Which Financial Shocks Drive the Business Cycle?[*]

Andrea Ajello, Federal Reserve Board

Jonathan Goldberg, Federal Reserve Board

Ander Perez-Orive[†], Federal Reserve Board

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Abstract

We develop a monetary dynamic general equilibrium model with a rich corporate finance structure to study which financial shocks drive the business cycles and how. Entrepreneurs optimally choose dividend payouts, long-term nominal debt, and real investment in a setting with idiosyncratic risk and strategic default. We model segmented asset markets and introduce sentiment shocks to the demand for corporate bonds, for corporate equity, and for default-free government bonds. On the supply side of the corporate credit market, we include an idiosyncratic entrepreneurial risk shock. We estimate the model on US data on corporate financial flows, asset prices, and standard indicators of economic activity. Sentiment shocks generate plausible business cycle responses and can explain around 20 percent of investment and employment fluctuations, comparable to the role played by the risk shock. Allowing for strategic default and an endogenous capital structure significantly amplifies the effects of a positive equity sentiment shock by lowering leverage and default risk. In contrast, entrepreneurs' use of long-term debt reduces the effect of a positive bond sentiment shock because a large fraction of the benefits accrue to existing bondholders.

JEL codes: E21, E22, E32, G12, G31, G32

1   Introduction

Which financial shocks matter, and how? To address this question we develop a monetary dynamic general equilibrium model in which entrepreneurs optimally choose dividend payouts, long-term nominal debt, and real investment in a setting with idiosyncratic risk and strategic default. We introduce sentiment shocks to the demand for corporate bonds, for corporate equity, and for default-free government bonds. On the supply side of the corporate credit market, we include an idiosyncratic entrepreneurial risk shock as well as "news" shocks about future idiosyncratic risk. We find that sentiment shocks are important drivers of the business cycle. For example, sentiment shocks together explain around 20 percent of investment and employment fluctuations, comparable to the share of these fluctuations explained by the risk shock and the risk news shocks. We also find that endogenous changes in leverage and default risk, as well as the long-term nature of corporate debt, are important in explaining the response to financial shocks, amplifying the effect of the shock to demand for equities but dampening the effect of the shock to demand for corporate debt.

Our paper builds on earlier empirical work showing that financial shocks account for an important fraction of macroeconomic fluctuations (*e.g.* Gilchrist and Zakrajsek (2012), Lopez-Salido, Stein and Zakrajsek (2015), Gertler and Karadi (2015), and Caldara and Herbst (2016)). The effort to provide a structural underpinning to these financial shocks has so far taken the form of estimated dynamic stochastic general equilibrium models featuring typically just one financial shock (*e.g.* Jermann and Quadrini (2012), Christiano, Motto and Rostagno (2014), and Ajello (2016)).

Our model extends Smets and Wouters (2007) along two key lines. First, households experience a shock to their preference for government bonds, a la Krishnamurthy and Vissing-Jorgensen (2012), a shock to their debt maturity preference, and a shock to their preference for holding equity. Second, firms fund themselves by choosing the appropriate mix of equity securities and multi-period defaultable nominal debt. Firms experience an idiosyncratic disturbance to their return on capital, the variance of which is stochastic as in Christiano, Motto, and Rostagno (2014). In the event of a sufficiently bad idiosyncratic shock, the equity injection that would be required to make debt payments is so large that equity holders choose to default; the default threshold is endogenous. Firms choose how much debt to issue each period taking into account the tax benefits of debt, as well as distress costs induced by debt and the market prices of debt and equity. Debt is priced fairly by households, who take into account default and inflation risk, their preference for government debt and debt maturity, and the outstanding real value of financial assets that they are required to hold. In total, our model contains five types of financial shocks: three portfolio preference shocks, a shock

to idiosyncratic risk, and "news" shocks about future idiosyncratic risk.

We use Bayesian methods to estimate a log-linearized version of the model, using data on 11 aggregate variables from 1982Q1 to 2017Q3. The estimated model yields several insights. First, we find that the three portfolio preference shocks together explain about 20 percent of investment and employment fluctuations, comparable to the total share explained by the risk shock and risk news shocks. Among the portfolio preference shocks, the shock to preference for equities is most important in explaining investment and hours, and contributes to the pro-cyclicality of investment, consumption, net worth, and wages and the counter-cyclicality of credit spreads and expected equity returns. Second, we find that endogenous changes in leverage and default risk, as well as the long-term nature of corporate debt, play an important role in amplifying some financial shocks and dampening others.

Endogenous changes in leverage and default risk emerge as important transmission channels for financial shocks because entrepreneurs issue long-term debt and default strategically. For example, a positive shock to the demand for equity leads to a reduction in default risk, partly due to a reduction in leverage as firms shift toward cheaper equity financing. The reduction in default risk is amplified by reduced strategic default as, even holding leverage constant, equity holders become more inclined to refinance firms on the margin of default. The resulting decline in default losses implies a reduction in the compensation corporate bond holders require for expected default. Correspondingly, investment and firm net worth boom, reflecting the decline in the equity risk premium and the corporate credit spread. As a result, a shock to the preference for equity is able to generate procyclical output, investment, consumption, net worth, wages, and expected equity returns, and a countercyclical credit spread.

We assume that firms issue long-term debt, as in Gomes, Jermann, and Schmid (2016). The long-term nature of corporate debt in the model plays an important role in the transmission of financial shocks. A rise in the demand for corporate debt leads to a decline in credit spreads, as the shock reduces the risk premium households require to own corporate debt. However, the resulting capital gains accrue to existing bondholders, dampening the effect of the lower credit risk premium on firm net worth and investment.[1] In addition, increased demand for corporate debt alters households' consumption-saving decision, leading to a decline in aggregate demand that pushes down consumption and investment. Overall, investment rises but consumption falls. Correspondingly, the model attributes only a very small share of fluctuations in aggregate quantities to the corporate bond preference shock.

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[1]While long-term debt also attenuates the effects of the endogenous decline in credit spreads from a preference shock for equity, that decline in credit spreads is not the only mechanism through which the equity preference shock boosts investment.

These results shed light on the reduced-form empirical literature showing that fluctuations in corporate bond spreads are robust predictors of economic growth, while stock returns and measures of stock valuations are not. Indeed, our results may at first seem at odd with these earlier findings: we find that shocks to household demand for stocks can generate plausible business cycle co-movements and account for a large share of fluctuations in investment and hours, but shocks to the demand for corporate debt do not. However, it is important to note that correlations between real activity and financial market prices do not have a structural interpretation on their own: financial market prices and real activity are driven by a wide variety of real and financial shocks. We find that the equity preference shock explains a meaningful share of fluctuations in credit spreads, and generates co-movement between aggregate quantities and credit spreads, thereby helping to explain the predictive power of corporate bond spreads for real activity.

To bolster our confidence in our finding that sentiment shocks are as important drivers of business cycles fluctuations as entrepreneurial risk, we align our modeling strategy of risk shocks as closely as possible to Christiano Motto and Rostagno (2014). In addition to the contemporaneous effect of unexpected changes in entrepreneurial risk, we chose to allow for the presence of correlated news shocks on future idiosyncratic risk available up to 8 quarters ahead, as in CMR's baseline model. In contrast, we do not allow for news shocks to affect any of the sentiment wedges.

The portfolio preference shocks are meant to capture several empirical facts. First, a significant share of variation in dividend yields, corporate bond spreads, and the term structure of interest rates appears to be driven by discount rates, rather than expectations for dividend growth, corporate defaults, or changes in the short-term interest rate. [2] Second, part of the variation in discount rates appears to arise because of market segmentation. Investors may not participate in all financial markets and risk might not be shared across investors active in different markets. As a result, asset prices fluctuate for reasons that cannot be explained by a representative consumer. For example, mortgage prepayment risk is a wash in the aggregate, but is priced in the market for mortgage-backed securities (Gabaix, Krishnamurthy, and Vigneron, 2007). There also appear to be clienteles for certain asset class, such as pension funds who need long-term assets to match long-term liabilities; the demand for those assets can vary for reasons unrelated to aggregate consumption dynamics. Segmented markets or clienteles can lead to "downward-sloping demand" for asset classes, for example, because of limited risk capacity of the investor base of a given asset class. Third, part of the variation in discount rates appears to arise because of differences in market or funding liquidity. Fourth, discount rates can vary because investors value attributes of a given asset

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[2]Cochrane (2011).

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classes, such as the easily understood safety of government bonds, for reasons unrelated to the payoffs of those assets. Rather than model the underlying frictions or taking a stand as to which of these frictions is most important, we model the household as experiencing exogenous shocks to its preference for government bonds, corporate bonds, and equity.

Literature Review

A large body of work in the empirical macroeconomics literature has shown that financial shocks are important drivers of the business cycle. Gilchrist and Zakrajsek (2012) show that their index of corporate bond credit spreads (the "GZ credit spread") has a strong predictive power for economic activity. Furthermore, they find that a large fraction of the predictive power of credit spreads is unrelated to issuer default risk and, instead, reflects credit market "sentiment" that affects investor demand for corporate bonds (the Excess Bond Premium (EBP)). Lopez-Salido, Stein and Zakrajsek (2017) provide further evidence that investor sentiment in credit markets can help explain macroeconomic fluctuations, and show that investor sentiment in bond and equity markets is weakly correlated, suggesting an important degree of sentiment segmentation. Brunnermeier, Palia, Sastry, and Sims (2017) confirm the importance of the GZ spread for macroeconomic fluctuations and show that shocks to the TED spread also have strong real effects but in a markedly different way. Taken together, this literature shows not just that financial shocks have important macroeconomic implications, but that there are distinct, imperfectly correlated, sources of financial shocks and that these different shocks affect the economy in diverse ways. Our structural estimation approach is designed to provide a better understanding of how different financial shocks propagate in the economy.

A related strand of the empirical literature focuses on the interaction between monetary policy and financial shocks, focusing mostly on the effect of monetary policy on financial conditions (Gertler and Karadi (2015), Caldara and Herbst (2016)). More closely related to our work is Bassetto, Benzoni, and Serrao (2016), who show that innovations to the Chicago Fed Financial Conditions Index (NFCI) and to the EBP have similar strong real effects as monetary policy shocks, but that the effects of financial shocks die out relatively fast, potentially because monetary policy responds to financial shocks. One key takeaway from this literature is that financial shocks and monetary policy interact with each other in important ways. Our model is able to identify and interpret these interactions clearly.

A recent literature has tried to provide a structural underpinning to these financial shocks. Jermann and Quadrini (2012) introduce a shock to firms short-term borrowing constraint, Christiano, Motto and Rostagno (2014) consider, instead, shocks to the volatility of entrepreneurial risk and to entrepreneurial net worth, and Ajello (2016) studies the effect of a shock to financial intermediation. These estimated dynamic stochastic general equilibrium

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frameworks typically find that the shocks they model are able to account for a large fraction of aggregate fluctuations. Our contribution relative to these papers is three-fold. First, we introduce a large number of financial shocks that capture factors from both the demand- and supply-side of financial assets and that cover all the major financial asset classes: bonds, stocks, and short- and long-term government debt. Second, we model a corporate sector in which both debt and equity financing play a role. Third, we include an expanded set of financial variables in our set of observable variables used in the estimation.

Our methodology resembles that of Chari, Kehoe, and McGrattan (2007). They propose the introduction of time-varying wedges that distort the equilibrium decisions of agents to gain an insight into which frictions or amplification mechanisms matter for aggregate fluctuations. One can interpret our preference shocks and our corporate debt supply shocks as time-varying wedges that can be measured using the structure of the model. In that sense, our results provide a unified accounting of business and financial cycles, along the lines of the "business cycle accounting" of Chari, Kehoe, and McGrattan (2007), which abstracted from financial shocks and corporate finance variables.

2 The Model

2.1 The Household

There is a large number of identical and competitive households. In period t, each household consumes C_t and provides every type of differentiated labor, $n_{i,t}$. Employment agencies combine differentiated labor inputs into homogeneous work hours. The household derives utility from consumption and disutility from supplying labor; the household also derives utility from its real holdings of financial assets .

2.1.1 Financial assets and budget constraint

The household holds four financial assets representing inter-temporal claims on the government or the corporate sector: Treasury bills; long-term Treasury bonds; long-term corporate debt; and equity. All debt is nominal and corporate debt is defaultable. Households also make an intra-temporal loan (i.e., a loan for working capital) to intermediate goods producers. The households also own equity claims issued by the entrepreneurs.

In period t, the household chooses real holdings of Treasury bills (TB_t^1), long-term government debt $(TB_t^{\frac{1}{\lambda}})$ and long-term corporate debt $(B_t^{\frac{1}{\lambda}})$. Denote the price of final consumption goods by P_t, so that inflation between periods $t-1$ and t is $\pi_t = P_t/P_{t-1}$.

Each Treasury bill held in period $t-1$ represents a claim to one dollar in period t. Thus, if the household chooses real holdings of Treasury bills in period $t-1$ equal to TB^1_{t-1}, then in period t the household will receive the equivalent of $\frac{TB^1_{t-1}}{\pi_t}$ consumption goods when its Treasury bill holdings are redeemed at par.

