

# Simple Analytics of Expectations-Driven Liquidity Traps\*

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### Abstract

We analytically characterize a sunspot equilibrium in a model where exogenous changes in agents' confidence give rise to occasional liquidity trap episodes. The key elements of the model are price rigidities, a discretionary policymaker and a lower bound on nominal interest rates. Episodes of low confidence—featuring a binding lower bound and subdued economy activity—have to be sufficiently rare and persistent for the sunspot equilibrium to exist. During episodes of high confidence—featuring a policy rate above the lower bound—the possibility of a decline in confidence gives rise to a policy trade-off between output and inflation stabilization. The setup is used to revisit the desirability of three institutional configurations known to improve welfare in models of fundamental-driven liquidity trap events: inflation conservatism, a positive inflation target and fiscal activism. Expectations-driven liquidity traps render policy design more complicated. We show that the welfare-maximizing weight on inflation relative to output stabilization in the policymaker's objective function can be smaller or larger than society's weight. Likewise, the optimal inflation target can be negative or positive. Allowing the policymaker to use government spending as an additional policy tool is welfare-reducing, but the appointment of a sufficiently fiscally-activist policymaker eliminates the sunspot equilibrium.

*Keywords:* Effective Lower Bound, Sunspot Equilibria, Monetary Policy, Fiscal Policy, Discretion, Policy Delegation

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# 1 Introduction

In recent years, monetary policy in many industrialized countries has been constrained by a binding lower bound on nominal interest rates. While central banks during these prolonged episodes invoked other policy instruments such as asset purchases to compensate for the unavailability of further interest rate cuts, inflation rates typically remained stubbornly below target. In light of this rather disappointing state of affairs, some central banks have recently decided to review not only their toolkit but also their policy frameworks (see e.g. Federal Reserve Board of Governors, 2018; Wilkins, 2018; Clarida, 2019). In so doing, central banks can resort to a by now rich literature on the design of monetary policy in the presence of a lower bound on nominal interest rates. Most of these studies base their analysis on models where liquidity trap events are caused by a change in the fundamentals of the economy. Liquidity trap episodes can, however, also arise as a consequence of a decline in agents' confidence that is unrelated to fundamentals, as was first shown by Benhabib et al. (2001). These so-called expectations-driven liquidity traps have received much less attention in the literature.

The aim of this paper is to enhance our understanding of expectations-driven liquidity traps and their implications for the design of monetary and fiscal policy frameworks. In particular, we revisit the desirability of three institutional configurations that have been shown to improve allocations and welfare in models of fundamental-driven liquidity trap events: inflation conservatism, a positive inflation target and fiscal activism. The analysis is based on a tractable macroeconomic model that can be solved in closed form.<sup>1</sup> The main model ingredients are sticky prices, a two-state confidence or 'sunspot' shock and a discretionary policymaker who sets the one-period nominal interest rate subject to a lower bound, and, in the fiscal policy extension, the level of government consumption. The policymaker's objective function is designed by society and taken as given by the policymaker. Throughout the analysis, results obtained for the benchmark setup with the sunspot shock are compared to those for an alternative version of the model where the sunspot shock is replaced with a two-state fundamental shock that affects the economy's natural real rate of interest.

We find that a sunspot equilibrium where the lower bound constraint is binding when agents' confidence is low and slack when confidence is high exists if and only if the low-confidence state is sufficiently persistent—liquidity trap episodes are long-lived—and the probability to move from the high to the low-confidence state is sufficiently small—liquidity trap episodes are rare events. This property of the sunspot equilibrium seems to be consistent with the highly persistent liquidity trap episodes recently observed in Japan, the United States and the euro area, with policy rates staying at their lower bounds for several years. The version of the model with the fundamental shock, instead, is unable to replicate such long-lived lower bound episodes.

In the sunspot equilibrium, when confidence is low and the lower bound is binding, inflation and the output gap are both strictly negative. Intuitively, the central bank responds to agents pessimistic expectations by lowering the policy rate to its lower bound at which point these pessimistic

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<sup>1</sup>More specifically, the model can be solved in closed form under the three considered institutional configurations. We deliberately leave out policy regimes that prevent a closed-form solution.

expectations become self-fulfilling. However, even when confidence is high and the policy rate is above its lower bound, the risk of a future decline in confidence induces forward-looking households and firms to reduce desired consumption and prices. These private sector incentives give rise to a policy trade-off between inflation and output gap stabilization. In equilibrium, inflation is negative and the output gap is strictly positive in the high-confidence state.

Can allocations and welfare be improved by designing a central bank objective function that differs from society’s objective function? We first consider the desirability of inflation conservatism, that is, the assignment of a central bank objective function that puts more weight on inflation stabilization relative to output gap stabilization than society as a whole does. Inflation conservatism has been shown to be welfare-improving in the model with fundamental-driven liquidity traps by mitigating the inflation shortfall both at and away from the lower bound (Nakata and Schmidt, 2018). In our model with expectations-driven liquidity traps, inflation conservatism also raises inflation when the lower bound constraint is slack since, all else equal, a central bank that is more concerned with inflation stabilization is more willing to tolerate a positive output gap to achieve its inflation objective. But at the same time inflation conservatism exacerbates the decline in inflation and the output gap when confidence is low and the nominal interest rate bound is binding. These opposing effects of inflation conservatism on low-state and high-state stabilization outcomes render the design of the central bank objective function in the sunspot equilibrium more complicated than in models of fundamental-driven liquidity trap episodes. In particular, the welfare-maximizing relative weight on inflation stabilization in the central bank objective function can be lower or higher than society’s weight, depending on the model’s structural parameters and the transition probabilities of the sunspot shock. Strict inflation conservatism, that is, a central bank objective function that is only concerned with inflation stabilization, is, however, never optimal in the sunspot equilibrium.<sup>2</sup>

Next, we consider the desirability of assigning a non-zero inflation target to the central bank. In models of fundamental-driven liquidity trap episodes, the optimal inflation target is strictly positive (Coibion et al., 2012; Nakata and Schmidt, 2018). Instead, in the sunspot equilibrium, the sign of the optimal inflation target is ambiguous, for reasons similar to those mentioned above in the context of inflation conservatism. Indeed, in our model, any allocation that is attainable under an inflation-conservative discretionary policymaker is also attainable by means of an appropriately designed central bank inflation target. Quantitatively, we find that if the optimal inflation target is strictly positive, it stays close to zero. In particular, a strictly positive inflation rate in the high-confidence state—which is attainable in case of a sufficiently high inflation target—is never a feature of the sunspot equilibrium under the optimal inflation target.

The final part of the paper extends the analysis to fiscal policy. In models of fundamental-driven liquidity trap events, allowing the discretionary policymaker to use government spending as an additional stabilization tool is welfare-improving (Eggertsson, 2006; Schmidt, 2013; Nakata, 2016; Werning, 2011). Moreover, because of a time inconsistency problem, welfare can be increased

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<sup>2</sup>Strict inflation conservatism is optimal in our model with the two-state natural real rate shock.

further by assigning an objective function to the monetary-fiscal policymaker that puts less relative weight on the stabilization of government expenditures than society's objective function does (Schmidt, 2017). Such a fiscally-activist policymaker adjusts government spending more elastically in response to changes in economic conditions. In the sunspot equilibrium of our model, instead, providing the policymaker with the fiscal stabilization tool is welfare-reducing. In the sunspot equilibrium, as in the fundamental equilibrium, the discretionary policymaker raises government spending when the economy moves from the high state to the low state and keeps government spending at the elevated level for as long as the economy stays in the low state. In so doing, the discretionary policymaker fails to internalize the detrimental effects that the systematic fiscal policy response has on previous period's private sector expectations in the sunspot equilibrium. In equilibrium, the increase in government spending reduces inflation and the output gap in the low-confidence state and exacerbates the stabilization trade-off in the high-confidence state.

The implications for the design of fiscal policy then seem straightforward. Conditional on the economy being in the sunspot equilibrium, it is optimal for society to put infinitely large weight on government spending stabilization in the monetary-fiscal policymaker's objective function, thereby removing any incentive for the policymaker to use government spending as a stabilization tool. However, we also show that the appointment of a sufficiently fiscally-activist policymaker—i.e. one who puts a sufficiently small relative weight on government spending stabilization—eliminates the sunspot equilibrium in the model with the sunspot shock. In this case, the prevailing equilibrium is one where the sunspot shock has no effect on agents decisions and the economy is perfectly stabilized.

Our paper is related to several strands of the literature on the lower bound on nominal interest rates. Studies on optimal monetary policy in models of fundamental-driven liquidity traps include Eggertsson and Woodford (2003), Jung et al. (2005), Adam and Billi (2006, 2007), and Nakov (2008). Optimal fiscal policy is analyzed by e.g. Eggertsson and Woodford (2006), Eggertsson (2006), Werning (2011) Schmidt (2013), Nakata (2016) and Bouakez et al. (2016), among others. Billi (2017), Schmidt (2017), Nakata and Schmidt (2018, 2019) and Mertens and Williams (2019) study alternative monetary and fiscal policy delegation schemes aimed at improving the equilibrium under discretionary policy, some of which we revisit in this paper.

This paper builds on the seminal work by Benhabib et al. (2001) who showed that accounting for the zero lower bound gives rise to two deterministic steady states in a model where monetary policy is governed by an interest-rate feedback rule. Besides the conventional steady state where inflation is at target and the policy rate is strictly positive, there exists a second steady state where the lower bound constraint is binding and inflation is below target. Furthermore, there exist infinitely many perfect-foresight equilibria that converge to the second unintended steady state. Other papers that consider permanent expectations-driven liquidity trap equilibria include Hursey and Wolman (2010), Armenter (2018), and Nakata and Schmidt (2018). Armenter (2018) and Nakata and Schmidt (2018) show that expectations-driven liquidity traps can also arise under optimal discretionary policy, and Armenter (2018) furthermore shows that price-level and nominal-

GDP targeting regimes also fail to rule out expectations-driven traps.

Temporary expectations-driven liquidity trap episodes are studied in Mertens and Ravn (2014), Schmitt-Grohé and Uribe (2017), Aruoba et al. (2018), Jarociński and Maćkowiak (2018), Bilbiie (2018), and Coyle and Nakata (2018). Mertens and Ravn (2014) show that the size of government spending multipliers at the lower bound critically depends on whether the economy is in a fundamental-driven or an expectations-driven liquidity trap. They use a two-state Markov shock structure similar to the one we use in this paper but assume that the high-confidence state is an absorbing state. Schmitt-Grohé and Uribe (2017) show that a model with downward nominal wage rigidities and a sunspot shock can mimic the economic dynamics of a recessionary lower bound episode that is followed by a jobless recovery. Aruoba et al. (2018) conduct a model-based empirical assessment to shed light on the type of liquidity trap events recently experienced by the US economy and the Japanese economy and conclude that Japan transitioned in the late 1990s to an expectations-driven liquidity trap state. Jarociński and Maćkowiak (2018) use a sticky price model with a sunspot shock to conduct counterfactual simulations of the euro area economic downturn in 2008-2015. Coyle and Nakata (2018) consider a model where the economy can either be in a fundamental-driven or an expectations-driven liquidity trap and show that the optimal inflation target is lower than in the case where lower bound events arise solely as a result of fundamental shocks.

Our paper is closely related to Bilbiie (2018). Bilbiie uses a tractable New Keynesian model to analytically characterize key properties of expectations-driven liquidity traps and contrasts them with fundamental-driven traps. However, several differences underline the complementary nature of the two studies. First, in Bilbiie’s model, expectations-driven liquidity traps are non-recurring events, i.e. once the economy is out of the trap it will never hit the nominal interest rate bound again. In our model, the possibility that confidence might decline (again) in the future affects agents’ behavior even when the economy is in the high-confidence state and the lower bound constraint is slack. The two studies also differ in their specification of monetary policy. In Bilbiie’s model, monetary policy is governed by an interest-rate feedback rule whereas we consider a discretionary central bank that optimizes an assigned objective function. Most importantly, Bilbiie (2018) analyzes the effects of exogenous policy interventions such as an exogenous change in the policy rate path or in government spending whereas we focus on the design of the systematic components of monetary and fiscal policy that give rise to endogenous changes in the policy instruments.

