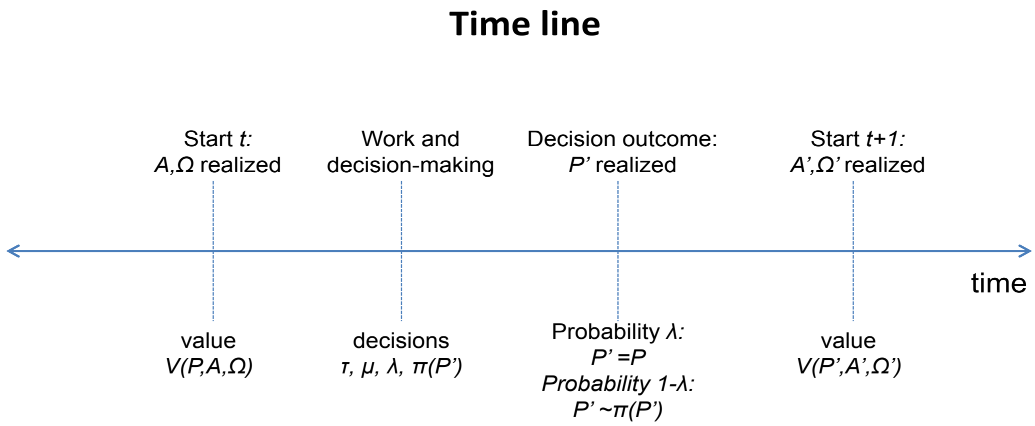
2.2 Monopolistic firms

Each firm j produces output Y_{jt} under a constant returns technology $Y_{jt} = A_{jt} N_{jt}$. Efficiency units of labor, denoted N_{jt}, are the only input. A_{jt} represents an idiosyncratic productivity process that follows a time-invariant Markov process on a bounded set, $A_{jt} \in \Gamma^A \subseteq [\underline{A}, \overline{A}]$. Productivity innovations are *iid* across firms. Thus, A_{jt} is correlated with $A_{j,t-1}$, but it is uncorrelated with other firms' shocks. Firm j is a monopolistic competitor that sets a price P_{jt}, facing the demand curve $Y_{jt} = C_t P_t^{\epsilon} P_{jt}^{-\epsilon}$. We assume each firm must fulfill all demand at its chosen price. Since firms are infinitesimal, each firm j ignores the effect of its own price P_{jt} on the aggregate price level P_t. It hires labor at wage rate W_t, generating profits

$$U_{jt} = P_{jt} Y_{jt} - W_t N_{jt} = \left(P_{jt} - \frac{W_t}{A_{jt}} \right) C_t P_t^{\epsilon} P_{jt}^{-\epsilon} \tag{7}$$

[4]We use an abbreviated notation here for the sake of brevity. The time subscript on the household's decision variables should not be interpreted as indicating deterministic dependence on time; instead, it indicates dependence on the stochastic aggregate state of the economy.

Figure 1: Sequencing of firms' decisions within the period.



per period. Firms are owned by the household, so they discount nominal income between times t and $t+1$ at the rate $\beta \frac{P_t u'(C_{t+1})}{P_{t+1} u'(C_t)}$, consistent with the household's marginal rate of substitution.

It will help to distinguish value functions at several different points in time. First, let $V_t(P, A)$ be the value of a firm that begins period t with nominal price P and productivity A, prior to any time t decisions, and prior to time t output (see the timeline). We assume that choices take time, so if the firm decides in period t to adjust its price, the new price only becomes effective at time $t+1$.[5] Next, let $O_t(P, A)$ be the firm's continuation value, net of current profits, when it still has the option to adjust prices. That is,[6]

$$V_t(P, A) \;=\; \left(P - \frac{W_t}{A}\right) C_t P_t^{\epsilon} P^{-\epsilon} + O_t(P, A) \tag{8}$$

The continuation value $O_t(P, A)$ incorporates the value of the firm's two possible time-t decisions: whether to adjust its price, and if so, which new price P' to set for period $t+1$. The firm may make errors in either of these choices. We discuss these two decisions in turn, beginning with the latter.

2.2.1 Choosing a new price

Our model formalizes the idea that nominal rigidities may derive primarily from the costs of decision-making. While one might assume that by paying a fixed cost, the firm can make the optimal choice, this would amount to imposing a corner solution with perfect precision. We find it more appealing and more realistic to assume that firms can devote more or less time and resources to decision-making, in order to choose more or less precisely. In equilibrium in our framework firms will typically prefer to make choices with an interior degree of precision. Therefore their chosen action will not always be the one

[5] A one-period lag would be unrealistic if the time period were very long. But when we calibrate the model, we will impose a monthly time period, so that a one-period lag is not excessively restrictive.

[6] Again, we use succinct notation, where time subscripts on the value functions represent dependence on the aggregate state. Thus, if the aggregate state of the economy is Ω_t, we define $V_t(P, A) \equiv V(P, A, \Omega_t)$ and $O_t(P, A) \equiv O(P, A, \Omega_t)$, Time-subscripted variables in equation (8) represent aggregate quantities: $P_t \equiv P(\Omega_t)$ is the aggregate price level, $W_t \equiv W(\Omega_t)$ is the aggregate wage, and $C_t \equiv C(\Omega_t)$ is aggregate consumption demand.

that would have been optimal in the absence of decision costs; instead, most choices will involve some degree of "error".

Consistent with this general description, we adopt the "control cost" approach from game theory (see van Damme, 1991, Chapter 4). A key feature of this approach is that we model the price decision indirectly: the firm's problem is written "as if" it chooses a probability distribution over prices, rather than choosing the price *per se*.[7] The problem incorporates a cost function that increases with precision: concentrating greater probability on a smaller range of prices increases costs. Many measures of precision could be used to define this cost function; we choose a definition based on relative entropy, also known as Kullback-Leibler divergence, which is a measure of the difference between one probability distribution and another. For two possible distributions $\pi_1(x)$ and $\pi_2(x)$ of some random variable x with support on set $\mathcal{X}$, the Kullback-Leibler divergence $\mathcal{D}(\pi_1||\pi_2)$ of π_1 relative to π_2 is defined by[8]

$$\mathcal{D}(\pi_1||\pi_2) = \int_{\mathcal{X}} \pi_1(x) \ln \left(\frac{\pi_1(x)}{\pi_2(x)} \right) dx. \tag{9}$$

Following Stahl (1990) and Mattsson and Weibull (2002), we assume that the decision cost is proportional to the Kullback-Leibler divergence of the chosen distribution, relative to an exogenous benchmark distribution. Thus, if no decision costs are paid, the action x is distributed according to the benchmark distribution. But by putting more effort into the decision process, the decision-maker can shrink the distribution of the action towards the most desirable alternatives.

We assume that decision costs are denominated in units of time, since we regard managers' time as the main input to decision-making. The only control variable that the firm must manage is its nominal price. We regard each adjustment of the nominal price as a costly decision; hence when the firm sets a new nominal price $\tilde{P}$, this remains constant in nominal terms until the firm again chooses to make an adjustment. We benchmark the cost of the decision process against an exogenous benchmark distribution $\eta_t(\widetilde{P})$ with support Γ_t^P. The time subscripts on η_t and Γ_t^P allow the benchmark price distribution to change over time, which allows the economy to have a nominal trend; later we detrend the model by restating it in real terms.

Assumption 1. The time cost of choosing a distribution $\pi(\widetilde{P})$ over nominal prices $\widetilde{P} \in \Gamma_t^P$ is $\kappa_\pi \mathcal{D}(\pi||\eta_t)$, where $\kappa_\pi > 0$ is a constant, and $\eta_t(\widetilde{P})$ is an exogenously-given benchmark distribution with support Γ_t^P.

Here κ_π represents the marginal cost of entropy reduction, in units of labor time. The cost function described in Assumption 1 is nonnegative and convex.[9] The upper bound on the cost function is associated with a distribution that places all probability on a single price $\widetilde{P}$ (concretely, costs are maximized when all probability is placed on one price that minimizes the benchmark probability $\eta_t(\widetilde{P})$). The lower bound on this cost function is zero, associated with choosing the distribution $\pi(\widetilde{P})$ equal to the benchmark distribution $\eta_t(\widetilde{P})$.

Now consider the pricing decision under this cost function. If the firm sets a new nominal price $\widetilde{P}$ at time t, this new price only becomes effective at $t + 1$, so the value of setting $\widetilde{P}$ at t is

$$V_t^e(\widetilde{P}, A) \equiv E_t \left[\beta \frac{P_t u'(C_{t+1})}{P_{t+1} u'(C_t)} V_{t+1}(\widetilde{P}, A') \middle| A \right], \tag{10}$$

[7]Luce (1959) and Machina (1985) are early advocates of analyzing decisions in terms of a probability distribution over alternatives; this approach is also adopted by Sims (2003). See Chapter 2 of Anderson *et al.* (1992) for discussion.

[8]While we write (9) with an integral, we can be agnostic at this point about whether $\mathcal{X}$ is a discrete or continuous set. If it is a continuous set, then π_1 and π_2 should be interpreted as density functions. If it is a discrete set, then π_1 and π_2 should be interpreted as vectors of probabilities, and the integral in (9) should be interpreted as a sum.

[9]Cover and Thomas (2006), Theorem 2.7.2.

where $E_t[\bullet|A]$ represents an expectation over the time $t+1$ variables $\Omega' \equiv \Omega_{t+1}$ and $A' \equiv A_{j,t+1}$ conditional on the time t aggregate state Ω_t and firm j's time t productivity $A_{j,t} = A$. Following the control costs methodology, we assume the firm maximizes its value by allocating probability across possible nominal prices $\widetilde{P}$, taking account of decision costs, as follows:

$$\tilde{V}_t(A) = \max_{\pi(\widetilde{P})} \int \pi(\widetilde{P}) V_t^e(\widetilde{P}, A) d\widetilde{P} - \kappa_\pi W_t \int \pi(\widetilde{P}) \ln\left(\frac{\pi(\widetilde{P})}{\eta_t(\widetilde{P})}\right) d\widetilde{P} \quad \text{s.t.} \quad \int \pi(\widetilde{P}) d\widetilde{P} = 1 \tag{11}$$

Note that the decision costs in (11) are converted to nominal units by multiplying by the wage rate. We write the nominal value of the pricing decision as $\tilde{V}_t(A)$, where $A \equiv A_{jt}$ is the firm's current productivity.

The first-order condition for $\pi(\widetilde{P})$ in problem (11) is[10]

$$V_t^e(\widetilde{P}, A) - \kappa_\pi W_t \left(1 + \ln\left(\frac{\pi(\widetilde{P})}{\eta_t(\widetilde{P})}\right)\right) - \mu = 0,$$

where μ is the multiplier on the constraint. Some rearrangement yields a weighted multinomial logit formula:

$$\pi_t(\widetilde{P}|A) \equiv \frac{\eta_t(\widetilde{P}) \exp\left(\frac{V_t^e(\widetilde{P},A)}{\kappa_\pi W_t}\right)}{\int_{\Gamma^P} \eta_t(P') \exp\left(\frac{V_t^e(P',A)}{\kappa_\pi W_t}\right) dP'} \tag{12}$$

The parameter κ_π in the logit function can be interpreted as the degree of noise in the decision process; in the limit as $\kappa_\pi \to 0$, (12) converges to the policy function under full rationality, so that the optimal price is chosen with probability one. Plugging the logarithm of π_t into the objective, we can also derive an analytical formula for the value function:

$$\tilde{V}_t(A) = \kappa_\pi W_t \ln\left(\int \eta_t(\widetilde{P}) \exp\left(\frac{V_t^e(\widetilde{P}, A)}{\kappa_\pi W_t}\right) d\widetilde{P}\right). \tag{13}$$

This formula gives the firm's nominal value when adjusting its current price, net of decision costs.

Some interpretive comments may be helpful at this point. First, although we write the firm's problem "as if" it chooses a probability distribution over prices, this should not be taken literally— actually choosing a distribution would be a complex, costly diversion from the true task of choosing the price itself. Rather, we define the decision as a choice of a mixed strategy because this is a way to incorporate errors into the model. And we describe it as an optimization problem because this disciplines the errors; it amounts to assuming that the firm devotes time and effort to avoiding especially costly mistakes. Aspects of the model that we do take seriously include (a) making decisions is costly in terms of time and other resources; (b) therefore decision-makers do not always take the action that would otherwise be optimal; (c) *ceteris paribus*, more valuable actions are more probable; (d) in a retail pricing context, these considerations apply to the timing of price adjustment, in addition to the actual price chosen, as we will see in the next subsection.

Second, the problem is written conditional on the true expected discounted values $V_t^e(\widetilde{P}, A)$ of the possible nominal prices $\widetilde{P}$, instead of conditioning on a prior, as a "rational inattention" model would. This reflects the fact that we are *not* assuming imperfect information. But this is different from saying that the firm "knows" the true values $V_t^e(\widetilde{P}, A)$. Instead, our interpretation is that the firm has sufficient

[10]Note that if we take future values $V_t^e(\widetilde{P}, A)d\widetilde{P}$, problem (11) maximizes a concave objective subject to a linear constraint. Therefore a unique maximum exists for any given backwards induction step.

information to calculate $V_t^e(\widetilde{P}, A)$. Even so, drawing correct conclusions from that information, and acting accordingly, may be costly.[11]

2.2.2 Choosing the timing of price adjustment

We next analyze, in an analogous manner, the decision whether or not to adjust at time t. As in Sec. 2.2.1, we define costs relative to a benchmark probability distribution over possible actions. But for this decision, at any t, there are only two options: adjust now, or not. Since the probabilities of these two alternatives must sum to one, effectively the relevant benchmark is just a single number, which we can interpret as an exogenous default hazard rate.

We suppose the time period is sufficiently short so that we can ignore multiple adjustments within a single period. If the firm chooses not to adjust its current price P, then its nominal price next period will be unchanged: $\widetilde{P}' = P$; the expected value of this unchanged price, from the point of view of period t, is $V_t^e(P, A)$. If instead the firm adjusts its price at time t, then its expected value is $\tilde{V}_t(A)$, as given by (11) and (13). Now suppose it adjusts its price with probability λ. We measure the cost of this adjustment probability in terms of Kullback-Leibler divergence, relative to some arbitrary Poisson process with arrival rate $\bar{\lambda}$:

> **Assumption 2.** The time cost incurred in period t by setting the price adjustment hazard $\lambda \in [0, 1]$ in period t is $\kappa_\lambda \mathcal{D}((\lambda, 1-\lambda)||(\bar{\lambda}, 1-\bar{\lambda}))$, where $\kappa_\lambda > 0$ and $\bar{\lambda} \in [0, 1]$ are exogenous parameters.

Here κ_λ is the marginal cost of entropy reduction in the timing decision, which might or might not equal the corresponding parameter κ_π from the pricing decision.

Rewriting this cost function using definition (9), the optimal adjustment probability at time t solves the following Bellman equation:

$$O_t(P, A) \;=\; \max_\lambda \, (1-\lambda)V_t^e(P, A) + \lambda \tilde{V}_t(A) - \kappa_\lambda W_t \left[\lambda \ln\left(\frac{\lambda}{\bar{\lambda}}\right) + (1-\lambda)\ln\left(\frac{1-\lambda}{1-\bar{\lambda}}\right)\right]. \tag{14}$$

Recall that $O_t(P, A)$ represents the continuation value of the firm, net of decision costs, when it still has the option to adjust, or not to do so. The first order condition from (14) is

$$V_t^e(P, A) - \tilde{V}_t(A) = \kappa_\lambda W_t \left[\ln \lambda + 1 - \ln \bar{\lambda} - \ln(1-\lambda) - 1 + \ln(1-\bar{\lambda})\right]. \tag{15}$$

Rearranging, we can solve (15) to obtain[12]

$$\lambda_t(P, A) \;=\; \frac{\bar{\lambda}\exp\left(\frac{\tilde{V}_t(A)}{\kappa_\lambda W_t}\right)}{\bar{\lambda}\exp\left(\frac{\tilde{V}_t(A)}{\kappa_\lambda W_t}\right) + (1-\bar{\lambda})\exp\left(\frac{V_t^e(P,A)}{\kappa_\lambda W_t}\right)} \tag{16}$$

$$\;=\; \frac{\bar{\lambda}}{\bar{\lambda} + (1-\bar{\lambda})\exp\left(\frac{-D_t(P,A)}{\kappa_\lambda W_t}\right)}\,, \tag{17}$$

[11] Since economists are accustomed to models of perfect rationality, they often equate observing a given information set with knowing all quantities that can be calculated from that information set. But when rationality is less than perfect, we cannot equate these two assumptions. Here, we assume firms can observe all relevant shocks and state variables, but we do not equate this with actually knowing $V_t^e(\widetilde{P}, A)$ or knowing the optimal action, and therefore we do not equate it with implementing the optimal action with probability one.

[12] Note also that (16) has a well-defined continuous-time limit. If $\bar{\lambda}$ is a continuous-time constant hazard against which we benchmark the costs of a time-varying hazard λ_t, then the continuous-time analogue of (16) is $\lambda_t(P, A) = \bar{\lambda}\exp\left(\frac{D_t(P,A)}{\kappa_\lambda W_t}\right)$.

where $D_t(P, A)$ is the expected gain from adjustment:

$$D_t(P, A) \equiv \tilde{V}_t(A) - V_t^e(P, A). \tag{18}$$

The weighted binary logit hazard (16) was also derived by Woodford (2008) from a model with a Shannon constraint.[13] The free parameter $\bar{\lambda}$ measures the rate of decision making; concretely, the probability of adjustment in one discrete time period is $\bar{\lambda}$ when the firm is indifferent between adjusting and not adjusting (*i.e.* when $D_t(P, A) = 0$).[14]

2.2.3 Deriving the Bellman equation

Next, to calculate the value function $V_t(P, A)$, we concatenate the two decision steps described in Secs. 2.2.1-2.2.2. If the firm starts period t with nominal price P, then its value $V_t(P, A) \equiv V_t(P, A, \Omega_t)$ at the beginning of t satisfies:

$$\begin{aligned} V_t(P, A) \;=\; \max_{\lambda, \pi(\widetilde{P})} & \left(P - \frac{W_t}{A}\right) C_t P_t^{\epsilon} P^{-\epsilon} + (1-\lambda) V_t^e(P, A) + \lambda \int \pi(\widetilde{P}) V_t^e(\widetilde{P}, A) d\widetilde{P} \\ & - \; \lambda \kappa_\pi W_t \int \pi(\widetilde{P}) \ln\left(\frac{\pi(\widetilde{P})}{\eta_t(\widetilde{P})}\right) d\widetilde{P} \; - \; \kappa_\lambda W_t \left[\lambda \ln\left(\frac{\lambda}{\bar{\lambda}}\right) + (1-\lambda)\ln\left(\frac{1-\lambda}{1-\bar{\lambda}}\right)\right] \\ & \text{s.t.} \quad \int \pi(\widetilde{P}) d\widetilde{P} = 1. \end{aligned} \tag{19}$$

This Bellman equation subtracts off the two cost functions seen in the previous subsections.[15] There is a time cost associated with monitoring whether or not a price adjustment is required, which we will call

$$\mu_t(P, A) \equiv \kappa_\lambda \left[\lambda \ln\left(\frac{\lambda}{\bar{\lambda}}\right) + (1-\lambda)\ln\left(\frac{1-\lambda}{1-\bar{\lambda}}\right)\right]. \tag{20}$$

The time cost of choosing which new price to set is

$$\tau_t(P, A) \equiv \lambda \kappa_\pi \int \pi(\widetilde{P}) \ln\left(\frac{\pi(\widetilde{P})}{\eta_t(\widetilde{P})}\right) d\widetilde{P}. \tag{21}$$

Finally, the time devoted to the actual production of goods will be written as

$$N_t(P, A) \equiv \frac{C_t}{A}\left(\frac{P_t}{P}\right)^{\epsilon}. \tag{22}$$

Hence, the firm's total demand for labor hours is $N_t(P, A) + \mu_t(P, A) + \tau_t(P, A)$.

[13] Woodford's (2009) paper only states a first-order condition like (15); his (2008) manuscript points out that the first-order condition implies a logit hazard of the form (16).

[14] This model nests Calvo price adjustment as a special case. If we set $\kappa_\pi = 0$ and $\kappa_\lambda = \infty$, then the firm always sets the optimal price, conditional on adjustment, and adjustment occurs with a constant probability $\bar{\lambda}$.

[15] For expositional transparency, we described pricing and timing above as two separate decisions, each associated with its own cost function. However, these two steps can equivalently be rewritten as a single decision, subject to a single cost function, encompassing the alternatives of non-adjustment or of adjustment to any $\widetilde{P} \in \Gamma_t^P$. For details, see CN18, Sec. 2.2.1. We will see below that the worker's problem must generally be written as a single combined decision, except in the special case of linear labor disutility.