Each period, the holder of a long-term government bond with a face amount of one dollar receives a coupon payment of c dollars. In addition, λ fraction of the principal is repaid at par, while the remaining fraction $(1-\lambda)$ remains outstanding; thus, long-term debt has a half-life of $1/\lambda$. Long-term corporate bonds that are not in default are also repaid at rate λ and provide a coupon payment c. From the firms that do default, corporate-bondholders' recovery rate is rec_t, taken as given by the representative household and defined later in (36).

The household also owns equity claims issued by the entrepreneurs. The share count s_t is normalized to one each period. For each share owned in period $t-1$, the household receives an aggregate real dividend D_t in period t. The cum-dividend real value of the firm is denoted by J_t. Thus, the real price of one share in period t is the ex-dividend value, $J_t - D_t$.

In addition, households directly own the intermediate goods producers and capital producers and receive profits Π^{int}_t from the intermediate goods producers and Π^{cap}_t from the capital goods producers. Moreover, the households make an intra-temporal loan, with real value B^1_t, to the corporate sector. The gross return on the intra-temporal loan is $\tilde{R}_t$.

Denote the real wage by W_t and the real lump-sum transfer from the government by T_t. Then, the budget constraint is:

$$C_t + B^1_t + Q^{TB,1}_t TB^1_t + Q^{TB,\frac{1}{\lambda}}_t TB^{\frac{1}{\lambda}}_t + Q^{B,\frac{1}{\lambda}}_t B^{\frac{1}{\lambda}}_t + s_t(J_t - D_t) \leq$$

$$W_t N_t + s_{t-1} J_t + \frac{B^{\frac{1}{\lambda}}_{t-1}}{\pi_t}\left((1 - \Phi(z^*_t))(c + \lambda + (1-\lambda)Q^d_t) + \Phi(z^*_t)\text{rec}_t\right)$$

$$+\tilde{R}_t B_t + \frac{TB^1_{t-1}}{\pi_t} + (c+\lambda)\frac{TB^{\frac{1}{\lambda}}_{t-1}}{\pi_t} + (1-\lambda)Q^{TB,\frac{1}{\lambda}}_t \frac{TB^{\frac{1}{\lambda}}_{t-1}}{\pi_t} + \Pi^{cap}_t + \Pi^{int}_t + T_t. \quad (1)$$

2.1.2 Preferences

The household's preferences are as follows:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{(C_t - hC_{t-1})^{1-\psi}}{1-\psi} - \omega\frac{N^{1+\nu}_t}{1+\nu} + \Phi'_t U_{fin,t}\right] \quad (2)$$

where

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\Phi_t = \begin{bmatrix} \phi_t^{TB} \\ \phi_t^{TB} + \phi_t^{TP} \\ \phi_t^{B} + \phi_t^{TP} \\ \phi_t^{J} \end{bmatrix} , \text{ and } U_{fin,t} = \begin{bmatrix} u(Q_t^{TB,1} TB_t^1) \\ u(Q_t^{TB,\frac{1}{\lambda}} TB_t^{\frac{1}{\lambda}}) \\ u(Q_t^{B,\frac{1}{\lambda}} B_t^{\frac{1}{\lambda}} s_{b,t}) \\ u(J_t s_{e,t}) \end{bmatrix} .
$$

The first term in the household utility function reflects utility from consumption, with habit parameter h and elasticity of intertemporal substitution ψ. The second term reflects disutility from supplying labor, with Frisch elasticity ν. The last term, $\Phi_t' U_{fin,t}$ reflects the household utility from its real holdings of financial assets. Φ_t is a vector capturing preference shocks to the demand for holding different types of assets. $U_{fin,t}$ is a vector with each element an increasing and concave function $u()$ of the real holdings of a given type of financial asset. We assume u() has constant elasticity to the real holdings of each asset, with $u(x) = \frac{1}{1-\kappa} x^{1-\kappa}$ and $\kappa > 0$.

The shock ϕ_t^{TB} reflects stochastic demand for default-free government debt; this is meant to capture the convenience yield described by Krishnamurthy and Vissing-Jorgensen (2012). This shock is also akin to the Smets and Wouters (2007) "liquidity" shock that increases the desire of households to hold the risk-free asset, rather than physical capital (Fisher, 2015). In their model, as a result of nominal rigidities, a liquidity shock generates a demand-driven contraction in activity.

Whereas Smets and Wouters (2007) contains only one risky asset (physical capital) and features a zero net supply of the single-period risk-free asset, here we have a richer array of financial assets. Thus, the shock to the preference for government bonds affects both the demand for one-quarter T-bills as well as the demand for long-term government debt. In addition, we also introduce a shock to the preference for corporate equity (ϕ_t^S) and for corporate bonds (ϕ_t^B), since households do not invest directly in physical capital. Finally, since we are interested in how the preference for debt maturity affects corporate financing decisions, we introduce a shock to the preference for long-term debt (ϕ_t^{TP}) that affects demand for long-term government bonds as well as long-term corporate bonds.

In summary, the household makes consumption, labor and investment decisions,

$$
(C_t, N_t, TB_t^1, TB_t^{\frac{1}{\lambda}}, B_t^{\frac{1}{\lambda}}, s_t)_{t=0}^\infty
$$

subject to the budget constraint (1) to maximize utility (2).[3]

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[3]Note that in the model used for estimation, there is positive steady-state technological progress. This requires appropriate normalization of the household's problem, which we abstract from here; the full set of equations is available upon request.

2.1.3 Asset pricing

The first order conditions of the household's problem imply that the household uses a different stochastic discount factor to price each financial asset. Define the period-t marginal utility of consumption, adjusted for habits, as Λ_t:

$$\Lambda_t = (C_t - hC_{t-1})^{-\psi} - E_t\beta h(C_{t+1} - hC_t)^{-\psi}. \tag{3}$$

The price $Q_t^{TB,1}$ of a one-period Treasury bill is given by:

$$Q_t^{TB,1} = \beta E_t\left[SDF_{t,t+1}^{TB,1}\frac{1}{\pi_{t+1}}\right] \tag{4}$$

where

$$SDF_{t,t+1}^{TB,1} = \frac{1}{1 - \frac{1}{\Lambda_t}\phi_t^{TB}(Q_t^{TB,1}TB_t^1)^{-\kappa}}\frac{\Lambda_{t+1}}{\Lambda_t} \tag{5}$$

The real redemption payment from holding a nominal T-bill is eroded by inflation, hence the T-bill price (4) is the expectation of the real redemption value $\frac{1}{\pi_{t+1}}$ times the stochastic discount factor that prices T-bills. If ϕ_t^{TB} is zero, as in a standard model, then the SDF that prices T-bills (5) is, as usual, the ratio of the marginal utilities of consumption. However, an increase in ϕ_t^{TB} raises the period-t price of the T-bill, holding all else equal.

The stochastic discount factors that price long-term government debt $(SDF_{t,t+1}^{TB,\frac{1}{\lambda}})$, corporate debt $(SDF_{t,t+1}^{B,\frac{1}{\lambda}})$, and equity $(SDF_{t,t+1}^J)$ are similarly given by:

$$SDF_{t,t+1}^{TB,\frac{1}{\lambda}} = \frac{1}{1 - \frac{1}{\Lambda_t}(\phi_t^{TB} + \phi_t^{TP})(Q_t^{TB,\frac{1}{\lambda_t}}TB_t^{\frac{1}{\lambda_t}})^{-\kappa}}\frac{\Lambda_{t+1}}{\Lambda_t}$$

$$SDF_{t,t+1}^{B,\frac{1}{\lambda}} = \frac{1}{1 - \frac{1}{\Lambda_t}(\phi_t^{B} + \phi_t^{TP})(Q_t^{B,\frac{1}{\lambda_t}}B_t^{\frac{1}{\lambda_t}})^{-\kappa}}\frac{\Lambda_{t+1}}{\Lambda_t} \tag{6}$$

$$SDF_{t,t+1}^{J} = \frac{1}{1 - \frac{1}{\Lambda_t}\phi_t^{J}(J_ts_t)^{-\kappa}}\frac{\Lambda_{t+1}}{\Lambda_t} \tag{7}$$

The corresponding asset prices are, for long-term government debt,

$$Q_t^{TB,\frac{1}{\lambda}} = \beta E_t[SDF_{t,t+1}^{TB,\frac{1}{\lambda}}(c + \lambda + (1 - \lambda)Q_{t+1}^{TB,\frac{1}{\lambda}})]$$

for corporate bonds,

$$Q_t^{B,\frac{1}{\lambda}} = \beta E_t[SDF_{t,t+1}^{TB,\frac{1}{\lambda}}(((1 - \Phi(z_t^*))(c + \lambda + (1 - \lambda)Q_t^d) + \Phi(z_t^*)\text{rec}_t))]$$

and for equities,

$$J_t = D_t + \beta E_t[SDF_{t,t+1}^J(1 - \Phi(z_{t+1}^*))J_{t+1}.]$$

2.2 The Productive Sector

2.2.1 Final Good Producers

The final consumption good, y_t, is produced by perfectly competitive producers using the following CES production function:

$$y_t = \left(\int_0^1 y_t(i)^{\frac{1}{1+\theta_p}} di \right)^{1+\theta_p}, \tag{8}$$

where $0 \leq \theta_p < \infty$. The final good producer seeks to maximize profits, Π_t:

$$\max_{y_t, y_t(i) \forall i} \Pi_t = P_t y_t - p_t(i) y_t(i), \tag{9}$$

subject to the production function (8), where P_t is the price of the final good and $p_t(i)$ is the per-unit price of intermediate goods $y_t(i)$.

Profit maximization, taking prices $p_t(i)$ and P_t as given, implies the following demand for intermediate good $y_t(i)$ conditional on y_t:

$$y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\frac{1+\theta_p}{\theta_p}} y_t. \tag{10}$$

The usual price aggregator is derived by plugging the demand function (10) into the production function (8):

$$P_t = \left(\int_0^1 p_t(i)^{-\frac{1}{\theta_p}} di \right)^{-\theta_p} \tag{11}$$

2.2.2 Intermediate Goods Producers

Each intermediate-goods producer employs hours worked, $h_t(i)$, and capital, K_{t-1}, to produce goods $y_t(i)$ according to the production function:

$$y_t(i) = A_t^{(1-\alpha)} h_t(i)^{(1-\alpha)} K_{t-1}^{\alpha}. \tag{12}$$

where A_t is aggregate TFP and the growth rate of TFP z_t follows an AR(1) process:

$$\gamma_t = \log \frac{A_t}{A_{t-1}} = \rho_\gamma \gamma_{t-1} + \sigma_\gamma \epsilon_t^\gamma$$

and $\epsilon_t^\gamma \sim N(0,1)$.

Producers pay workers a real nominal hourly wage, $w_t = \frac{W_t}{P_t}$, and have to borrow resources from the household at the beginning of the period to pay the workers' wage bill. We assume that this type of working capital loan is paid back before the end of the period, and that producers are charged an interest rate, $\tilde{R}_t$ that is equal to the the 1-quarter risk-free rate, R_t, augmented by a fraction ϕ_{WK} of the default spread paid by corporate bond markets in compensation for aggregate default risk.

Intermediate goods are partial substitutes and intermediate good producers act in regime of monopolistic competition. In every period t, they observe the demand for their good, (10), and select the $p_t(i)$ that maximizes their profits under minimum costs. Producer i^{th} wishes to minimize total real costs, $TC_t(i)$:

$$TC_t(i) = (R_t + \phi_{WK} Sp_t) w_t h_t(i) + r_t^K K_{t-1}(i) = \tilde{R}_t w_t h_t(i) + r_t^K K_{t-1}(i), \quad \forall t \qquad (13)$$

subject to the production function (12), to which we assign a multiplier $mc_t(i)$. The FOCs are:

$$h_t(i) : \tilde{R}_t w_t = mc_t(i)(1-\alpha) A_t^{1-\alpha} h_t(i)^{-\alpha} K_{t-1}(i)^\alpha \qquad (14)$$

$$K_{t-1}(i) : r_t^K = mc_t(i)\alpha A_t^{1-\alpha} h_t(i)^{1-\alpha} K_{t-1}(i)^{\alpha-1} \qquad (15)$$

Taking the ratio of these last two equations we obtain that the ratio of labor to capital inputs is homogeneous across producers and is proportional to the ratio of their remuneration rates:

$$\frac{\tilde{R}_t W_t}{R_t^K} = \frac{(1-\alpha)K_{t-1}}{\alpha H_t} \qquad (16)$$

Multiplying (14) by $h_t(i)$ and equation (15) by $K_{t-1}(i)$ and summing them together, we obtain:

$$\tilde{R}_t w_t h_t(i) + r_t^K K_{t-1}(i) = TC_t(i) = mc_t(i) y_t(i) \qquad (17)$$

where $mc_t(i)$, by definition, is the real marginal cost of firm i. Solving for $h_t(i)$ and $K_{t-1}(i)$ from (14) and (15) and substituting in the production function (12), we can solve for the marginal cost:

$$mc_t = \left(\frac{r_t^K}{\alpha}\right)^\alpha \left(\frac{\tilde{R}_t w_t}{A_t(1-\alpha)}\right)^{1-\alpha}$$

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Note that the marginal cost $mc_t(i)$ is the same across producers i so that we can drop reference to index i.