Finally, our paper also makes contact with studies on how to avoid expectations-driven liquidity traps, including Benhabib et al. (2002), Sugo and Ueda (2008), Schmitt-Grohé and Uribe (2014), Schmidt (2016) and Roulleau-Pasdeloup (2019).

The remainder of the paper is organized as follows. Section 2 presents the model, describing the private sector behavioral constraints, monetary policy and the shock structure, and defines the equilibria of interest. Section 3 presents results on equilibrium existence, stabilization outcomes and comparative statics. Section 4 assesses the desirability of inflation conservatism in the sunspot equilibrium and Section 5 the desirability of a non-zero central bank inflation target. Section 6

extends the model by introducing government consumption and explores the implications of fiscal activism for equilibrium existence, allocations, and welfare. Section 7 concludes.

## 2 Model

We use a tractable New Keynesian infinite-horizon model that can be solved in closed form. The economy is inhabited by identical households who consume and work, goods-producing firms that act under monopolistic competition and are subject to price rigidities, and a government. More detailed descriptions of the model can be found in Woodford (2003) and Galí (2015). Time is discrete and indexed by  $t$ .

### 2.1 Private sector behavior and welfare

Aggregate private sector behavior is described by a Phillips curve and a consumption Euler equation

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t \pi_{t+1} \quad (1)$$

$$y_t = \mathbb{E}_t y_{t+1} - \sigma (i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \quad (2)$$

The private sector behavioral constraints have been (semi) log-linearized around the intended zero-inflation steady state.  $\pi_t$  is the inflation rate between periods  $t - 1$  and  $t$ ,  $y_t$  denotes the output gap,  $i_t$  is the *level* of the riskless nominal interest rate between periods  $t$  and  $t + 1$ ,  $r_t^n$  is the exogenous natural real rate of interest, and  $\mathbb{E}_t$  is the rational expectations operator conditional on information available in period  $t$ . The parameters are defined as follows:  $\beta \in (0, 1)$  is the households' subjective discount factor,  $\sigma > 0$  is the inverse of the elasticity of the marginal utility of consumption with respect to output, and  $\kappa$  represents the slope of the Phillips curve.<sup>3</sup>

Households' welfare at time  $t$  is given by the expected discounted sum of current and future utility flows. A second-order approximation to household preferences leads to

$$V_t = -\frac{1}{2} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j [\pi_{t+j}^2 + \bar{\lambda} y_{t+j}^2], \quad (3)$$

where  $\bar{\lambda} = \kappa/\theta$ .<sup>4</sup>

### 2.2 Monetary policy

At the beginning of time, society delegates monetary policy to a central bank. The central bank does not have a commitment technology, that is, it acts under discretion. The monetary policy

<sup>3</sup> $\kappa$  is itself a function of several structural parameters of the economy:  $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\eta\theta)}(\sigma^{-1} + \eta)$ , where  $\alpha \in (0, 1)$  denotes the share of firms that cannot reoptimize their price in a given period,  $\eta > 0$  is the inverse of the labor-supply elasticity, and  $\theta > 1$  denotes the price elasticity of demand for differentiated goods.

<sup>4</sup>See Woodford (2003). We assume that the steady state distortions arising from monopolistic competition are offset by a wage subsidy.

objective is given by

$$V_t^{CB} = -\frac{1}{2} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[ (\pi_{t+j} - \pi^*)^2 + \lambda y_{t+j}^2 \right], \quad (4)$$

where  $\lambda \geq 0$  and  $\pi^*$  are policy parameters to be set by society when designing the central bank's objective function. For  $\lambda = \bar{\lambda}$  and  $\pi^* = 0$ , the central bank's objective function coincides with society's objective function (3).

The policy problem of a generic central bank is as follows. Each period  $t$ , she chooses the inflation rate, the output gap, and the nominal interest rate to maximize its objective function (4) subject to the behavioral constraints of the private sector (1)–(2), and the lower bound constraint  $i_t \geq 0$ , with the policy functions at time  $t + 1$  taken as given.

In this case, interest rate policy is governed by the following targeting rule

$$[\kappa(\pi_t - \pi^*) + \lambda y_t] i_t = 0, \quad (5)$$

where  $\kappa(\pi_t - \pi^*) + \lambda y_t = 0$  whenever  $i_t > 0$  and  $\kappa(\pi_t - \pi^*) + \lambda y_t < 0$  when the lower bound constraint is binding,  $i_t = 0$ . In words, each period the central bank aims to stabilize a weighted sum of current period's inflation rate (in deviation from target) and the output gap.

### 2.3 Benchmark setup: Sunspot shock

For the benchmark setup, we assume that there is no uncertainty regarding the economy's fundamentals. Specifically,  $r_t^n = r^n = 1/\beta - 1$  for all  $t$ . However, agents' expectations may be affected by a non-fundamental sunspot or 'confidence' shock  $\xi_t$ . The sunspot shock follows a two-state Markov process,  $\xi_t \in (\xi_L, \xi_H)$ . We refer to state  $\xi_L$  as the low-confidence state and to state  $\xi_H$  as the high-confidence state. The transition probabilities are given by

$$\text{Prob}(\xi_{t+1} = \xi_H | \xi_t = \xi_H) = p_H \quad (6)$$

$$\text{Prob}(\xi_{t+1} = \xi_L | \xi_t = \xi_L) = p_L \quad (7)$$

In words,  $p_H \in (0, 1]$  is the probability of being in the high-confidence state in period  $t+1$  conditional on being in the high-confidence state in period  $t$ , and can be interpreted as the persistence of high confidence. Note that while we allow the high-confidence state to be an absorbing state we do not restrict our analysis to this special case.  $p_L \in (0, 1)$  is the probability of being in the low-confidence state in period  $t + 1$  when the economy is in the low-confidence state in period  $t$ , and can be interpreted as the persistence of low confidence.<sup>5</sup>

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<sup>5</sup>Mertens and Ravn (2014), Schmidt (2016), Aruoba et al. (2018) and Bilbie (2018) also consider a sunspot shock that follows a two-state Markov process. However, Mertens and Ravn (2014), Schmidt (2016) and Bilbie (2018) assume that the high-confidence state is an absorbing state, that is,  $p_H = 1$ . Aruoba et al. (2018) allow for recurring declines in confidence and assume that conditional on being in the high-confidence state agents attach a 1% probability to the possibility of ending up in the low-confidence state in the next period. Formally, in the context of our setup they impose  $p_H = 0.99$ .

Let  $x_s$ ,  $s \in \{L, H\}$  be the equilibrium value of some variable  $x$  in state  $\xi_s$ . *Sunspots matter* if there is an equilibrium in which  $\{\pi_L, y_L, i_L, V_L\} \neq \{\pi_H, y_H, i_H, V_H\}$ . We are interested in a sunspot equilibrium where the economy is subject to occasional and temporary liquidity trap episodes. We associate the occurrence of these liquidity trap events with the low-confidence state.

**Definition 1** *The sunspot equilibrium in the model with the sunspot shock is given by a vector  $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$  that solves the following system of linear equations*

$$y_H = [p_H y_H + (1 - p_H) y_L] + \sigma [p_H \pi_H + (1 - p_H) \pi_L - i_H + r^n] \quad (8)$$

$$\pi_H = \kappa y_H + \beta [p_H \pi_H + (1 - p_H) \pi_L] \quad (9)$$

$$0 = \kappa(\pi_H - \pi^*) + \lambda y_H \quad (10)$$

$$y_L = [(1 - p_L) y_H + p_L y_L] + \sigma [(1 - p_L) \pi_H + p_L \pi_L - i_L + r^n] \quad (11)$$

$$\pi_L = \kappa y_L + \beta [(1 - p_L) \pi_H + p_L \pi_L] \quad (12)$$

$$i_L = 0, \quad (13)$$

and satisfies the following two inequality constraints

$$i_H > 0 \quad (14)$$

$$\kappa(\pi_H - \pi^*) + \lambda y_H < 0. \quad (15)$$

## 2.4 Alternative setup: Fundamental shock

Throughout the paper, we contrast results for the benchmark model—an economy that is subject to a sunspot shock—with those for an economy that is subject to a fundamental shock instead of a sunspot shock but is otherwise identical to the benchmark economy. In this alternative model, the natural real rate is assumed to be stochastic.

To keep the model setup as close as possible to the one with the sunspot shock,  $r_t^n$  is assumed to follow a two-state Markov process. In the high-fundamental state, the natural real rate is strictly positive  $r_H^n = r^n > 0$ , and in the low-fundamental state it is strictly negative  $r_L^n < 0$ . The transition probabilities for the natural real rate shock are given by

$$\text{Prob}(r_{t+1}^n = r_H^n | r_t^n = r_H^n) = p_H^f \quad (16)$$

$$\text{Prob}(r_{t+1}^n = r_L^n | r_t^n = r_L^n) = p_L^f, \quad (17)$$

and are distinguished from the transition probabilities of the sunspot shock via the superscript  $f$ . The fundamental equilibrium in the model with the natural real rate shock is defined as follows.

**Definition 2** *The fundamental equilibrium in the model with the fundamental shock is given*



by a vector  $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$  that solves

$$y_H = \left[ p_H^f y_H + (1 - p_H^f) y_L \right] + \sigma \left[ p_H^f \pi_H + (1 - p_H^f) \pi_L - i_H + r^n \right] \quad (18)$$

$$\pi_H = \kappa y_H + \beta \left[ p_H^f \pi_H + (1 - p_H^f) \pi_L \right] \quad (19)$$

$$y_L = \left[ (1 - p_L^f) y_H + p_L^f y_L \right] + \sigma \left[ (1 - p_L^f) \pi_H + p_L^f \pi_L - i_L + r_L^n \right] \quad (20)$$

$$\pi_L = \kappa y_L + \beta \left[ (1 - p_L^f) \pi_H + p_L^f \pi_L \right] \quad (21)$$

as well as (10) and (13), and satisfies inequality constraints (14) and (15).

This fundamental equilibrium has been analyzed in detail in Nakata and Schmidt (2018).<sup>6</sup> To keep the exposition parsimonious, we will refer to this companion paper for the proofs related to the fundamental equilibrium whenever applicable.

### 3 Basic properties of the sunspot equilibrium

This section presents some basic properties of the sunspot equilibrium and contrasts them with those of the fundamental equilibrium. In particular, we analyze the conditions for equilibrium existence, allocations/prices, and how they are affected by some of the key non-policy parameters.

#### 3.1 Equilibrium existence

The following proposition establishes a necessary and sufficient condition for existence of the sunspot equilibrium.

**Proposition 1** *The sunspot equilibrium exists if and only if*

$$p_L - (1 - p_H) - \frac{1 - p_L + 1 - p_H}{\kappa \sigma} (1 - \beta p_L + \beta(1 - p_H)) > 0, \quad (22)$$

and

$$\pi^* > -\frac{\kappa^2 + \lambda(1 - \beta)}{\kappa^2} r^n. \quad (23)$$

**Proof:** See Appendix A.

Three observations are in order. First, for the sunspot equilibrium to exist, the two confidence states have to be sufficiently persistent. Second, prices have to be sufficiently flexible, i.e.  $\kappa$  has to be sufficiently large. Third, the central bank's inflation target must be higher than some strictly

<sup>6</sup>Nakata and Schmidt (2018) show that there exists a second equilibrium where the economy is stuck in a permanent liquidity trap. While not his focus, Nakata (2018) provides a numerical analysis of this equilibrium in the Appendix. Here, we do not consider this equilibrium.

negative lower bound. Conditional on the inflation target not being too low, equilibrium existence does not depend on the policy parameters  $\lambda$  and  $\pi^*$ .