2.3 Labor market

We next construct a model of nominal wage rigidity analogous to our treatment of nominal price rigidity. We suppose each worker i is the monopolistic supplier of a specific type of labor H_{it}, sold at wage W_{it} per unit of time. The productivity of worker i's labor H_{it} is shifted by a shock process Z_{it}, which follows a time-invariant Markov process on a bounded set, $Z_{it} \in \Gamma^Z \subset [\underline{Z}, \overline{Z}]$. We will define $N_{it} = Z_{it}H_{it}$ as the "effective labor" of worker i. By this definition, we can say that the price of effective labor is $\frac{W_{it}}{Z_{it}}$. The idiosyncratic shock process Z_{it} represents worker-specific productivity dynamics, which may include various forms of human capital accumulation.

Firm j's labor input into goods production, N_{jt}, is defined as a CES aggregate across varieties of effective labor i, with elasticity of substitution ϵ_n. That is,

$$N_{jt} = \left\{ \int_0^1 N_{ijt}^{\frac{\epsilon_n - 1}{\epsilon_n}} di \right\}^{\frac{\epsilon_n}{\epsilon_n - 1}}. \tag{23}$$

It is straightforward to show that under this demand structure, the firm's optimal hiring satisfies

$$H_{ijt} \equiv \frac{N_{ijt}}{Z_{it}} = Z_{it}^{\epsilon_n - 1} \left(\frac{W_{it}}{W_t} \right)^{-\epsilon_n} N_{jt}, \tag{24}$$

when we define the wage index

$$W_t \equiv \left\{ \int_0^1 \left(\frac{W_{it}}{Z_{it}} \right)^{1-\epsilon_n} di \right\}^{\frac{1}{1-\epsilon_n}}. \tag{25}$$

Firm j's nominal wage bill for goods production is then

$$\int_0^1 W_{it} H_{ijt} di = W_t N_{jt}. \tag{26}$$

We assume that firms use the same CES mix of labor for decision making that they use for goods production. Then (24) implies that total demand for worker i's time is $H_{it} = H_t(W_{it}, Z_{it})$, defined by

$$H_{it} = Z_{it}^{\epsilon_n - 1} \left(\frac{W_{it}}{W_t} \right)^{-\epsilon_n} N_t \equiv H_t(W_{it}, Z_{it}), \tag{27}$$

where N_t represents aggregate labor demand by all firms. N_t includes labor demand for goods production, given by (22), and labor demand for decision making, given by (20)-(21).

The worker adjusts her nominal wage W_{it} intermittently to maximize the value of labor income net of labor disutility. She faces control costs, both on her timing decision, and on the choice of which wage to set. We assume workers act in the interest of the households of which they form part, and that their consumption is fully insured by the household; hence they discount future income at the same rate $\beta \frac{P_t u'(C_{t+1})}{P_{t+1} u'(C_t)}$ that applies to the household and firm. Now let $L_t(W, Z)$ be the nominal value of a worker with wage W and productivity Z at the beginning of period t, before supplying labor, and before making any decisions. As in the case of price decisions, we assume that a wage adjustment in period t becomes effective in period $t+1$. Therefore the value of setting the nominal wage to an arbitrary new value $\widetilde{W}$ is

$$L_t^e(\widetilde{W}, Z) \equiv E_t \left[\beta \frac{P_t u'(C_{t+1})}{P_{t+1} u'(C_t)} L_{t+1}(\widetilde{W}, Z') \middle| Z \right].$$

We make two assumptions about workers' decision costs that are analogous to our assumptions about firms.

Assumption 3. The time cost of choosing a distribution $\pi^W(\widetilde{W})$ over nominal wages $\widetilde{W} \in \Gamma_t^W$ is $\kappa_w \mathcal{D}(\pi^W || \eta_t^W)$, where $\kappa_w > 0$ is a constant, and $\eta_t^W(\widetilde{W})$ is an exogenously-given benchmark distribution with support Γ_t^W.

Assumption 4. The time cost incurred in period t by setting the wage adjustment hazard $\rho \in [0,1]$ in period t is $\kappa_\rho \mathcal{D}((\rho_t, 1-\rho_t)||(\bar{\rho}, 1-\bar{\rho}))$, where $\kappa_\rho > 0$ and $\bar{\rho} \in [0,1]$ are exogenous parameters.

Now, let τ^w be the (expected) amount of time dedicated in period t to setting a new wage, let μ^w be the time dedicated to monitoring whether it is a good moment to reset the wage. We can then write the worker's wage setting problem in a form analogous to the pricing problem (19):

$$
\begin{aligned}
L_t(W,Z) &= \max_{\tau^w,\mu^w,\rho,\pi^W(\widetilde{W})} WH_t(W,Z) - \frac{P_t}{u'(C_t)} X(H_t(W,Z) + \tau^w + \mu^w) + (1-\rho) L_t^e(Z,W) + \rho \int \pi^W(\widetilde{W}) L_t^e(\widetilde{W}, Z) d\widetilde{W} \\
&\text{s.t.} \quad \int \pi^W(\widetilde{W}) d\widetilde{W} = 1, \\
&\quad \rho\kappa_w \int \pi^W(\widetilde{W}) \ln\left(\frac{\pi^W(\widetilde{W})}{\eta_t^W(\widetilde{W})}\right) d\widetilde{W} \;=\; \tau^w, \\
&\quad \kappa_\rho \left[\rho \ln\left(\frac{\rho}{\bar{\rho}}\right) + (1-\rho)\ln\left(\frac{1-\rho}{1-\bar{\rho}}\right)\right] \;=\; \mu^w.
\end{aligned} \tag{28}
$$

Notice that (28) allows for a nonlinear labor disutility function X; this function is scaled by the factor $P_t/u'(C_t)$ to express the whole Bellman equation in nominal units.

Recall now that we stated the firm's decision in two separate steps, (14) and (11), representing the decision of whether or not to adjust prices, and the decision of what price to set conditional on adjustment, respectively. This decomposition was possible because we assumed the firm could hire any quantity of labor at the (aggregate) wage rate W_t, making its labor costs a linear function of its labor demand. But imposing a linear cost function for a worker's time use would be highly restrictive. We will compute an example with a linear labor disutility function $X(h) = \chi h$ in Sec. 3.1, but we will find that a more general, nonlinear specification $X(h) = \chi \frac{h^{1+\varsigma}}{1+\varsigma}$ is needed to match wage adjustment data. But therefore we cannot simply condition on a given, constant marginal cost of labor: time supplied to firms affects the marginal cost of time used for each type of decision-making, so the two decisions are analyzed simultaneously in the wage setting problem (28).

Nonetheless, the policy functions for wage setting and wage adjustment timing resemble the policy functions from the firm's problem. Following our previous calculations, we find that if the worker adjusts, she chooses the following density over nominal wages $\widetilde{W}$:

$$
\pi_t^W(\widetilde{W}|W,Z) \;\equiv\; \frac{\eta_t^W(\widetilde{W}) \exp\left(\frac{L_t^e(\widetilde{W},Z)}{\kappa_w x_t(W,Z)}\right)}{\int \eta_t^W(W') \exp\left(\frac{L_t^e(W',Z)}{\kappa_w x_t(W,Z)}\right) dW'}, \tag{29}
$$

where $x_t(W,Z)$ denotes the marginal disutility of time in period t:

$$
x_t(W,Z) \;\equiv\; \frac{P_t}{u'(C_t)} X'(H_t^{tot}(W,Z)). \tag{30}
$$

This depends on the worker's total time use $H_t^{tot}(W,Z)$:

$$H_t^{tot}(W,Z) \equiv H_t(W,Z) + \tau_t^w(W,Z) + \mu_t^w(W,Z), \tag{31}$$

which sums the labor hours $H_t(W,Z)$ demanded by employers, plus the two components of time implied by the worker's wage decision process, $\tau_t^w(W,Z)$ and $\mu_t^w(W,Z)$. Note also that (30) rescales disutility to nominal units, for commensurability with the value function L^e.

Likewise, if the worker's beginning-of-period wage and productivity are W and Z, her optimal adjustment probability must satisfy:

$$\rho_t(W,Z) = \frac{\bar{\rho}\exp\left(\frac{\tilde{L}_t(W,Z)}{\kappa_\rho x_t(W,Z)}\right)}{\bar{\rho}\exp\left(\frac{\tilde{L}_t(W,Z)}{\kappa_\rho x_t(W,Z)}\right) + (1-\bar{\rho})\exp\left(\frac{L_t^e(W,Z)}{\kappa_\rho x_t(W,Z)}\right)} \tag{32}$$

$$= \frac{\bar{\rho}}{\bar{\rho} + (1-\bar{\rho})\exp\left(\frac{-D_t^W(W,Z)}{\kappa_\rho x_t(W,Z)}\right)}, \tag{33}$$

where

$$D_t^W(W,Z) \equiv \tilde{L}_t(W,Z) - L_t^e(W,Z) \tag{34}$$

represents the gain in value from adjusting rather than leaving the nominal wage unchanged. The value of adjusting (net of decision costs) has an analytical solution analogous to (13):

$$\tilde{L}_t(W,Z) = \kappa_w x_t(W,Z) \ln\left(\int \eta_t^W(\widetilde{W}) \exp\left(\frac{L_t^e(\widetilde{W},Z)}{\kappa_w x_t(W,Z)}\right) d\widetilde{W}\right). \tag{35}$$

The key to solving the worker's equations is to calculate the marginal disutility of time, $x_t(W,Z)$. Note that if the aggregate variables P_t, W_t, C_t, and N_t are known, then the labor demand function $H_t(W,Z)$ is known from (27). Then, in a context of backwards induction, where the function $L_t^e(W,Z)$ is known, we can use a fixed-point calculation to find $x_t(W,Z)$. By guessing the function $x_t(W,Z)$, we can construct the probabilities and the hazard rate from (29) and (32), and then calculate the decision time components $\tau_t^w(W,Z)$ and $\mu_t^w(W,Z)$ from the constraints on problem (28). This then gives us total time use $H_t^{tot}(W,Z)$, so we can update the function $x_t(W,Z)$ using (30).[16]

While this fixed point calculation suffices to find $x_t(W,Z)$ and thereby solve the worker's problem, it can be avoided in the linear disutility case, where the marginal value of time $x_t = P_t\chi/u'(C_t)$ is independent of the idiosyncratic state (W,Z). This makes the worker's problem much easier to solve under linear disutility than it is in the general nonlinear case.[17] For this reason, in Sec. 3.1 we first compute an example with linear disutility, before attempting the higher-dimensional calculation of the nonlinear case in Sec. 3.2.

[16] As we showed earlier for the worker's problem, problem (28) can be rewritten in terms of a single entropy cost term (a convex function) and a linear objective function. Since labor disutility is also convex, a unique well-defined solution exists for the maximization problem involved in a single backwards induction step. This allows us to conclude that the algorithm described here to calculate $x_t(W,Z)$ has a unique fixed point, which characterizes the marginal value of time in problem (28).

[17] While the worker's logit formulas (29) and (32) look superficially similar to the firm's logits (12) and (17), the worker's problem is generally much harder, because the value of time varies with the worker's idiosyncratic state (W,Z). Assuming a constant marginal disutility reduces the worker's problem to make it similar to the firm's problem, since the firm's marginal cost of time, W_t, is independent of the firm's idiosyncratic state (P,A).

2.4 Detrending

Before we describe the dynamics of the distributions of firms and workers, it is helpful to remove the model's nominal trend. If we choose the default distributions for nominal prices and wages, $\eta_t^P(\widetilde{P})$ and $\eta_t^W(\widetilde{W})$, so that they can be interpreted as unchanging distributions $\eta^p(\widetilde{p})$ and $\eta^w(\widetilde{w})$ of *real* prices and wages, then the firms' and workers' decision problems are homogeneous of degree one in nominal prices, so their Bellman equations can be stated in real rather than nominal terms.

Let Ω_t be a nominal aggregate state variable for this economy at time t. This implies that there exist functions P and W that define the nominal price level and the nominal wage level as a function of Ω_t:

$$P_t = P(\Omega_t), \tag{36}$$
$$W_t = W(\Omega_t). \tag{37}$$

We will define real variables by dividing by the aggregate price level, and we will treat all idiosyncratic real variables in logs. In particular, we define the following idiosyncratic quantities:

$$p_{jt} \equiv \ln P_{jt} - \ln P(\Omega_t), \tag{38}$$
$$\widetilde{p}_{jt} \equiv \ln \widetilde{P}_{jt} - \ln P(\Omega_t), \tag{39}$$
$$a_{jt} \equiv \ln A_{jt}, \tag{40}$$
$$w_{it} \equiv \ln W_{it} - \ln P(\Omega_t), \tag{41}$$
$$\widetilde{w}_{it} \equiv \ln \widetilde{W}_{it} - \ln P(\Omega_t), \tag{42}$$
$$z_{it} \equiv \ln Z_{it}, \tag{43}$$
$$\xi_{it} \equiv x(W_{it}, Z_{it}, \Omega_t)/P(\Omega_t). \tag{44}$$

Defining the default distributions of *real* prices and wages to be time invariant places obvious restrictions on the default distributions of nominal variables. In particular, for any $\widetilde{P} \equiv P(\Omega_t)e^{\widetilde{p}}$, we must have $\eta_t^P(\widetilde{P}) = \widetilde{P}^{-1}\eta^p(\widetilde{p})$. Likewise, given $\widetilde{W} \equiv P(\Omega_t)e^{\widetilde{w}}$, we must have $\eta_t^W(\widetilde{W}) = \widetilde{W}^{-1}\eta^w(\widetilde{w})$.[18]

Now let Ξ_t be the real variable constructed by replacing all nominal state variables that are included in Ω_t by their log real counterparts, and by likewise replacing any distributions of nominal idiosyncratic state variables that are included in Ω_t by the corresponding distributions of log real state variables.[19] It is reasonable to conjecture that Ξ_t is a valid real aggregate state variable for this economy at time t. If so, there must exist functions m, w, and i that determine the real money supply, the real aggregate wage, and the inflation rate in terms of Ξ:

$$m_t \equiv M_t/P(\Omega_t) = m(\Xi_t), \tag{45}$$
$$w_t \equiv W(\Omega_t)/P(\Omega_t) = w(\Xi_t), \tag{46}$$
$$i_t \equiv \ln P(\Omega_t) - \ln P(\Omega_{t-1}) = i(\Xi_t, \Xi_{t-1}). \tag{47}$$

Likewise, aggregate consumption and labor must be functions of the real state, so that

$$c(\Xi_t) = C_t \equiv C(\Omega_t), \tag{48}$$
$$n(\Xi_t) = N_t \equiv N(\Omega_t), \tag{49}$$

[18] To see this, when we say that there is an unchanging distribution of $\widetilde{p}$, we mean that $cdf_t^P(\widetilde{P}) = cdf^p(\widetilde{p})$, evaluated at the point $\widetilde{P} = P_t e^{\widetilde{p}}$. Using the chain rule, this implies $\frac{\partial cdf_t^P}{\partial P}(\widetilde{P})P_t e^{\widetilde{p}} = \frac{\partial cdf^p}{\partial p}(\widetilde{p})$. Then since $\eta_t^P(\widetilde{P}) \equiv \frac{\partial cdf_t^P}{\partial P}(\widetilde{P})$ and $\eta^p(\widetilde{p}) \equiv \frac{\partial cdf^p}{\partial p}(\widetilde{p})$ we obtain $\eta_t^P(\widetilde{P}) = \widetilde{P}^{-1}\eta^p(\widetilde{p})$.

[19] Here we are not yet describing which variables are included in the real state Ξ. We will identify a candidate for the real state Ξ in the next subsections, as we describe the real distributional dynamics.

and firm-specific labor demand can be written as

$$h(w, z, \Xi_t) \;\equiv\; H(P(\Omega_t)e^w, e^z, \Omega_t) \;=\; e^{z(\epsilon_n - 1)} n(\Xi_t) w(\Xi_t)^{\epsilon_n} e^{-\epsilon_n w}. \tag{50}$$

Now, given the real state variable Ξ, the Bellman equations of the firms and workers can be rewritten in terms of real value functions v and v^e that satisfy the identities

$$v(p, a, \Xi) \;\equiv\; \frac{V(P(\Omega)e^p, e^a, \Omega)}{P(\Omega)}, \tag{51}$$

$$v^e(p, a, \Xi) \;\equiv\; \frac{V^e(P(\Omega)e^p, e^a, \Omega)}{P(\Omega)} \;=\; \beta E\left\{ \frac{u'(c(\Xi_{t+1}))}{u'(c(\Xi_t))} v(p - i_{t+1}, a', \Xi_{t+1}) \middle| a, \Xi_t \right\}. \tag{52}$$

We see in (52) that, absent any nominal price adjustment, a log real price p at time t becomes $p - i_{t+1}$ at time $t+1$.[20] Now, the Bellman equation (19) becomes:

$$\begin{aligned} v(p, a, \Xi_t) \;=\; & \max_{\lambda, \pi^p(\tilde{p})} \left(e^p - \frac{w(\Xi_t)}{e^a}\right) c(\Xi_t) e^{-\epsilon p} + (1-\lambda) v^e(p, a, \Xi_t) + \lambda \int \pi^p(\tilde{p}) v^e(\tilde{p}, a, \Xi_t) d\tilde{p} \\ & - \; \lambda \kappa_\pi w(\Xi_t) \int \pi^p(\tilde{p}) \ln\left(\frac{\pi^p(\tilde{p})}{\eta^p(\tilde{p})}\right) d\tilde{p} \; - \; \kappa_\lambda w(\Xi_t) \left[\lambda \ln\left(\frac{\lambda}{\bar{\lambda}}\right) + (1-\lambda)\ln\left(\frac{1-\lambda}{1-\bar{\lambda}}\right)\right] \\ & \text{s.t.} \quad \int \pi^p(\tilde{p}) d\tilde{p} = 1. \end{aligned} \tag{53}$$

Obviously, the worker's Bellman equation (28) can be detrended in analogy with that of the firm. To do so, we postulate real value functions l and l^e that satisfy the identities

$$l(w, z, \Xi) \;\equiv\; \frac{L(P(\Omega)e^w, e^z, \Omega)}{P(\Omega)}, \tag{54}$$

$$l^e(w, z, \Xi) \;\equiv\; \frac{L^e(P(\Omega)e^w, e^z, \Omega)}{P(\Omega)} \;=\; \beta E\left\{ \frac{u'(c(\Xi_{t+1}))}{u'(c(\Xi_t))} l(w - i_{t+1}, z', \Xi_{t+1}) | z, \Xi_t \right\}. \tag{55}$$

The worker's Bellman equation can then be rewritten in real terms as follows:

$$\begin{aligned} l(w, z, \Xi_t) \;=& \max_{\tau^w, \mu^w, \rho, \pi^w(\tilde{w})} e^w h(w, z, \Xi_t) - \frac{X(h(w, z, \Xi_t) + \tau^w + \mu^w)}{u'(c(\Xi_t))} + (1-\rho) l^e_t(w, z, \Xi_t) + \rho \int \pi^w(\tilde{w}) l^e(\tilde{w}, z, \Xi_t) d\tilde{w} \\ & \text{s.t.} \quad \int \pi^w(\tilde{w}) d\tilde{w} = 1, \\ & \rho \kappa_w \int \pi^w(\tilde{w}) \ln\left(\frac{\pi^w(\tilde{w})}{\eta^w(\tilde{w})}\right) d\tilde{w} \;=\; \tau^w, \\ & \kappa_\rho \left[\rho \ln\left(\frac{\rho}{\bar{\rho}}\right) + (1-\rho)\ln\left(\frac{1-\rho}{1-\bar{\rho}}\right)\right] \;=\; \mu^w. \end{aligned} \tag{56}$$

[20] To derive (52) step by step, note that

$$\begin{aligned} P(\Omega) v^e(p, a, \Xi_t) \equiv V^e(P(\Omega)e^p, e^a, \Omega) \;&=\; E\left\{ \beta \frac{P(\Omega_t) u'(C(\Omega_{t+1}))}{P(\Omega_{t+1}) u'(c(\Omega_t))} V(P(\Omega_t)e^p, A', \Omega_{t+1}) \middle| A, \Omega_t \right\} \\ &= E\left\{ \beta \frac{P(\Omega_t) u'(C(\Omega_{t+1}))}{P(\Omega_{t+1}) u'(C(\Omega_t))} V(P(\Omega_{t+1}) e^{p - i_{t+1}}, A', \Omega_{t+1}) \middle| A, \Omega_t \right\} \\ &= P(\Omega_t) E\left\{ \beta \frac{u'(c(\Xi_{t+1}))}{u'(c(\Xi_t))} v(p - i_{t+1}, a', \Xi_{t+1}) \middle| a, \Xi_t \right\}. \end{aligned}$$

Analyzing (56), it is straightforward to show that the chosen distribution of wages takes the form

$$\pi_t^w(\widetilde{w}|w,z) \;\equiv\; \frac{\eta^w(\widetilde{w})\exp\left(\frac{l_t^e(\widetilde{w},w)}{\kappa_w \xi_t(w,z)}\right)}{\int \eta^w(w')\exp\left(\frac{l_t^e(w',z)}{\kappa_w \xi_t(w,z)}\right)dw'}, \tag{57}$$

where

$$\xi_t(w,z) \;\equiv\; \frac{X'(h_t(w,z)+\tau_t^w(w,z)+\mu_t^w(w,z))}{u'(C_t)} \tag{58}$$

is the worker's marginal disutility of time spent working, expressed in units of consumption goods. Similarly, using the first-order condition for ρ, we derive the following adjustment hazard:

$$\rho_t(w,z) \;=\; \frac{\bar{\rho}\exp\left(\frac{\tilde{l}_t(w,z)}{\kappa_\rho \xi_t(w,z)}\right)}{\bar{\rho}\exp\left(\frac{\tilde{l}_t(w,z)}{\kappa_\rho \xi_t(w,z)}\right)+(1-\bar{\rho})\exp\left(\frac{l_t^e(w,z)}{\kappa_\rho \xi_t(w,z)}\right)}. \tag{59}$$

Thus, the decision noise in both the timing choice and the wage-setting choice is proportional to the worker's marginal disutility of labor.