Following Rotemberg (1992), we now assume that firms are free to change their nominal price, $p_t(i)$, every period, but that they incur a real cost:

$$\frac{\psi}{2}\left(\frac{p_t(i)}{p_{t-1}(i)\pi}-1\right)^2,\tag{18}$$

which is quadratic in the deviation from 1 of the ratio of producer's i inflation rate, $\frac{p_t(i)}{p_{t-1}(i)}$, from steady state inflation, π.

The intermediate firm wishes to maximize the present discounted value of real profits at time t:

$$\max_{p_t(i)}\sum_{t=0}^{\infty}\beta^t E_t\left[\frac{\Lambda_t}{\Lambda_0}\left(\left(\frac{p_t(i)}{P_t}-mc_t\right)y_t(i)-\frac{\psi}{2}\left(\frac{p_t(i)}{p_{t-1}(i)\pi}-1\right)^2 y_t\right)\right]\tag{19}$$

In a symmetric equilibrium $p_t(i) = p_t(j)\quad \forall i,j\in[0,1]$, the first order condition of the producers' problem will give rise to the Phillips curve:

$$1+(1+\theta_p)mc_t-\psi\left(\frac{\pi_t}{\pi}-1\right)\frac{\pi_t}{\pi}+\beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\psi\left(\frac{\pi_{t+1}}{\pi}-1\right)\frac{\pi_{t+1}}{\pi}\frac{y_{t+1}}{y_t}=0\tag{20}$$

where Λ_t is marginal utility of household consumption defined in (3).

[XXX SHOULDN'T (20) BE INSTEAD:

$$-\frac{1}{\theta_p}+\frac{1+\theta_p}{\theta_p}mc_t-\psi\left(\frac{\pi_t}{\pi}-1\right)\frac{\pi_t}{\pi}+\beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\psi\left(\frac{\pi_{t+1}}{\pi}-1\right)\frac{\pi_{t+1}}{\pi}\frac{y_{t+1}}{y_t}=0\tag{21}$$

CODE HAS A DIFFERENT EQUATION:

$$-1+(1+\theta_p)mc_t-\psi\left(\frac{\pi_t}{\pi}-1\right)\frac{\pi_t}{\pi}+\beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\psi\left(\frac{\pi_{t+1}}{\pi}-1\right)\frac{\pi_{t+1}}{\pi}\frac{y_{t+1}}{y_t}=0\tag{22}$$

We also define aggregate dividends of intermediate good producers as:

$$D_t^p = Y_t - w_t h_t - r_t^K K_{t-1}$$

2.2.3 Capital Producers

At the beginning of each period, capital producers buy the aggregate stock of old depreciated capital $(1-\delta)\bar{K}_{t-1}$ from the population of entrepreneurs. The capital producers buy an amount I_t of final goods, combine them with the old capital stock, and build new capital

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stock, $\bar{K}_t$. Their profit maximization problem is:

$$\max_{I_t} E_t \sum_{s=0}^{\infty} \beta^s SDF^J_{t,t+s} \left\{ Q_{t+s} \left(\bar{K}_{t+s} - (1-\delta)\bar{K}_{t+s-1} \right) - P_{t+s}I_{t+s} \right\}$$

subject to the physical capital accumulation technology:

$$\bar{K}_t = \mu^I_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] I_t + (1-\delta)\bar{K}_{t-1}$$

where δ is the depreciation rate, and μ^I_t is an exogenous shock to the marginal efficiency of investment and follows a process:

$$\log(\mu^I_t) = rho_{\mu^I} \log(\mu^I_{t-1}) + \epsilon_{\mu^I}.$$

The function S captures the presence of adjustment costs in the accumulation of capital. The steady-state properties of the function S are standard: $S(\gamma) = 0$, $S'(\gamma) = 0$ and $S''(\gamma) > 0$, and characterize adjustment costs that are zero at the steady state growth rate of investment, while positive and convex at any other $\frac{I_t}{I_{t-1}}$. The first order condition of the capital producers' problem is:

$$Q_t = \frac{1 - E_t SDF^J_{t,t+1} Q_{t+1} \left(\mu^I_{t+1} S'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right)}{\mu^I_t \left[1 - \left(S'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} + S\left(\frac{I_t}{I_{t-1}} \right) \right) \right]}$$

2.2.4   Employment Agencies

Employment agencies hire differentiated labor inputs, $n_{i,t}$ from households at monopolistic wages $\tilde{W}_{i,t}$ and transform them into homogenous hours worked by means of the CES technology:

$$N_t = \left[\int_0^1 n_{i,t}^{\frac{1}{1+\theta_w}} di \right]^{1+\theta_w}$$

so that the demand of any differentiated labor input, $n_{i,t}$, is:

$$N_{i,t} = \left(\frac{W_{i,t}}{W_t} \right)^{-\frac{1+\theta_w}{\theta_w}} N_t \tag{23}$$

Household i is the monopolistic supplier or labor inputs of kind $n_{i,t}$. In every period t the households set wages that maximize their welfare. In similarity with the price-setting decision of intermediate firms, intertemporal adjustments in the monopolistic wage rate

generate a cost per unit of the aggregate nominal wage bill, $W_t L_t$:

$$\frac{\psi_w}{2}\left(\frac{W_{i,t}}{W_{i,t-1}\pi\mu_z}-1\right)^2$$

Households then re-optimize monopolistic wages $\tilde{W}_{i,t}$ by maximizing the difference between the real consumption value of its wage bill in every period $t+s$, $\Lambda_t \tilde{W}_{i,t+s} n_{i,t+s}$, where Λ_t is the marginal utility granted by an additional unit of income, and the disutility induced by labor supply, $n_{i,t+s}$, minus the real consumption value of the adjustment cost:

$$\max_{W_{i,t+s}} E_t\left\{\sum_{s=0}^{\infty}\beta^s\left[\Lambda_t\frac{W_{i,t+s}}{P_{t+s}}n_{i,t+s}-\chi_0 Z_{t+s}^{1-\varphi}\frac{n_{i,t+s}^{1+\chi}}{1+\chi}-\Lambda_t\frac{\psi_w}{2}\left(\frac{W_{i,t+s}}{W_{i,t+s-1}\pi\mu_z}-1\right)^2\frac{W_{t+s}}{P_{t+s}}N_{t+s}\right]\right\}$$

subject to labor demand from employment agencies, (23).

In a symmetric equilibrium, the maximization problem gives rise to a standard wage Phillips curve:

$$-\left((1+\theta_w)u_t^L+w_t\right)-\psi_w\left(\frac{\pi_t^W}{\pi\mu_z}-1\right)\pi_t^W+E_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}\psi_w\left(\frac{\pi_{t+1}^W}{\pi\gamma}-1\right)\frac{(\pi_{t+1}^W)^2}{\pi_{t+1}}\frac{N_{t+1}}{N_t}=0$$

where w_t is the real hourly wage and u_t^L is the per-period marginal rate of substitution between hours worked and consumption and:

$$\pi_t^W=\frac{W_t}{W_{t-1}}\pi_t.$$

wage inflation is the product of the growth rate of real wages times the rate of price inflation.

2.2.5 Entrepreneurs

There is a continuum of entrepreneurs indexed by e. They face a classic capital structure choice which trades off the tax advantage of debt and the costs of financial distress associated with a high leverage.

Technology

Each entrepreneur buys installed capital $\overline{K}_{t-1,e}$ at price $Q_{t-1}^{k,\$}$ per unit from the capital producers at the end of period $t-1$. Nominal variables are indicated with a $\$$ superscript. In the next period (period t) she rents capital to intermediate good producing firms, earning a rental rate $R_t^{k,\$}$ per unit of effective capital. In period t an idiosyncratic shock $z_{t,e}$ may increase or shrink entrepreneurs' capital. The shock $z_{t,e}$ has a mean of 1 and follows a

lognormal distribution. Denote the standard deviation of the log of $z_{t,e}$ as $\sigma_{z,t}$. This standard deviation is one of the aggregate shocks we consider and is similar to the risk shock in Christiano, Motto and Rostagno (2014).

After observing the shock, the entrepreneur chooses a level of capital utilization $u_{t,e}$ by paying a cost in terms of general output equal to $a(u_{t,e})$ per-unit-of-capital. At the end of period t the entrepreneur sells the depreciated capital to the capital producers.

Entrepreneurs' revenues Π_t net of the utilization cost in period t are:

$$\Pi_t^{\$} = \left[R_t^{k,\$} u_{t,e} - P_t a(u_{t,e}) + (1-\delta) Q_t^{k,\$} \right] z_{t,e} \overline{K}_{t-1,e}, \tag{24}$$

where δ is the capital depreciation rate. Since the choice of the utilization rate, given by $R_t^{k,\$}/P_t = a\prime(u_{t,e})$, is independent of the amount of capital purchased and of the z_t shock, we drop the index e from the return $\widetilde{R}_t^{k,\$}$ in what follows. For convenience we define post-tax real profits as:

$$\Pi_t^{pt}(z_t) = \left\{ (1-\tau) \left[R_t^k u_{t,e} - a(u_{t,e}) \right] + (1-\delta) Q_t^k \right\} z_t \overline{K}_{t-1} \tag{25}$$

where τ is the corporate tax rate.

Financing

Entrepreneurs can obtain external funds by issuing bonds and equity.

Debt takes the form of nominal long-term defaultable debt. We assume–as in Gomes, Jermann, and Schmid (2016)–that in every period a fraction λ of the stock of outstanding debt $B_{t-1}^{\$}$ is paid back, while the remaining $(1-\lambda)$ remains outstanding. The firm is also required to pay a periodic nominal coupon c per unit of outstanding debt, which is tax deductible. The tax deductibility of interest increases the attractiveness of debt relative to equity. This benefit of debt, however, is compensated by the presence of financial distress and bankruptcy costs, which increase, in expectation, as the firm issues more debt. We, thus, capture the classic trade-off theory of capital structure. The endogenous ex-post bankruptcy costs and the reduced form ex-ante financial distress costs (captured by $\varrho(B_t)$) are described in detail later.

Firms face costs of adjusting both their debt and their equity toward their desired capital structure. These costs are important to capture the stickiness in leverage of firms. We assume that a firm that chooses a real amount of debt B_t in period t incurs the cost

$$\upsilon(B_t) = \frac{\zeta_B}{2} \left(\frac{B_t}{B} - 1 \right)^2 B_t, \tag{26}$$

where $\zeta_B > 0$ and B is the steady state value of corporate debt. Our specification implies that υ and its derivative are equal to zero in the deterministic steady state.

The entrepreneurial firms are owned by the households. Real dividends paid to them are

$$d_t = \Pi_t^{pt}(z_t) - Q_t^k \overline{K}_t - (c+\lambda)\frac{B_{t-1}}{\pi_t} + \tau(c+\lambda(1-Q_t^d))\frac{B_{t-1}}{\pi_t} + Q_t^d \Delta B_t - \upsilon(B_t) - \varrho(B_t) - \Theta(K_t, B_t),$$
(27)

where $\tau(c + \lambda(1 - Q_t^d))B_{t-1}$ is the deduction granted on interest payment ("tax shield"), $\Delta B_t = B_t - (1-\lambda)\frac{B_{t-1}}{\pi_t}$ is the amount of new debt issued (if positive) or the amount of old debt retired early (if negative), and $\Theta(K_t, B_t)$ are dividend adjustment costs. Negative dividends represent equity issues.

Aggregate real dividends obtained from a perfectly diversified portfolio of shares, D_t, are:

$$D_t = \int_{z_t^*}^{\infty} d_t(z_t) d\Phi(z_t),$$
(28)

which takes into account that defaulting firms (those with a realization of the idiosyncratic shock z_t below a threshold z_t^*, as will be described later) do not pay any dividends.

Equity flows are subject to costs of adjustment given by

$$\Theta(K_t, B_t) = \frac{\zeta_S}{2}\left(\frac{D_t^*}{D} - 1\right)^2 D_t^*,$$
(29)

where D_t^* are current period aggregate dividends substituting aggregate equilibrium values of K_t and B_t with an entrepreneur's individual choice of her capital and debt stocks. [4]

Optimal Choices

At the beginning of the period the shock z_t is realized, and the firm decides whether or not to default. If it does not default, then it produces, pays wages, suffers the depreciation of capital, and decides dividends payments and debt issuances. The real value of the firm to its shareholders at the beginning of a period, denoted J_t, is equal to

$$J(B_{t-1}/\pi_t, \overline{K}_{t-1}, z_t, \mathbf{S}_t) = \max_{\sigma_t}[0, \Pi_t^{pt}(z_t) - \left\{(1-\tau)c + \left[1 - \tau(1-Q_t^d)\right]\lambda\right\}\frac{B_{t-1}}{\pi_t} + V(B_{t-1}/\pi_t, \mathbf{S}_t)],$$
(30)

where the value of the firm conditional on no default, $V(B_{t-1}/\pi_t, \mathbf{S}_t)$, is

---

[4]This specification of dividend adjustment costs ensures simple aggregation of the entrepreneurial sector by preventing the need to keep track of the distribution of individual dividend payments. Under our assumptions, entrepreneurs choose the same level of dividends, debt, and investment every period.

$$V(B_{t-1}/\pi_t, \mathbf{S}_t) = \max_{B_t, \overline{K}_t} Q_t^d \Delta B_t - \upsilon(B_t) - \varrho(B_t) - \Theta(K_t, B_t) - Q_t^k \overline{K}_t$$

$$+ E_t[\beta SDF_{t,t+1}^J \int_0^\infty J(B_t, \overline{K}_t, z_{t+1}, \mathbf{S}_{t+1}) d\Phi(z_{t+1})], \tag{31}$$

and $\sigma_t = \{0, 1\}$ is a choice variable that takes value 1 if the firm decides to default and 0 otherwise.[5] The vector $\mathbf{S}_t$ captures the aggregate state variables. The rate at which firms discount future nominal dividends is the equity-specific household stochastic discount factor $SDF_{t,t+1}^J$ defined in (7).