These conditions for existence of the sunspot equilibrium differ quite a bit from the conditions for existence of the fundamental equilibrium. In particular, for the fundamental equilibrium to exist, the low-fundamental state must not be too persistent (see Nakata and Schmidt, 2018). Hence, the fundamental equilibrium stipulates an upper bound on the average duration of liquidity traps whereas the sunspot equilibrium stipulates a lower bound.

Figure 1 provides a numerical example, plotting the region of existence for the sunspot equilibrium in the  $(p_H, p_L)$  space (black area), and the region of existence for the fundamental equilibrium in the  $(p_H^f, p_L^f)$  space (gray area).<sup>7</sup> One period corresponds to one quarter. The values used for the model parameters are listed in Table 1 and follow those in Eggertsson and Woodford (2003).<sup>8</sup>

Table 1: Parameter values for the numerical example

Parameter	Value	Economic interpretation
$\beta$	0.9975	Subjective discount factor
$\sigma$	0.5	Intertemporal elasticity of substitution in consumption
$\eta$	0.47	Inverse labor supply elasticity
$\theta$	10	Price elasticity of demand
$\alpha$	0.8106	Share of firms per period keeping prices unchanged
$\lambda$	$\bar{\lambda}$	Policy parameter: Relative weight on output term
$\pi^*$	0	Policy parameter: Inflation target
$r_L^n$	-0.005	Low-state natural real rate in model with fundamental shock

Note: This parameterization implies  $r^n = 0.0025$ ,  $\kappa = 0.0194$ ,  $\bar{\lambda} = 0.0019$ .

### 3.2 Allocations and prices

The allocations and prices in the sunspot equilibrium can be solved for in closed form. For now, we assume that the central bank has the same objective function as society as a whole. The signs of inflation and the output gap in the two states of confidence are then unambiguously determined.

**Proposition 2** *Suppose  $\lambda = \bar{\lambda}$  and  $\pi^* = 0$ . In the sunspot equilibrium,  $\pi_L < 0$ ,  $y_L < 0$ ,  $\pi_H \leq 0$ ,  $y_H \geq 0$ . When  $p_H < 1$ , then  $\pi_H < 0$ ,  $y_H > 0$ .*

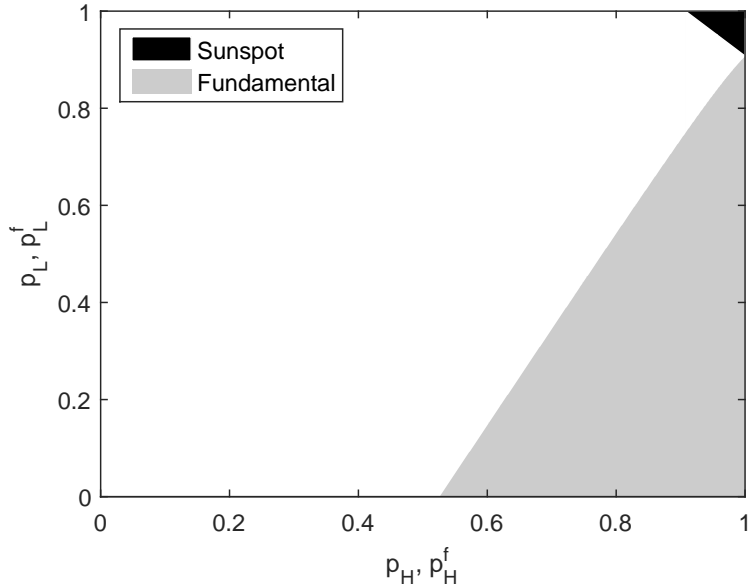
**Proof:** See Appendix A

When confidence is low, agents expect persistently low future income, and therefore increase desired saving at the expense of lower desired consumption. Due to the presence of price rigidities, prices do not fully adjust immediately and output falls. The central bank lowers the policy rate to equate desired saving to zero, but if agents are sufficiently pessimistic, the lower bound on the

<sup>7</sup>In case of the fundamental equilibrium, the condition for equilibrium existence depends on the value of the natural real rate in the low-fundamental state,  $r_L^n$ . The region of existence is shrinking in the absolute value of  $r_L^n$ .

<sup>8</sup>We choose a higher value for the subjective discount factor  $\beta$  than Eggertsson and Woodford (2003), consistent with recent empirical evidence indicating a decline in the long-run natural real rate of interest.

Figure 1: Existence regions for sunspot equilibrium and fundamental equilibrium



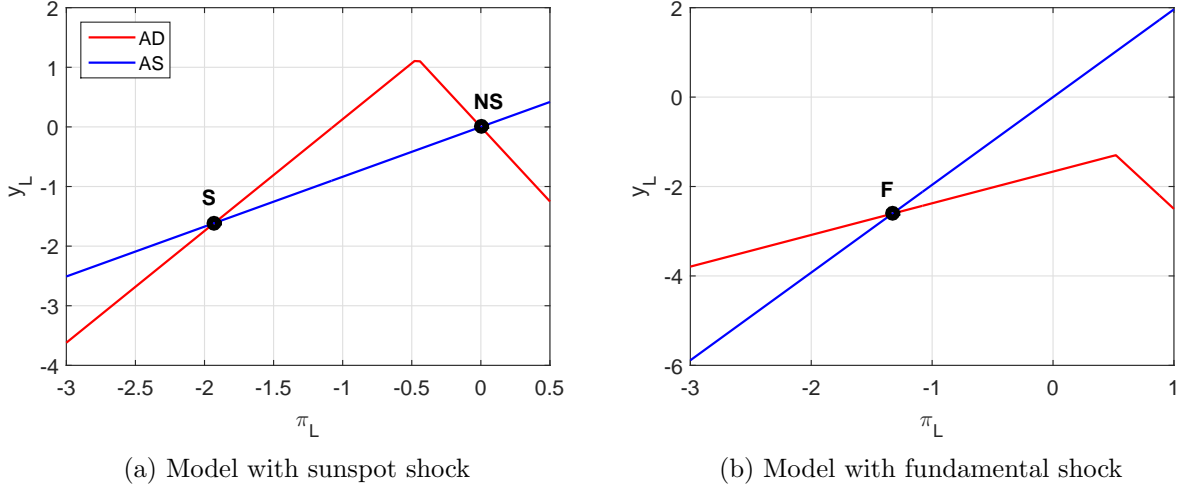
policy rate becomes binding. At the lower bound, to equate desired saving to zero, output has to fall, validating agents' pessimistic expectations. The lower bound is binding, and inflation and the output gap both settle below target.

When confidence is high, the policy rate is strictly positive but if  $p_H < 1$  the risk of a future decline in confidence creates a monetary policy trade-off between inflation and output gap stabilization. Specifically, the possibility that confidence might fall in the future while the price set by a firm reoptimizing today is still in place provides an incentive for forward-looking firms to set a lower price than they would in the absence of any risk of a future drop in confidence. To counteract these deflationary forces, the central bank allows for a positive output gap, that is, it sets the policy rate in the high-confidence state such that the ex-ante real interest rate is below the constant natural real rate. In equilibrium, the high-confidence output gap is thus positive and inflation is below target.

The signs of output and inflation in the fundamental equilibrium are identical to those in the sunspot equilibrium. Output and inflation are negative in the low-fundamental state, and output (inflation) is positive (negative) in the high-fundamental state (see Nakata and Schmidt, 2018). However, in the fundamental equilibrium, it is the temporarily negative natural real rate of interest in the low-fundamental state that leads to the decline in output and inflation in the low state.

In order to better understand low-state outcomes in the model with the sunspot shock and in the model with the fundamental shock, and how the models' parameters affect them, we make use of aggregate demand (AD) and aggregate supply (AS) curves. The low-state AD curve is the set of pairs  $\{\pi_L, y_L\}$  consistent with Euler equation (2) where the policy rate set in line with target criterion (5), and the low-state AS curve is the set of pairs  $\{\pi_L, y_L\}$  consistent with Phillips curve

Figure 2: Aggregate demand and aggregate supply in the low state



Note: In the left panel,  $S$  marks the sunspot equilibrium and  $NS$  the no-sunspot equilibrium. In the right panel,  $F$  marks the fundamental equilibrium. Inflation is expressed in annualized terms.

(1). Specifically, for the sunspot-shock model the two curves are given by

$$\mathbf{AD-sunspot:} \quad y_L = \min \left[ \left( y_H + \sigma \pi_H + \frac{\sigma}{1 - p_L} r^n \right) + \frac{\sigma p_L}{1 - p_L} \pi_L, \frac{\kappa}{\lambda} (\pi^* - \pi_L) \right] \quad (24)$$

$$\mathbf{AS-sunspot:} \quad y_L = -\frac{\beta(1 - p_L)}{\kappa} \pi_H + \frac{1 - \beta p_L}{\kappa} \pi_L, \quad (25)$$

where in each equation we distinguish between terms that are multiplied by  $\pi_L$ —the slope coefficient—and the other terms—the intercept. For the fundamental-shock model, the two curves are given by

$$\mathbf{AD-fundamental:} \quad y_L = \min \left[ \left( y_H + \sigma \pi_H + \frac{\sigma}{1 - p_L^f} r^n \right) + \frac{\sigma p_L^f}{1 - p_L^f} \pi_L, \frac{\kappa}{\lambda} (\pi^* - \pi_L) \right] \quad (26)$$

$$\mathbf{AS-fundamental:} \quad y_L = -\frac{\beta(1 - p_L^f)}{\kappa} \pi_H + \frac{1 - \beta p_L^f}{\kappa} \pi_L. \quad (27)$$

Figure 2 plots these AD-AS curves for the model with the sunspot shock (left panel) and for the model with the fundamental shock (right panel), assuming that the high state in both models is an absorbing state. The parameters capturing the persistence of the low state are calibrated such that the sunspot equilibrium exists in the model with the sunspot shock and the fundamental equilibrium exists in the model with the fundamental shock.<sup>9</sup> For  $\pi_H, y_H = 0$ , the intercept terms in the AS curves are zero, whereas the intercept terms in the AD curves are positive (sunspot-shock model) and negative (fundamental-shock model), respectively.

<sup>9</sup>Specifically, we set  $p_L = 0.9375$  in the model with the sunspot shock, implying an average duration of lower bound episodes of 4 years. In the model with the fundamental shock, we set  $p_L^f = 0.85$ , implying an average duration of lower bound episodes of 1 1/2 years. The other parameter values are summarized in Table 1.

The low-state AD-AS curves in the two models have several common features. First, due to the lower bound constraint, the AD curve has a kink. To the left of the kink, the lower bound constraint is binding and to the right of the kink the lower bound constraint is slack. Second, the AD curve is upward-sloping to the left of the kink, and downward-sloping to the right of the kink. Third, the AS curve is monotonically upward-sloping and goes through the origin.

In the model with the sunspot shock, the AD curve is steeper than the AS curve. This is a necessary—and in case of  $\pi^* = 0$  sufficient—condition for existence of the sunspot equilibrium.<sup>10</sup> Otherwise there would be no intersection of the AD and AS curves to the left of the kink. Consistent with Proposition 2, when confidence is low, output and inflation are strictly negative in the sunspot equilibrium as reflected by the intersection point  $S$ . Besides the sunspot equilibrium, there is a second equilibrium—represented by intersection point  $NS$ —where the lower bound constraint on the policy rate is not binding and low-state output and inflation are at target. In this ‘no-sunspot’ equilibrium, the sunspot shock does not affect agents expectations and decisions, i.e. *sunspots do not matter*.