For purposes of backwards induction, to characterize the worker's decision in a given state (w,z,Ξ), it suffices to find the unique value of $\xi_t(w,z)$ that solves (58). The time allocations to the timing decision and the wage-setting decision are

$$\mu_t^w(w,z) \;= \kappa_\rho\left[\rho_t(w,z)\ln\left(\frac{\rho_t(w,z)}{\bar{\rho}}\right)+(1-\rho_t(w,z))\ln\left(\frac{1-\rho_t(w,z)}{1-\bar{\rho}}\right)\right], \tag{60}$$

$$\tau_t^w(w,z) \;= \kappa_w \rho_t(w,z)\int \pi^w(\widetilde{w}|w,z)\ln\left(\frac{\pi^w(\widetilde{w}|w,z)}{\eta^w(\widetilde{w})}\right)d\widetilde{w}. \tag{61}$$

These can be calculated using (57) and (59); their sum is strictly decreasing as a function of ξ. Since marginal disutility increases strictly with total time use (and since $h_t(w,z)$ does not depend on ξ), the right-hand side of (58) can be viewed as a strictly decreasing function of ξ. Therefore (58) can be solved by bisection to give a unique solution $\xi_t(w,z)\geq 0$ in any given state (w,z,Ξ_t).

2.5 Distributional dynamics

The distribution of firms' prices and productivities, and likewise that of workers' wages and productivities, evolves over time as firms and workers respond to idiosyncratic and aggregate shocks. We first state the equations governing the dynamics of the distribution across firms.

We continue to use the notation P_{jt} to refer to the nominal price at which firm j produces in period t, prior to adjustment. This may of course differ from its price $\widetilde{P}_{jt}$ at the end of t, when price adjustments are realized. Therefore we will distinguish the beginning-of-period distribution of prices and log productivities, $\Phi_t(P_{jt},a_{jt})$, from the distribution of prices and log productivities at the end of t, $\widetilde{\Phi}_t(\widetilde{P}_{jt},a_{jt})$. But instead of tracking nominal prices P_{jt}, it is simpler to focus on log real prices p_{jt}. Therefore, in analogy to the nominal distributions, we define $\Psi_t(p_{jt},a_{jt})$ as the real distribution at the beginning of t, when production takes place, and $\widetilde{\Psi}_t(\widetilde{p}_{jt},a_{jt})$ as the real distribution at the end of t. Finally, we also use lower-case letters to represent the joint densities associated with these distributions, which we write as $\phi_t(P_{jt},a_{jt})$, $\widetilde{\phi}_t(\widetilde{P}_{jt},a_{jt})$, $\psi_t(p_{jt},a_{jt})$, and $\widetilde{\psi}_t(\widetilde{p}_{jt},a_{jt})$, respectively.[21]

[21]Our notation in this section assumes that all densities are well-defined on a continuous support, but we do not actually impose this assumption on the model. With slightly more sophisticated notation we could allow explicitly for distributions with mass points, or with discrete support.

Two stochastic processes drive the dynamics of the distribution. First, there is the Markov process for firm-specific log productivity, which we can write in terms of the following *c.d.f.*:

$$S(a'|a) \;=\; prob(a_{j,t} \leq a'|a_{j,t-1} = a), \tag{62}$$

or in terms of the corresponding density function:

$$s(a'|a) \;=\; \frac{\partial}{\partial a'} S(a'|a). \tag{63}$$

Thus, suppose that the density of nominal prices and log productivities at the end of period $t-1$ is $\tilde{\phi}_{t-1}(\tilde{P}, a)$. The density at the beginning of t, after productivity shocks, will therefore be

$$\phi_t(\tilde{P}, a') \;=\; \int s(a'|a)\widetilde{\phi}_{t-1}(\tilde{P}, a)da. \tag{64}$$

But this equation conditions on a given nominal price $\tilde{P}$. Holding fixed a firm's nominal price, its real log price is changed by inflation, from $\widetilde{p}_{i,t-1}$ to $p_{i,t} \equiv \widetilde{p}_{i,t-1} - i_t$. Therefore the density of real log prices and log productivities at the beginning of t is given by

$$\psi_t\left(\widetilde{p} - i_t, a'\right) \;=\; \int s(a'|a)\widetilde{\psi}_{t-1}(\widetilde{p}, a)da, \tag{65}$$

and hence the cumulative distribution at the beginning of t, in real terms, is

$$\Psi_t(p, a') \;=\; \int^p \int^{a'} \left(\int s(b|a)\widetilde{\psi}_{t-1}\left(q + i_t, a\right) da \right) db\, dq. \tag{66}$$

The second stochastic process that determines the dynamics is the process of real price updates, which we have defined in terms of a conditional density of logit form in (12). A firm with real log price p and log productivity a at the beginning of period t adjusts its price with probability $\lambda\left(\frac{d_t(p,a)}{\kappa_\lambda w_t}\right)$, where

$$d_t(p, a) \;\equiv\; \widetilde{v}_t(a) - v^e_t(p, a). \tag{67}$$

Upon adjustment, its new real log price is distributed according to $\pi_t(\widetilde{p}|a)$. Therefore, if the density of firms at the beginning of t is $\psi_t(p, a)$, the density at the end of t is given by

$$\widetilde{\psi}_t(\widetilde{p}, a) \;=\; \left(1 - \lambda\left(\frac{d_t(\widetilde{p}, a)}{\kappa_\lambda w_t}\right)\right)\psi_t(\widetilde{p}, a) + \int \lambda\left(\frac{d_t(p, a)}{\kappa_\lambda w_t}\right)\pi_t(\widetilde{p}|a)\psi_t(p, a)dp. \tag{68}$$

The cumulative distribution at the end of t is simply given by integrating up this density:

$$\widetilde{\Psi}_t(p, a) \;=\; \int^{\widetilde{p}} \int^{a} \widetilde{\psi}_t(q, b)db\, dq. \tag{69}$$

The dynamics of wages and worker productivities is analogous, except that an individual worker may die and be replaced by a new worker with probability $1 - \beta_D$ per period. It suffices to go directly to the real log dynamics, without developing notation for the nominal dynamics. Let $\Psi^w_t(w_{it}, z_{it})$ be the distribution of real log prices and log worker productivities at the beginning of the period, when production takes place, and let $\widetilde{\Psi}^s_t(\widetilde{w}_{it}, z_{it})$ be the corresponding distribution of *surviving* workers at the end of the period. We write the densities associated with these distributions as $\psi_t^w(w_{it}, z_{it})$ and $\widetilde{\psi}_t^s(\widetilde{w}_{it}, z_{it})$, respectively.

Now, consider a worker with real log wage w and log productivity z at the beginning of period t; she adjusts her wage with probability $\rho\left(\frac{d_t^w(w,z)}{\kappa_\rho \xi_t(w,z)}\right)$, where

$$d_t^w(w,z) \;\equiv\; \widetilde{l}_t(w,z) - l_t^e(w,z). \tag{70}$$

Upon adjustment, her new real log wage is distributed according to $\pi_t^w(\tilde{w}|w,z)$. Therefore, if the density of workers at the beginning of t is $\psi_t^w(w,z)$, the density at the end of t is given by

$$\widetilde{\psi}_t^w(\widetilde{w},z) \;=\; \left(1-\rho\left(\frac{d_t^w(\widetilde{w},z)}{\kappa_\rho \xi_t(\widetilde{w},z)}\right)\right)\psi_t^w(\widetilde{w},z) + \int \rho\left(\frac{d_t^w(w,z)}{\kappa_\rho \xi_t(w,z)}\right)\pi_t^w(\widetilde{w}|w,z)\psi_t^w(w,z)dw. \tag{71}$$

The cumulative distribution at the end of t integrates up this density:

$$\widetilde{\Psi}_t^w(\widetilde{w},z) \;=\; \int^{\tilde{w}}\int^{z} \psi_t(q,b)db\,dq. \tag{72}$$

A worker alive in period t survives to period $t+1$ with probability β_D. The worker's productivity, conditional on survival, is driven by the Markov process S^z:

$$S^z(z'|z) \;=\; prob(z_{i,t+1} \leq z'|z_{i,t} = z), \tag{73}$$

with the following density function:

$$s^z(z'|z) \;=\; \frac{\partial}{\partial z'}S(z'|z). \tag{74}$$

Meanwhile, holding fixed a worker's nominal wage, her real log wage is changed by inflation, from $\widetilde{w}_{i,t}$ at the end of t, to $w_{i,t+1} \equiv \widetilde{w}_{i,t} - i_{t+1}$. Therefore the density of real log wages and log worker productivities among surviving workers at the beginning of $t+1$ is given by

$$\psi_{t+1}^s\left(\widetilde{w} - i_{t+1}, z'\right) \;=\; \int s^z(z'|z)\tilde{\psi}_t^w(\widetilde{w},z)dz. \tag{75}$$

Hence the cumulative distribution at the beginning of t integrates up the density in (75) and adds on the component of new-born workers, who have distribution Ψ_t^0:

$$\Psi_{t+1}^w(w,z) \;=\; \beta_D\int^{w}\int^{z}\left(\int s^z(b|y)\tilde{\psi}_t^w\left(q+i_{t+1},y\right)dy\right)db\,dq + (1-\beta_D)\Psi_{t+1}^0(w,z). \tag{76}$$

Taking account of birth and death matters here because it allows us to impose a productivity process that has an upward trend over the course of an individual's working life: a worker typically ends her career at a wage higher than the one she started with. We find that this upward trend is important for matching the distribution of wage adjustments. We denote the distribution of wages and productivity for newborn workers at time t by Ψ_t^0.

For simplicity, we assume that the wage of a newborn worker is the wage that she would set, conditional on her productivity, if her wage were costlessly flexible at all times. We make this simplifying assumption to avoid modeling an initial decision-making state prior to beginning life as a worker. Since our analysis only addresses the properties of wage changes, ignoring the level of the initial wage, this simplifying assumption has a negligible impact on the empirical properties we will document here.

2.6 Aggregate consistency and monetary policy

When supply equals demand for each good j, total supply and demand of effective labor satisfy

$$N_t - \mu_t - \tau_t = \int_0^1 \frac{C_{jt}}{A_{jt}} dj = C_t \int\int \psi_t(p,a)\exp(-\epsilon p - a) da\, dp \equiv \Delta_t C_t. \tag{77}$$

Here μ_t is total time devoted to deciding whether to adjust prices, and τ_t is total time devoted to choosing which price to set by firms that adjust:

$$\mu_t = \int\int \psi_t(p,a)\mu_t(p,a) da dp \tag{78}$$

$$\tau_t = \int\int \psi_t(p,a)\tau_t(p,a) da dp \tag{79}$$

where firm-specific decision times are given by (20)-(21). Equation (77) also defines a measure of price dispersion, $\Delta_t \equiv P_t^{\epsilon} \int_0^1 P_{jt}^{-\epsilon} A_{jt}^{-1} dj$, weighted to allow for heterogeneous productivity. As in Yun (2005), an increase in Δ_t decreases the goods produced per unit of labor, effectively acting like a negative aggregate productivity shock.

In nominal terms, the price level and wage level are given as follows

$$\int\int P^{1-\epsilon}\phi_t(P,A) dA\, dP = P(\Omega_t)^{1-\epsilon}. \tag{80}$$

$$\int\int \left(\frac{W}{Z}\right)^{1-\epsilon_N} \phi_t^W(W,Z) dZ\, dW = W(\Omega_t)^{1-\epsilon_N}. \tag{81}$$

Given (80), the real price level is one by definition:

$$\int\int \exp((1-\epsilon)p)\psi_t(p,a) da\, dp = 1. \tag{82}$$

The real wage level satisfies

$$\int\int \exp((1-\epsilon_N)(w-z))\psi_t^W(w,z) dz\, dw = w(\Xi_t)^{1-\epsilon_N}. \tag{83}$$

On the policy side, we consider a monetary authority that generates an exogenous process for the money growth rate. We assume the nominal money supply is affected by an AR(1) shock process g,[22]

$$g_t = \phi_g g_{t-1} + \epsilon_t^g, \tag{84}$$

where $0 \leq \phi_g < 1$ and $\epsilon_t^g \sim i.i.d.N(0, \sigma_g^2)$. Here g_t represents the time t rate of money growth:

$$M_t/M_{t-1} \equiv \mu_t = \mu^* \exp(g_t). \tag{85}$$

Seigniorage revenues are paid to the household as a lump sum transfer T_t^M, and the government budget is balanced each period, so that $M_t = M_{t-1} + T_t^M$.

[22] In related work (Costain and Nakov 2011) we have studied state-dependent pricing when the monetary authority follows a Taylor rule. Our conclusions about the degree of state-dependence, microeconomic stylized facts, and the real effects of monetary policy were not greatly affected by the type of monetary policy rule considered. Therefore we focus here on the simple, transparent case of a money growth rule.

To describe the aggregate state of the economy, we must take into account aggregate shocks and the distribution of idiosyncratic states. Since nominal prices are predetermined under the timing we have assumed here, it is natural to conjecture that the nominal state of the economy can be summarized by the following objects:

$$\Omega_t \ \equiv \ (M_t, g_t, \Phi_t, \Phi^w_t). \tag{86}$$

Since the model is homogeneous of degree one in nominal variables, the corresponding real state variable would be:

$$\Xi_t \ \equiv \ (g_t, \Psi_t, \Psi^w_t). \tag{87}$$

We will show that this is a valid state variable for the economy by constructing an equilibrium in terms of Ξ.

3 Results

3.1 Special case: linear labor disutility

As we discussed in Sec. 2.3, our model is much simpler to compute when labor disutility is linear; therefore we will explore the linear case before moving on to a nonlinear specification in Sec. 3.2. We simulate and compare several versions of the model with varying degrees of noise in the pricing and wage-setting processes. At the micro level, we study how decision costs affect the frequency and the distribution of price and wage adjustments; at the macro level, we study which noise margin contributes most to the non-neutrality of monetary shocks.

3.1.1 Parameters

Utility from consumption and money holdings, and disutility from labor, are $u(C) = \frac{1}{1-\gamma}(C^{1-\gamma} - 1)$, $v(m) = \nu \ln(m)$, and $X(h) = \frac{\chi}{1+\zeta} h^{1+\zeta}$, respectively; we initially set $\zeta = 0$ to study the linear case. Following Golosov and Lucas (2007), we set $\gamma = 2$, $\nu = 1$, $\chi = 6$, and $\epsilon = 7$, and we set the same the elasticity of substitution across varieties of labor as that across goods: $\epsilon_N = 7$. The discount factor is set to $\beta = 0.9967$ (a four percent annual discount rate).

We simulate the model at monthly frequency on a discrete grid. The productivity processes for firms and workers are assumed to follow discretized approximations of the following AR(1) processes:

$$a_{jt} = \rho_a a_{jt-1} + \epsilon^a_t, \tag{88}$$

$$z_{it} = \rho_z z_{it-1} + \epsilon^z_t, \tag{89}$$

where ϵ^a_t and ϵ^z_t are *i.i.d.* normal shocks with mean zero. Thus the variances of a_{jt} and z_{it} are $\sigma^2_a = \frac{\sigma^2_{\epsilon a}}{1-\rho^2_a}$ and $\sigma^2_z = \frac{\sigma^2_{\epsilon z}}{1-\rho^2_z}$, where $\sigma^2_{\epsilon a}$ and $\sigma^2_{\epsilon z}$ are the variances of the innovations ϵ^a_t and ϵ^z_t, respectively.

Note that a linear disutility specification severely limits our ability to match the wage distribution, because it means the wage is invariant to idiosyncratic productivity shocks. Therefore we postpone estimating the productivity processes until we study the nonlinear specification. Instead, we simply fix the standard deviations of the productivity shocks to $\sigma_a = 0.06$ for firms and $\sigma_z = 0.04$ for workers; both productivity processes are assumed to have monthly autocorrelation 0.8. We assume two percent annual money growth in steady state, consistent with our retail pricing data (discussed below).

To analyze the micro and macro implications of decision costs, we compare six calibrations (listed in Table 1) that vary the noise levels κ_π, κ_λ, κ_w, and κ_ρ. All six calibrations are variations on the

Table 1: Adjustment parameters for linear disutility simulations.

	V1	V2	V3	V4	V5	V6
$\kappa_\pi = \kappa_\lambda$	0.017	0.0017	0. 00017	0.017	0.017	0.00017
$\kappa_w = \kappa_\rho$	0.017	0.017	0.017	0.0017	0.00017	0.00017

Note: Baseline noise $\kappa_0 \equiv 0.017$ is estimated in CN18 by fitting retail price change data.

benchmark case V1, in which the four noise parameters are set to $\kappa_\pi = \kappa_\lambda = \kappa_w = \kappa_\rho = \kappa_0 \equiv 0.017$, implying substantial stickiness both for prices and for wages. The benchmark noise level $\kappa_0 = 0.017$ is the estimate of CN18, who found that this value, together with $\bar{\lambda} = 0.2$, gave the best fit to data on the frequency and distribution of retail price adjustments under the constraint $\kappa_\pi = \kappa_\lambda$. Following CN18, we set both $\bar{\lambda} = 0.2$ and $\bar{\rho} = 0.2$ in all versions V1-V6.

Versions V2-V6 vary the noise parameters while fixing all remaining parameters. Versions V2 and V3 reduce price stickiness relative to the benchmark V1, lowering κ_π and κ_λ first to $\kappa_0/10 = 0.0017$ and then to $\kappa_0/100 = 0.00017$, which makes prices almost perfectly flexible. Specifications V4 and V5 instead reduce wage stickiness relative to the benchmark V1, lowering both κ_w and κ_ρ first to $\kappa_0/10$ and then to $\kappa_0/100$, making wages almost perfectly flexible. Version V6 assumes both margins are very flexible, setting all noise parameters to $\kappa_0/100$.

3.1.2 Data

Table 2, Figure 2, and subsequent results will compare the various calibrations of our model to microdata on price and wage adjustments. As in CN18, our pricing data come from the Dominick's supermarket dataset documented by Midrigan (2011).[23] These data represent weekly regular price changes, excluding temporary sales, and are displayed (in logs) as a blue-shaded histogram in the left column of Fig. 2. We aggregate weekly adjustment rates to monthly rates for comparability with most related studies. We exclude sales because recent literature has shown that monetary nonneutrality depends primarily on the frequency of "regular" or "non-sale" price changes (see for example Eichenbaum *et al.*, 2011; Guimaraes and Sheedy, 2011; or Kehoe and Midrigan, 2014).

Our wage change data are from the International Wage Flexibility Project (IWFP), and are shown as a blue-shaded histogram in the right column of Fig. 2. These data are taken from Fig. 2a of Dickens *et al.* (2007), which documents the results of the IWFP. The figure aggregates histograms of wage adjustments across multiple countries. While most of the underlying national data are drawn from surveys of firms, they refer to annual nominal wage changes of individual workers who remain employed by the same firm. The IWFP focused on annual changes because it observed a widespread tendency for wages to change once a year for many workers in many countries, which in turn means that much of the available survey data addresses annual changes. Clearly this makes our data on wage changes less than perfect for comparison with our price change data, which are at weekly frequency. Nonetheless, to try to get a quantitative benchmark for our theoretical model, we will take the IWFP data at face value.[24] Therefore

[23]We are grateful to Virgiliu Midrigan for making his price data available to us, and to the James M. Kilts Center at the Univ. of Chicago GSB, which is the original source of those data.