The value function $J(B_{t-1}, \overline{K}_{t-1}, z_t, \mathbf{S}_t)$ is bounded at zero due to limited liability, which means that we can define a threshold z_t^* for the idiosyncratic shock below which the firm chooses to default. This threshold is given by

$$0 = \Pi_t^{pt}(z_t^*) - (c + \lambda)\frac{B_{t-1}}{\pi_t} + \tau(c + \lambda(1 - Q_t^d))\frac{B_{t-1}}{\pi_t} + V(B_{t-1}/\pi_t, \mathbf{S}_t). \tag{32}$$

We can substitute (30) and (32) into (31) to get

$$V(B_{t-1}/\pi_t, \mathbf{S}_t) = \max_{B_t, \overline{K}_t} Q_t^d \Delta B_t - \upsilon(B_t) - \varrho(B_t) - \Theta(K_t, B_t) - Q_t^k \overline{K}_t$$

$$+ E_t[\beta SDF_{t,t+1}^J \int_{z_{t+1}^*}^\infty [\Pi_{t+1} - (c + \lambda)\frac{B_t}{\pi_{t+1}} + \tau(c + \lambda(1 - Q_{t+1}^d))\frac{B_t}{\pi_{t+1}} + V(B_t/\pi_{t+1}, \mathbf{S}_{t+1})] d\Phi(z_{t+1})], \tag{33}$$

Financial Distress and Default

Default occurs before period t production occurs. In default, incumbent shareholders lose their ownership of shares in the firm and bondholders take over and become the sole owners. As new owners, the bondholders are entitled to collect any claims to the firm assets, including current profits, the recovery value of capital, the outstanding debt liabilities, and the proceeds from the sale of the equity in the firm. The restructuring ends when bondholders sell the restructured firm to new equityholders at price $V(\frac{B_{t-1}}{\pi_t}, \mathbf{S}_t)$. In the process, bond investors lose a fraction $(1 - \xi_t)$ of profits and the continuation value of the defaulting entrepreneur's assets. The recovery share ξ_t follows an AR(1) process.

---

[5] Note that this formulation assumes that the λ fraction of debt that is scheduled to mature is paid back in full, whereas any additional retirements over and above that amount are paid back at price Q_{t-1}^d. This implicit assumption is also in Gomes, Jermann, and Schmid (2016) but is not discussed.

Given these assumptions, the price of debt can be obtained as

$$
\begin{aligned}
Q_t^{d,\frac{1}{\lambda}}B_t = E_t\Bigg[\beta SDF_{t,t+1}^B\Bigg[&(1-\Phi(z_{t+1}^*))(c+\lambda+(1-\lambda)Q_{t+1}^{d,\frac{1}{\lambda}})\frac{B_t^{\frac{1}{\lambda}}}{\pi_{t+1}}+\xi_t\left(\int_0^{z_{t+1}^*}\Pi_{t+1}^{pt}(z_{t+1})d\Phi(z)\right.\\
&+\Phi(z_{t+1}^*)(V(B_t^{\frac{1}{\lambda}}/\pi_{t+1},\mathbf{S}_{t+1})+(1-\lambda)Q_{t+1}^{B,\frac{1}{\lambda}}\frac{B_t^{\frac{1}{\lambda}}}{\pi_{t+1}})\Bigg]\Bigg]
\end{aligned}
\tag{34}
$$

where $SDF_{t,t+1}^B$ is the stochastic discount factor used by the household to price corporate bonds, defined in (6).

Real dividends from a perfectly diversified portfolio of shares, D_t, are:

$$
D_t = \int_{z_t^*}^{\infty} d_t(z_t)d\Phi(z_t),
\tag{35}
$$

which takes into account that defaulting firms do not pay any dividends.

We compute the recovery rate rec_t as the real amount recovered in period t by debtholders of defaulted firms as a share of defaulted firm's aggregate real debt in period $t-1$ adjusted for inflation π_t:

$$
\mathrm{rec}_t\frac{B_{t-1}^{\frac{1}{\lambda}}}{\pi_t}\Phi(z_t^*) = \int_0^{z_t^*}\Pi_t^{pt}(z_t)d\Phi(z)+\Phi(z_t^*)(V(\frac{B_{t-1}^{\frac{1}{\lambda}}}{\pi_t},\mathbf{S}_t)+(1-\lambda)Q_t^{B,\frac{1}{\lambda}}\frac{B_{t-1}^{\frac{1}{\lambda}}}{\pi_t}),
\tag{36}
$$

This recovery value appears in the household budget constraint (1) and is taken as given by the representative household in choosing its real holdings of financial assets. When the restructuring process is complete, a defaulting firm is indistinguishable from a nondefaulting firm with the same debt level. All losses take place in the current period and are absorbed by the creditors. Since all idiosyncratic shocks are independent and there are no adjustment costs, default has no further consequences. As a result, both defaulting and nondefaulting firms adopt the same optimal policies and look identical at the beginning of the next period.

In addition to these endogenous ex-post bankruptcy costs, the entrepreneur suffers ex-ante costs of financial distress. We assume that a firm with an real amount of debt B_t going into period $t+1$ incurs agency costs in period t equal to $\varrho(B_t)$, in real terms, where ϱ is an increasing and convex function given by:

$$
\varrho(B_t) = \frac{\varkappa_B}{2}\left(\frac{B_t}{Q^K\overline{K}}\right)^2 Q^K\overline{K},
\tag{37}
$$

Notice that ϱ and its derivative are positive in the deterministic steady state. These costs of financial distress are modelled in the spirit of Miao and Wang (2010) and Quadrini and

Sun (2015), and capture several indirect ex ante costs of high indebtedness. [6]

Optimality Conditions

We begin by combining (24), (32), and (33) to express the value function conditional on not having defaulted and before making the optimal choices for debt and investment as

$$V(B_{t-1}/\pi_t, \mathbf{S}_t) = \max_{B_t, \overline{K}_t} Q_t^d \Delta B_t - \upsilon(B_t) - \varrho(B_t) - \Theta(K_t, B_t) - Q_t^k \overline{K}_t \tag{38}$$

$$+ E_t \left[\beta SDF_{t,t+1}^J ((1-\tau)[R_{t+1}^k u_{t+1} - a(u_{t+1})] + (1-\delta)Q_{t+1}^k)\overline{K}_t \int_{z_{t+1}^*}^{\infty} (z_{t+1} - z_{t+1}^*) d\Phi(z_{t+1}) \right],$$

The first-order necessary conditions with respect to investment and borrowing are given by

$$Q_t^k - \frac{\partial Q_t^d}{\partial \overline{K}_t} \Delta B_t + \frac{\partial \Theta(K_t, B_t)}{\partial \overline{K}_t}$$

$$= E_t \left[\beta SDF_{t,t+1}^J ((1-\tau)[R_{t+1}^k u_{t+1} - a(u_{t+1})] + (1-\delta)Q_{t+1}^k) \int_{z_{t+1}^*}^{\infty} (z_{t+1} - z_{t+1}^*) d\Phi(z_{t+1}) \right]$$

$$- E_t \left[\beta SDF_{t,t+1}^J ((1-\tau)[R_{t+1}^k u_{t+1} - a(u_{t+1})] + (1-\delta)Q_{t+1}^k)\overline{K}_t [1 - \Phi(z_{t+1}^*)] \frac{\partial z_{t+1}^*}{\partial \overline{K}_t} \right], \tag{39}$$

and

$$Q_t^d + \frac{\partial Q_t^d}{\partial B_t} \Delta B_t - \upsilon'(B_t) - \varrho'(B_t) - \frac{\partial \Theta(K_t, B_t)}{\partial B_t} = \tag{40}$$

$$E_t \left[\beta SDF_{t,t+1}^J ((1-\tau)[R_{t+1}^k u_{t+1} - a(u_{t+1})] + (1-\delta)Q_{t+1}^k)\overline{K}_t [1 - \Phi(z_{t+1}^*)] \frac{\partial z_{t+1}^*}{\partial \overline{B}_t} \right],$$

Where

$$\frac{\partial \Theta(K_t, B_t)}{\partial B_t} = \tag{41}$$

$$\zeta_S (1 - \Phi(z_t^*)) \left[Q_t^d - \frac{\partial \upsilon(B_t)}{\partial B_t} - \frac{\partial \varrho(B_t)}{\partial B_t} - \frac{\partial \Theta(K_t, B_t)}{\partial B_t} \right] \left[\left(\frac{D_t^*}{D} - 1 \right) \frac{D_t^*}{D} + \frac{1}{2} \left(\frac{D_t^*}{D} - 1 \right)^2 \right],$$

---

[6]In models of the trade-off theory of capital structure it is easy to end up with two possible equilibria: a stable and an unstable one. The unstable one is associated with high leverage and very high default rates: it is an equilibrium in which the payoff to diluting old bondholders is very high compared to the continuation value of the firm for shareholders. Once youve reached that point with near certain default in the near future, shareholders just want to issue as much debt as possible and sell the recovery value of old diluted bondholders to new bondholders. Their only alternative is to reduce debt drastically, which is very expensive. The way you get rid of this other equilibrium is by having an additional reduced form cost of high leverage.

$$\frac{\partial \Theta(K_t, B_t)}{\partial K_t} = \tag{42}$$

$$\zeta_S(1 - \Phi(z_t^*)) \left[-Q_t^k + \frac{\partial Q_t^d}{\partial K_t} \Delta B_t - \frac{\partial \Theta(K_t, B_t)}{\partial K_t} \right] \left[\left(\frac{D_t^*}{D} - 1 \right) \frac{D_t^*}{D} + \frac{1}{2} \left(\frac{D_t^*}{D} - 1 \right)^2 \right],$$

$$\upsilon'(B_t) = \zeta_B \left(\frac{B_t}{B} - 1 \right) \frac{B_t}{B} + \frac{\zeta_B}{2} \left(\frac{B_t}{B} \gamma_t - \gamma \right)^2, \tag{43}$$

and

$$\varrho'(B_t) = \varkappa_B \frac{B_t}{Q^K \overline{K}}. \tag{44}$$

Equation (39) equates the marginal cost of one additional unit of investment (LHS) to the marginal benefit (RHS). One unit of investment costs Q_t^k and affects the cost of issuing debt. The marginal return on investment is adjusted by the change it causes on the default threshold.

Equation (41) equates the marginal proceeds of one additional unit of debt (LHS) to the marginal cost (RHS). One unit of debt can be sold for Q_t^d, affects the price of debt, affects the adjustment costs of debt and equity, and generates additional ex-ante costs of financial distress. An increase in borrowing increases the default threshold ($\frac{\partial z_{t+1}^*}{\partial B_t} > 0$) making default more likely.

Note that firm optimal choices are not a function of the idiosyncratic shock z. The intuition is that equity issuance frictions affect aggregate dividends and not idiosyncratic dividends, which means that equity funds flow across firms to equalize the marginal product of capital and to optimize the debt-equity ratio.