In the model with the fundamental shock, the AD curve is flatter than the AS curve, which is a necessary condition for the fundamental equilibrium to exist. In the fundamental equilibrium, marked by intersection point  $F$  in the right panel, low-state output and inflation are negative, again in line with analytical results.

### 3.3 Comparative statics

This subsection explains how allocations and prices in the sunspot equilibrium depend on the persistence of the two confidence states and on the degree of price flexibility.

#### 3.3.1 Persistence of the low state

We begin with the effects of a marginal increase in the persistence of the low state.

**Proposition 3** *Suppose  $\lambda = \bar{\lambda}$  and  $\pi^* = 0$ . In the sunspot equilibrium,  $\frac{\partial \pi_L}{\partial p_L} > 0$ ,  $\frac{\partial y_L}{\partial p_L} > 0$ ,  $\frac{\partial \pi_H}{\partial p_L} \geq 0$ ,  $\frac{\partial y_H}{\partial p_L} \leq 0$ . When  $p_H < 1$ , then  $\frac{\partial \pi_H}{\partial p_L} > 0$ ,  $\frac{\partial y_H}{\partial p_L} < 0$ .*

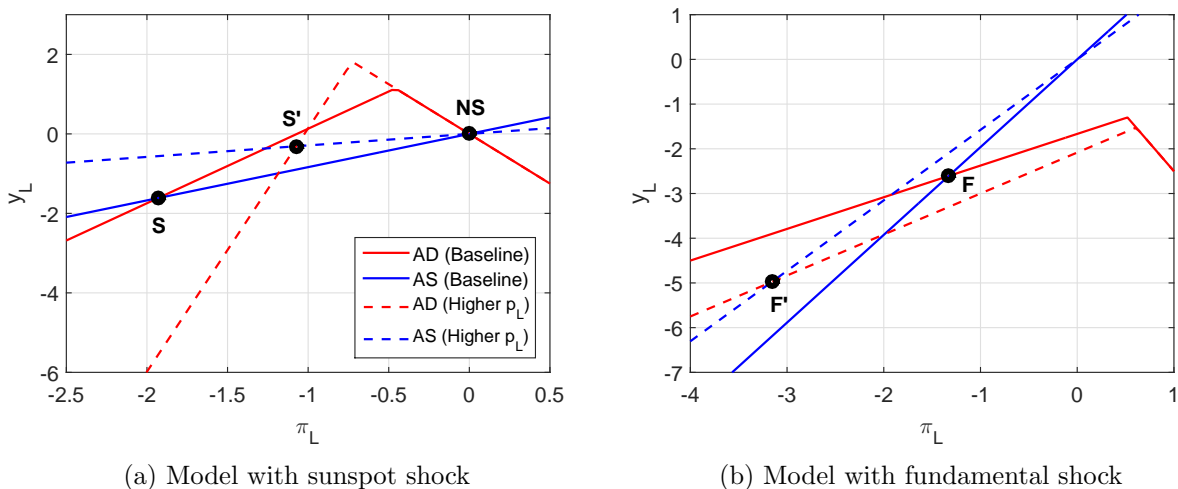
**Proof:** See Appendix A.

The first part of the proposition states that the more persistent the low-confidence state, the smaller the target shortfall of inflation and output in that state. The second part states that the more persistent the low-confidence state, the smaller the deviation of high-state output and high-state inflation from target.

The left panel of Figure 3 depicts how an increase in the persistence of the low-confidence state affects the low-state AD and AS curves (24)–(25) of the sunspot-shock model, assuming  $p_H = 1$ . The baseline curves are represented by the two solid lines, and the new curves are represented by

<sup>10</sup>See the condition for existence of the sunspot equilibrium (22) with  $p_H = 1$ .

Figure 3: The effect of an increase in the persistence of the low state



Note: Solid lines:  $p_L = 0.9375$  ( $p_L^f = 0.85$ ); dashed lines:  $p_L = 0.98$  ( $p_L^f = 0.88$ ). In the left panel,  $S$  marks the sunspot equilibrium in the baseline and  $S'$  marks the sunspot equilibrium in the case of a higher  $p_L$ .  $NS$  marks the no-sunspot equilibrium. In the right panel,  $F$  marks the fundamental equilibrium in the baseline and  $F'$  in the case of a higher  $p_L^f$ . Inflation is expressed in annualized terms.

the two dashed lines. An increase in the persistence of the low state leads to a flattening of the AS curve. Intuitively, when firms expect the low-confidence state to persist for longer, they adjust prices more elastically to a change in low-state demand conditions. Since prices are sticky, the inflation rate adjusts more elastically as well. The AD curve shifts upward—the intercept term increases—and becomes steeper to the left of the kink. The steepening of the AD curve reflects the fact that households adjust their desired consumption more elastically to a change in the rate of inflation when that change is assumed to persist for longer. To understand the increase in the intercept term of the AD curve, note that so long as the lower bound is binding, the *nominal* interest rate is below the natural real rate. An increase in the persistence of the low-confidence state implies that this negative gap between the actual nominal rate and the natural real rate—which abstracts from inflation—is expected to persist for longer. All else equal, this pushes up desired consumption. In the low state of the sunspot equilibrium, inflation is, however, lower than the negative of the natural real rate and hence there is a positive gap between the ex-ante *real* interest rate and the natural real rate. At a given equilibrium rate of inflation, an increase in the persistence of the low state will thus always result in a decline in households' desired consumption. Together, the shifts in the AD and AS curves imply that at the inflation rate consistent with the sunspot equilibrium in the baseline, marked by point  $S$ , there is now excess supply. Since the AD curve is steeper than the AS curve, excess supply is a declining function of the inflation rate as long as the lower bound is binding.<sup>11</sup> Hence, to restore equilibrium, inflation (and output) have to increase. The new intersection point  $S'$  lies to the north-east of the baseline intersection point  $S$ .

The analysis of the AD and AS curves also makes clear why the sunspot equilibrium does not

<sup>11</sup>This property of models of expectations-driven liquidity traps is also emphasized by Mertens and Ravn (2014).

exist when  $p_L$  is sufficiently low. When  $p_L$  is sufficiently low, the AD curve is flatter than the AS curve, and the only intersection point is the one associated with the no-sunspot equilibrium, denoted  $NS$ .

Having understood why low-state output and inflation are increasing in  $p_L$ , the effects of an increase in  $p_L$  on high-state output and inflation in the more general case of  $p_H < 1$  are relatively straightforward to understand. Since the target criterion is satisfied in the high-confidence state what matters for high-state output and inflation is how conditional inflation expectations are affected by the change in  $p_L$ . An increase in  $p_L$  raises low-state inflation and thereby mitigates the downward bias in conditional inflation expectations in the high-confidence state. Better anchored inflation expectations improve the output-inflation stabilization trade-off in the high state. In equilibrium, high-state output is decreasing and high-state inflation is increasing in  $p_L$ .

In the fundamental equilibrium, instead, an increase in the persistence of the low-fundamentals state  $p_L^f$  makes the downturn in low-state output and inflation more severe, and deteriorates the stabilization trade-off in the high-fundamentals state. See Appendix B. The right panel of Figure 3 illustrates graphically how the low-state AD and AS curves (26)–(27) in the fundamental-shock model are affected by an increase in  $p_L^f$ . As before, for the graphical analysis  $p_H^f = 1$ . Like in the sunspot-shock model, an increase in the persistence of the low state flattens the AS curve and steepens the AD curve. Unlike in the sunspot-shock model, the AD curve shifts downwards to the left of the kink, since the gap between the nominal interest rate and the natural real rate at the lower bound is positive in the low-fundamentals state. In the model with the fundamental shock the AD curve has to be flatter than the AS curve for the fundamental equilibrium to exist, and hence the intersection point in case of higher persistence of the low-fundamental state  $F'$  lies to the south-west of the intersection point for the baseline  $F$ . For sufficiently high  $p_L^f$  the AD curve becomes steeper than the AS curve and the fundamental equilibrium fails to exist.

### 3.3.2 Persistence of the high state

The next proposition sheds light on the effects of a change in the persistence of the high state on allocations and prices.

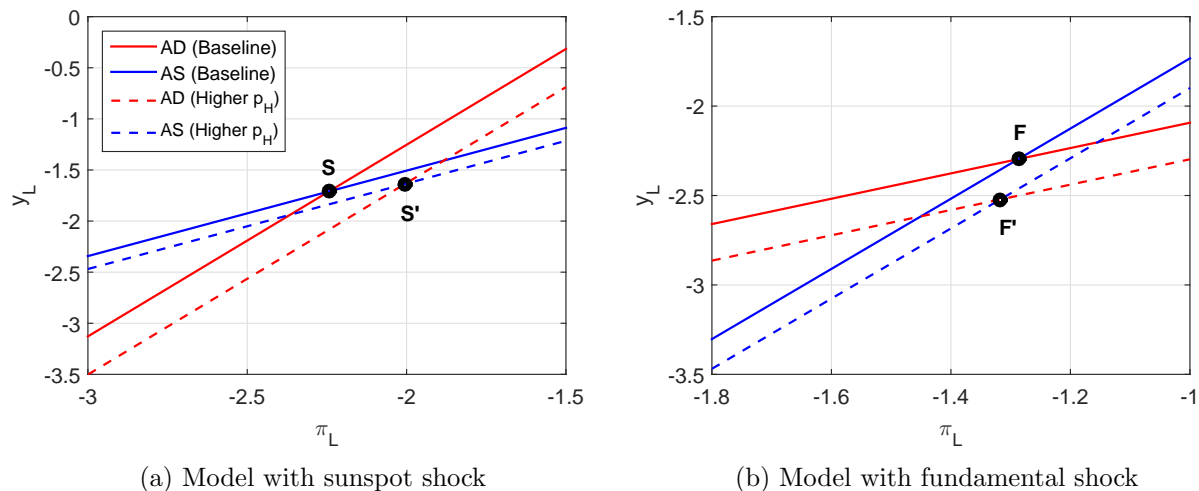
**Proposition 4** *Suppose  $\lambda = \bar{\lambda}$  and  $\pi^* = 0$ . In the sunspot equilibrium,  $\frac{\partial \pi_H}{\partial p_H} > 0$ ,  $\frac{\partial y_H}{\partial p_H} < 0$ . Moreover,  $\frac{\partial \pi_L}{\partial p_H} > 0$  if and only if  $p_L > 1 - \frac{\kappa^2 - \kappa \sigma \lambda}{\beta \lambda}$ , and  $\frac{\partial y_L}{\partial p_H} > 0$  if and only if  $p_L < \frac{\kappa^2 - \kappa \sigma \lambda}{\beta \kappa^2}$ .*

**Proof:** See Appendix A.

Consider, first, output and inflation in the high-confidence state. The lower the conditional probability of a future decline in confidence, the more benign is the trade-off between inflation and output gap stabilization in the high-confidence state for given low-confidence state outcomes. This partial equilibrium effect turns out to determine the signs of the overall effects on high-state outcomes, i.e. high-state output is decreasing and high-state inflation is increasing in  $p_H$ .

Instead, the effects of an increase in  $p_H$  on stabilization outcomes in the low-confidence state are ambiguous. The left panel of Figure 4 shows how the low-confidence-state AD and AS curves

Figure 4: The effect of an increase in the persistence of the high state



Note: Solid lines:  $p_H, p_H^f = 0.98$ ; dashed lines:  $p_H, p_H^f = 0.995$ . In the left panel,  $S$  marks the sunspot equilibrium in the baseline and  $S'$  marks the sunspot equilibrium in the case of a higher  $p_H$ . In the right panel,  $F$  marks the fundamental equilibrium in the baseline and  $F'$  in the case of a higher  $p_H^f$ . Inflation is expressed in annualized terms.