[24]Grigsby, Hurst, and Yildirmaz (2018) study wage adjustment using higher-frequency data more comparable to those from the retail price adjustment literature. In U.S. data from a large payroll data processing firm, they find a wage adjustment probability of 26.0% quarterly and 72.7% annually; the mean absolute wage change, conditional on adjustment, is 10.7%.

Table 2: Evaluating the linear disutility model with different values of κ_π, κ_λ, κ_w and κ_ρ

	Data		Sticky		Decreasing price stickiness				Decreasing wage stickiness				Flexible	
			V1		V2		V3		V4		V5		V6	
Consumption			0.3496		0.3508		0.3514		0.3501		0.3489		0.3522	
Labor			0.3530		0.3493		0.3481		0.3535		0.3523		0.3488	
Wage			0.8576		0.8638		0.8666		0.8576		0.8576		0.8666	
	Prices	Wages	Prices	Wages	Prices	Wages	Prices	Wages	Prices	Wages	Prices	Wages	Prices	Wages
Freq. of change, %/mo.	10.2	8.3	10.1	6.02	22.5	6.03	54.4	6.04	10.1	6.41	10.41	7.28	54.4	6.95
Mean change, %	1.60	5.10	1.68	2.83	0.76	2.82	0.31	2.82	1.68	2.66	1.68	2.34	0.31	2.45
Mean abs(change), %	9.90	6.47	8.57	6.14	6.80	6.16	4.76	6.16	8.57	2.70	8.57	1.98	4.76	2.29
Std. of changes, %	13.2	6.52	10.6	8.53	7.50	8.53	5.30	8.52	10.6	2.10	10.6	1.26	5.26	1.79
Skewness of changes	-0.42	0.35	-0.11	-0.41	-0.15	-0.41	-0.08	-0.41	-0.11	-0.91	-0.11	-1.06	-0.08	-1.26
Kurtosis of changes	4.81	4.39	3.20	10.0	1.82	9.77	1.94	9.66	3.20	5.21	3.20	3.62	1.94	3.33
Percent increases	65.1	86.5	58.5	72.1	55.5	72.0	54.0	71.9	58.5	92.2	58.5	99.1	54.0	92.0
Changes $\leq$5%	35.5	43.0	28.8	48.1	27.0	47.9	60.7	47.9	28.8	91.7	28.8	100	60.8	99.9
Changes $\leq$2.5%	12.0	11.8	14.2	24.7	8.90	24.6	19.9	24.6	14.2	57.4	14.2	81.1	20.0	61.6
Std. of prices, wages, %			5.26	3.07	5.63	3.08	6.06	3.08	5.26	1.21	5.26	0.75	6.06	0.96
Resetting cost, % rev.*			0.43	0.33	0.08	0.34	0.01	0.34	0.43	0.06	0.43	0.01	0.01	0.00
Timing cost, % rev.*			0.34	0.38	0.05	0.38	0.01	0.39	0.34	0.05	0.34	0.01	0.01	0.01
Loss relative to flex, %†			1.67	0.92	0.38	0.95	0.05	0.94	1.67	0.13	1.67	0.01	0.05	0.01

**Note:* Costs μ, μ^w, τ, and τ^w are expressed as percentages of average revenues (for firms) or average labor income (for workers).

†*Note:* Gain accruing to a single firm or worker not constrained by decision costs ($\kappa = 0$), relative to constrained, as % of average revenues (firms) or labor income (workers).

in Table 2 we report that the monthly frequency of nominal wage adjustment is 1/12=0.083, and we calculate statistics about nominal wage changes directly from the IWFP histogram.

3.1.3 Steady-state results: linear disutility

Table 2 compares steady-state statistics on price and wage adjustments as the noise parameters vary. Note first that as we move from the benchmark V1 to the low-noise specification V6, aggregate consumption increases while total labor hours decrease. This is to be expected, as the overall efficiency of the economy increases when there are less frictions. The price adjustment frequency more than quintuples, rising from 10.1% to 54.4% per month. The wage adjustment frequency instead rises only slightly, from 6.02% to 6.95% monthly.

There are also some small cross effects, from price rigidity to wage adjustment, and *vice versa*. While decreasing wage rigidity slightly increases the price adjustment frequency (it rises from 10.1% in V4 to 10.4% in V5), a decrease in price rigidity may instead *decrease* the frequency of wage changes (which falls from 7.28% in V5 to 6.95% in version V6).

As price rigidity decreases (comparing V1, V2, and V3), the absolute size of price changes falls from 8.57% to 4.76%. Likewise, their standard deviation falls from 10.6% to 5.30%, and their kurtosis decreases from 3.20 to 1.94. Price resetting and timing costs (μ and τ) fall as a fraction of revenues, and the losses relative to the fully flexible case fall precipitously, to only a few basis points. The effects of decreased wage rigidity are analogous. Comparing V1 with V4 and V5, the absolute size of wage changes falls from 6.14% to 1.98%; their standard deviation falls dramatically, from 8.53% to 1.26%, and their kurtosis likewise falls from 10.0 to 3.62. The costs associated with wage resetting and wage reset timing (μ^w and τ^w) almost vanish, as a fraction of labor income, in specifications V5 and V6.

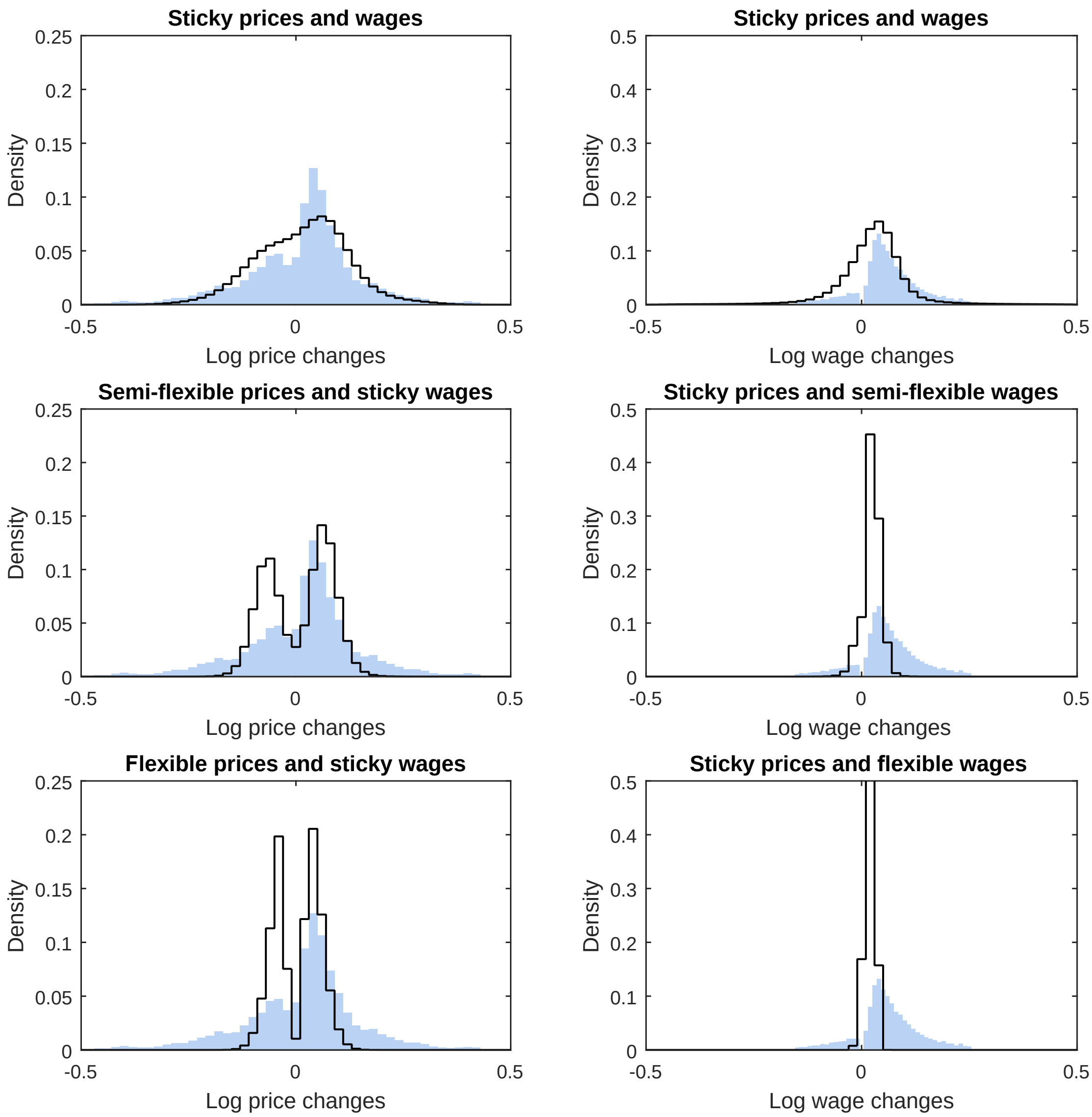
Again, there are contrasting cross-effects between prices and wages. Decreased wage rigidity has no observable effect on the absolute size of price adjustments. Decreased price rigidity is instead observed to *increase* the size of wage changes (compare V5 and V6).

The statistics in Table 2 are drawn from the steady-state distributions of nonzero log price and wage changes, which are plotted as histograms in Fig. 2. Black lines represent the predicted distributions from the various model versions; the blue shaded areas show the distributions from microdata. The left column of the figure shows how the histogram of nonzero log price changes varies as we decrease the noise in the price decision, comparing versions V1, V2, and V3. The variance of the price change distribution in the benchmark version V1 is similar to that of the data, though the model-generated distribution is smoother and less bimodal than the data. As κ_π and κ_λ decrease, the model histogram becomes much more bimodal than the data, displaying two sharp spikes like those from the menu cost model of Golosov and Lucas (2007). In contrast, the right column of the figure shows that the distribution of nonzero log wage changes remains unimodal, becoming ever more concentrated around a single sharp peak as we decrease the noise in wages from version V1 to V4 and V5. The peak of the wage change histogram lies above zero, reflecting the nominal trend in our simulations; likewise, the mean price change is positive.

Thus, as decision noise decreases, price adjustments increasingly resemble the familiar (S, s) behavior associated with a menu cost model. Errors in pricing and timing smooth out the distribution of changes under calibration V1, but as noise is reduced, the preponderance of price changes occur around two upper and lower thresholds. Very small changes are rare, because it is not worth paying the cost of changing the price when it is already near its target value. The chosen decision cost $\kappa_\pi \mathcal{D}(\pi||\eta)$ decreases

Considering job stayers only, they find a 66.3% annual wage adjustment probability, with a mean absolute change of 6.34%. While their data support a somewhat higher degree of wage variation than the IWFP data in our graphs, nonetheless the order of magnitude is similar.

Figure 2: Distribution of nonzero price and wage changes: varying stickiness ($\zeta = 0$).



Notes: Black lines: model versions with linear labor disutility ($\zeta = 0$). Blue shaded areas: Data.
Left column: Effect of decreasing price stickiness (versions V1, V2, V3) on distribution of nonzero price adjustments.
Right column: Effect of decreasing wage stickiness (versions V1, V4, V5) on distribution of nonzero wage adjustments.

as κ_π declines; this is why the distance between the two peaks of the price change histogram decreases as we move down the left panels of Fig. 2 from version V1 to V2 and V3.

Linear labor disutility is the reason why the wage change histogram behaves differently than the price change histogram in these simulations. Given linear disutility, if decisions were perfectly costless, labor supply would respond elastically to productivity at the wage $w_t = \frac{\epsilon_N \chi}{\epsilon_N - 1}/u'(C_t)$: worker i would respond to a positive shock to $z_{i,t}$ by supplying all the additional labor demanded, instead of setting a higher wage. Error-prone choice spreads wages out around this frictionless optimum, as we see in version V1 (top, right panel of Fig. 2). But as the noise in wage adjustment decreases (moving down the right panels from V1 to V4 and V5), wage changes are ever more tightly concentrated around a single peak slightly above zero.

The sharp peak of the wage change histogram in case V5 corresponds to small intermittent upward adjustments in response to the nominal trend of the model. Although the worker faces idiosyncratic shocks, it is not optimal to respond to them by adjusting the wage (given linear disutility). Similar behavior may occur in a fixed menu cost model, if there is positive trend inflation but no idiosyncratic shocks: although there are implicitly two "(S,s) bands", the only observed adjustments are the upward bumps that occur when the nominal trend drives the worker's real wage down past its lower threshold.

This analysis points to a possible way forward for better modeling the behavior of wages. On one hand, it will be crucial to allow for nonlinear labor disutility, so that workers have an incentive to vary the wages in response to idiosyncratic shocks, which will spread out the distribution and possibly make it bimodal, as is the case in the data. On the other hand, it will also be useful to allow for a trend in labor productivity over the life cycle. The wage change histogram from the IWFP data shows far more upward than downward adjustments. Likewise, those data imply average monthly wage growth of 0.43% for continuing workers, while our retail price data imply that prices rise by only 0.16% per month on average. Imposing demographic turnover on the model, so that the expected wage growth of continuing workers can exceed that of the workforce as a whole, due to an expected positive idiosyncratic productivity trend over the lifetime, will help match both of these facts.

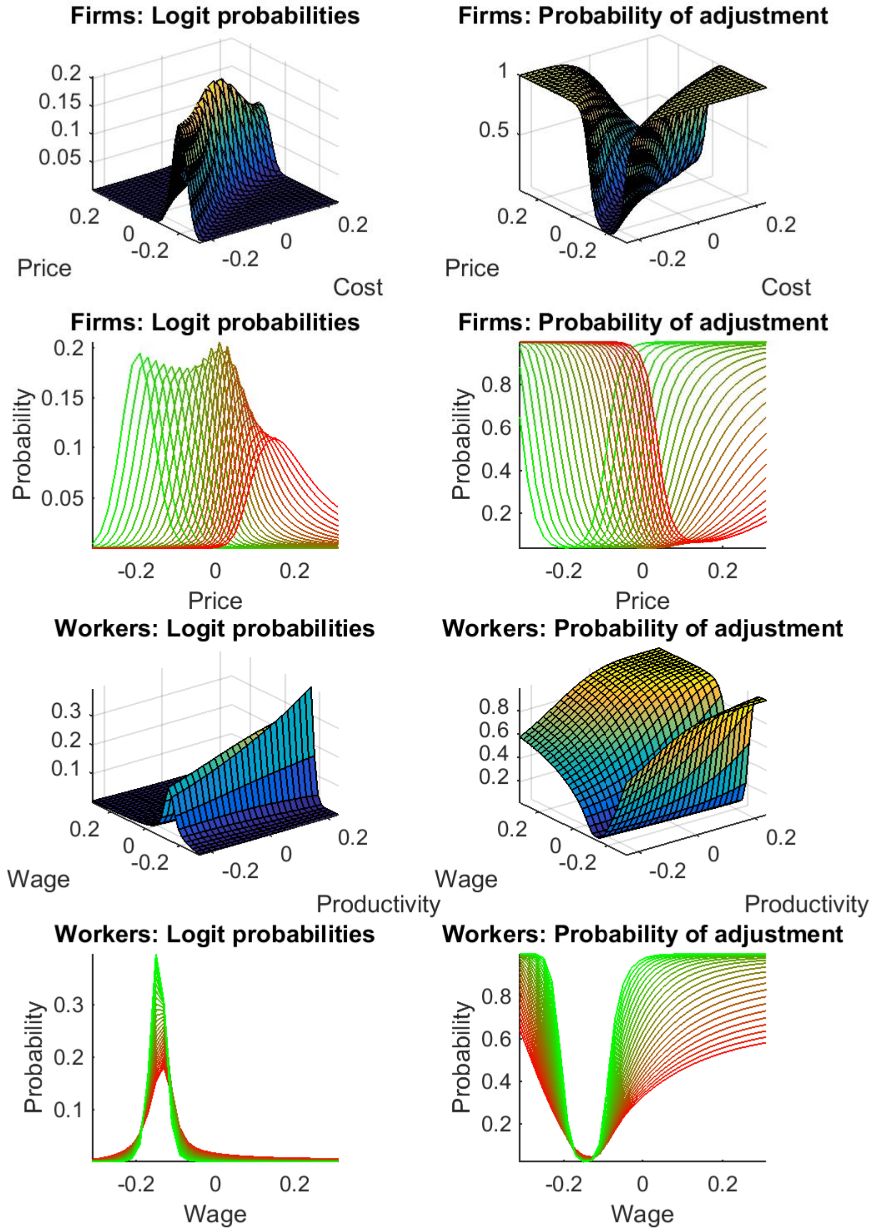
Figure 3 further documents adjustment behavior in our model by graphing the logit policy functions from the benchmark case V1. The left panels display the logit probabilities of each price (wage), conditional on cost, while the right panels show the adjustment probabilities conditional on the current price (wage) and cost. Each function is graphed in two ways, for greater clarity: as a surface plot (first and third rows) and as multiple overlaid cross-sections (second and fourth rows). The upper left panel of the graph shows a surface plot of the logit probabilities $\pi(p|a)$ as a function of the firm's cost shock $-a$ and its possible prices p. Just below this, in the second row, we see the smooth, bell-shaped probability distributions $\pi(p|a)$ corresponding to each possible productivity level a. If the firm's cost shock is high (*i.e.* a is low, shown in red in the graph) then its chosen probability distribution shifts towards higher prices. Looking to the right column, we see that the adjustment probability $\lambda(p, a)$ approaches zero for any p that is near the modal value of $\pi(p|a)$.

The bottom panels of Fig. 3 are analogous, but instead show the worker's policy functions $\pi^w(w|z)$ and $\rho(w, z)$. Notice that the worker's logit probabilities $\pi^w(w|z)$ are concentrated around *the same* w, regardless of z (see the bottom left panel). Regardless of her productivity shock, the worker prefers the same real wage, which explains the tight unimodal distribution of wage changes seen earlier in Fig. 2.

3.1.4 Dynamic results: linear disutility

Next, we turn to the macroeconomic implications of the model, comparing impulse responses to money shocks across versions V1, V3, V5, and V6 in Figure 4. The figure shows the impulse responses to a 1%

Figure 3: Adjustment behavior. Benchmark model (V1) with sticky prices and sticky wages ($\zeta = 0$).



Notes: Distribution of adjustments and adjustment probability for prices (top four panels) and wages (bottom four panels) under linear labor disutility ($\zeta = 0$).

Left panels: 3d plots show price (wage) choice probabilities, conditional on cost (productivity).

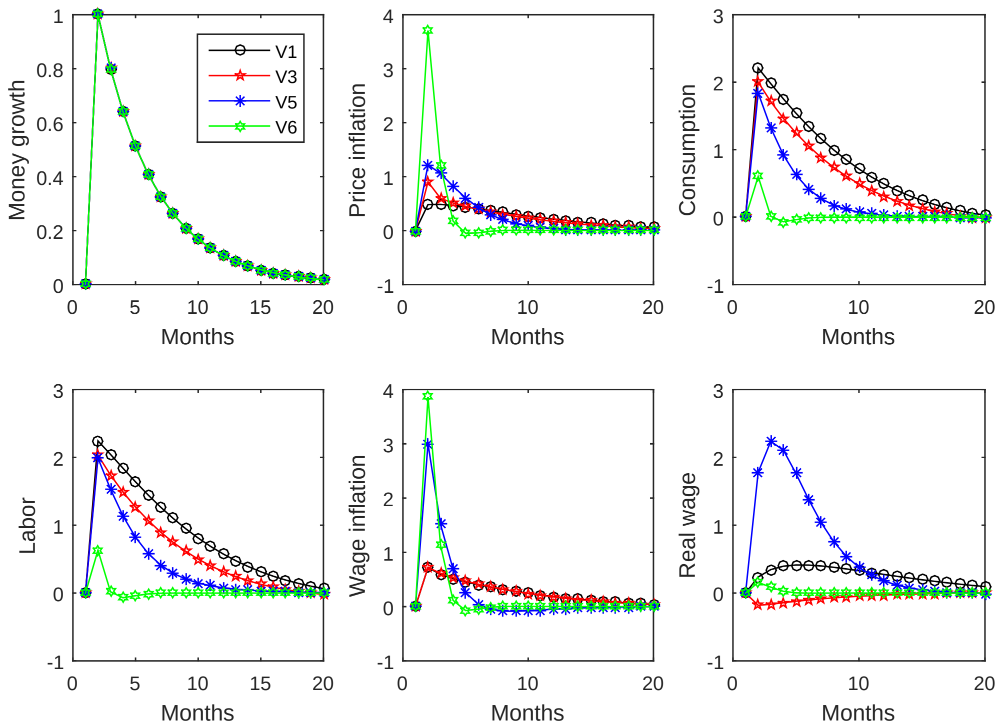
Left panels: 2d plots show price (wage) choice probabilities, conditional on each possible cost (productivity).