To solve for $\frac{\partial z_{t+1}^*}{\partial B_t}$, $\frac{\partial z_{t+1}^*}{\partial \overline{K}_t}$, $\frac{\partial Q_t^d}{\partial B_t}$, and $\frac{\partial Q_t^d}{\partial K_t}$ we need to solve for these four unknowns in the following four equations, which are the derivatives of the incentive default boundary constraint (32) and of the household's Euler equation for bond-holdings (34) with respect to B_t and $\overline{K}_t$:

$$Q_t^d + \frac{\partial Q_t^d}{\partial B_t} B_t = E_t \left(\beta SDF_{t,t+1}^B \left[1 - \Phi\left(z_{t+1}^*\right)\right] \left[c + \lambda + (1-\lambda) Q_{t+1}^d\right] \frac{1}{\pi_{t+1}} \right)$$

$$+ E_t \left(\beta SDF_{t,t+1}^B \frac{\partial z_{t+1}^*}{\partial B_t} \phi\left(z_{t+1}^*\right) \left[\xi_{t+1} V\left(B_t/\pi_{t+1}, \mathbf{S}_{t+1}\right) - \left((c+\lambda) + (1-\xi_{t+1})(1-\lambda)\right) Q_{t+1}^d \frac{B_t^{\frac{1}{\lambda}}}{\pi_{t+1}} \right] \right)$$

$$+ E_t \left(\beta SDF_{t,t+1}^B \Phi\left(z_{t+1}^*\right) \xi_{t+1} \left[\frac{\partial V\left(B_t/\pi_{t+1}, \mathbf{S}_{t+1}\right)}{\partial B_t} + (1-\lambda) Q_{t+1}^d \frac{1}{\pi_{t+1}} \right] \right)$$

$$+ E_t \left(\beta SDF_{t,t+1}^B \xi_{t+1} \left[(1-\tau)\left(R_t^k u_t - a(u_t)\right) + (1-\delta) Q_{t+1}^k \right] z_{t+1}^* \overline{K}_t \phi\left(z_{t+1}^*\right) \frac{\partial z_{t+1}^*}{\partial B_t} \right)$$

$$+ E_t \left(\beta SDF_{t,t+1}^B \left(1 - \Phi(z_{t+1}^*) + \xi_t \Phi(z_{t+1}^*)\right)(1-\lambda) \frac{B_t^{\frac{1}{\lambda}}}{\pi_{t+1}} \frac{\partial Q_{t+1}^{d,\frac{1}{\lambda}}}{\partial B_{t+1}^{\frac{1}{\lambda}}} \frac{\partial B_{t+1}^{\frac{1}{\lambda}}}{\partial B_t^{\frac{1}{\lambda}}} \right) \tag{45}$$

$$\frac{\partial Q_t^d}{\partial \overline{K}_t} B_t = E_t \beta SDF_{t,t+1}^B \frac{\partial z_{t+1}^*}{\partial \overline{K}_t} \phi(z_{t+1}^*) \left[\xi_{t+1} V\left(\frac{B_t}{\pi_{t+1}}, \mathbf{S}_{t+1}\right) - \left((c+\lambda) + (1-\xi_{t+1})(1-\lambda) Q_{t+1}^{d,\frac{1}{\lambda}}\right) \frac{B_t^{\frac{1}{\lambda}}}{\pi_{t+1}} \right]$$

$$+ E_t \beta SDF_{t,t+1}^B \xi_{t+1} \left[(1-\tau)\left(R_t^k u_t - a(u_t)\right) + (1-\delta) Q_{t+1}^k \right] z_{t+1}^* \overline{K}_t \phi\left(z_{t+1}^*\right) \frac{\partial z_{t+1}^*}{\partial \overline{K}_t}$$

$$+ E_t \beta SDF_{t,t+1}^B \xi_{t+1} \left[(1-\tau)\left(R_t^k u_t - a(u_t)\right) + (1-\delta) Q_{t+1}^k \right] \int_{-\infty}^{z_{t+1}^*} z_{t+1} d\Phi\left(z_{t+1}\right), \tag{46}$$

$$\frac{\partial z_t^*}{\partial \overline{K}_{t-1}} = -\frac{z_t^*}{\overline{K}_{t-1}}, \tag{47}$$

and

$$\frac{\partial z_t^*}{\partial B_{t-1}} = \frac{\frac{1}{\pi_t}\left((c+\lambda) - \tau(c + \lambda(1 - Q_t^d))\right) - \frac{\partial V(B_{t-1}/\pi_t, \mathbf{S}_t)}{\partial B_{t-1}}}{((1-\tau)[R_t^k u_t - a(u_t)] + (1-\delta) Q_t^k) \overline{K}_{t-1}}, \tag{48}$$

where, applying the envelope condition:

$$\frac{\partial V(B_{t-1}/\pi_t, \mathbf{S}_t)}{\partial B_{t-1}} = -(1-\lambda) Q_t^d \frac{1}{\pi_t}. \tag{49}$$

The entrepreneurial sector equations that are needed to compute the competitive equilibrium and solve for the policy functions for $\overline{K}_t$, B_t, Q_t^d, z_t^*, $\frac{\partial z_{t+1}^*}{\partial B_t}$, $\frac{\partial z_{t+1}^*}{\partial \overline{K}_t}$, $\frac{\partial Q_t^d}{\partial B_t}$, $\frac{\partial Q_t^d}{\partial \overline{K}_t}$, and $J_t^\$$ are (32), (39), (41), (45), (46), (47), and (48). [7] [8]

---

[7]There is an additional variable that needs to be solved for in (XX), which is $\frac{\partial B_{t+1}^{\frac{1}{\lambda}}}{\partial B_t^{\frac{1}{\lambda}}}$. We follow the approach of Gomes, Jermann, and Schmid (2016) to generate an additional equation that allows us to solve for this variable. We discuss our approach in detail in Appendix XX

[8]Notice that in (XX) we assume, for simplicity and without loss of generality, that $\frac{\partial \Theta(K_t, B_t)}{\partial \overline{B}_{t-1}} = 0$. In

2.3 The Monetary Policy Authority

To close a baseline version of the model, we assume that the monetary policy authority sets the nominal rate of interest, R_t, by means of the Taylor-type rule:

$$\log(R_t) = \rho_R \log(R_{t-1}) + (1-\rho_R)(\log(r_t) + \log(\pi_t) + \phi_\pi(\log(\pi_t) - \log(\pi_{ss})) + \phi_y(\log(Y_t) - \log(Y_{ss}))) + \sigma^R \epsilon_t^R; \tag{50}$$

where ϵ_t^R is a standard normal innovation.

2.4 The Fiscal Authority and the Consolidated Government Budget Constraint

The model features segmented asset markets and the demand for Treasury bonds is downward sloping. It is important to be explicit about what drives the supply of government bonds in the economy. We assume that short-term Treasury bills are in zero net supply, while the totality of outstanding government bonds is issued in long-term notes, TB_t^λ.

The government collects tax revenues from entrepreneurs and issues long-term bonds at their market price. It uses proceeds to finance government spending, pay coupon and principal on maturing debt, and to fund a lump-sum transfer to households, T_t.

$$g_t Y_t + TB_{t-1}^1 + (c+\lambda)TB_{t-1}^\lambda + T_t = \tag{51}$$
$$Q_t^{TB,1} TB_t^1 + Q_t^{TB,\lambda}(TB_t^\lambda - (1-\lambda)TB_{t-1}^\lambda) + \tau(R_t^k K_{t-1} u_t - a_t(u_t) - (c+\lambda(1-Q_{t-1}^{TB,\lambda})TB_{t-1}^\lambda)$$

where:

$$g_t = (1-\rho_g)g_{ss} + \rho_g g_{t-1} + \epsilon_t^g \tag{52}$$

and we assume that the stock of government debt evolves according to

$$TB_t^\lambda - TB_{ss}^\lambda = \rho_{TB}(TB_{t-1}^\lambda - TB_{ss}^\lambda) - \tau_{TB}(\log(TB_t^\lambda) - \log(TB_{t-1}^\lambda)) - \tau_y(\log(Y_t) - \log(Y_{t-1})) \tag{53}$$

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other words: the entrepreneur internalizes the cost of current aggregate dividends deviating from previous period's aggregate dividends, but does not internalize that it will affect aggregate dividends today for the purpose of next period's adjustment costs.

2.5 Aggregation and Market Clearing

Capital market clearing:

$$\hat{K}_t = \left[1 - S\left(\frac{\hat{I}_t}{\hat{I}_{t-1}} \gamma_t \right) \right] \hat{I}_t + (1 - \delta)\hat{K}_{t-1}/\gamma_t \tag{54}$$

Goods market clearing:

$$\hat{Y}_t = \hat{C}_t + \hat{I}_t + \hat{G}_t + \frac{\psi}{2}\left(\frac{\pi_t}{\pi} - 1 \right)^2 \hat{Y}_t + \frac{\psi_w}{2}\left(\frac{\pi_{t+1}^W}{\pi\gamma} - 1 \right)^2 \tag{55}$$

2.6 Shocks Processes

The economy is buffeted by a rich set of shocks.[9] We include shocks to the growth rate of total factor productivity γ_t:

$$\hat{\gamma}_t = \rho_\gamma \hat{\gamma_{t-1}} + \sigma_\gamma \epsilon_t^\gamma$$

to the price and wage mark-ups λ_t^p and λ_t^w:

$$\hat{\lambda_t^p} = \rho_p \hat{\lambda_{t-1}^p} + \sigma_p \epsilon_t^p$$

$$\hat{\lambda_t^w} = \rho_w \hat{\lambda_{t-1}^w} + \sigma_w \epsilon_t^w$$

to the marginal efficiency of investment technology, μ_t^I:

$$\mu_t^I = \rho_I \mu_{t-1}^I + \sigma_I \epsilon_t^I$$

to government spending share of GDP g_t:

$$\hat{g}_t = \rho_g \hat{g_{t-1}} + \sigma_g \epsilon_t^g$$

to the discount factor β_t:

$$\hat{\beta}_t = \rho_\beta \hat{\beta_{t-1}} + \sigma_\beta \epsilon_t^\beta$$

to the dispersion of entrepreneurial risk σ_t^z (and up to 8-quarter-ahead news thereof as in Christiano, Motto, Rostagno (2014)):

$$\hat{\sigma_t^z} = \rho_{\sigma_z} \hat{\sigma_{t-1}^z} + \sigma_{\sigma_{z,1}} \epsilon_t^{\sigma_z} + \Sigma_{k=1}^8 \sigma_{\sigma_{z,2}} \epsilon_{t+k}^{\sigma_z} + \Sigma_{k=1,l=1}^8 corr(\sigma_{\sigma_{z,2}} \epsilon_{t+k}^{\sigma_z}, \sigma_{\sigma_{z,2}} \epsilon_{t+l}^{\sigma_z})$$

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[9] All exogenous shocks are expressed in deviation from steady state (denoted for a generic variable x by $\hat{x}$) and follow the AR(1) processes. All innovations ϵ_t are modeled as standard normals.

and an Gaussian shock to the monetary policy rule ϵ_t^R. We add to these standard shocks, by including shocks to the household's preferences for specific classes of assets. In particular we model shocks to the preference for Treasury bonds (both long-term and short-term) as ϕ_t^{TB}:

$$\hat{\phi}_t^{TB} = \rho_{TB}\hat{\phi}_{t-1}^{TB} + \sigma_{TB}\epsilon_t^{TB}$$

shocks to the preference for long-maturity assets ϕ_t^{TP}:

$$\hat{\phi}_t^{TP} = \rho_{TP}\hat{\phi}_{t-1}^{TP} + \sigma_{TP}\epsilon_t^{TP}$$

shocks to the preference for corporate bonds:

$$\hat{\phi}_t^{B} = \rho_{B}\hat{\phi}_{t-1}^{B} + \sigma_{B}\epsilon_t^{B}$$

and shocks to the preference for equity holdings, ϕ_t^S:

$$\hat{\phi}_t^{S} = \rho_{S}\hat{\phi}_{t-1}^{S} + \sigma_{S}\epsilon_t^{S}$$

3   Data, Calibration, and Estimation

We solve the model using first-order perturbation methods of its first order conditions around their deterministic steady state.[10] We calibrate the set of parameters that affect the steady state of the economy, excluding the steady-state distribution of idiosyncratic entrepreneurial risk,σ_{ss}^z. The choices are rather standard in the macro literature. We choose the IES coefficient to be equal to 0.55 and the habit preference to be 0.8. We calibrate both the quarterly rate of TFP and the rate of inflation at 0.5%. We choose the discount factor $\beta = 0.9935$ to set the steady state level of the risk free rate of 4.6% when the convenience yield ϕ^{TB} is equal to zero. We choose the steady state level of the asset preference wedges so that they deliver a steady-state nominal risk-free rate of 3.6%. We fix the demand elasticities for all assets in the household's portfolio (Treasuries, Corporate Bonds, and Stocks) to $\kappa = 1$. The maturity of long-term Treasuries and Corporate bonds is set to 10 years, by imposing $\lambda = \frac{1}{40}$.

In steady state government spending expressed as a share of GDP is set to be equal to 17% and we assume that supply of Treasuries is 70% of GDP, divided equally between short-term and long-term bonds, B_t^1 and $B_t^{1/\lambda}$. The capital share of income α is equal to 0.36, while the steady-state mark-ups for prices and wages, λ_p and λ_w are both set at 0.15.

We calibrate the corporate tax rate τ to be 30% and the average recovery of the value

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[10]For the full set of first order conditions of the model, see the Appendix.

of defaulted firm ξ to be 40%. These values, together with an estimate of the steady-state standard deviation of idiosyncratic risk $\sigma_{ss}^z = 0.05$ deliver an average corporate spread of around 2% per annum and a default probability of 0.45% per quarter, broadly in line with similar targets in the modeling literature (Christiano, Motto, and Rostagno (2014), Bernanke Gertler Gilchrist (1999)).