(24)–(25) are shifted in response to an increase in  $p_H$  for our baseline calibration. Unlike in the previous AD-AS analysis, we now assume  $p_H < 1$  and set  $y_H$  and  $\pi_H$  equal to their equilibrium values associated with the sunspot equilibrium.<sup>12</sup> For ease of exposition we focus on pairs  $\{\pi_L, y_L\}$  for which the lower bound constraint is binding.

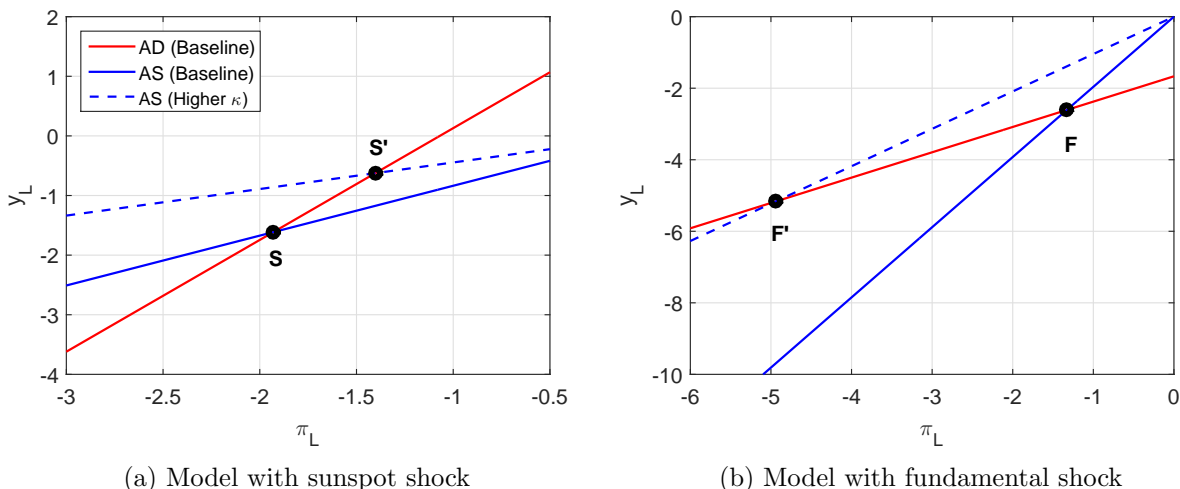
According to Proposition 4, we know that an increase in  $p_H$  leads to a decline in high-state output and to an increase in high-state inflation. According to equation (25), a rise in high-state inflation shifts down the low-state AS curve. According to equation (24), the overall effect of the changes in high-state output and inflation on the AD curve are ambiguous. In our numerical example, the AD curve is shifted downward, and the shift is big enough such that not only low-state inflation but also low-state output increases in response to the increase in  $p_H$ .

In the fundamental equilibrium, the effect of an increase in the persistence of the high-fundamental state on stabilization outcomes is ambiguous. The right panel of Figure 4 shows how the low-fundamental-state AD and AS curves (26)–(27) are shifted in response to an increase in  $p_H$  for our baseline calibration. Assuming  $p_H^f < 1$ , we set  $y_H$  and  $\pi_H$  equal to their equilibrium values associated with the fundamental equilibrium. Since an increase in  $p_H$  leads to an increase in high-state inflation, the AS curve shifts downward. The effect of a change in  $p_H^f$  on the AD curve is ambiguous due to the opposing responses of high-state output and inflation. In our numerical example, the AD curve shifts downward, and the shift is big enough such that not only low-state output but also low-state inflation declines.

<sup>12</sup>Hence, when moving *along* the AD and AS curves we ignore the feedback effect from output and inflation in the low-confidence state to output and inflation in the high-confidence state.



Figure 5: The effect of an increase in price flexibility



Note: Solid lines:  $\kappa = 0.0194$ ; dashed lines:  $\kappa = 0.0364$ . The increase in  $\kappa$  is obtained by a reduction in the parameter  $\alpha$  from 0.8106 to 0.75. In the left (right) panel,  $S$  ( $F$ ) marks the sunspot (fundamental) equilibrium in the baseline and  $S'$  ( $F'$ ) marks the sunspot (fundamental) equilibrium in the case of a higher  $\kappa$ . Inflation is expressed in annualized terms.

### 3.3.3 Degree of price flexibility

Finally, a parameter of particular interest is  $\kappa$ , the slope coefficient of the Phillips curve. Fundamental-driven liquidity traps are prone to the so-called ‘paradox of flexibility’. That is, when the lower bound is binding more price flexibility amplifies the contraction in output. In contrast, there is in general no ‘paradox of flexibility’ in the sunspot equilibrium.

**Proposition 5** *Suppose  $\lambda = \bar{\lambda}$ ,  $\pi^* = 0$ , and  $p_H = 1$ . In the sunspot equilibrium,  $\frac{\partial \pi_L}{\partial \kappa} > 0$ ,  $\frac{\partial y_L}{\partial \kappa} > 0$ .*

**Proof:** See Appendix A.

In Appendix A, we provide expressions for  $\frac{\partial \pi_L}{\partial \kappa}$ ,  $\frac{\partial y_L}{\partial \kappa}$  for the more general case of  $p_H \leq 1$  and find that their signs depend on parameter values. However, for reasonable parameterizations, we find that an increase in price flexibility increases output and inflation in the low-confidence state. Appendix B shows that the sign of  $\frac{\partial \pi_L}{\partial \kappa}$ ,  $\frac{\partial y_L}{\partial \kappa}$  in the fundamental equilibrium is in principle also ambiguous. However, for reasonable parameterizations, we find that an increase in price flexibility reduces output and inflation in the low-fundamental state. Moreover, for  $p_H^f = 1$ , the signs are unambiguously negative.

Figure 5 shows how an increase in the degree of price flexibility affects the AD and AS curves in the low state of the model with the sunspot shock (left panel) and the model with the fundamental shock (right panel). Increasing the degree of price flexibility does not affect the AD curve in the region where the lower bound is binding. The AS curve, however, becomes flatter. Since in the model with the sunspot shock the AD curve is steeper than the AS curve, a flattening of the AS

curve raises equilibrium output and inflation—from point  $S$  to point  $S'$ —in the low-confidence state of the sunspot equilibrium. The graphical analysis also helps to understand why prices have to be sufficiently flexible for the sunspot equilibrium to exist. The less flexible prices are, the steeper is the AS curve. When prices are sufficiently sticky, the AS curve is steeper than the AD curve and the sunspot equilibrium fails to exist.

In the model with the fundamental shock, instead, the increase in price flexibility leads to a decline in equilibrium output and inflation—from point  $F$  to point  $F'$ . When prices become sufficiently flexible, the AS curve becomes flatter than the AD curve and the fundamental equilibrium ceases to exist.

## 4 Inflation conservatism

Having understood the basic properties of the sunspot equilibrium, we now explore the desirability of alternative policy frameworks in the sunspot equilibrium. We begin by analyzing the desirability of inflation conservatism. We then explore the desirability of a non-zero inflation target in the next section. In the section that follows, we extend the model to allow for government spending and examine the desirability of fiscal activism.

An inflation-conservative central banker is a policymaker who puts a higher relative weight on inflation stabilization than society as a whole ( $\lambda < \bar{\lambda}$ ). In models with occasional fundamental-driven liquidity trap episodes, the appointment of an inflation-conservative policymaker improves welfare relative to the case where the policymaker has the same objective function as society as a whole (Nakata and Schmidt, 2018). Specifically, if the only source of uncertainty is a natural real rate shock—as assumed for the fundamental-shock model—then it is optimal to appoint a strictly inflation-conservative policymaker, i.e.  $\lambda = 0$ .

Let us now turn to the model with the sunspot shock. We first establish how a change in the central bank’s relative weight on output stabilization  $\lambda$  affects allocations and prices in the sunspot equilibrium and then explore the welfare implications. To focus on the role of inflation conservatism, we assume  $\pi^* = 0$  throughout this section.

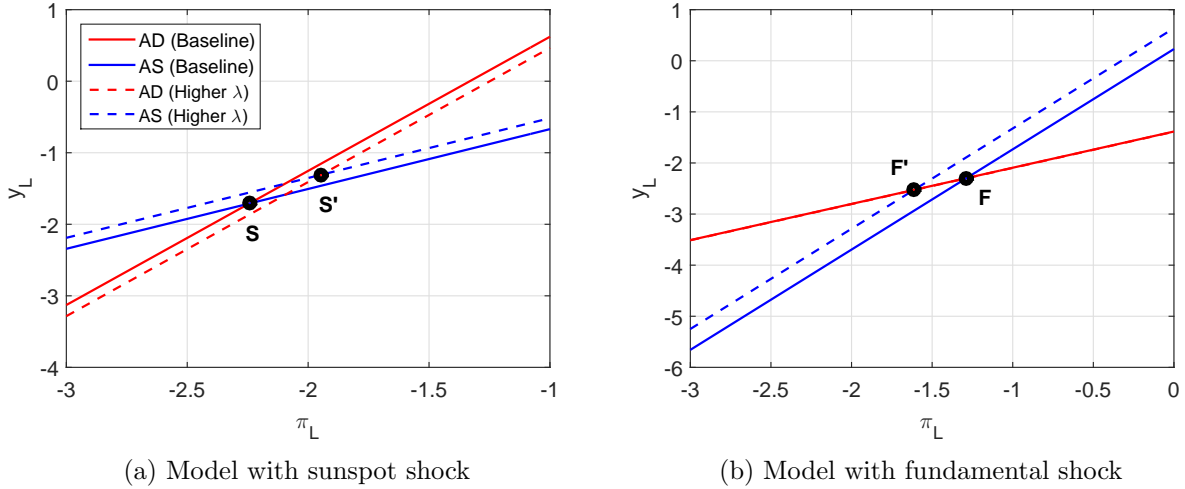
**Proposition 6** *Suppose  $\pi^* = 0$  and  $p_H < 1$ . In the sunspot equilibrium,  $\frac{\partial \pi_L}{\partial \lambda} > 0$ ,  $\frac{\partial y_L}{\partial \lambda} > 0$ ,  $\frac{\partial \pi_H}{\partial \lambda} < 0$ ,  $\frac{\partial y_H}{\partial \lambda} < 0$ .*

**Proof:** See Appendix A.

Consider first the high-confidence state. A change in  $\lambda$  affects the policymaker’s perceived trade-off between inflation and output stabilization. If the policymaker becomes more concerned with output stabilization (i.e.  $\lambda$  increases), she is ceteris paribus less willing to tolerate a positive output gap in order to mitigate the rate of deflation. In equilibrium, an increase in  $\lambda$  therefore reduces output and inflation in the high-confidence state.

Now consider the low-confidence state. To understand why low-state output and inflation are increasing in  $\lambda$ , we turn again to the AD-AS framework. The left panel of Figure 6 shows graphically

Figure 6: The effect of an increase in the central bank’s relative weight on output stabilization



Note: Solid lines:  $\lambda = \bar{\lambda} = 0.0019$ ; dashed lines:  $\lambda = 0.005$ . In the left (right) panel,  $S$  ( $F$ ) marks the sunspot (fundamental) equilibrium in the baseline and  $S'$  ( $F'$ ) marks the sunspot (fundamental) equilibrium in the case of a higher  $\lambda$ . Inflation is expressed in annualized terms.

how the low-confidence-state AD and AS curves (24)–(25) are shifted in response to an increase in  $\lambda$ . As in the case of Figure 4, we assume  $p_H < 1$ , and set  $y_H$  and  $\pi_H$  equal to their equilibrium values associated with the sunspot equilibrium. The intersection point  $S$  of the two solid lines marks the sunspot equilibrium for the baseline calibration. From Proposition 6, we know that an increase in  $\lambda$  leads to a decline in high-state output and inflation. According to equation (24), lower high-state output and inflation shifts down the low-state AD curve. Intuitively, households’ desired consumption declines in response to a drop in expected future output and inflation. According to equation (25), lower high-state inflation shifts up the low-state AS curve. Firms want to lower prices in response to a decline in expected future inflation. The new AD and AS curves are represented by the two dashed lines. At the equilibrium inflation rate prevailing in the baseline, there is now excess supply. As pointed out earlier, when the lower bound is binding excess supply is decreasing in the rate of inflation in the sunspot-shock model. Consistent with Proposition 6, the intersection point for the case of a higher  $\lambda$ , marked  $S'$ , lies to the north-east of the intersection point for the baseline.