Right panels: 3d plots show adjustment probabilities, conditional on current price (wage) and cost (productivity).

Right panels: 2d plots show adjustment probabilities, conditional on each possible cost (productivity).

Colors in 2d plots: For firms, green represents low cost (high a). For workers, green represents high productivity (high z).

Figure 4: Money growth shock: effects of nominal rigidity. Error-prone adjustment, $\zeta = 0$.



Notes:

Impulse responses of inflation and consumption to money growth shock with autocorrelation 0.8 (monthly), under linear labor disutility ($\zeta = 0$).

Black: Benchmark (V1), both prices and wages sticky. Red: V3, flexible prices and sticky wages.

Blue: V5, sticky prices and flexible wages. Green: V6: both prices and wages flexible.

money growth shock, with monthly autocorrelation 0.8, on consumption, labor, price and wage inflation, and the real wage. In the benchmark specification V1 (black with circles), consumption and labor rise by more than 2% on impact, then revert smoothly and gradually with a half-life of roughly six months. Price inflation and wage inflation both rise persistently to a rate of roughly 0.5%. Wage inflation is slightly higher than price inflation, causing the real wage to peak at roughly 0.4% above steady state after four months.

In contrast, in the flexible specification V6 (green), both price and wage inflation spike on impact, with an 4% jump in prices and wages. Consumption and labor increase by half a percent in the period of impact only, then return to their steady state levels. Thus, current and expected money growth feeds rapidly into prices, and its real impact is small and transitory.

It is particularly interesting to compare specifications V3 (red, with sticky wages but flexible prices) and V5 (blue, with sticky prices but flexible wages). The key takeaway is seen in the response of consumption – version V3, with wage stickiness only, comes very close to the baseline model V1 with both price and wage stickiness. The reason is that wage stickiness keeps firms' marginal costs from adjusting rapidly, so even though prices are much more flexible in version V3 than V1, the impulse response of price inflation is quite similar in both cases. Both wages and prices adjust gradually in version V3, giving a real impact on consumption and output that is almost as large and persistent as we saw in case V1.

In contrast, specification V5, with sticky prices and flexible wages, implies an immediate burst of wage inflation when the money supply shock hits– wages rise 3% on impact.[25] In spite of price stickiness, this rise in nominal marginal costs also causes prices to increase by 1.2% on impact, more than they do in cases V1 and V3. Overall, the effect is a large increase in real wages, which discourages labor demand (firms' profits fall sharply upon monetary stimulus) and thus drives down the persistence of the real effects of the money shock.

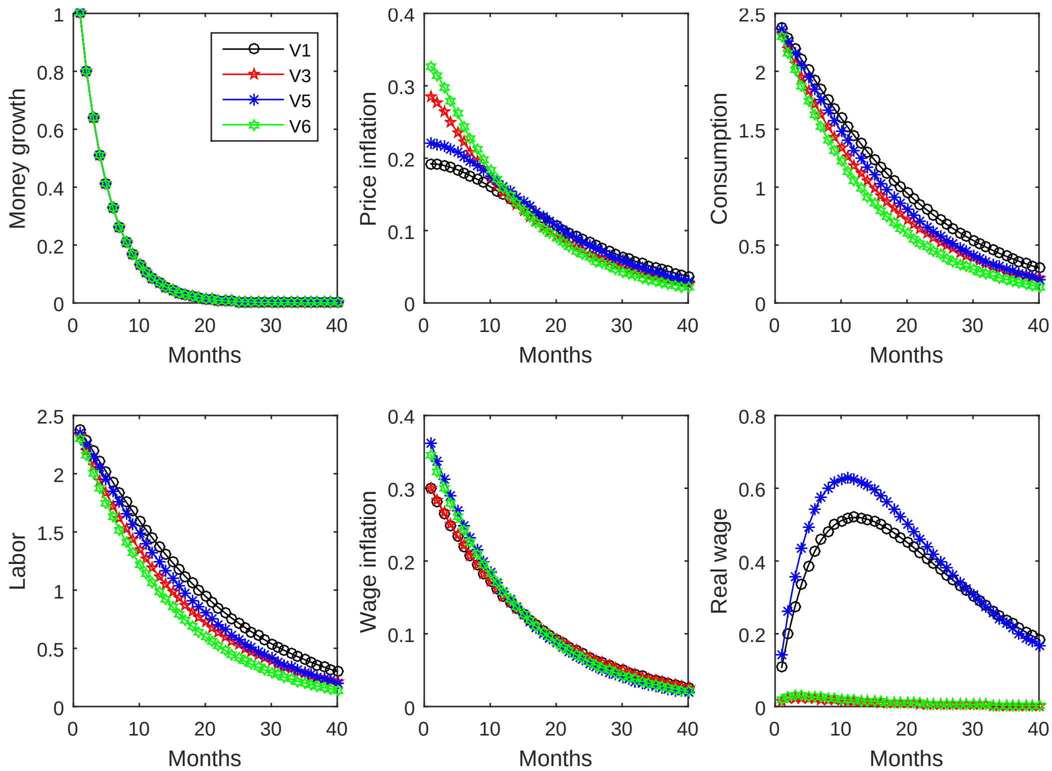
Summarizing, wage stickiness is substantially more important for monetary non-neutrality than price stickiness alone. Sticky wages imply that firms' marginal costs only adjust slowly in response to the shock, which slows down firms' price adjustments even if prices are relatively flexible. The importance of wage rigidity for propagation of nominal shocks to real variables provides support for New Keynesian mechanisms in the light of empirical evidence against procyclical markups of price over marginal cost (Nekarda and Ramey, 2013). On the other hand, these findings do not offer any strong macroeconomic reason to favor the benchmark specification V1, with both rigidities, versus version V3, where only wages are rigid. Empirical studies rarely find a significantly nonzero response of the real wage to monetary policy shocks (see for example Christiano *et al.*, 2005; McCallum and Smets, 2006; Olivei and Tenreyro, 2007; Christiano *et al.*, 2016). Thus it is easy to reject the strongly procyclical real wage (and countercyclical profits) of specification V5, but both versions V1 (with a mildly positive real wage response) and V3 (with a mildly negative response) lie within the range of behavior consistent with macroeconomic evidence.

Finally, to isolate the effects of state-dependence in prices and wages, we compare the impulse responses of Fig. 4 to those of an otherwise identical economy (same model, same parameters, and same finite grid approximation) in which firms' price changes and workers' wage changes are governed by the Calvo (1983) mechanism. That is, firms (workers) reset their prices (wages) with a constant, exogenously-fixed probability per month, and the new price (wage) is optimally chosen (it is optimal taking into account the fact that future adjustments will take place at random times in the future). For comparability with the simulations reported previously, we impose the adjustment hazards found in our state-dependent versions V1-V6 on the Calvo versions V1C-V6C. In other words, although the hazard is exogenously fixed in each Calvo simulation, we vary the hazard across specifications V1C-V6C.

The Calvo simulation results are displayed in Figure 5. Three findings stand out. First, the Calvo model generates much more persistence than the state-dependent models seen in Fig. 4; the half-life of the impulse responses of consumption and labor rises to roughly 15 months (the time span on the horizontal axis of Fig. 5 is twice as long as that in Fig. 4). This highlights the importance of "selection effects" in nominal adjustments: in our logit framework, firms (workers) that face a more costly deviation between their current and desired prices (wages) are more likely to adjust, which speeds up aggregate adjustment relative to the Calvo framework. Second, the impulse responses of the Calvo specifications V1C, V3C, V5C, and V6C are quantitatively quite similar (except in their implications for real wages). This reflects the fact that the frequency of wage adjustment changes very little in our state-contingent simulations V1-V6, and therefore the wage adjustment hazards we plug into our Calvo model do not differ much across simulations V1C-V16. Again, wage stickiness is the more important form of nominal rigidity, so versions V1C-V6C behave similarly even though they reflect very different degrees of price stickiness. Finally, the qualitative behavior of (all) the Calvo specifications resemble that of our benchmark state-dependent model V1: consumption, labor, price inflation, and wage inflation all jump

[25]Note that the impulse responses for case V5 are quantitatively very similar to those reported in our earlier paper, CN18, which studied a model with price stickiness only. The decision cost parameters for price adjustment in model V5 are taken from CN18, so specification V5 essentially reproduces our previous paper's results.

Figure 5: Money growth shock: effects of nominal rigidity. Calvo adjustment, $\zeta = 0$.



Notes:

Impulse responses of inflation and consumption to money growth shock with autocorrelation 0.8 (monthly), under Calvo adjustment with linear labor disutility ($\zeta = 0$).

Black: Benchmark (V1C), both prices and wages sticky. Red: V3C, flexible prices and sticky wages.

Blue: V5C, sticky prices and flexible wages. Green: V6C: both prices and wages flexible.

on impact after a money supply shock, and then smoothly revert to their means. The real wage rises, but by much less than it does in state-dependent version V5. Thus, the dynamic predictions of the Calvo framework are in many ways consistent with those of a state-dependent model, as long as we adjust hazards appropriately and bear in mind that the Calvo setup exaggerates aggregate nominal persistence.

3.2 Main results: Nonlinear disutility

3.2.1 Parameter estimation

As we discussed above, generating a nontrivial wage distribution will require nonlinear disutility of labor. Therefore, we now compute a nonlinear specification, setting $X(h) = \frac{\chi}{1+\zeta} h^{1+\zeta}$, with $\zeta = 0.5$. We estimate parameters for this model version by matching the steady-state model-generated adjustment hazards and adjustment histograms to the Dominick's pricing data and IWFP wage adjustment data discussed earlier; the estimates are stated in Table 3. Thus, we seek to match the observed average price and wage adjustment frequencies λ_{Dom} and ρ_{IFWP} in the Dominick's and IWFP data, as well as the corresponding histograms of nonzero log price changes and nonzero log wage changes, which we denote

Table 3: Parameters for nonlinear disutility simulations.

Parameter estimates for nonlinear disutility benchmark model (V1N):	*Firms*	*Workers*
Adjustment parameters		
Noise level	$\kappa_\pi \equiv \kappa_\lambda = 0.0177$	$\kappa_w \equiv \kappa_\rho = 0.0275$
Default hazard (monthly)	$\bar{\lambda} = 0.2707$	$\bar{\rho} = 0.2317$
Productivity processes		
Persistence (monthly)	$\rho_a = 0.6441$	$\rho_z = 0.9700$
Standard deviation	$\sigma_a = 0.0703$	$\sigma_z = 0.0574$

Adjustment parameters for nonlinear disutility versions:	V1N	V2N	V3N	V4N	V5N	V6N
$\kappa_\pi = \kappa_\lambda$	0.0177	0.00177	0. 000177	0.0177	0.0177	0.000177
$\kappa_w = \kappa_\rho$	0.0275	0.0275	0.0275	0.00275	0.000275	0.000275

Calibrated utility parameters common to all versions:	
Discount rate (monthly):	$1 - \beta = 0.0033$
Death probability (monthly):	$1 - \beta_D = 0.0021$
Log productivity at birth:	$z_0 = -0.6$
Intertemporal elasticity of substitution:	$\gamma = 2$
Coefficient on utility of money:	$\nu = 1$
Coefficient on disutility of labor:	$\chi = 6$
Inverse Frisch elasticity:	$\zeta = 1/2$
Elasticities of substitution across varieties:	$\epsilon = \epsilon_N = 7$

by $\vec{h}_{Dom}$ and $\vec{h}^w_{IWFP}$, respectively. The histograms are vectors representing the observed frequencies of price changes lying in $\#_{Dom}$ fixed bins, and of wage changes lying in $\#_{IFWP}$ fixed bins; thus the $\#_{Dom}$ elements of vector $\vec{h}_{Dom}$ sum to one, as do the $\#_{IFWP}$ elements of $\vec{h}^w_{IWFP}$. The estimation routine minimizes the following criterion:

$$\text{distance} = \sqrt{\#_{Dom}}\,||\lambda_{model} - \lambda_{Dom}|| + ||\vec{h}^w_{model} - \vec{h}_{Dom}|| + \sqrt{\#_{IWFP}}\,||\rho_{model} - \rho_{IWFP}|| + ||\vec{h}^w_{model} - \vec{h}^w_{IWFP}||, \tag{90}$$

where λ_{model}, ρ_{model}, $\vec{h}_{model}$, and $\vec{h}^w_{model}$ are the adjustment frequencies and adjustment histograms generated by our model, and $|| \bullet ||$ represents the Euclidean norm. We scale the component related to adjustment hazards by the square root of the length of the histograms so that the hazards and the histograms are similarly weighted in our minimization routine.

The parameters we estimate are the default hazard rates $\bar{\lambda}$ and $\bar{\rho}$; the noise parameters $\kappa_\pi \equiv \kappa_\lambda$ and $\kappa_w \equiv \kappa_\rho$, and the parameters of the productivity processes of firms and workers, ρ_a, σ^2_a, ρ_z, and σ^2_z.[26] The noise parameters and default hazard rates all rise moderately compared with the values assumed in our earlier linear simulations. Our estimates suggest that workers' productivity is much more persistent than we assumed earlier (and it is much more persistent than the productivity of firms); in fact, the parameter hits the boundary value, 0.97, which we imposed on the estimation routine.[27] The estimated

[26] Since minimizing (90) involves computing the model's steady-state only, our estimation strategy is computationally feasible when run in FORTRAN.

[27] We are obliged to place an upper bound on the persistence of productivity in order to keep the processes inside the finite

version is called V1N; we then vary the stickiness of prices and the stickiness of wages, as before, defining the versions V2N - V6N described in the table. Calibrated utility parameters are as before ($\gamma = 2$, $\nu = 1$, $\chi = 6$), taken originally from Golosov and Lucas (2007), except that the Frisch elasticity of labor supply is now $\zeta^{-1} = 2$. The overall discount rate is set to $1 - \beta = 0.0033$, which combines pure time discounting with the probability of death. The monthly death probability is $1 - \beta_D = 0.0021$, implying an expected working life of forty years. The log productivity of newborn workers is set to -0.6; since the productivity process (89) converges to zero over time, workers expect a 60% productivity gain over their life cycles.

3.2.2 Steady-state results: convex disutility

Table 4 reports steady-state statistics for this parameterization, comparing versions with different combinations of noise parameters (V1N-V6N) as we did previously in Table 2. As in the linear case, decreased noise in price setting or wage setting makes adjustment more frequent. Crucially, the rise in the frequency of monthly wage adjustment is now very large, from 8.34% in version V1N to 30.8% in version V6N, while in the linear case this frequency only rose by one percentage point between versions V1 and V6. Relatedly, lower noise implies smaller absolute price and wage changes, a lower standard deviation and kurtosis of price and wage changes, and more of the smallest changes (less than 5% or less than 2.5%). Price adjustment and especially wage adjustment are significantly more costly here than they were in the linear case, both because of the higher estimated noise parameters and because convex labor disutility means that some adjustments are particularly costly, on the margin.[28]

Overall, the behavior of price and wage statistics in the nonlinear specification is similar to that in the linear case. The main difference is visible in the histograms shown in Figure 6. When prices and wages are sticky, both histograms are smooth and display rather fat tails; price adjustments are mildly left-skewed while wage adjustments are mildly right-skewed. As prices (wages) become more flexible, the price (wage) adjustment histogram becomes sharply bimodal. This contrasts with our earlier linear specification, in which the wage adjustment histogram collapsed to a single sharp peak, reflecting the absence of incentives to adjust wages in response to idiosyncratic productivity shocks.

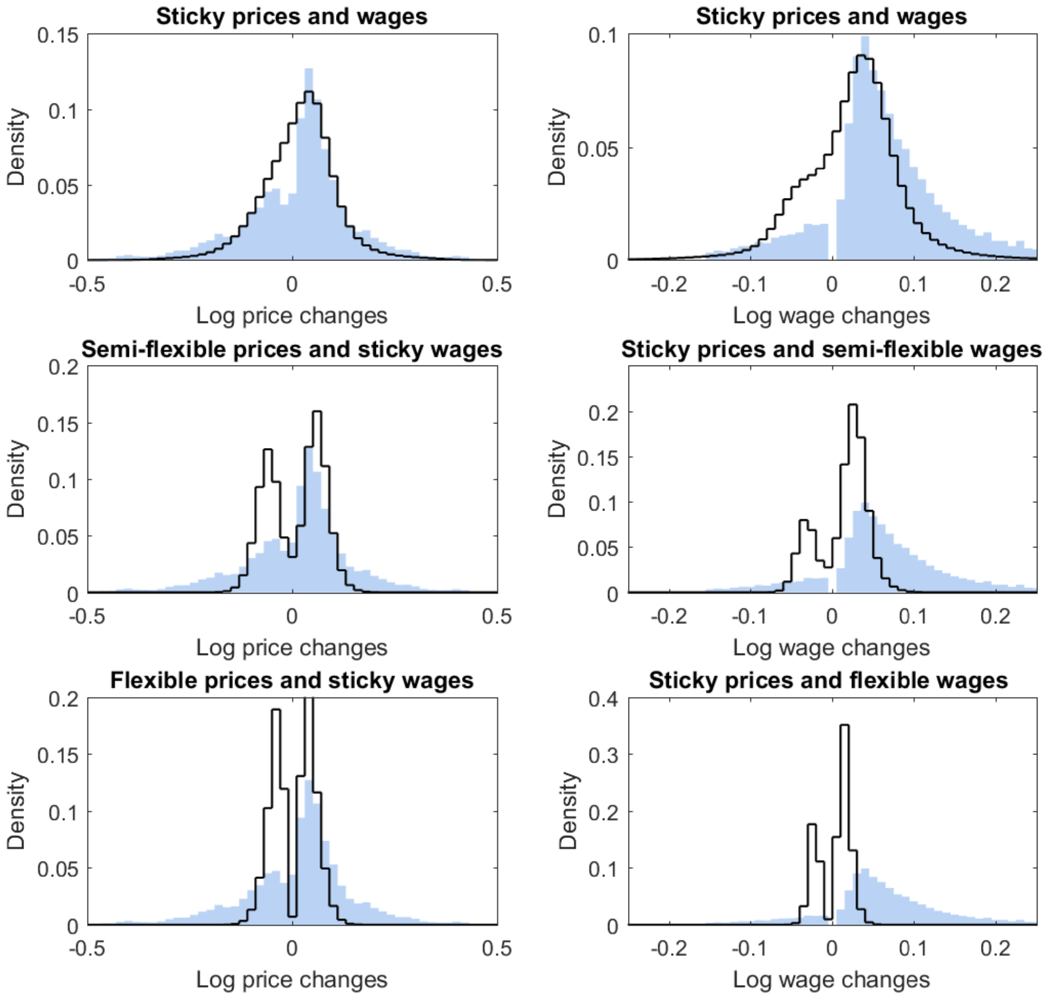
Likewise, by comparing Figure 7 with our previous Fig. 3, we see that the policy functions of the nonlinear case are similar to those from the linear case, except that the worker in the nonlinear version sometimes desires large idiosyncratic wage changes. The figure shows the logit probabilities governing price resets and wage resets (left panels) and firms' and workers' adjustment hazards (right panels). In each case the probabilities are shown as functions of the price-cost (resp. wage-productivity) pairs. As in Fig. 3, firms prefer higher prices when costs are higher, and the probability of adjustment rises smoothly as firms deviate from the prices they prefer (conditional on costs). In contrast with Fig. 3, we now see that workers also set substantially higher wages as their productivity rises. The preferred wage now varies by roughly $\pm 30\%$ as worker productivity varies between its maximum and minimum values in the Tauchen (1986) grid approximation, which differ by $\pm 45\%$.

By estimating parameters for the nonlinear case, we have also improved the model's fit to the wage data in the baseline parameterization V1N. In Fig. 2, the wage adjustment histogram was smooth and almost symmetric, but now in Fig. 6, the histogram of wage adjustments has a more complex shape, with heavy tails. Likewise, the IWFP histogram (blue shaded area) has a lot of weight in the tails. Most of the mass is concentrated on small positive wage adjustments, but there is a fat right tail and a long,

grid on which we perform the simulations.

[28] In particular, a high productivity worker with an excessively low wage will have a high marginal disutility of time, making it costly to set a new wage precisely.

Figure 6: Distribution of nonzero price and wage changes: varying stickiness ($\zeta = 0.5$).