We estimate the remaining model parameters by Bayesian methods. We use the model solution in state-space form to fit a panel of US macro and financial variables at quarterly frequency, relying on the Kalman filter to build the likelihood function. The set of observables include the standard set of macro time-series as in Christiano, Eichenbaum, and Evans (2005): the growth rates of per-capita GDP, investment, consumption and real wages, the log of per-capita hours worked, the inflation rate, the federal funds rate. We also add the set of financial variables in Christiano, Motto, and Rostagno (2014): the growth rate of corporate credit, the quarterly growth rate of the stock market as a measure of entrepreneurial net worth, the spread between the BAA corporate bond yield and the 10-year Treasury yield, the term spread between the 10-year Treasury yield and the 3-month Treasury rate. At the current stage we fit the model to data from 1985:Q1 to 2010:Q2 to allow for the maximum comparability with the results in Christiano, Motto, and Rostagno (2015).[11]

We impose priors on the parameter values that are largely consistent with those chosen by Christiano, Motto, and Rostagno, listed in table 4. We impose loose priors on the Rotemberg adjustment costs for nominal prices and wages, as well as on the standard deviations and autoregressive coefficients of the sentiment shocks, to let the data freely determine their role in shaping business cycle fluctuations. We maximize the posterior function with respect to the parameter values and use a Metropolis Hasting algorithm to explore its surface and compute credible sets for the parameters, as well as for the model-implied second moments of the observables reported in tables 1 and 2 and the variance decomposition in table 3. Parameter estimates are rather standard. Notably, the Rotemberg adjustment costs in table 4 suggest that the economy features a moderate degree of wage rigidities and a low degree of price rigidities. The scarce prevalence of price rigidities in our model estimates provides suggestive evidence that the presence of nominal long-term debt plays a role in the transmission of demand shocks to the rest of the economy, as in Gomes, Jermann and Schmidt (2016).

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[11]We use BEA seasonally adjusted series for GDP, Investment (defined as the sum of fixed private investment and consumption in durable goods), and Consumption (defined as the sum of personal consumption expenditures in non-durable goods and services). Each series is deflated by its implicit deflator. We define inflation as the quarterly rate of change in the GDP deflator. Corporate credit growth is defined as the rate of change of liabilities of the corporate sector from the Flow of Funds data, while changes in net worth are matched to the quarterly growth rate of the Wilshire 5000 stock market index. We use a interpolated population series from annual OECD data to compute per-capita quantities.

Tables 1 reports in the first row the standard deviation of GDP growth in the data and the median and 90% credible sets implied by the model under the posterior parameters. The model-implied GDP volatility is slightly higher than the data realization, but in line with historical measures of volatility of aggregate activity. The remaining rows in the table report the volatilities of the other observable variables relative to the volatility of GDP. The evidence in the table suggests that the estimated model is able to generate aggregate fluctuations that are largely in line with the historical experience of the U.S. economy.

Table 2 shows the first-order autocorrelation of the observable data series, and their model-implied counterparts. The model estimates are largely in line with data evidence, with the exception of the autocorrelation of credit growth, which is high and positive in the data, while model estimates suggest that the moment can plausibly be zero. [12]

4 The Financial Shocks

In this section we explore which financial shocks matter for the business cycle, and why.

Table 3 displays the independent contribution of each shock to the variance of the observable variables at business cycle frequencies. The table shows the median variance decomposition and the 90 percent credible sets produced by the exploration of the model posterior. Our five financial shocks combined are the second most important drivers of the business cycle fluctuations in the data, explaining together more than 21 percent of the unconditional variance of GDP growth and close to 39 percent of the volatility of investment growth. Figure 1 displays the time series of GDP that results from feeding only the estimated financial shocks to the model and compares it with the time series of GDP in the data. Financial shocks emerge as strongly pro-cyclical drivers of output growth. Table XXX also shows that the financial shocks are particularly important for the financial variables. More than 90 percent of the volatility of the credit spread and of net worth and nearly half of the variation of credit flows and the slope of the term structure are accounted for by the financial shocks.

Out of the five financial shocks, the shock to the preference for holding equity, the shock to the preference for holding government debt, and the entrepreneurial risk shock (including news shocks) are the most important for business cycles. They explain, respectively, 6, 4, and 11 percent of the variance of GDP growth. The three shocks have in common that they barely explain consumption variations–despite the moderate degree of nominal rigidities–but explain a significant fraction of investment volatility (38 percent).

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[12]In the current version of the model the entrepreneur does not face adjustment costs when adjusting the stock of debt. We plan to release this assumption in future estimation attempts and include updated results in future drafts of the paper.

Why does our estimation assign an important role to these three financial shocks? Overall, disturbances to ϕ^{TB}, ϕ^J, and σ_z trigger responses in our model that broadly resemble business cycles observed in the data. Figure 3 displays the impulse response functions to a one standard deviation shock to the preference for government debt. Following a positive shock to ϕ^{TB}, households consume less and rebalance their savings toward government debt and out of stocks and corporate bonds. As a result, the credit spread and the required return on equity increase. Thus, entrepreneurs acquire less raw capital and investment falls. Thus, with the decline in the aggregate demand for consumption goods and capital, an ϕ^{TB} shock generates declines in consumption, investment, output and employment. Finally, the overall decline in economic activity results in a decline in the marginal cost of production and, thus, a decline in inflation. So, according to the model, the liquidity preference shock implies a countercyclical credit spread and pro-cyclical investment, consumption, employment, inflation, and stock market. These implications of the model correspond well to the analogous features of US business cycle data. This shock closely resembles the risk premium shock in Smets and Wouters (2007). Figure 4 displays the impulse response functions to a one standard deviation shock to the preference for stocks. Disturbances to ϕ^J trigger responses in our model that resemble most, but not all, business cycle properties observed in the data. Following a positive shock to η^J, households rebalance their savings toward stocks and out of government and corporate bonds. The required rate of return on equity drops, and so does entrepreneurs' strategic default incentive. This causes corporate bond spreads to fall and credit to entrepreneurs and entrepreneurial equity capitalization to increase. It follows that investment increases. Despite the boost in economic activity, employment and inflation fall. So, according to the model, the stock preference shock implies a countercyclical credit spread, inflation, and employment and procyclical investment, consumption, stock market, and credit. These different implications allow the model to separately identify shocks to ϕ^J and ϕ^{TB}. Finally, Figure 5 shows the impulse response functions to a shock to entrepreneurial risk. Shocks to σ_z overall produce fluctuations in key macroeconomic variables that resemble actual business cycles, except in the case of net worth and employment. Our model estimation still assigns a large explanatory power to the risk shock, as in Christiano, Motto, and Rostagno (2014), but the shocks to ϕ^{TB} and ϕ^J crowd out part of the relevance of this shock.[13]

Figure 1 shows the time series representation of the evolution of quarterly GDP growth in the data and decomposes each quarterly realization into the positive (above the x axis)

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[13]Note that one of the differences between CMR and our framework is that the entrepreneur chooses dividend payouts optimally, whereas in CMR the flow of dividends is exogenous. This distinction can result in very different dynamics of the net worth of entrepreneurs.

and negative (below the x axis) contributions of the fundamental shocks in the model, listed in the legend on the right-hand side of the graph. Shocks to the preference for government debt (η^{TB}) display a markedly procyclical but only moderately strong role in most of the expansions and downturns in our sample. Shocks to the preference for equity (η^J) played an important role in the downturns of 1991 and 2008-09 and in their recoveries. They do not, however, play any meaningful role in the late 1990s boom and in the dot-com bubble crash of 2000-01. The risk shock, on the other hand, displays a markedly pro-cyclical pattern throughout most of the sample, contributing strongly to all recessions and moderately to all expansions.

The variance decomposition in Table 3 indicates that shocks to the demand for corporate bonds and shocks to debt maturity preference have a minor role in explaining real variables and are only important to explain a subset of the financial variables. Consistent with this observation, the historical decomposition of the GDP growth series shown in Figure 1 suggests that the corporate bond preference shock (η^B) does not possess a clear cyclical pattern-driving the economic expansions following the recessions of 1991 and 2008-09, but exerting countercyclical pressure in the expansion of the mid-2000s and during the recession of 1991. The term premium shock, on the other hand, has a minimal role, even though its contribution is moderately pro-cyclical. A key reason that these shocks have little effect on business cycles is that our model features long-term corporate debt. Because debt is long-term, a decline in the yield demanded by investors to own corporate debt generates large capital gains for existing bond holders, but has little effect on firms' net worth and investment decisions.

5   Extensions (in progress)

In this section, we extend our analysis to evaluate the presence of a common component in the financial shocks, to study the effects of central bank asset purchases in our framework, and to shed more light on the mechanisms through which the different financial shocks affect economic activity.

5.1   The Presence of a Common Component in the Financial Shocks (in progress)

There might be important commonality across all, or different subsets of, the financial shocks. To study this possibility, we test for the presence of a common 'sentiment' or 'headwinds' shock that affects all of the risk-appetite shocks. We also consider the possibility of a

component that is common to both the risk-appetite shocks and structural shocks.

5.2 Policy Analysis: The Effect of Central Bank Asset Purchases (in progress)

Our model features downward-sloping demand for government debt, corporate debt, and equity, which means that central bank balance-sheet policy actions (large-scale asset purchases (LSAP), maturity transformation (operation twist),...) can have real effects. Large-scale purchases of corporate bonds, while not directly relevant for the U.S., have been carried out by other countries and can be studied within our framework as well.

5.3 Sensitivity to Corporate Debt Maturity and to Debt-Equity Substitutability (in progress)

Two factors that have an important effect on the relative importance of some of the financial shocks (particularly the shocks to the preference for corporate bonds and equity, respectively η^{TB} and η^{J}) are the long-term nature of corporate debt and the degree of substitutability of debt and equity. We explore first how much restricting the ability to issue equity dampens the effects of a positive equity sentiment shock. Second, we assess if shortening the maturity of corporate debt substantially is able to restore the effect of a positive bond sentiment shock by reducing the fraction of the benefits that accrue to existing bondholders.

6 Conclusion (TO BE COMPLETED)

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Appendix A - Solution Method

A.1 - Solving for $\frac{\partial B_{t+1}^{\frac{1}{\lambda}}}{\partial B_t^{\frac{1}{\lambda}}}$

The solution of the model is complicated by the presence of the derivative $\frac{\partial B_{t+1}^{\frac{1}{\lambda}}}{\partial B_t^{\frac{1}{\lambda}}}$ in 45. This is a derivative of an unknown function and both the function and its derivative need to be solved for simultaneously. The computation of the steady state and of the dynamic equations require different approximations to deal with this problem, and we follow Klein, Krussell, and Rios-Rull (2008) and Gomes, Jermann, and Schmid (2016) to deal with this.

Tables and Figures

Table 1: Model Fit : Standard Deviations

Observables	Data	Model Median	[	5%	-	95%	]
Stdev $\Delta \log GDP$	0.71	1.33	[	1.09	—	1.61	]
Stdev $\Delta \log C$	0.72	0.64	[	0.51	—	0.78	]
Stdev $\Delta \log I$	2.91	2.46	[	2.12	—	2.83	]
Stdev $\log H$	6.47	5.35	[	3.87	—	7.99	]
Stdev $\Delta \log w$	0.94	0.64	[	0.51	—	0.80	]
Stdev π	0.54	0.56	[	0.42	—	0.76	]
Stdev FFR	1.26	0.56	[	0.37	—	0.89	]
Stdev $Corp.Spread$	0.23	0.15	[	0.10	—	0.23	]
Stdev $\Delta \log Credit$	1.68	2.15	[	1.69	—	2.77	]
Stdev $\Delta \log NetWorth$	11.83	14.73	[	11.79	—	18.57	]
Stdev $TermSpread$	0.53	0.43	[	0.31	—	0.64	]

Table 2: Model Fit : Autocorrelations of Order 1

Observables	Data	Model Median	[	5%	-	95%	]
AC(1) $\Delta \log GDP$	0.50	0.57	[	0.39	—	0.70	]
AC(1) $\Delta \log C$	0.51	0.66	[	0.50	—	0.78	]
AC(1) $\Delta \log I$	0.53	0.66	[	0.52	—	0.77	]
AC(1) $\log H$	0.98	0.59	[	0.26	—	0.81	]
AC(1) $\Delta \log w$	0.11	0.65	[	0.50	—	0.76	]
AC(1) π	0.62	0.75	[	0.62	—	0.84	]
AC(1) FFR	0.99	0.91	[	0.82	—	0.96	]
AC(1) $Corp.Spread$	0.90	0.86	[	0.73	—	0.94	]
AC(1) $\Delta \log Credit$	0.76	0.06	[	-0.17	—	0.27	]
AC(1) $\Delta \log NetWorth$	0.07	-0.06	[	-0.23	—	0.13	]
AC(1) $TermSpread$	0.92	0.85	[	0.73	—	0.92	]

Table 3: Posterior Variance Decomposition - Independent Wedges (benchmark case)