How does this compare to the fundamental equilibrium? In the fundamental equilibrium, an increase in  $\lambda$  reduces inflation in the high state whereas the effect on output is ambiguous (Nakata and Schmidt, 2018). The logic underlying the effect on inflation is very similar to the one in the sunspot equilibrium. However, the effect on low-fundamental state outcomes differs from the one in the sunspot equilibrium. In the fundamental equilibrium, an increase in  $\lambda$  reduces output and inflation in the low-fundamental state. This is the case because lower inflation in the high-fundamental state lowers conditional inflation expectations in the low-fundamental state and thereby aggravates the downward pressure on output and inflation. This also explains why the effect of an increase in  $\lambda$  on high-state output is ambiguous. From a partial equilibrium perspective, an increase in  $\lambda$  should

lead to a reduction in high-state output (moving it closer to its target). However, if the low-state stabilization outcomes deteriorate sufficiently in response to the increase in  $\lambda$ , then the stabilization trade-off in the high state worsens sufficiently such that high-state output has to increase.

The right panel of Figure 6 depicts how the low-fundamental-state AD and AS curves (26)–(27) are shifted in response to an increase in  $\lambda$ . Assuming  $p_H^f < 1$ , we set  $y_H$  and  $\pi_H$  equal to their equilibrium values associated with the fundamental equilibrium. The intersection point  $F$  of the two solid lines marks the fundamental equilibrium for the baseline calibration. Now consider an increase in  $\lambda$ . Since an increase in  $\lambda$  leads to a decline in high-state inflation, the AS curve shifts upward. The effect on the AD curve is ambiguous due to the ambiguous response of high-state output. In our numerical example, the AD curve shifts slightly downwards in response to the increase in  $\lambda$ . In the fundamental equilibrium,  $\frac{\partial y_L}{\partial \lambda} < 0$  and hence even if the AD curve was shifting upwards in response to the increase in  $\lambda$ , the upward shift would have to be small enough so that the new intersection point  $F'$  of the two dashed lines lies to the south-west of the intersection point for the baseline.

In summary, while a change in  $\lambda$  shifts the AS curve in the sunspot-shock model and in the fundamental-shock model in the same direction, and in most cases shifts the AD curve in both models in the same direction, the sign of the effect on low-state output and inflation is mirror inverted. As can be seen from Figure 6, the reason is again that the relative slopes of the AD and AS curves in the two models differ.

We are now ready to assess the welfare implications of inflation conservatism in the model with the sunspot shock.

**Proposition 7** *Suppose  $\pi^* = 0$  and  $p_H < 1$ . Let  $\lambda^*$  denote the value of  $\lambda \in [0, \infty]$  that maximizes households' unconditional welfare  $EV_t$  where  $V_t$  is defined in equation (3). In the sunspot equilibrium,  $\lambda^* > 0$ .*

**Proof:** See Appendix A.

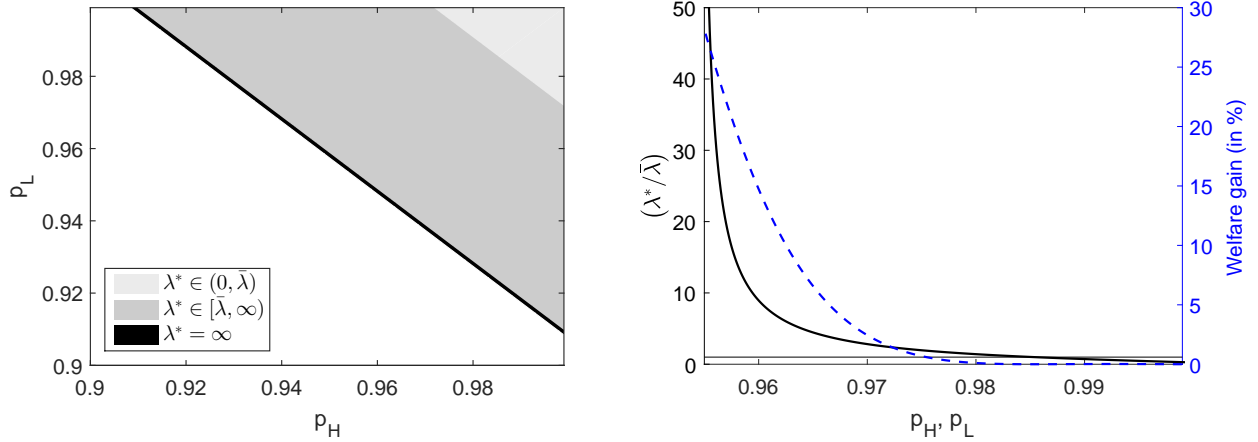
In words, strict inflation conservatism is not desirable in the sunspot equilibrium. In the Appendix, we show that  $\lambda^*$  can be either smaller or bigger than households' relative weight on output gap stabilization  $\bar{\lambda}$  and provide the corresponding necessary and sufficient conditions.<sup>13</sup>

To understand the ambiguity with regard to the optimal  $\lambda$ , note that society's relative weight on output stabilization  $\bar{\lambda}$  is typically very small so that the effect of a change in  $\lambda$  on society's welfare primarily depends on its effect on inflation. In the sunspot equilibrium, an increase in  $\lambda$  has a positive effect on low-state inflation (moving low-state inflation closer to target), and a negative effect on high-state inflation (moving high-state inflation further away from target), leading to the ambiguity. As mentioned earlier, this result is in contrast to the result for the fundamental equilibrium that the optimal  $\lambda$  is unambiguously zero.

The left panel of Figure 7 shows how  $\lambda^*$  in the sunspot-shock model depends on  $p_H$  and  $p_L$ , the persistence of the high and the low state, respectively. The parameterization follows Table 1. The

<sup>13</sup>In fact, it can be optimal to assign a pure output gap stabilization objective to the central bank,  $\lambda^* = \infty$ .

Figure 7: Optimal relative weight on output stabilization in model with sunspot shock



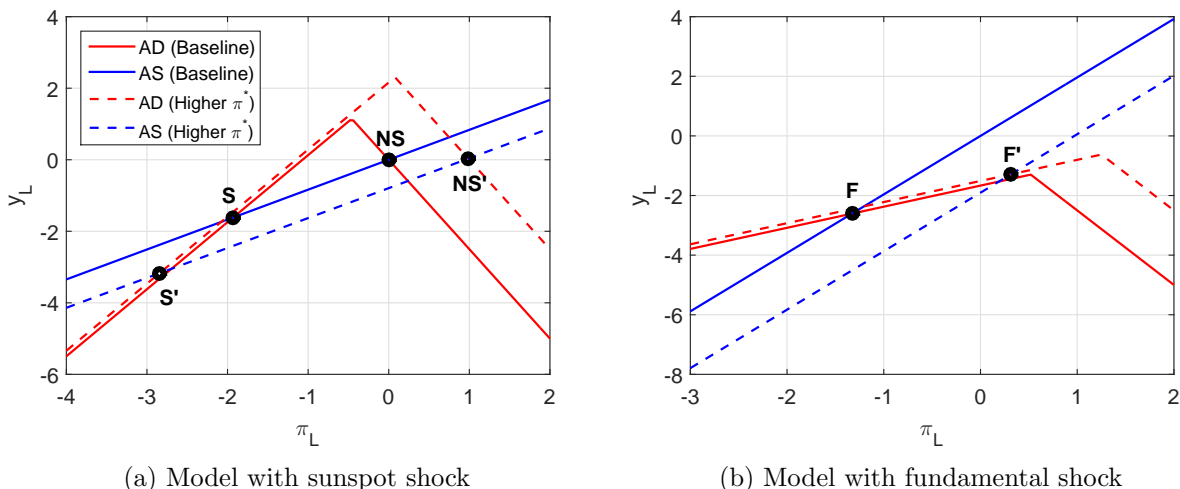
Note: The parameterization is summarized in Table 1. In the right panel,  $p_H = p_L$ . The dashed horizontal line in the right panel indicates  $\lambda^*/\bar{\lambda} = 1$ .

figure distinguishes three cases: i.  $\lambda^* \in (0, \bar{\lambda})$  (light gray-shaded area), ii.  $\lambda^* \in [\bar{\lambda}, \infty)$  (gray-shaded area), and iii.  $\lambda^* = \infty$  (black-shaded area). The white-shaded area represents pairs of  $p_H$  and  $p_L$  for which the sunspot equilibrium does not exist. A clear pattern emerges. For pairs  $\{p_H, p_L\}$  that are just high enough to satisfy the conditions for existence of the sunspot equilibrium, it is optimal to assign a pure output stabilization objective to the central bank. When increasing the persistence of the two confidence states, the optimal relative weight on output stabilization becomes finite but is larger than society's output weight. Most pairs  $\{p_H, p_L\}$  that are consistent with equilibrium existence fall into this second category. Finally, if both confidence states are highly persistent, then the optimal relative weight on output stabilization is smaller than society's weight but strictly positive.

The right panel of Figure 7 plots the ratio of the optimal relative weight on output stabilization  $\lambda^*$  to society's weight  $\bar{\lambda}$  (left vertical axis, solid black line) and the welfare gain from assigning  $\lambda^*$  instead of  $\bar{\lambda}$  to the central bank (right vertical axis, blue dashed line) as a function of the persistence of the two confidence states, assuming  $p_H = p_L$ . The optimal relative weight on output gap stabilization is monotonically declining in the persistence of the confidence states. For values of  $p_H, p_L$  for which the solid line lies below the thin horizontal line, the optimal relative weight on output stabilization in the central bank's objective function is smaller than society's weight. The welfare gain from assigning an optimized relative output weight is most elevated when the persistence parameters take on the smallest possible values that are still consistent with equilibrium existence, that is, when inflation conservatism is undesirable.

In summary, whether or not some degree of inflation conservatism is optimal in the sunspot equilibrium depends on the parameterization. Strict inflation conservatism ( $\lambda = 0$ )—the optimal institutional configuration in the fundamental equilibrium—is never optimal in the sunspot equilibrium.

Figure 8: The effect of increasing the central bank's inflation target



Note: Solid lines:  $\pi^* = 0$ ; dashed lines:  $\pi^* = 1/400$ . In the left (right) panel,  $S$  ( $F$ ) marks the sunspot (fundamental) equilibrium in the baseline and  $S'$  ( $F'$ ) marks the sunspot (fundamental) equilibrium in the case of a higher  $\pi^*$ .  $NS$  marks the no-sunspot equilibrium in the baseline, and  $NS'$  marks the no-sunspot equilibrium in the case of a higher  $\pi^*$ . Inflation is expressed in annualized terms.

## 5 A non-zero inflation target

An alternative institutional configuration that has been shown to improve society's welfare in the fundamental equilibrium is the imposition of a strictly positive central bank inflation target (Nakata and Schmidt, 2018). This section explores whether society's welfare in the sunspot equilibrium can be increased by assigning to the central bank a non-zero inflation target. Throughout this section, we assume  $\lambda = \bar{\lambda}$ .