Notes: Black lines: model versions with nonlinear labor disutility ($\zeta = 0.5$). Blue shaded areas: Data.
Left column: Effect of decreasing price stickiness (versions V1N, V2N, V3N) on distribution of nonzero price adjustments.
Right column: Effect of decreasing wage stickiness (versions V1N, V4N, V5N) on distribution of nonzero wage adjustments.

Table 4: Evaluating the nonlinear LPW model with different values of κ_π, κ_λ, κ_w and κ_ρ

	Data		Sticky		Decreasing price stickiness				Decreasing wage stickiness				Flexible	
			V1N (κ_0)		V2N ($\kappa_0/10$)		V3N ($\kappa_0/100$)		V4N ($\kappa_0/10$)		V5N ($\kappa_0/100$)		V6N ($\kappa_0/100$)	
Consumption			0.5012		0.5035		0.5043		0.5049		0.5058		0.5090	
Labor			0.5078		0.5039		0.5026		0.5115		0.5125		0.5073	
Wage			0.8544		0.8590		0.8609		0.8544		0.8544		0.8609	
	Prices	Wages	Prices	Wages	Prices	Wages	Prices	Wages	Prices	Wages	Prices	Wages	Prices	Wages
Freq. of change, %/mo.	10.2	8.33	10.2	8.34	24.8	8.33	59.51	8.33	10.2	13.4	10.2	30.8	59.7	30.7
Mean change, %	1.60	5.10	1.67	3.00	0.69	3.01	0.29	3.01	1.67	1.86	1.67	0.81	0.29	0.81
Mean abs(change), %	9.90	6.47	6.94	5.50	6.06	5.50	4.53	5.50	6.92	3.17	6.92	1.95	4.52	1.96
Std. of changes, %	13.2	6.52	8.96	6.74	6.70	6.73	5.03	6.72	8.94	2.99	8.93	1.95	5.03	1.95
Skewness of changes	-0.42	0.35	-0.12	0.43	-0.15	0.17	-0.06	0.17	-0.12	-0.56	-0.12	-0.46	-0.06	-0.46
Kurtosis of changes	4.81	4.39	4.60	11.9	1.85	11.8	2.01	11.7	4.60	2.56	4.60	2.00	2.01	2.00
Percent increases	65.1	86.5	56.5	70.6	53.9	70.6	52.4	70.6	56.5	73.2	56.5	66.8	52.4	66.8
Changes $\leq$5%	35.5	43.0	45.0	60.8	36.4	60.8	65.2	60.8	45.0	93.4	45.0	99.9	65.3	99.9
Changes $\leq$2.5%	12.0	11.8	27.3	25.2	13.8	25.2	25.7	25.2	27.3	33.2	27.3	80.2	25.8	80.0
Std. of prices, wages, %			3.73	7.86	4.10	7.86	4.57	7.86	3.72	7.90	3.72	7.94	4.57	7.94
Resetting cost, % rev.*			0.50	1.09	0.20	1.09	0.07	1.10	0.49	0.27	0.49	0.08	0.07	0.08
Timing cost, % rev.*			0.48	0.94	0.10	0.95	0.03	0.95	0.48	0.14	0.48	0.03	0.03	0.03
Loss relative to flex, %†			2.49	2.77	1.39	2.78	1.01	2.79	2.48	0.75	2.48	0.29	1.01	0.29

**Note:* Costs μ, μ^w, τ, and τ^w are expressed as percentages of average revenues (for firms) or average labor income (for workers).

†Note: Gain accruing to a single firm or worker not constrained by decision costs ($\kappa = 0$), relative to constrained, as % of average revenues (firms) or labor income (workers).

thin left tail, and there seems to be some "missing mass" of small negative adjustments. This pattern is usually taken to indicate downward nominal rigidity. It is interesting that our model, in which rigidities are entirely symmetric, also seems to show some "missing mass" just below zero, although this effect is weaker than it is in the IWFP data. While downward adjustments are no more costly than upward adjustments in our model, workers have little incentive to make small negative adjustments because they expect their productivity to grow as they age, and because nominal prices have an inflationary trend. Thus, while workers have an incentive to set a higher wage when they become more productive, they can react to small negative productivity shocks by waiting for price inflation to reduce their real wage.

3.2.3 Dynamic results: convex disutility

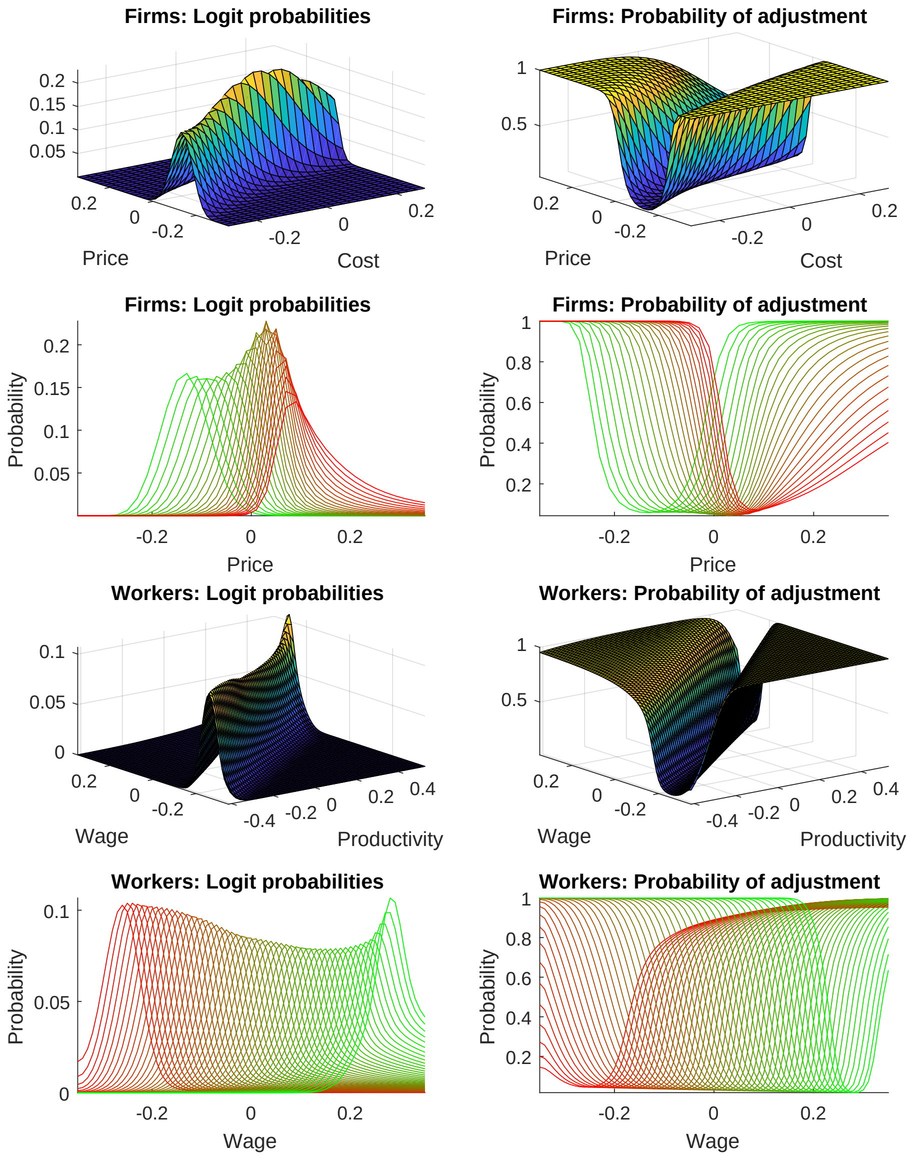
We now return to the issue of monetary non-neutrality. Figure 8 shows the effects of an autocorrelated money growth shock with monthly persistence 0.8. The figure compares the responses of price and wage inflation, consumption, hours and the real wage as price and wage stickiness vary, across models V1N, V3N, V5N and V6N. As before, the sticky-price, sticky-wage specification implies substantial real effects: consumption and labor rise 2.5% on impact, with a half-life of seven months. The version with reduced wage stickiness (V5N) has similar real effects on impact, but much lower persistence, because it implies a large and persistent increase in real wages that offsets firms' incentive to demand more labor. As expected the smallest real impact comes from version V6N, which has very low persistence, as in the Golosov-Lucas (2007) menu cost model. Again, we find that the real effects of money shocks are large as long as wages are sticky. Version V3N (sticky wages and flexible prices) has almost the same consumption response as V1N, and lies substantially above V5N (flexible wages and sticky prices). So again, the key takeaway is that wage rigidity matters more than price rigidity for the overall degree of monetary nonneutrality in this model.

Qualitatively similar results are found under a Calvo specification. Note that our state-dependent model generates substantially different adjustment hazards across model versions, with the wage adjustment hazard rising above 30% in versions V5N and V6N. Figure 9 shows impulse responses under Calvo specifications in which we change the adjustment hazards to reflect the hazards obtained from the state-dependent model versions shown in Figure 8. Therefore, unlike what we found in our linear disutility exercise, the real effects now differ substantially across the Calvo specifications. In fact, this makes our Calvo simulations resemble our state-dependent simulations on many dimensions. The big difference between the state-dependent simulations and the Calvo simulations is that the latter have substantially greater persistence: the half-life of the consumption response is more than twice as long in the V1CN simulation as it is in the estimated state-dependent case V1N.

3.2.4 Nonlinearities in inflation dynamics

Next, we discuss several aspects of our state-dependent model's dynamics that are highly nonlinear. Figure 10 shows that as money supply shocks become larger, their impact falls proportionally more on inflation and less on the real economy. The figure compares the impact of one-time, permanent, uncorrelated shocks to the money supply varying from two to sixteen percentage points. A two-percent jump in the money supply causes a small, persistent rise in inflation, and a persistent increase in consumption, peaking at 0.8% on impact. The impact effect on consumption increases to 1.4% (1.8%) for a four (six) percent jump in the money supply; but the persistence of the real effects drops rapidly with the size of the shock, so the cumulative real change is actually smaller for a six-percent money shock than it is for a four-percent shock. The reason is that larger shocks give firms and workers ever stronger incentives

Figure 7: Adjustment behavior. Benchmark model (V1N) with sticky prices and sticky wages ($\zeta = 0.5$).



Notes: Distribution of adjustments and adjustment probability for prices (top four panels) and wages (bottom four panels) under nonlinear labor disutility ($\zeta = 0.5$).

Left panels: 3d plots show price (wage) choice probabilities, conditional on cost (productivity).

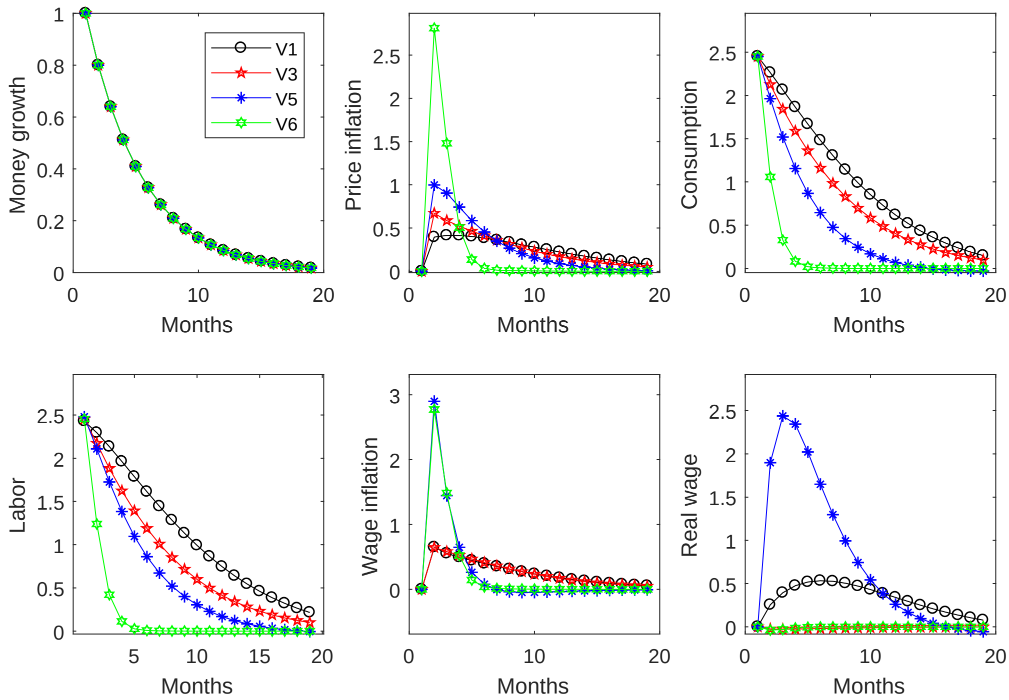
Left panels: 2d plots show price (wage) choice probabilities, conditional on each possible cost (productivity).

Right panels: 3d plots show adjustment probabilities, conditional on current price (wage) and cost (productivity).

Right panels: 2d plots show adjustment probabilities, conditional on each possible cost (productivity).

Colors in 2d plots: For firms, green represents low cost (high a). For workers, green represents high productivity (high z).

Figure 8: Money growth shock: effects of nominal rigidity. Error-prone pricing, $\zeta = 0.5$.



Notes:

Impulse responses of inflation and consumption to money growth shock with autocorrelation 0.8 (monthly), under nonlinear labor disutility ($\zeta = 0.5$).

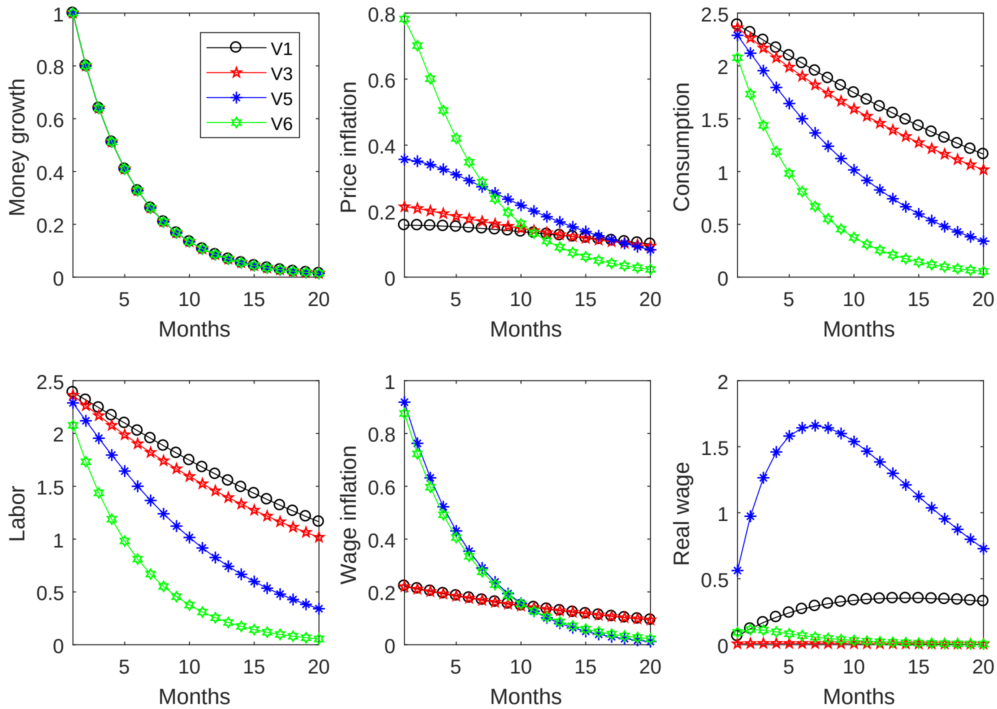
Black: Benchmark (V1N), both prices and wages sticky. Red: V3N, flexible prices and sticky wages.

Blue: V5N, sticky prices and flexible wages. Green: V6N: both prices and wages flexible.

to adjust prices and wages immediately (a stronger selection effect). Thus, most of the nominal reaction occurs immediately, making the real effects smaller. Indeed, for money supply shocks larger than 8%, the real stimulus on impact shrinks, and the brief initial rise is followed by a prolonged slump in consumption and labor due to inflationary distortions.

Finally, in the context of the current prolonged episode of low inflation, it is interesting to ask how our model's behavior changes with trend inflation. Figure 11 and Table 5 document some of the differences across annual trend inflation rates from -1% to 10%. The figure compares the impulse responses of our estimated benchmark model V1N to a 1% money supply shock (with monthly autocorrelation 0.8, as before) as trend inflation varies. The largest real effects are obtained when trend inflation is zero (orange); they are slightly smaller at either plus or minus one percent trend inflation (yellow and light blue respectively). While there is little difference in the contemporaneous impact of money on consumption across trend inflation rates, higher trend inflation rapidly lowers the persistence of the real effects. The half-life of the consumption response falls from 10 months at 0% trend inflation to seven months in the baseline simulation (purple), which features a 2% trend inflation rate, and the half-life of consumption falls to four months at a ten percent trend inflation rate. On the other hand, these moderate changes in trend inflation have a big impact on the response of inflation to a monetary shock: inflation rises more

Figure 9: Money growth shock: effects of nominal rigidity. Calvo pricing, $\zeta = 0.5$.



Notes:

Impulse responses of inflation and consumption to money growth shock with autocorrelation 0.8 (monthly), under Calvo adjustment with nonlinear labor disutility ($\zeta = 0.5$).

Black: Benchmark (V1CN), both prices and wages sticky. Red: V3CN, flexible prices and sticky wages.

Blue: V5CN, sticky prices and flexible wages. Green: V6CN: both prices and wages flexible.

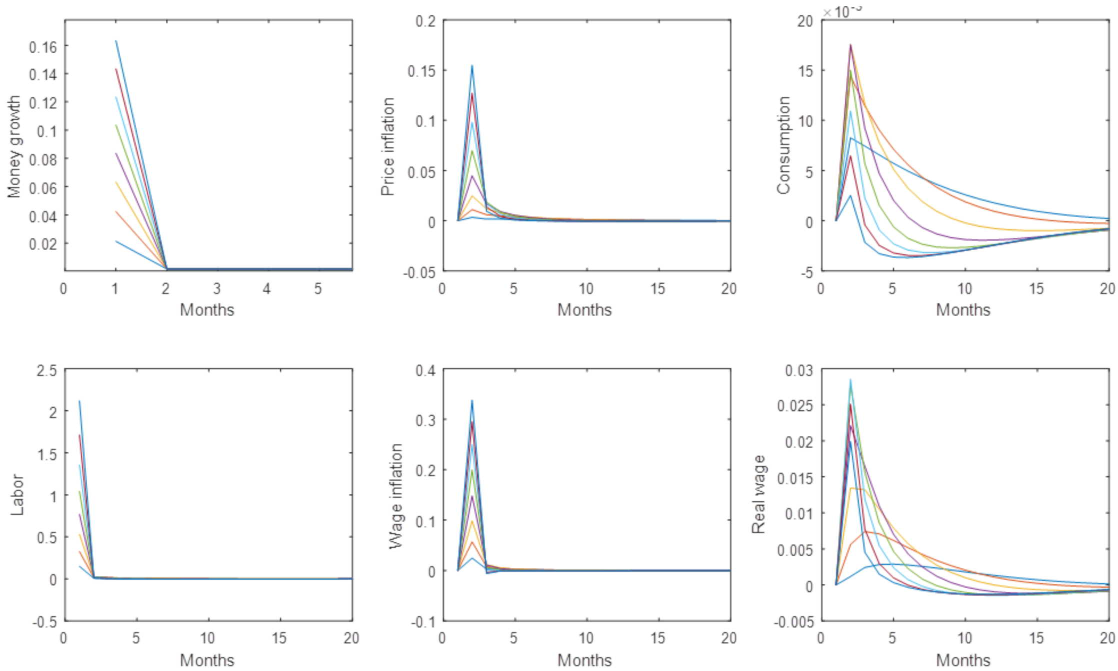
than twice as much on impact, starting from 5% trend inflation, as it does after the same shock in the absence of a nominal trend.

Stated differently, if we define the "Phillips multiplier" as the ratio of the change in inflation to the change in log employment on impact, then Table 5 shows that this multiplier is more than doubled, from 0.108 to 0.232, as trend inflation rises from 0% to 5%. Alternatively, we could define this multiplier as the ratio of the area under the inflation impulse response to the area under the log employment impulse response; this is reported in the table as the "Long-run Phillips multiplier",[29] which rises from 0.147 at zero trend inflation to 0.394 at 5% annual inflation.

These results suggest that our model may help explain the notably flat Phillips curve that has been observed in recent years. Indeed, the flattening of the Phillips curve is especially pronounced as trend inflation falls from 2% to 1% annually. This is particularly interesting because many papers have argued that downward nominal wage rigidity causes a flattening of the Phillips curve at low inflation (Benigno and Ricci, 2011; Lindé and Trabandt, 2018). But our framework does not feature any asymmetry between the costs of upward and downward adjustments of wages or prices. Instead, the flattening of the Phillips

[29] Barnichon and Meesters (2018) propose directly estimating this multiplier to measure the tradeoff between inflation and unemployment.