	MP	MEI	P M-up	W M-up	TFP	Beta	Govt	Risk	TB	TP	S	B
GDP	2.8	2.9	8.1	12.4	24.9	7.5	15.4	11.3	3.5	0.2	5.8	0.4
	[2.0 - 3.7]	[1.4 - 4.8]	[5.2 - 11.5]	[8.1 - 19.2]	[17.5 - 33.8]	[3.1 - 10.1]	[12.4 - 19.1]	[7.9 - 15.0]	[2.1 - 5.0]	[0.1 - 0.2]	[3.5 - 8.4]	[0.2 - 0.8]
C	0.3	0.4	0.5	5.5	30.8	31.9	2.3	0.5	0.4	0.0	0.4	0.0
	[0.2 - 0.4]	[0.2 - 0.8]	[0.3 - 0.7]	[2.4 - 17.7]	[22.3 - 38.0]	[14.1 - 39.3]	[1.7 - 3.1]	[0.3 - 0.9]	[0.2 - 0.5]	[0.0 - 0.0]	[0.2 - 0.6]	[0.0 - 0.0]
I	4.8	8.2	15.0	13.6	16.0	0.4	0.3	21.3	6.1	0.2	10.5	0.8
	[3.4 - 6.1]	[4.0 - 14.0]	[9.6 - 20.3]	[8.5 - 19.3]	[8.3 - 23.2]	[0.1 - 0.5]	[0.2 - 0.4]	[15.2 - 27.9]	[3.8 - 8.3]	[0.1 - 0.3]	[6.2 - 14.9]	[0.4 - 1.5]
w	1.5	0.1	36.9	12.8	33.5	7.6	0.5	0.7	0.9	0.0	0.3	0.0
	[1.1 - 2.0]	[0.1 - 0.2]	[26.7 - 48.3]	[7.6 - 18.0]	[22.2 - 44.2]	[2.9 - 10.3]	[0.3 - 0.6]	[0.3 - 0.9]	[0.6 - 1.3]	[0.0 - 0.0]	[0.2 - 0.4]	[0.0 - 0.0]
H	1.3	2.1	10.4	22.9	14.8	4.4	1.6	6.8	1.8	0.2	14.2	2.0
	[0.8 - 1.7]	[1.0 - 3.6]	[7.0 - 14.5]	[14.8 - 33.6]	[9.1 - 22.3]	[1.6 - 6.1]	[1.2 - 2.0]	[4.8 - 8.7]	[1.1 - 2.6]	[0.1 - 0.3]	[11.7 - 16.5]	[1.5 - 2.5]
π	5.2	9.8	19.4	6.7	7.5	1.3	0.6	8.3	23.1	0.2	6.5	0.5
	[3.3 - 7.1]	[5.0 - 16.0]	[13.7 - 25.9]	[3.9 - 9.8]	[2.9 - 12.6]	[0.5 - 1.9]	[0.4 - 0.8]	[5.8 - 10.9]	[17.2 - 30.4]	[0.1 - 0.3]	[4.7 - 8.7]	[0.4 - 0.7]
FFR	14.2	12.5	6.5	2.6	6.3	0.6	0.9	8.3	26.2	0.2	6.4	0.6
	[10.8 - 17.0]	[6.7 - 20.0]	[4.2 - 8.7]	[1.4 - 4.1]	[3.7 - 9.2]	[0.2 - 0.8]	[0.7 - 1.2]	[5.8 - 10.8]	[19.1 - 33.7]	[0.2 - 0.4]	[4.7 - 8.7]	[0.4 - 0.8]
Spread	0.1	0.3	0.3	2.2	0.4	0.1	0.3	39.7	8.0	43.1	4.1	1.1
	[0.1 - 0.1]	[0.1 - 0.7]	[0.2 - 0.4]	[0.1 - 3.4]	[0.1 - 0.8]	[0.0 - 0.1]	[0.2 - 0.4]	[29.0 - 50.5]	[5.4 - 11.3]	[35.2 - 50.3]	[2.2 - 6.3]	[0.3 - 1.8]
Credit	4.3	8.1	11.3	25.9	1.2	0.7	1.6	32.1	1.5	0.0	0.9	10.1
	[3.0 - 5.6]	[3.5 - 14.9]	[7.2 - 15.6]	[14.3 - 34.0]	[0.1 - 4.3]	[0.3 - 1.3]	[1.0 - 2.3]	[22.3 - 42.0]	[0.5 - 3.1]	[0.0 - 0.0]	[0.4 - 2.1]	[7.5 - 12.7]
NW	0.8	0.1	1.4	1.5	2.6	0.6	0.7	66.7	0.9	0.1	20.0	5.1
	[0.5 - 1.1]	[0.0 - 0.1]	[0.8 - 1.9]	[0.9 - 2.4]	[1.5 - 4.8]	[0.3 - 1.0]	[0.5 - 1.0]	[49.7 - 83.9]	[0.5 - 1.3]	[0.1 - 0.2]	[16.4 - 23.3]	[1.3 - 9.1]
Slope	16.5	9.8	7.8	2.0	9.4	0.7	0.5	9.7	21.2	6.8	7.3	0.8
	[12.8 - 19.6]	[4.6 - 15.6]	[5.0 - 10.6]	[1.0 - 3.6]	[5.4 - 13.8]	[0.3 - 1.0]	[0.3 - 0.6]	[6.7 - 13.0]	[14.1 - 26.9]	[4.7 - 8.7]	[5.4 - 9.7]	[0.5 - 1.0]

Variance Decomposition of the observables, periodic component with cycles between 6 and 32 quarters. Median values and 90% confidence intervals reported. Posterior percentiles obtained from 3 chains of 100,000 draws generated using a Random Walk Metropolis algorithm. Acceptance rate 19%. Observations retained: one in every 4 draws. Variables are the growth rate of real per-capita GDP, Consumption (C), Investment (I), real hourly wages (w), the log of hours worked (H), inflation (π), federal funds rate (FFR), BAA - 10-year Treasury spread (Spread), the growth rate of real credit (Credit), the rate of growth of net worth (NW), and the slope of the term structure of Treasury yields (Slope). The shocks are monetary policy shock (MP), marginal efficiency of investment shock (MEI), price and wage mark-up shock (P M-up, and W M-up), TFP shock, beta shock (Beta), Government spending shock (Govt), risk shock (Risk), convenience yield shock (TB), term premium shock (TP), equity sentiment shock (S), and bond sentiment shock (B). Values are percentages. Rows may not sum up to 100% due to rounding error.

Table 4: Calibrated Values, Priors and Posterior Estimates for the Model Parameters

parameters	prior mean	post. mean	5%	95%	prior	pstdev
$\sigma 1_z$	0.500	0.2006	0.1785	0.2218	invg2	2.0000
$\sigma 2_z$	0.500	0.0877	0.0726	0.1056	invg2	2.0000
$corr_\sigma$	0.000	-0.5020	-0.6799	-0.3361	norm	0.5000
ρ_i	0.500	0.9608	0.9426	0.9830	beta	0.2000
ρ_w	0.500	0.9612	0.9296	0.9971	beta	0.2000
ρ_γ	0.500	0.6746	0.6041	0.7510	beta	0.2000
ρ_ξ	0.500	0.2929	0.0530	0.4652	beta	0.2000
ρ_{TB}	0.500	0.9178	0.9012	0.9322	beta	0.2000
ρ_{TP}	0.500	0.8014	0.7388	0.8567	beta	0.2000
ρ_s	0.500	0.3548	0.2454	0.4723	beta	0.2000
ρ_b	0.500	0.9862	0.9796	0.9926	beta	0.2000
τ_p	0.500	0.4791	0.3015	0.6955	beta	0.2000
τ_w	0.500	0.1268	0.0119	0.2280	beta	0.2000
ψ	40.000	6.5849	6.1985	6.9284	norm	20.0000
ψ_W	40.000	40.3043	40.0927	40.4813	norm	20.0000
θ_i	3.000	2.0498	1.7611	2.2777	norm	3.0000
ψ_{sp}	0.500	0.4590	0.2431	0.6855	invg2	0.2000
ρ_r	0.750	0.8340	0.8121	0.8552	beta	0.1000
ϕ_π	0.500	0.6614	0.5672	0.7455	norm	0.1000
ϕ_y	0.500	0.6700	0.5272	0.8099	norm	0.1000
$\rho^t b_\lambda$	0.700	-0.2094	-0.4605	0.0989	norm	0.4000
ϕ_y^{tb}	0.500	0.2832	-0.0245	0.4998	norm	0.4000
σ_{ss}^z	-1.350	-2.9424	-2.9524	-2.9301	norm	1.0000
ϵ_{mp}	0.002	0.0066	0.0059	0.0073	invg2	0.0033
ϵ_i	0.002	0.0137	0.0094	0.0179	invg2	0.0033
ϵ_p	0.002	0.0101	0.0079	0.0123	invg2	0.0033
ϵ_w	0.002	0.0484	0.0354	0.0626	invg2	0.0033
ϵ_γ	0.002	0.0067	0.0053	0.0084	invg2	0.0033
ϵ_β	0.002	0.0338	0.0226	0.0403	invg2	0.0033
ϵ_g	0.002	0.0258	0.0229	0.0284	invg2	0.0033
ϵ_{TB}	2.000	0.5740	0.4331	0.6906	invg2	3.0000
ϵ_{TP}	2.000	0.2311	0.1665	0.2891	invg2	3.0000
ϵ_S	2.000	9.3736	9.0557	9.7220	invg2	3.0000
ϵ_B	2.000	14.0127	13.6722	14.4615	invg2	3.0000

Figure 1: GDP historical decomposition
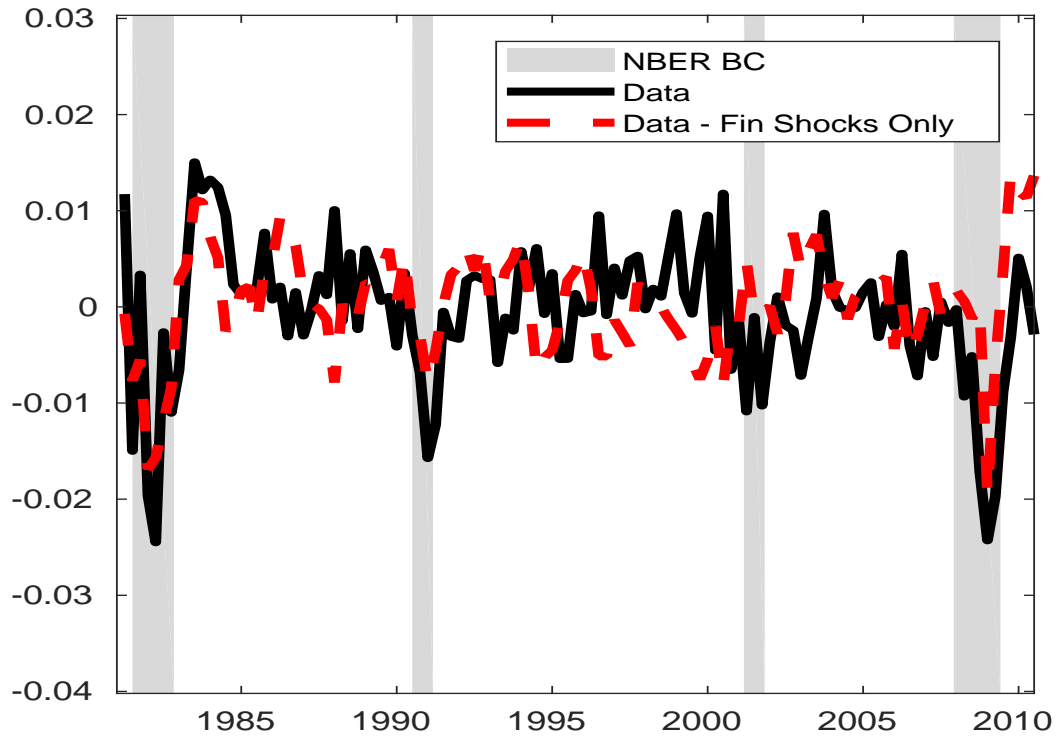
NOTE: This figure shows the time series representation of the evolution of quarterly GDP growth in the data and the time series of GDP that results from feeding only the estimated financial shocks to the model. DATA SOURCE: Authors' calculations.