While the sign of allocations and prices is sensitive to the quantitative value of the central bank's inflation target, the effects of a marginal change in the target on allocations and prices is unambiguously determined.

**Proposition 8** *In the sunspot equilibrium,  $\frac{\partial \pi_L}{\partial \pi^*} < 0$ ,  $\frac{\partial y_L}{\partial \pi^*} < 0$ ,  $\frac{\partial \pi_H}{\partial \pi^*} > 0$ ,  $\frac{\partial y_H}{\partial \pi^*} > 0$ .*

**Proof:** See Appendix A.

In the sunspot equilibrium, a marginal increase in the inflation target lowers output and inflation in the low-confidence state and raises output and inflation in the high-confidence state. Qualitatively, the effects are thus the same as those of a marginal reduction in  $\lambda$ , see Proposition 6. The left panel of Figure 8 depicts how the low-confidence state AD and AS curves (24)–(25) are shifted in response to an increase in the central bank's inflation target, assuming that the high state is an absorbing state. An increase in the inflation target shifts the AD curve upwards, since, all else equal, agents increase their desired consumption given higher expected inflation. At the same time, the AS curve shifts downwards, since firms' desired price increases in light of higher expected inflation for given current demand. Hence, at the inflation rate consistent with the sunspot

equilibrium in the baseline, marked by intersection point  $S$ , there is now excess demand. In the model with the sunspot shock, excess demand is increasing in the inflation rate as long as the lower bound is binding. To restore equilibrium, low-state inflation and output thus have to decline. The new intersection point  $S'$  lies to the south-west of the baseline intersection point  $S$ .<sup>14</sup>

In the fundamental equilibrium, a marginal increase in the inflation target also raises high-state inflation. The effects on low-state outcomes, however, differ from those in the sunspot equilibrium. Higher inflation in the high-fundamental state lowers the conditional ex-ante real interest rate in the low-fundamental state. This stimulates aggregate demand and leads to an increase in low-state output and inflation (Nakata and Schmidt, 2018). The right panel of Figure 8 depicts how in the model with the fundamental shock the low-state AD and AS curves (26)–(27) are shifted in response to an increase in the inflation target.

For the characterization of the welfare-maximizing inflation target in the sunspot-shock model, it is also useful to show that there exists an inflation target such that inflation in the high-confidence state is perfectly stabilized.

**Lemma 1** *There exists a  $\pi^0 > 0$  such that in the sunspot equilibrium  $\pi_H = 0$  if  $\pi^* = \pi^0$ .*

**Proof:** See Appendix A.

One can then establish the following result concerning the welfare-maximizing inflation target.

**Proposition 9** *Suppose  $\lambda = \bar{\lambda}$  and  $p_H < 1$ . Let  $\pi^{**}$  denote the value of  $\pi^* > -\frac{\kappa^2 + \lambda(1-\beta)}{\kappa^2} r^n$  that maximizes households' unconditional welfare  $EV_t$  where  $V_t$  is defined in equation (3). In the sunspot equilibrium,  $\pi^{**} < \pi^0$ .*

**Proof:** See Appendix A.

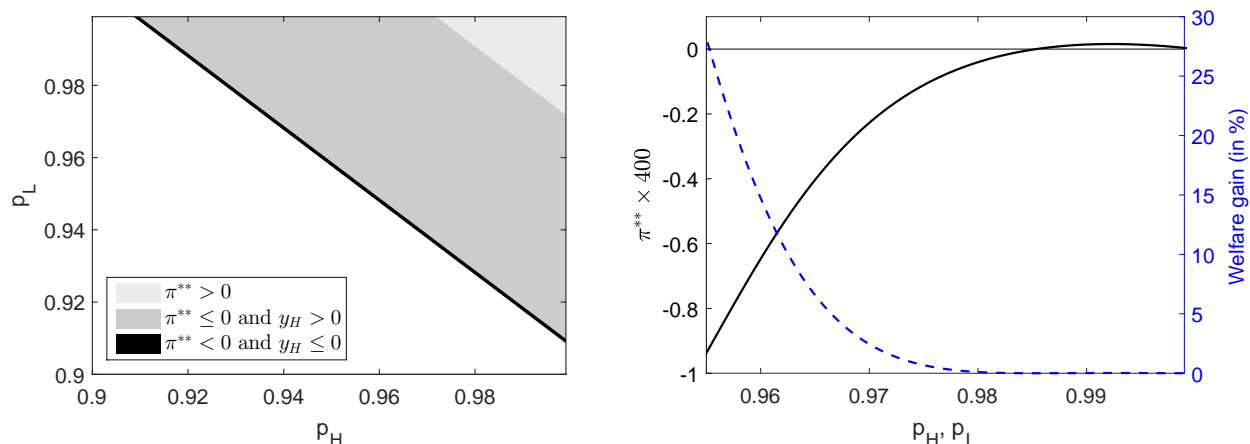
The optimal inflation target can be negative or positive. However, according to the above proposition, even if it is positive, it will be below the level needed to engineer strictly positive inflation in the high-confidence state. The reason for the ambiguity concerning the sign of the optimal target is similar to the reason for why the optimal relative weight on output stabilization—analyzed in the previous section—can be higher or lower than society's weight. In the sunspot equilibrium, an increase in  $\pi^*$  has a negative effect on low-state inflation (moving low-state inflation further into negative territory), and a positive effect on high-state inflation (moving high-state inflation closer to target as long as  $\pi^* < \pi^0$ ).<sup>15</sup>

The left panel of Figure 9 shows how  $\pi^{**}$  in the sunspot-shock model depends on  $p_H$  and  $p_L$ , the persistence of the high and the low state, respectively. The figure distinguishes three cases: i.  $\pi^{**} > 0$  (light gray-shaded area), ii.  $\pi^{**} \leq 0$  and  $y_H > 0$  (gray-shaded area), and iii.  $\pi^{**} < 0$  and  $y_H \leq 0$  (black-shaded area). The white-shaded area represents pairs of  $p_H$  and  $p_L$  for which the sunspot equilibrium does not exist. When the two confidence states are highly persistent, the

<sup>14</sup>An increase in the inflation target also affects the no-sunspot equilibrium. With a non-zero inflation target, the central bank faces a trade-off between output stabilization and stabilization of inflation at target. In equilibrium, when the inflation target is positive, high-state inflation is slightly below target and the output gap is slightly positive.

<sup>15</sup>Appendix A provides a numerical example of how  $\pi^*$  affects allocations and welfare in the sunspot equilibrium.

Figure 9: Optimal inflation target in model with sunspot shock



Note: The parameterization is summarized in Table 1. In the right panel  $p_H = p_L$ .

optimal inflation target is strictly positive. When the two states are less persistent, the optimal inflation target is negative. Most pairs  $\{p_H, p_L\}$  that are consistent with equilibrium existence fall into this second category. If the pair of persistence parameters just marginally satisfies the conditions for equilibrium existence, the optimal inflation target is sufficiently negative to engineer a negative output gap in the high state. This last case is special in that a negative high-state output gap is unattainable under inflation conservatism.<sup>16</sup>

The right panel of Figure 9 plots the optimal inflation target (left vertical axis, solid black line) and the welfare gain from assigning the optimal target instead of  $\pi^* = 0$  (right vertical axis, dashed blue line) as a function of the persistence of the two confidence states, assuming  $p_H = p_L$ . For sufficiently low values of  $p_H, p_L$ , the optimal inflation target is negative and increasing in the persistence parameters. When  $p_H, p_L$  are high enough, the optimal inflation target is slightly positive. The welfare gain from assigning an optimized inflation target is most elevated when the persistence parameters take on the lowest possible values for which the sunspot equilibrium exists.

There is a close relationship between the assignment of an inflation target and inflation conservatism.

**Proposition 10** *Suppose  $p_H < 1$ . For any  $\hat{\lambda} \geq 0$ , there exists a  $\hat{\pi}^*$  such that the sunspot equilibrium under optimal discretionary policy associated with the inflation conservatism regime satisfying  $(\lambda = \hat{\lambda}, \pi^* = 0)$  is replicated by the inflation target regime satisfying  $(\lambda = \bar{\lambda}, \pi^* = \hat{\pi}^*)$ , where*

$$\hat{\pi}^* \equiv \frac{\beta(1-p_H)r^n}{\beta\hat{\lambda}(1-p_H) - (\kappa^2 + \hat{\lambda}(1-\beta))C} (\bar{\lambda} - \hat{\lambda}). \quad (28)$$

**Proof:** See Appendix A.

<sup>16</sup>The lowest value for high-state output attainable under inflation conservatism is  $y_H = 0$ , which requires  $\lambda = \infty$ .



The reverse is not true, since a sufficiently negative inflation target results in a strictly negative high-state output gap, an allocation that is unattainable under inflation conservatism for any  $\lambda \geq 0$ .<sup>17</sup> An interesting implication of equation (28) is that if the allocation under the optimal inflation target is attainable under inflation conservatism, then the optimal inflation target  $\pi^{**}$  is positive if and only if the optimal relative output weight  $\lambda^*$  is smaller than society's weight  $\bar{\lambda}$ .<sup>18</sup> This can also be seen by the fact that the boundary between the light gray and dark gray areas in Figure 9 is identical to the one in Figure 7.

## 6 Fiscal activism

This section extends the model to allow for a meaningful role of fiscal stabilization policy. To do so, we introduce government consumption into the baseline model, the level of which is chosen optimally by the discretionary policymaker together with the level of the policy rate. We first show how the introduction of fiscal policy affects equilibrium existence and allocations and then turn to the design of fiscal policy by asking how much relative weight the objective function of the discretionary policymaker should put on government spending stabilization.

### 6.1 The model with fiscal policy

The aggregate private sector behavioral constraints in the model with government spending are

$$\pi_t = \kappa x_t + \beta \mathbf{E}_t \pi_{t+1} \quad (29)$$

$$x_t = (1 - \Gamma)g_t + \mathbf{E}_t(x_{t+1} - (1 - \Gamma)g_{t+1}) - \sigma(i_t - \mathbf{E}_t \pi_{t+1} - r_t^n), \quad (30)$$

where  $g_t$  denotes government spending as a share of steady-state output, expressed in deviation from the steady-state ratio,  $x_t \equiv y_t - \Gamma g_t$  will be referred to as the modified output gap, and  $\Gamma = \frac{\sigma^{-1}}{\sigma^{-1} + \eta}$ .<sup>19</sup>

We assume that the provision of public goods provides utility to households and that utility is separable in private and public consumption. A second-order approximation to household preferences leads to<sup>20</sup>

$$V_t = -\frac{1}{2} \mathbf{E}_t \sum_{j=0}^{\infty} \beta^j (\pi_{t+j}^2 + \bar{\lambda} x_{t+j}^2 + \bar{\lambda}_g g_{t+j}^2). \quad (31)$$

The relative weight on government spending stabilization satisfies  $\bar{\lambda}_g = \bar{\lambda} \Gamma (1 - \Gamma + \frac{\sigma}{\nu}) > 0$ , where

<sup>17</sup>Likewise, a sufficiently positive inflation target results in a strictly positive high-state inflation rate, an allocation that is also unattainable under inflation conservatism for any  $\lambda \geq 0$ .

<sup>18</sup>To see this, note that  $\beta \hat{\lambda}(1 - p_H) - (\kappa^2 + \hat{\lambda}(1 - \beta))C > 0$  in the sunspot equilibrium.

<sup>19</sup>The public consumption good is assumed to be compiled based on the same aggregation technology as the private consumption good.

<sup>20</sup>See Schmidt (2013) for details.

$\nu$  denotes the inverse of the elasticity of the marginal utility of public consumption with respect to total output. As before,  $\bar{\lambda} = \kappa/\theta$ .