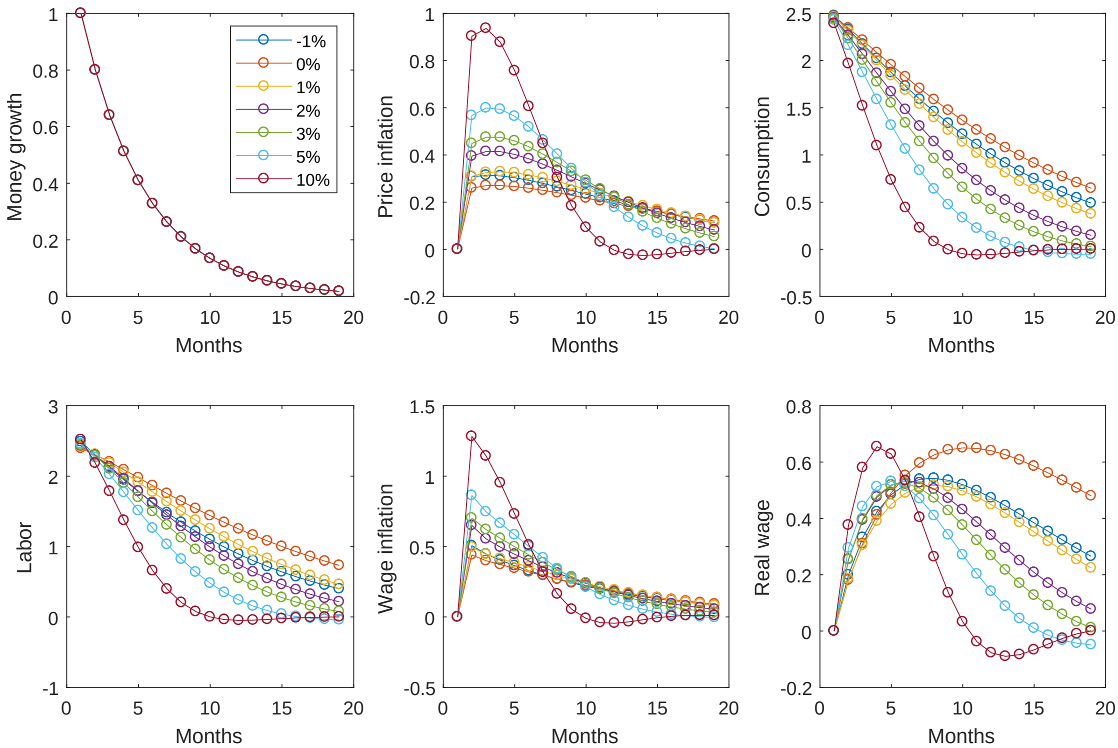
Figure 10: Comparing small and large money supply shocks. Benchmark model (V1N).



Notes:

Impulse responses to a permanent jump in the money supply of 2% (dark blue), 4% (red), 6% (yellow), 8% (purple), 10% (green), 12% (light blue), 14% (dark red), and 14% (dark blue).

Figure 11: Impulse responses at varying trend inflation rates. Benchmark model (V1N).



Notes:

Impulse responses to a 1 percentage point money supply shock (autocorrelation 0.8), starting from annual trend inflation rates of -1% (dark blue), 0% (red), 1% (yellow), 2% (baseline case, purple), 3% (green), 5% (light blue), and 10% (dark red).

Table 5: Evaluating the nonlinear LPW model, benchmark calibration V1N, at different trend inflation rates

	-1%		0%		1%		2%		3%		5%	
Consumption	0.5017		0.5019		0.5016		0.5012		0.5008		0.5001	
Labor	0.5081		0.5081		0.5080		0.5078		0.5076		0.5073	
Wage	0.8545		0.8548		0.8546		0.8544		0.8543		0.8540	
	Prices	Wages	Prices	Wages	Prices	Wages	Prices	Wages	Prices	Wages	Prices	Wages
Freq. of change, %/mo.	9.04	7.28	7.53	6.95	9.05	7.53	10.2	8.34	11.0	9.11	12.5	10.8
Mean change, %	-0.93	-0.05	0.00	1.15	0.92	2.17	1.67	3.00	2.23	3.58	3.25	4.53
Mean abs(change), %	6.50	4.91	6.18	4.93	6.59	5.20	6.94	5.50	7.21	5.76	7.72	6.27
Std. of changes, %	8.65	6.77	8.54	6.82	8.80	6.82	8.96	6.74	9.05	6.67	9.19	6.55
Skewness of changes	0.24	0.82	0.10	0.59	-0.03	0.35	-0.12	0.43	-0.19	0.06	-0.30	-0.11
Kurtosis of changes	4.88	12.1	5.12	12.3	4.80	12.0	4.60	11.9	4.49	12.0	4.36	12.0
Percent increases	37.8	42.7	43.5	53.7	51.4	63.4	56.5	70.6	59.7	75.0	64.8	80.8
Changes $\leq$5%	49.4	68.2	53.7	69.0	49.1	65.3	45.0	60.8	42.0	57.0	37.3	49.8
Changes $\leq$2.5%	30.8	29.1	35.7	30.5	30.7	28.0	27.3	25.2	25.1	22.9	21.9	19.1
Std. of prices, wages, %	3.39	7.80	2.93	7.73	3.40	7.79	3.73	7.86	3.96	7.92	4.35	8.04
Resetting cost, % rev.*	0.45	1.00	0.38	0.95	0.45	1.00	0.50	1.09	0.53	1.17	0.59	1.33
Timing cost, % rev.*	0.50	0.97	0.56	0.99	0.51	0.97	0.48	0.94	0.47	0.91	0.45	0.87
Loss relative to flex, %†	2.44	2.56	2.40	2.52	2.44	2.63	2.49	2.77	2.52	2.89	2.59	3.16
Phillips multiplier, impact⋆	0.124		0.105		0.126		0.161		0.184		0.233	
Phillips multiplier, long run⋆⋆	0.172		0.147		0.195		0.254		0.303		0.394	

**Note:* Costs μ, μ^w, τ, and τ^w are expressed as percentages of average revenues (for firms) or average labor income (for workers).

†*Note:* Gain accruing to a single firm or worker not constrained by decision costs ($\kappa = 0$), relative to constrained, as % of average revenues (firms) or labor income (workers).

†*Note:* Ratio of change in inflation to change in log employment, on impact of money shock.

†*Note:* Ratio of cumulative change in inflation to cumulative change in log employment, after money shock.

curve is a result of state-dependent adjustment. At low inflation, the frequencies of wage and price adjustments both decrease, falling from 10.8% and 12.5% per month at 5% trend inflation to 6.95% and 7.53% when trend inflation is zero. Likewise, the adjustments get smaller; the mean absolute wage and price changes are 6.27% and 7.72% at 5% inflation, falling to 4.93% and 6.18% at zero trend inflation. Since workers and firms are less reactive to shocks at low inflation, the overall price level also becomes less reactive (causing the real economy to become more reactive). Hence, the Phillips curve becomes substantially flatter.

4 Conclusions

We have developed a DSGE model with state-dependent price and wage rigidity, combining monopolistic competition in goods and labor (as in Erceg, Henderson, and Levin, 2000), with nominal rigidity due to costly decision-making (as in Costain and Nakov, 2018). Our heterogeneous-agents approach, with idiosyncratic shocks both to firms and to workers, allows us to fit our model to microdata on price and wage adjustments, but also permits us to calculate the dynamic effects of monetary policy shocks. Our model assumes that labor can be costlessly reallocated across firms at any time, so our study should be understood as documenting the interactions of nominal price stickiness with nominal wage stickiness, abstracting from matching frictions or any other forms of labor specificity.

At a microeconomic level, we compare different calibrations to see how nominal rigidities affect price and wage adjustment behavior. Assuming linear labor disutility makes the model much easier to solve, but implies that the wage never varies in response to individual productivity shocks; therefore our preferred specification has convex disutility of labor. We estimate the convex disutility specification to match hazard rates and adjustment histograms from price and wage microdata; our estimation is numerically feasible since it only requires computing the model's steady state. Our estimates match the frequency of adjustment from microdata, and produce a histogram somewhat smoother than that observed in the data. Firms in our estimated model spend less than one percent of revenues on decisions related to price setting, while workers devote approximately two percent of their time to decisions about wage setting. Allowing for a trend in idiosyncratic productivity over the life cycle implies that small negative wage changes are relatively infrequent; this helps explain a pattern which is often interpreted as evidence of downward nominal wage rigidity, in spite of the fact that there is no inherent downward rigidity in our framework.

Our model implies a policy-relevant degree of monetary nonneutrality. Money growth shocks have similar real effects on impact in our state-dependent framework, but only half the persistence, compared with the time-dependent framework of Calvo (1983). We find that wage stickiness is a stronger source of monetary nonneutrality than price stickiness; calibrations of our model with wage stickiness only produce almost as much non-neutrality as calibrations with wage and price stickiness together. This accords with the consensus from time-dependent models of nominal rigidity (Huang and Liu, 2002; Christiano et al., 2005); our study is the first to demonstrate this result in a state-dependent model. In contrast, calibrations of our model with price stickiness only have much reduced real effects of money shocks, and imply a strong, counterfactual rise in the real wage in response to monetary stimulus.

Monetary policy has a number of highly nonlinear effects in our framework. Larger money shocks cause adjustment hazards to rise, so inflation responds more quickly and real effects are proportionally smaller. Indeed, the absolute size of the cumulative real impact is maximized by a rise of roughly 5% in the money supply; money shocks larger than this have a predominantly negative impact on real variables. Decreasing the trend inflation rate causes adjustment hazards to fall, both for prices and wages. This alters the slope of the Phillips curve, as lower responsiveness of price-setting and wage-setting makes inflation less responsive to macro shocks too. The real effects of a money shock are largest at zero trend inflation, and decrease as the inflation trend becomes negative or positive. The effects on the slope of the Phillips are quantitatively significant: its slope more than doubles as trend inflation rises from 0% to 5% annually. A flatter Phillips curve at low trend inflation rates has often been explained by appealing to downward nominal wage rigiditiy, but in our context it is caused by state-dependent changes in adjustment frequencies, not by any downward asymmetry in the costs of adjustment.

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Appendix. Computation

Outline of algorithm

Computing this model is challenging due to heterogeneity. At any time t, firms face different productivity shocks A_{jt} and are stuck at different prices P_{jt}; likewise productivity and wages vary across workers. The Calvo model is popular because, up to a first-order approximation, only the average price matters for equilibrium. But this property does not hold in most models; here we must treat all equilibrium quantities as functions of the time-varying distribution of prices and productivity across firms.

We address this problem by implementing Reiter's (2009) solution method for dynamic general equilibrium models with heterogeneous agents and aggregate shocks. As a first step, the algorithm calculates the steady-state general equilibrium in the absence of aggregate shocks. Idiosyncratic shocks are still active, but are assumed to have converged to their ergodic distribution, so the real aggregate state of the economy is a constant, Ξ. The algorithm solves for a discretized approximation to this steady state, restricting all idiosyncratic state variables to discrete grids. That is, real log prices p_{jt} lie at all times on a fixed grid $\gamma^p \equiv \{p^1, p^2, ...p^{\#^p}\}$; real log wages w_{it} lie in $\gamma^w \equiv \{w^1, w^2, ...w^{\#^w}\}$; and likewise for log productivities of firms and workers: $a_{jt} \in \gamma^a \equiv \{a^1, a^2, ...a^{\#^a}\}$ and $z_{it} \in \gamma^z \equiv \{z^1, z^2, ...z^{\#^z}\}$. The four grids γ^p, γ^w, γ^a, and γ^z are all assumed to have constant step sizes (in logs) between grid points. Moreover, we assume (only for numerical convenience) that the step size in γ^w equals that in γ^p, and also that the number of grid points is the same in these two grids: $\#^w = \#^p$.

We can then view firms' steady state value function as a matrix $\mathbf{V}$ of size $\#^p \times \#^a$, comprising the values $v^{jk} \equiv v(p^j, a^k, \Xi)$ associated with prices and productivities $\left(p^j, a^k\right) \in \gamma^p \times \gamma^a$.[30] Similarly, the distribution of firms at the beginning (or end) of any given period can be viewed as a $\#^p \times \#^a$ matrix $\mathbf{\Psi}$ (or $\widetilde{\mathbf{\Psi}}$) in which the row j, column k element Ψ^{jk} (or $\widetilde{\Psi}^{jk}$) represents the fraction of firms in state (p^j, a^k) at the beginning (or end) of any given period. Likewise, the workers' steady-state value function and the beginning- and end-of-period distributions of workers can be represented by matrices $\mathbf{L}$, $\mathbf{\Psi^w}$ and $\widetilde{\mathbf{\Psi}}^\mathbf{w}$ of size $\#^w \times \#^z$. While these matrices are large objects, we can nonetheless solve for a steady-state general equilibrium as a low-dimensional root-finding problem. By guessing the steady-state values of C and N, we can set up the Bellman equations of the workers and firms, and solve for their fixed points $\mathbf{L}$ and $\mathbf{V}$; given optimal policies, we can describe the dynamics of the distributions, and thus solve for the steady-state distributions $\mathbf{\Psi^w}$, $\widetilde{\mathbf{\Psi}}^\mathbf{w}$, $\mathbf{\Psi}$, and $\widetilde{\mathbf{\Psi}}$; knowing the distributions, we will show that we can construct two scalar equations that suffice to check the values of C and N.

In a second step, Reiter's method constructs a linear approximation to the dynamics of the discretized model, by perturbing it around the steady state general equilibrium on a point-by-point basis. That is, the firms' value function is represented by a $\#^p \times \#^a$ matrix $\mathbf{V}_t$ with row j, column k element $v_t^{jk} \equiv v(p^j, a^k, \Xi_t)$, thus summarizing the time t values at all grid points $(p^j, a^k) \in \gamma^p \times \gamma^a$. Then, instead of viewing the Bellman equation as a functional equation that defines $v(p, a, \Xi)$ for all possible idiosyncratic and aggregate states p, a, and Ξ, we think of it as an expectational relation between the matrices $\mathbf{V}_t$ and $\mathbf{V}_{t+1}$. This amounts to a (large!) system of $\#^p\#^a$ first-order expectational difference equations that determine the dynamics of the $\#^p\#^a$ variables v_t^{jk}. In addition, there will be a relation between the workers' values $\mathbf{L}_t$ and $\mathbf{L}_{t+1}$ at times t and $t+1$, which can also be seen as a system of $\#^w\#^z$ scalar equations in $\#^w\#^z$ unknowns. Finally, the distribution of firms at time $t+1$, $\mathbf{\Psi}_{t+1}$ is derived from the distribution at time t, $\mathbf{\Psi}_t$, which amounts to $\#^p\#^a$ scalar equations; and the distributional dynamics of workers links the distributions $\mathbf{\Psi^w}_t$ and $\mathbf{\Psi^w}_{t+1}$ with a matrix equation that is equivalent to a system

[30] In this appendix, bold face indicates matrices, and (most) superscripts represent indices of matrices or grids.

of $\#^w\#^z$ scalar equations.[31]

We linearize these equations numerically (together with a handful of scalar equations, including first-order conditions for some aggregate variables). We then solve for the saddle-path stable solution of the linearized model using the QZ decomposition, following Klein (2000). It is crucial to note here that our problem is tractable because we have separated the two sticky decisions in our model between two different classes of decision-makers. In a model where a single decision-maker adjusted p and w in response to the shocks a and z, the value function and distributional dynamics would both have to be evaluated over $\#^p\#^w\#^a\#^z$ grid points. Solving for dynamic general equilibrium would require solving a system of slightly more than $2\#^p\#^w\#^a\#^z$ equations. Instead, since we have assumed prices and wages are set by different agents, we will have to solve slightly more than $2\#^p\#^a + 2\#^w\#^z$ equations, which is a vastly smaller problem.[32]

The discretized model

Firms' values are summarized by matrices $\mathbf{V}_t$ and $\mathbf{V}^e_t$, of size $\#^p \times \#^a$, and the vector $\tilde{\mathbf{v}}_t$, of length $\#^a$. Workers' values are described by the matrices, $\mathbf{L}_t$, $\mathbf{L}^e_t$, and $\widetilde{\mathbf{L}}_t$, of size $\#^w \times \#^z$. The elements of $\mathbf{V}_t$ are $v^{jk}_t \equiv v(p^j, a^k, \Xi_t)$, and the elements of $\mathbf{V}^e_t$ are $v^{e,jk}_t \equiv v^e(p^j, a^k, \Xi_t)$, for $\left(p^j, a^k\right) \in \gamma^p \times \gamma^a$. Likewise, $\mathbf{L}_t$ has elements $l^{jk}_t \equiv l(w^j, z^k, \Xi_t)$, and $\mathbf{L}^e_t$ has elements $l^{e,jk}_t \equiv l^e(w^j, z^k, \Xi_t)$, for $\left(w^j, z^k\right) \in \gamma^w \times \gamma^z$. The expected values of setting a new price or wage are given by vectors $\tilde{\mathbf{v}}_t$ and $\tilde{\mathbf{L}}_t$, with elements $\tilde{v}^k_t \equiv \tilde{v}(a^k, \Xi_t)$ and $\tilde{l}^{jk}_t \equiv \tilde{l}(w^j, a^k, \Xi_t)$.

Related matrices include the probability matrices of firms and workers, $\mathbf{\Lambda}_t$ and $\mathbf{R}_t$. The (j,k) elements of these matrices are given by[33]

$$\lambda^{jk}_t \equiv \lambda\left(\frac{\tilde{v}^k_t - v^{jk}_t}{\kappa_\lambda w_t}\right), \quad \rho^{jk}_t \equiv \rho\left(\frac{\tilde{l}^k_t - l^{jk}_t}{\kappa_\rho \xi^{jk}_t}\right). \tag{91}$$

Finally, we also define the logit probabilities $\mathbf{\Pi}_t$ (a matrix) and $\mathbf{\Pi^w}_t$ (a 3d array). The elements of these matrices are

$$\pi^{jk}_t = \pi_t(p^j|a^k) \equiv \frac{\eta^j \exp\left(v^{jk}_t/(\kappa_\pi w_t)\right)}{\sum_{n=1}^{\#p} \eta^n \exp\left(v^{nk}_t/(\kappa_\pi w_t)\right)}, \tag{92}$$

$$\pi^{w,jkn}_t = \pi^w_t(w^n|w^j, z^k) \equiv \frac{\eta^{w,n} \exp\left(l^{nk}_t/(\kappa_w \xi^{jk}_t)\right)}{\sum_{m=1}^{\#w} \eta^{w,m} \exp\left(l^{mk}_t/(\kappa_w \xi^{jk}_t)\right)}. \tag{93}$$

[31] Here we are assuming that we can substitute out the steps that define the end-of-period distributions $\widetilde{\mathbf{\Psi}}_t$ and $\widetilde{\mathbf{\Psi}}^{\mathbf{w}}_t$. If not, our system will contain an additional $2\#^w\#^z$ equations.

[32] In other words, computational complexity under our approach scales exponentially with the number of sticky decisions if these decisions are all taken by the same agent, but scales linearly in the number of sticky decisions if different decisions are controlled by different agents. (Actually, the same principle is true in models of fully flexible decisions, but the issue is more relevant here because *stickiness creates heterogeneity*— while prices and wages are jump variables in flexible models, in the presence of nominal rigidity they become state variables.)

[33] Actually, (91) is a simplified description of λ^{jk}_t. While (91) implies that λ^{jk}_t represents the function $\lambda(\bullet)$ evaluated at the log price grid point p^j and log productivity grid point a^k, in our computations λ^{jk}_t actually represents the *average* of $\lambda(\bullet)$ over all log prices in the interval $\left(\frac{p^{j-1}+p^j}{2}, \frac{p^j+p^{j+1}}{2}\right)$, given log productivity a^k. Calculating this average requires interpolating the function $d_t(p, a^k)$ between price grid points. Defining λ^{jk}_t this way ensures differentiability with respect to changes in the aggregate state Ξ_t.

Here π_t^{jk} is the probability that a firm which has decided to adjust its price at time t chooses real log price p^j, conditional on log productivity a^k; $\pi_t^{w,jkn}$ is a worker's corresponding probability of choosing the real log wage w^n, conditional on current log real wage w^j and log productivity z^k. The default probabilities for log real prices $p \in \gamma^p$ are $\boldsymbol{\eta} \equiv (\eta^1, \ldots, \eta^{\#^p}) \equiv (\eta(p^1), \ldots, \eta(p^{\#^p}))$, and $\boldsymbol{\eta}^{\mathbf{w}} \equiv (\eta^{w,1}, \ldots, \eta^{w,\#^w}) \equiv (\eta^w(w^1), \ldots, \eta^w(w,^{\#^w}))$ is the analogous vector for log real wages $w \in \gamma^w$.