Figure 2:   GDP historical decomposition
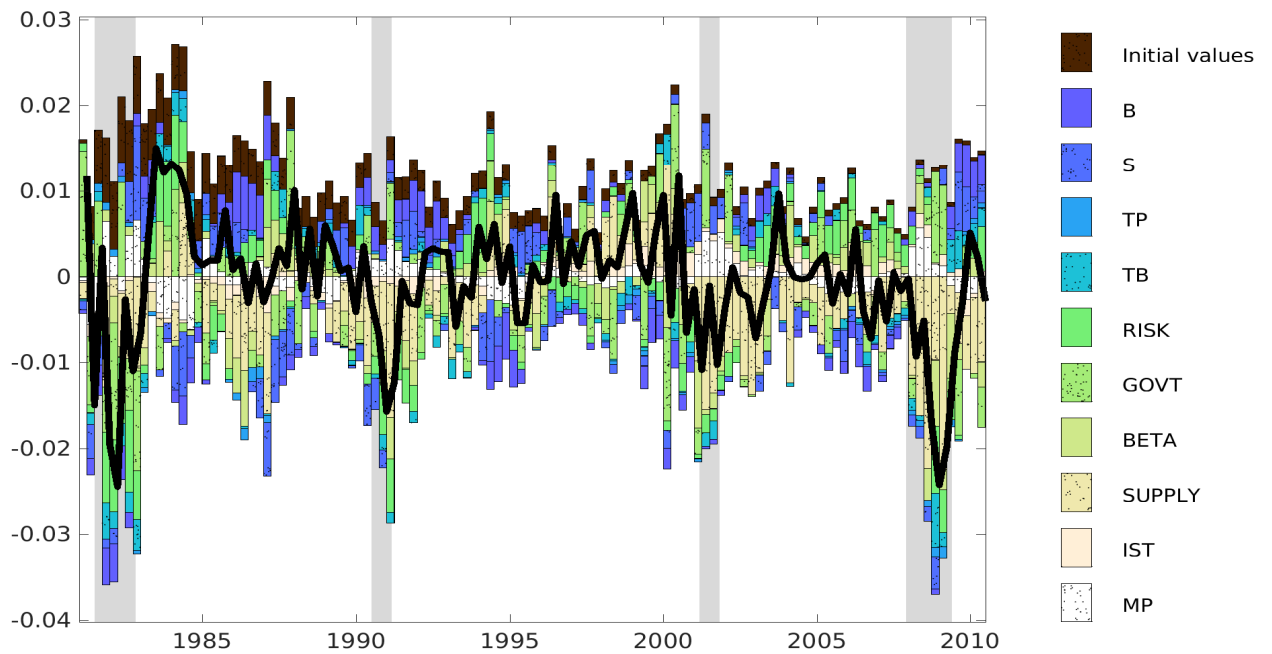
NOTE: This figure shows the time series representation of the evolution of quarterly GDP growth in the data and decomposes each quarterly realization into the positive (above the x axis) and negative (below the x axis) contributions of the fundamental shocks in the model, listed in the legend on the right-hand side of the graph.
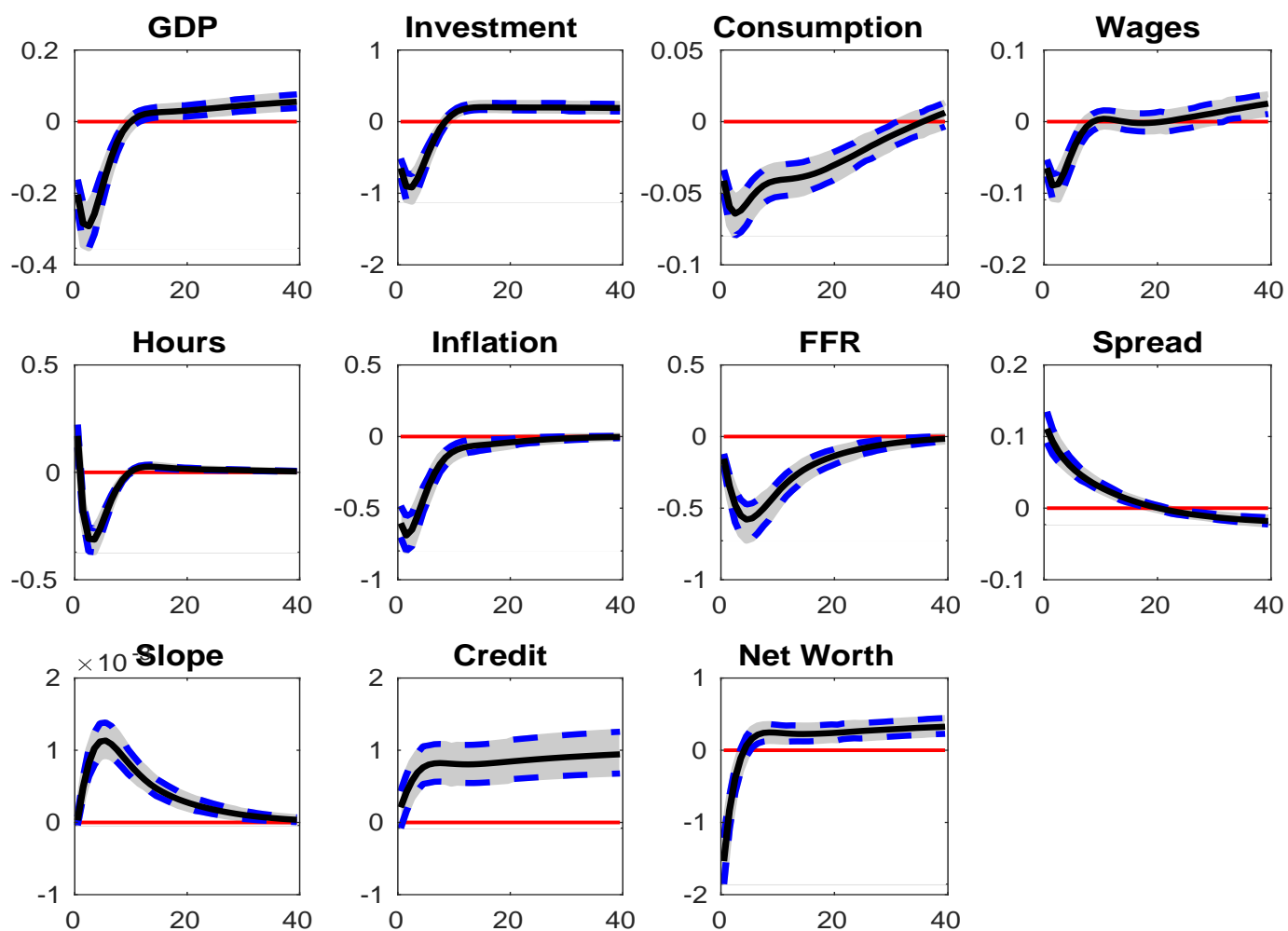
DATA SOURCE: Authors' calculations.

Figure 3: Impulse Response to a Positive Liquidity Preference Shock (η^{TB})

NOTE: This figure shows the impulse response function to a one standard deviation positive liquidity preference shock (η^{TB}).
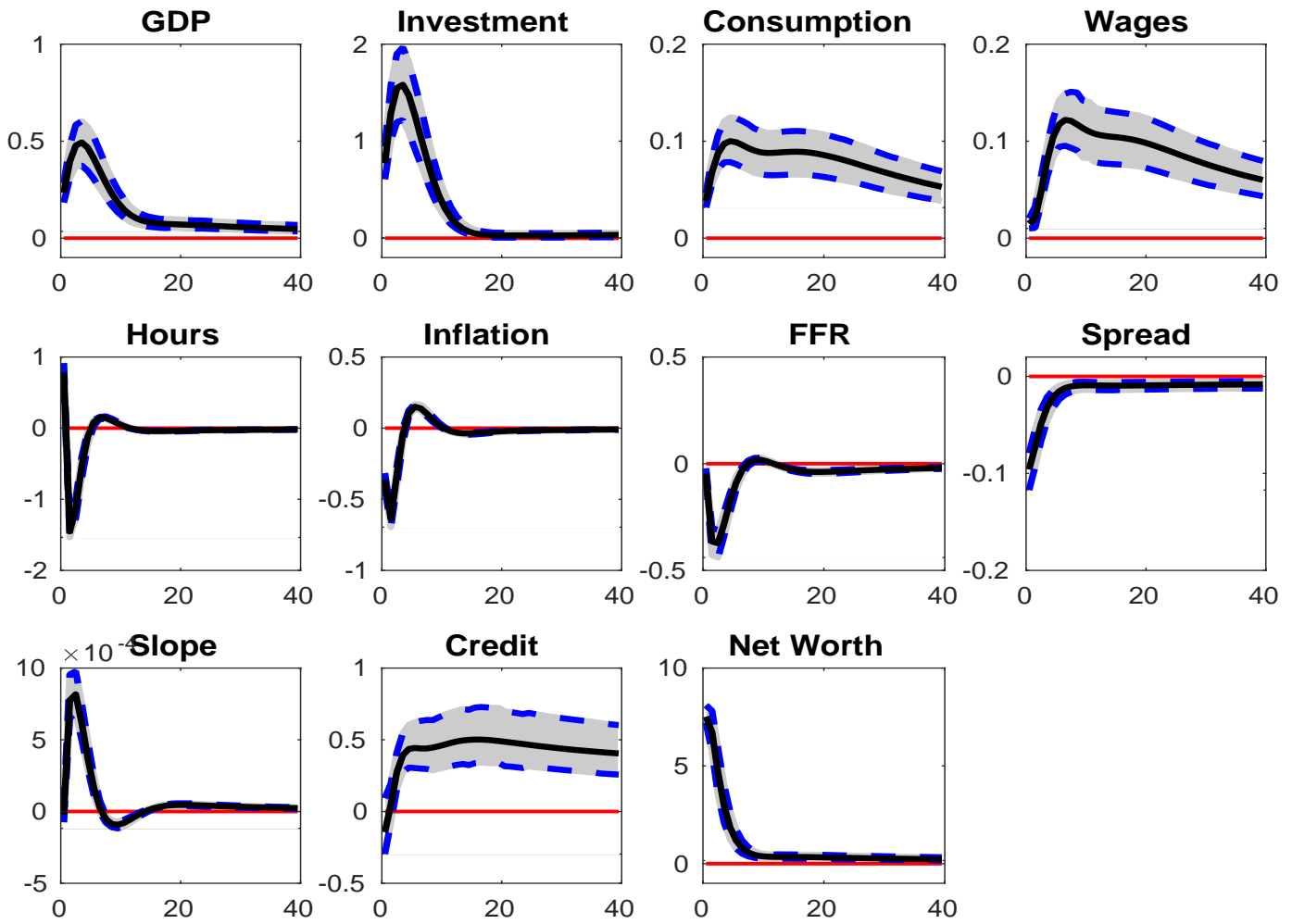DATA SOURCE: Authors' calculations.

Figure 4: Impulse Response to a Positive Equity Preference Shock (η^S)

NOTE: This figure shows the impulse response function to a one standard deviation positive equity preference shock (η^S).
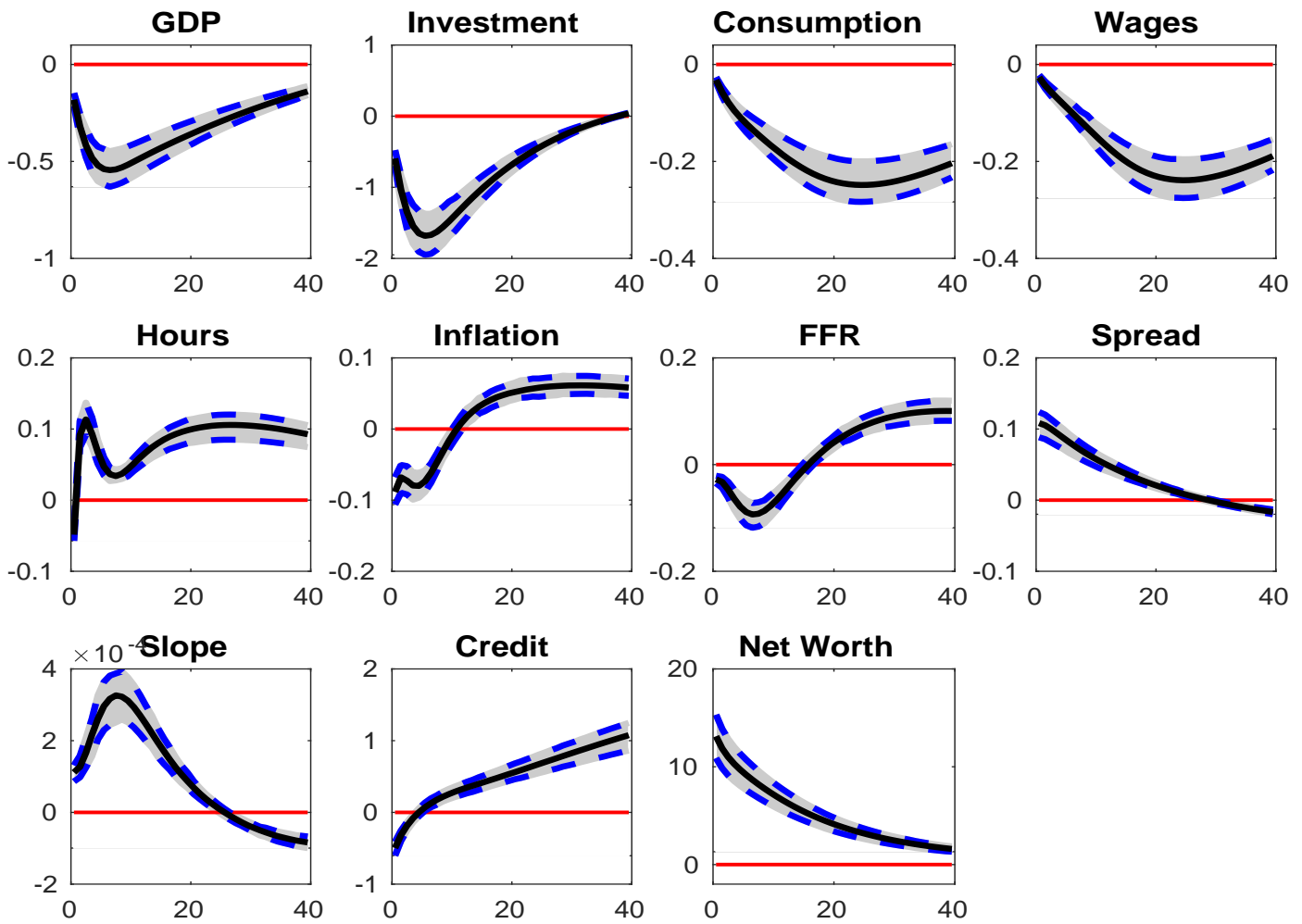DATA SOURCE: Authors' calculations.

Figure 5: Impulse Response to a Positive Risk Shock (σ_z)

NOTE: This figure shows the impulse response function to a one standard deviation positive risk shock (σ_z).
DATA SOURCE: Authors' calculations.