In this discrete representation, the productivity processes (62) and (73) can be summarized by matrices $\mathbf{S}$ and $\mathbf{S^z}$ of size $\#^a \times \#^a$ and $\#^z \times \#^z$. The (m, k) elements of these matrices represent the following transition probabilities, respectively:

$$S^{mk} = prob(a_{jt} = a^m | a_{j,t-1} = a^k), \quad S^{z,mk} = prob(z_{it} = z^m | z_{i,t-1} = z^k). \tag{94}$$

It is helpful to introduce analogous Markovian notation to describe the deflation of real prices and wages as the aggregate price level rises. Let $\mathbf{T}_t$ be a $\#^p \times \#^p$ Markov matrix in which the row m, column l element represents the probability that firm j's beginning-of-period log real price $\tilde{p}_{jt}$ equals $p^m \in \gamma^p$ if its log real price at the end of the previous period was $p^l \in \gamma^p$:

$$T_t^{ml} \equiv prob(\widetilde{p}_{jt} = p^m | p_{j,t-1} = p^l). \tag{95}$$

Generically, the deflated log price $\tilde{p}_{jt} \equiv p_{j,t-1} - i_t \equiv p_{j,t-1} - i(\Xi_t, \Xi_{t-1})$ will fall between two grid points; then the matrix $\mathbf{T}_t$ must round up or down stochastically. Also, if $p_{j,t-1} - i_t$ lies below the smallest or above the largest element of the grid, then $\mathbf{T}_t$ must round up or down to keep prices on the grid.[34] Therefore we construct $\mathbf{T}_t$ according to

$$T_t^{ml} = prob(\widetilde{p}_{jt} = p^m | p_{j,t-1} = p^l, i_t) = \begin{cases} 1 & \text{if } p^l - i_t \leq p^1 = p^m \\ \frac{p^l - i_t - p^{m-1}}{p^m - p^{m-1}} & \text{if } p^1 < p^m = \min\{p \in \Gamma^p : p \geq p^l - i_t\} \\ \frac{p^{m+1} - p^l + i_t}{p^{m+1} - p^m} & \text{if } p^1 \leq p^m = \max\{p \in \Gamma^p : p < p^l - i_t\} \\ 1 & \text{if } p^l - i_t > p^{\#^p} = p^m \\ 0 & \text{otherwise} \end{cases} \tag{96}$$

Furthermore, recall that we have assumed that the price and wage grids γ^p and γ^w have the same step size, and the same number of grid points. Note that in this case, the transition probabilities mapping real log wages from one period to the beginning of the next are the same as those for real log prices. In other words, for all m and l,

$$prob(\widetilde{w}_{it} = w^m | w_{j,t-1} = w^l) \ = \ prob(\widetilde{p}_{jt} = p^m | p_{j,t-1} = p^l) \ = \ T_t^{ml}. \tag{97}$$

Thus we can describe the distributional dynamics of wages using exactly the same matrix $\mathbf{T}_t$ that we used from prices.

Given this notation, we can now write the distributional dynamics in a more compact form. The time t distributions of firms and workers are derived from the distributions at the end of $t-1$ as follows:

$$\mathbf{\Psi}_t = \mathbf{T}_t \widetilde{\mathbf{\Psi}}_{t-1} \mathbf{S}', \qquad \mathbf{\Psi}_t^w = \beta_D \mathbf{T}_t \widetilde{\mathbf{\Psi}}_{t-1}^w (\mathbf{S^z})' + (1 - \beta_D) \mathbf{\Psi}_t^0. \tag{98}$$

[34] In other words, we assume that any nominal price that would have a real log value less than p^1 after inflation is automatically adjusted upwards to the real log value p^1 (and when computing examples with deflation we must adjust down any real log price exceeding $p^{\#^p}$). This assumption is made for numerical purposes only, and has a negligible impact on the equilibrium as long as we choose a sufficiently wide grid γ^p.

Note that exogenous shocks are represented from left to right in the matrices $\widetilde{\boldsymbol{\Psi}}_t$ and $\widetilde{\boldsymbol{\Psi}}_t^w$, so that their transitions can be treated by right multiplication, while sticky decision variables are represented vertically, so that transitions related to choice variables can be described by left multiplication. The workers' dynamics reflect the fact that a worker dies at the end of any period with probability $1-\beta_D$, being replaced by a newborn worker, whose wage and productivity are governed by the distribution $\boldsymbol{\Psi}_t^0$. Next, to calculate the effects of price adjustment on the distribution, let $\mathbf{1}_{pp}$, $\mathbf{1}_{pa}$, $\mathbf{1}_{ww}$, and $\mathbf{1}_{wz}$ be matrices of ones of size $\#^p \times \#^p$, $\#^p \times \#^a$, $\#^w \times \#^w$, and $\#^w \times \#^z$, respectively. After production occurs at time t, as new real prices are set, the price distribution adjusts as follows:

$$\widetilde{\boldsymbol{\Psi}}_t \quad = \quad (\mathbf{1}_{pa} - \boldsymbol{\Lambda}_t) \odot \boldsymbol{\Psi}_t + \boldsymbol{\Pi}_t \odot (\mathbf{1}_{pp}(\boldsymbol{\Lambda}_t \odot \boldsymbol{\Psi}_t)). \tag{99}$$

where the operator $\odot$ represents element-by-element multiplication (the Hadamard product). The matrix notation does not carry over to the wage dynamics, because the distribution of new wages varies with the *current* wage, so instead we state the dynamics one row at a time:

$$\widetilde{\boldsymbol{\Psi}}_t^{w,j} \quad = \quad (\mathbf{1}_z - \boldsymbol{\Lambda}_t^{w,j}) \odot \boldsymbol{\Psi}_t^{w,j} + \ \mathbf{1}_w'(\boldsymbol{\Pi}_t^{w,j} \odot \boldsymbol{\Lambda}_t^w \odot \boldsymbol{\Psi}_t^w). \tag{100}$$

Here $\widetilde{\boldsymbol{\Psi}}_t^{w,j}$, $\boldsymbol{\Psi}_t^{w,j}$, and $\boldsymbol{\Lambda}_t^{w,j}$ are the jth rows of $\widetilde{\boldsymbol{\Psi}}_t^w$, $\boldsymbol{\Psi}_t^w$, and $\boldsymbol{\Lambda}_t^w$, respectively, while $\boldsymbol{\Pi}_t^{w,j}$ is the matrix representing the probability of choosing wage w^j conditional on each possible state: $prob(w^j|w^i, z^k, \Xi_t)$. $\mathbf{1}_z$ and $\mathbf{1}_w$ are conformable row vectors of ones.

The same transition matrices $\mathbf{T}_t$, $\mathbf{S}$, and $\mathbf{S^z}$ show up when we write the Bellman equations in matrix form. The discounted values of choosing each possible real price $\tilde{p}$ are

$$\mathbf{V}_t^e = \beta E_t \left\{ \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \mathbf{T}_{t+1}' \mathbf{V}_{t+1} \mathbf{S} \right\}, \qquad \mathbf{L}_t^e = \beta E_t \left\{ \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \mathbf{T}_{t+1}' \mathbf{L}_{t+1} \mathbf{S^z} \right\}. \tag{101}$$

Here the expectation E_t refers only to the effects of the time $t+1$ aggregate shock g_{t+1}, because the dynamics of the idiosyncratic states (p_{jt}, a_{jt}) and (w_{it}, z_{it}) are completely described by the matrices $\mathbf{T}_{t+1}'$, $\mathbf{S}$, and $\mathbf{S^z}$.

Now, let $\mathbf{U}_t$ be the $\#^p \times \#^a$ matrix of current payoffs to the firm, with elements

$$u_t^{jk} \equiv \left(\exp(p^j) - \frac{w_t}{\exp(a^k)} \right) \frac{C_t}{\exp(\epsilon p^j)} \tag{102}$$

for $(p^j, a^k) \in \gamma^p \times \gamma^a$. The define the current payoffs of the workers, let $\mathbf{H}_t$ be the $\#^w \times \#^z$ matrix containing the elements $h_t^{jk} \equiv h_t(w^j, z^k)$, representing labor demand in state (w^j, z^k, Ξ_t). Also define $\mathbf{W}$ as a conformable matrix with all the elements of row j equal to $\exp w^j$, and $\mathbf{X}_t$ as a matrix containing the elements $\frac{X(h_t^{jk} + \tau_t^{jk} + \mu_t^{jk})}{u'(c(\Xi_t))}$ representing total disutility of time use in state (w^j, z^k, Ξ_t). Then we can calculate the value functions as

$$\mathbf{V}_t = \mathbf{U}_t + \boldsymbol{\Lambda}_t \odot (E^{\pi} \mathbf{V^e}_t - \mathbf{K}_t^{\pi}) + (\mathbf{1}_{pp} - \boldsymbol{\Lambda}_t) \odot \mathbf{V^e}_t - \mathbf{K}_t^{\lambda} \tag{103}$$

$$\mathbf{L}_t = \mathbf{W} \odot \mathbf{H}_t - \mathbf{X}_t + \mathbf{R}_t \odot E^{\pi} \mathbf{L^e}_t + (\mathbf{1}_{ww} - \mathbf{R}_t) \odot \mathbf{L^e}_t \tag{104}$$

In order to check labor market clearing it will be helpful to define several summary statistics related to labor time use. First, let K_t^{λ} and K_t^{π} and be total time use for choosing the timing of the price decision, and actually choosing prices:

$$K_t^\lambda = \sum_{j=1}^{\#p}\sum_{k=1}^{\#a}\psi_t^{jk}\left(\lambda_t^{jk}\ln\left(\frac{\lambda_t^{jk}}{\bar{\lambda}}\right)+(1-\lambda_t^{jk})\ln\left(\frac{1-\lambda_t^{jk}}{1-\bar{\lambda}}\right)\right), \tag{105}$$

$$K_t^\pi = \sum_{j=1}^{\#p}\sum_{k=1}^{\#a}\psi_t^{jk}\lambda_t^{jk}\left(\sum_{i=1}^{\#p}\pi_t^{ik}\ln\left(\frac{\pi_t^{ik}}{\eta^k}\right)\right), \tag{106}$$

$$\Delta_t = \sum_{j=1}^{\#p}\sum_{k=1}^{\#a}\psi_t^{jk}\exp(-\epsilon p^j - a^k). \tag{107}$$

Note that in the second equation, the time K_t^π devoted to choosing prices is weighted by the fraction adjusting, λ_t^{jk}. In the third equation, Δ_t represents a price dispersion measure that relates time devoted to production to total goods produced.

Next, we discuss how we apply the two steps of Reiter's (2009) method to this discrete model.

Step 1: steady state

In the aggregate steady state, aggregate shocks are zero; the distribution of firms takes some unchanging value $\mathbf{\Psi}$, and the distribution of workers takes some unchanging value $\mathbf{\Psi^w}$. Thus the aggregate state of the economy is constant: $\Xi_t \equiv (g_t, \Psi_{t-1}, \Psi_{t-1}^w) = (0, \mathbf{\Psi}, \mathbf{\Psi^w}) \equiv \Xi$. We indicate the steady state of all equilibrium objects by dropping the time subscripts and the function argument Ξ, so the steady state value function $\mathbf{V}$ has elements $v^{jk} \equiv v(p^j, a^k, \Xi)$.

Long run monetary neutrality in steady state implies that the rate of nominal money growth equals the rate of inflation:

$$\mu = \exp(i).$$

Thus, the steady-state transition matrix $\mathbf{T}$ is known, since it depends only on steady state inflation i. Morever, the Euler equation reduces to

$$\exp(i) = \beta R,$$

which simply serves to determine the nominal interest rate R.

We can then calculate general equilibrium as a three-dimensional root-finding problem, by guessing consumption C, labor demand N, and the aggregate wage level w. On one hand, knowing $c(\Xi)$ and $w(\Xi)$ we can construct the firm's profit function $u(p, a, \Xi) = (e^p - w(\Xi)e^{-a})c(\Xi)e^{-\epsilon p}$. Knowing the profit function, we can solve the firm's problem by backwards induction, which yields the value functions v, v^e, and $\tilde{v}$, and the policy functions λ and π. Given the firm's policy functions, we can calculate the distributional dynamics to find the steady-state distribution of prices and productivities, $\Psi(p, a)$. From the firm's problem and the steady-state distribution we can also calculate the time firms devote to decision-making (K_t^λ and K_t^π), and the efficiency wedge Δ.

On the other hand, knowing $n(\Xi)$ and $w(\Xi)$ we can construct the labor demand function $h(w, z, \Xi) = e^{z(\epsilon_n - 1)}n(\Xi)w(\Xi)^{\epsilon_n}e^{-\epsilon_n w}$, and given $c(\Xi)$ we can also calculate worker's utility value of labor income, $u'(c(\Xi))e^w h(w, z, \Xi)$. We can then solve the worker's Bellman equation by backwards induction. This yields the vaue functions l, l^e, and $\tilde{l}$, and the policy functions ρ, and π^w, as well as the time use function τ and μ, and the worker's marginal value of time ξ. Given the worker's policy functions, we can calculate the distributional dynamics to find the steady-state distribution of wages and productivities, $\Psi^w(w, z)$.

With these distributions in hand, we can then check whether the guessed values of C, N, and w are consistent with an equilibrium. Then we check the following three scalar equations:

$$1 = \sum_{j=1}^{\#^p}\sum_{k=1}^{\#^a} \psi^{jk} \exp\left((1-\epsilon)p^j\right), \tag{108}$$

$$w = \left\{\sum_{j=1}^{\#^p}\sum_{k=1}^{\#^a} \psi^{w,jk} \exp\left((1-\epsilon_n)(w^j - z^k)\right)\right\}^{\frac{1}{1-\epsilon_n}}. \tag{109}$$

$$N = \Delta C + \kappa_\pi K^\pi + \kappa_\lambda K^\lambda. \tag{110}$$

The first two equations are the aggregate price and wage identities; the last is the labor market clearing condition. If these three equations are satisfied with sufficient accuracy, then a steady-state general equilibrium has been found.

Step 2: linearized dynamics

We now conjecture that nominal and real state variables take the form $\Omega_t \equiv (M_t, g_t, \Phi_t, \Phi_t^w)$ and $\Xi_t \equiv (g_t, \Phi_t, \Phi_t^w)$, respectively. We will show that this is a valid state variable for the economy by constructing an equilibrium in terms of this state.

Given the steady state, the general equilibrium dynamics can be calculated by linearization. To reduce the size of the Jacobian, we will eliminate many variables from the equation system. Thus, we calculate the end-of-period distributions as an intermediate step, without explicitly counting them in the equation system:

$$\widetilde{\mathbf{\Psi}}_t = (\mathbf{1}_{pa} - \mathbf{\Lambda}_t) \odot \mathbf{\Psi}_t + \mathbf{\Pi}_t \odot (\mathbf{1}_{pp}(\mathbf{\Lambda}_t \odot \mathbf{\Psi}_t)) \tag{111}$$

$$\widetilde{\mathbf{\Psi}}_t^{w,j} = (\mathbf{1}_z - \mathbf{\Lambda}_t^{w,j}) \odot \mathbf{\Psi}_t^{w,j} + \mathbf{1}_w'(\mathbf{\Pi}_t^{w,j} \odot \mathbf{\Lambda}_t^w \odot \mathbf{\Psi}_t^w) \tag{112}$$

Having thus calculated $\widetilde{\mathbf{\Psi}}_t$ and $\widetilde{\mathbf{\Psi}}_t^w$, the following two equations can be counted as determining the dynamics of the distributions $\mathbf{\Psi}$ and $\mathbf{\Psi^w}$ from periods t to $t+1$:

$$\mathbf{\Psi}_{t+1} = \mathbf{T}_{t+1}\widetilde{\mathbf{\Psi}}_t\mathbf{S}', \tag{113}$$

$$\mathbf{\Psi}_{t+1}^w = \beta_D \mathbf{T}_{t+1}\widetilde{\mathbf{\Psi}}_t^w(\mathbf{S^z})' + (1-\beta_D)\mathbf{\Psi}_{t+1}^0. \tag{114}$$

Similarly, we do not count the expected values V^e and L^e explicitly in our equation system, but we evaluate them in an intermediate step as follows:

$$\mathbf{V}_t^e = \beta E_t \left\{\frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}}\mathbf{T}'_{t+1}\mathbf{V}_{t+1}\mathbf{S}\right\} \tag{115}$$

$$\mathbf{L}_t^e = \beta E_t \left\{\frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}}\mathbf{T}'_{t+1}\mathbf{L}_{t+1}\mathbf{S^z}\right\} \tag{116}$$

Given the expected values V_t^e and L_t^e, which can be used to calculate the probabilities Λ_t, Π_t, and so forth, we then count the following two Bellman equations, which determine the dynamics of the value functions V_t and L_t:

$$\mathbf{V}_t = \mathbf{U}_t + \mathbf{\Lambda}_t \odot (E^\pi \mathbf{V^e}_t - \mathbf{K}_t^\pi) + (1-\mathbf{\Lambda}_t) \odot \mathbf{V^e}_t - \mathbf{K}_t^\lambda \tag{117}$$

$$\mathbf{L}_t = \mathbf{W} \odot \mathbf{H}_t - \mathbf{X}_t + \mathbf{R}_t \odot E^{\pi} \mathbf{L^e}_t + (1 - \mathbf{R}_t) \odot \mathbf{L^e}_t \tag{118}$$

We also include the following six scalar equations in our system:

$$1 = \sum_{j=1}^{\#^p} \sum_{k=1}^{\#^a} \Psi_t^{jk} \exp((1-\epsilon)p^j) \tag{119}$$

$$w_t^{1-\epsilon_n} = \sum_{j=1}^{\#^p} \sum_{k=1}^{\#^a} \Psi_t^{w,jk} \exp((1-\epsilon_n)(w^j - z^k)) \tag{120}$$

$$N_t \ = \ \Delta_t C_t + \kappa_\pi K_t^\pi + \kappa_\lambda K_t^\lambda \tag{121}$$

$$\frac{\mu \exp(g_t)}{\exp i_t} = \frac{m_t}{m_{t-1}} \tag{122}$$

$$1 - \frac{\nu}{m_t C_t^{-\gamma}} = \beta E_t \left(\frac{C_{t+1}^{-\gamma}}{i_{t+1} C_t^{-\gamma}} \right) \tag{123}$$

$$g_{t+1} = \phi_g g_t + \epsilon_{t+1}^g \tag{124}$$

If we now collapse all the endogenous variables into a single vector

$$\overrightarrow{X}_t \equiv \left(vec\,(\mathbf{\Psi}_t)', \ vec\,(\mathbf{\Psi^w}_t)', \ \ m_{t-1}, \ w_t, \ i_t, \ vec\,(\mathbf{V}_t)', \ vec\,(\mathbf{L}_t)', \ C_t, \ N_t \right)'$$

then the four matrix equations (113), (114), (117), and (118), together with the six scalar equations (119)-(124), amount to first-order system of the following form:

$$E_t \mathcal{F} \left(\overrightarrow{X}_{t+1}, \overrightarrow{X}_t, g_{t+1}, g_t \right) = 0 \tag{125}$$

where E_t is an expectation conditional on g_t and all previous shocks.

Since the number of equations matches the number of variables included in the system $\mathcal{F}$, we can linearize the system numerically with respect to all its arguments to construct the Jacobian matrices $\mathcal{A} \equiv D_{\overrightarrow{X}_{t+1}} \mathcal{F}$, $\mathcal{B} \equiv D_{\overrightarrow{X}_t} \mathcal{F}$, $\mathcal{C} \equiv D_{g_{t+1}} \mathcal{F}$, and $\mathcal{D} \equiv D_{g_t} \mathcal{F}$. Thus we obtain the following first-order expectational difference equation system:

$$E_t \mathcal{A} \Delta \overrightarrow{X}_{t+1} + \mathcal{B} \Delta \overrightarrow{X}_t + E_t \mathcal{C} g_{t+1} + \mathcal{D} g_t = 0 \tag{126}$$

where Δ represents a deviation from steady state. This system has the form considered by Klein (2000), so we solve our model using his QZ decomposition method. When applying this method, note that $\mathbf{\Psi}_t$, $\mathbf{\Psi^w}_t$, m_{t-1}, w_t, and i_t are all predetermined at t, while $\mathbf{V}_t$, $\mathbf{L}_t$, C_t, and N_t are jump variables.