At the beginning of time, society delegates monetary and fiscal policy to a discretionary policymaker. The objective function of the policymaker is given by

$$V_t^{MF} = -\frac{1}{2} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j (\pi_{t+j}^2 + \bar{\lambda} x_{t+j}^2 + \lambda_g g_{t+j}^2), \quad (32)$$

where  $\lambda_g > 0$  is a policy parameter the value of which is chosen by society when designing the policymaker's objective function. For  $\lambda_g = \bar{\lambda}_g$ , the policymaker's objective function coincides with society's objective function. The policymaker's optimization problem and the first-order conditions are relegated to Appendix C.

As before, we focus on a sunspot equilibrium where the lower bound is binding in the low-confidence state and slack in the high-confidence state.

**Definition 3** *The sunspot equilibrium in the sunspot-shock model with fiscal policy is given by a vector  $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$  that solves the following system of linear equations*

$$x_H = p_H x_H + (1 - p_H) [x_L + (1 - \Gamma)(g_H - g_L)] + \sigma [p_H \pi_H + (1 - p_H) \pi_L - i_H + r^n] \quad (33)$$

$$\pi_H = \kappa x_H + \beta [p_H \pi_H + (1 - p_H) \pi_L] \quad (34)$$

$$\lambda_g g_H = -(1 - \Gamma) (\bar{\lambda} x_H + \kappa \pi_H) \quad (35)$$

$$0 = \bar{\lambda} x_H + \kappa \pi_H \quad (36)$$

$$x_L = p_L x_L + (1 - p_L) [x_H - (1 - \Gamma)(g_H - g_L)] + \sigma [(1 - p_L) \pi_H + p_L \pi_L - i_L + r^n] \quad (37)$$

$$\pi_L = \kappa x_L + \beta [(1 - p_L) \pi_H + p_L \pi_L] \quad (38)$$

$$\lambda_g g_L = -(1 - \Gamma) (\bar{\lambda} x_L + \kappa \pi_L) \quad (39)$$

$$i_L = 0, \quad (40)$$

and satisfies the following two inequality constraints

$$i_H > 0 \quad (41)$$

$$\bar{\lambda} x_L + \kappa \pi_L < 0. \quad (42)$$

The sunspot equilibrium is compared to a fundamental equilibrium in a setup where the two-state sunspot shock is replaced with a two-state natural real rate shock. As before, we consider a fundamental equilibrium where the lower bound constraint is slack in the high-fundamental state and binding in the low-fundamental state.

**Definition 4** *The fundamental equilibrium in the fundamental-shock model with fiscal policy is given by a vector  $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$  that solves the following system of linear equations*

$$x_H = p_H^f x_H + (1 - p_H^f) [x_L + (1 - \Gamma)(g_H - g_L)] + \sigma [p_H^f \pi_H + (1 - p_H^f) \pi_L - i_H + r^n] \quad (43)$$

$$\pi_H = \kappa x_H + \beta [p_H^f \pi_H + (1 - p_H^f) \pi_L] \quad (44)$$

$$x_L = p_L^f x_L + (1 - p_L^f) [x_H - (1 - \Gamma)(g_H - g_L)] + \sigma [(1 - p_L^f) \pi_H + p_L^f \pi_L - i_L + r_L^n] \quad (45)$$

$$\pi_L = \kappa x_L + \beta [(1 - p_L^f) \pi_H + p_L^f \pi_L] \quad (46)$$

as well as (35), (36), (39) and (40), and satisfies the inequality constraints (41) and (42).

## 6.2 Equilibrium existence and allocations

The following proposition establishes a necessary and sufficient condition for existence of the sunspot equilibrium in the model with fiscal policy.

**Proposition 11** *The sunspot equilibrium exists if and only if*

$$\lambda_g \Omega(p_L, p_H, \kappa, \sigma, \beta) - (1 - \Gamma)^2 \frac{1 - p_L + 1 - p_H}{\kappa \sigma} [\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H))] > 0, \quad (47)$$

where  $\Omega(p_L, p_H, \kappa, \sigma, \beta) \equiv p_L - (1 - p_H) - \frac{1 - p_L + 1 - p_H}{\kappa \sigma} (1 - \beta p_L + \beta(1 - p_H))$ .

**Proof:** See Appendix D.

From Proposition 1 we know that the sunspot equilibrium in the model without fiscal policy and a zero-inflation target exists if and only if  $\Omega(\cdot) > 0$ . In the model with fiscal policy,  $\Omega(\cdot) > 0$  is a necessary but not a sufficient condition for existence of the sunspot equilibrium. Importantly, the condition for equilibrium existence depends on the policy parameter  $\lambda_g$ . Suppose  $\Omega(\cdot) > 0$ . Then the sunspot equilibrium exists if and only if  $\lambda_g > \frac{(1 - \Gamma)^2}{\Omega(\cdot)} \frac{1 - p_L + 1 - p_H}{\kappa \sigma} [\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H))] > 0$ .

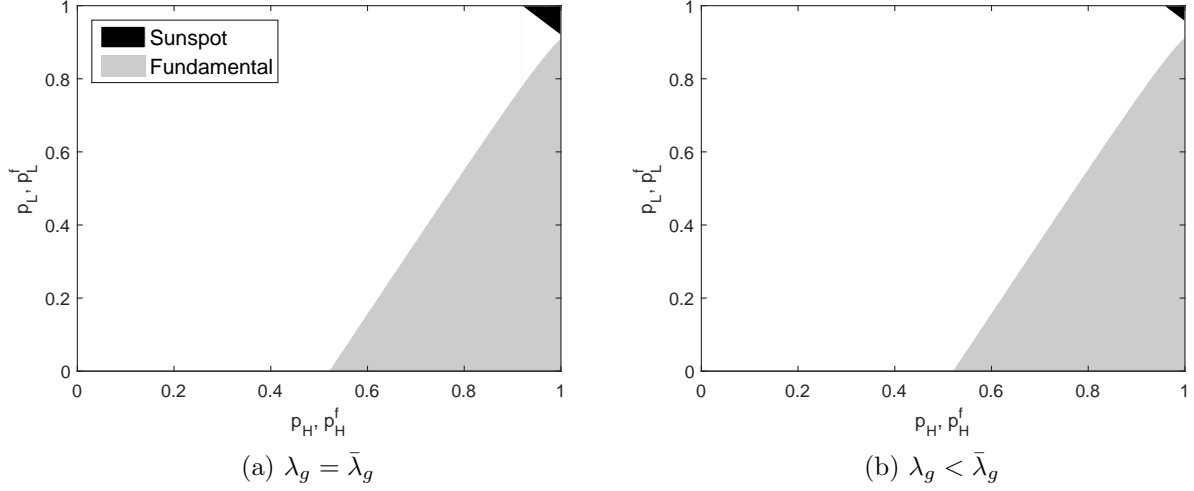
The condition for existence of the fundamental equilibrium in the model with the natural real rate shock is provided in Appendix E.

Figure 10 plots the region of existence for the sunspot equilibrium (black area), and the region of existence for the fundamental equilibrium (gray area). The left panel shows results for  $\lambda_g = \bar{\lambda}_g$  and the right panel for  $\lambda_g < \bar{\lambda}_g$ . The parameterization follows Table 1, except that we now account for a non-zero steady-state government spending to output ratio of 0.2, which implies that the inverse of the elasticity of the marginal utility of private consumption with respect to output  $\sigma$  becomes 0.4.<sup>21</sup> The inverse of the elasticity of the marginal utility of public consumption with respect to output  $\nu$  is set to 0.1.<sup>22</sup> This implies  $\bar{\lambda}_g = 0.0082$ . As in the case without government consumption, for the sunspot equilibrium to exist the two confidence states have to be sufficiently

<sup>21</sup> Assuming that the intertemporal elasticity of substitution in private consumption equals 0.5, as before, we have  $\sigma = 0.5 \times 0.8 = 0.4$ .

<sup>22</sup> This corresponds to the case in which the marginal utility of consumption of the public good decreases at the same rate as the marginal utility of consumption of the non-public good, i.e.  $\nu = 0.5 \times 0.2 = 0.1$ .

Figure 10: Existence regions in the model with fiscal policy



persistent, whereas for the fundamental equilibrium to exist the low-fundamental state must not be too persistent. Furthermore, in line with Proposition 11, the existence region for the sunspot equilibrium shrinks when  $\lambda_g$  is lowered.

Next, we characterize allocations and prices in the sunspot equilibrium.

**Proposition 12** *In the sunspot equilibrium,  $\pi_L < 0$ ,  $x_L < 0$ ,  $g_L > 0$ ,  $\pi_H \leq 0$ ,  $x_H \geq 0$  and  $g_H = 0$  for any  $\lambda_g > 0$ . When  $p_H < 1$ , then  $\pi_H < 0$ ,  $x_H > 0$ .*

**Proof:** See Appendix D.

The policymaker implements a government spending stimulus when the lower bound on nominal interest rates is binding, and keeps government spending at its steady state otherwise. The same holds true for the fundamental equilibrium. See Appendix E. Next, we explore how the availability of government spending as an additional policy tool affects society's welfare.

### 6.3 Welfare

As an intermediate step, we first establish how a marginal change in the policymaker's relative weight on government spending stabilization  $\lambda_g$  affects allocations and prices.

**Proposition 13** *In the sunspot equilibrium,  $\frac{\partial \pi_L}{\partial \lambda_g} > 0$ ,  $\frac{\partial x_L}{\partial \lambda_g} > 0$ ,  $\frac{\partial g_L}{\partial \lambda_g} < 0$ ,  $\frac{\partial \pi_H}{\partial \lambda_g} \geq 0$ ,  $\frac{\partial x_H}{\partial \lambda_g} \leq 0$ . If  $p_H < 1$ ,  $\frac{\partial \pi_H}{\partial \lambda_g} > 0$ ,  $\frac{\partial x_H}{\partial \lambda_g} < 0$ .*

**Proof:** See Appendix D.

In words, the higher the relative weight on government spending stabilization in the policymaker's objective function, the smaller the fiscal stimulus in the low-confidence state. At the same

time, an increase in  $\lambda_g$  raises the inflation rate in both confidence states as well as the modified output gap in the low-confidence state and lowers the modified output gap in the high-confidence state. Thus, the higher  $\lambda_g$  the closer to target is the economy.

In the fundamental equilibrium, instead, an increase in  $\lambda_g$  lowers the modified output gap and inflation in the low state as well as inflation in the high state and raises the high-state modified output gap. See Appendix E. The effect on government spending in the low-fundamental state is ambiguous. On the one hand, a higher relative weight on government spending stabilization implies a smaller fiscal stimulus for given values of low-state inflation and modified output gap, see equation (39). On the other hand, an increase in  $\lambda_g$  makes the decline in low-fundamental state inflation and modified output gap more severe, thereby raising the optimal level of government spending in the low-fundamental state.

It is instructive to show how a change in  $\lambda_g$  affects the low-state AD and AS curves in the two models. For the sunspot-shock model with fiscal policy and an absorbing high-confidence state the low-confidence-state AD and AS curves are given by

$$\text{AD-sunspot: } x_L = \min \left[ \frac{1}{\lambda_g + (1 - \Gamma)^2 \bar{\lambda}} \left( \frac{\sigma \lambda_g}{1 - p_L} r^n + \left( \frac{\sigma p_L \lambda_g}{1 - p_L} - (1 - \Gamma)^2 \kappa \right) \pi_L \right), -\frac{\kappa}{\bar{\lambda}} \pi_L \right] \quad (48)$$

$$\text{AS-sunspot: } x_L = \frac{1 - \beta p_L}{\kappa} \pi_L, \quad (49)$$

where  $\pi_H$  and  $x_H$  have been set equal to zero. For the fundamental-shock model with fiscal policy and an absorbing high-fundamental state, the low-fundamental-state AD and AS curves are given by

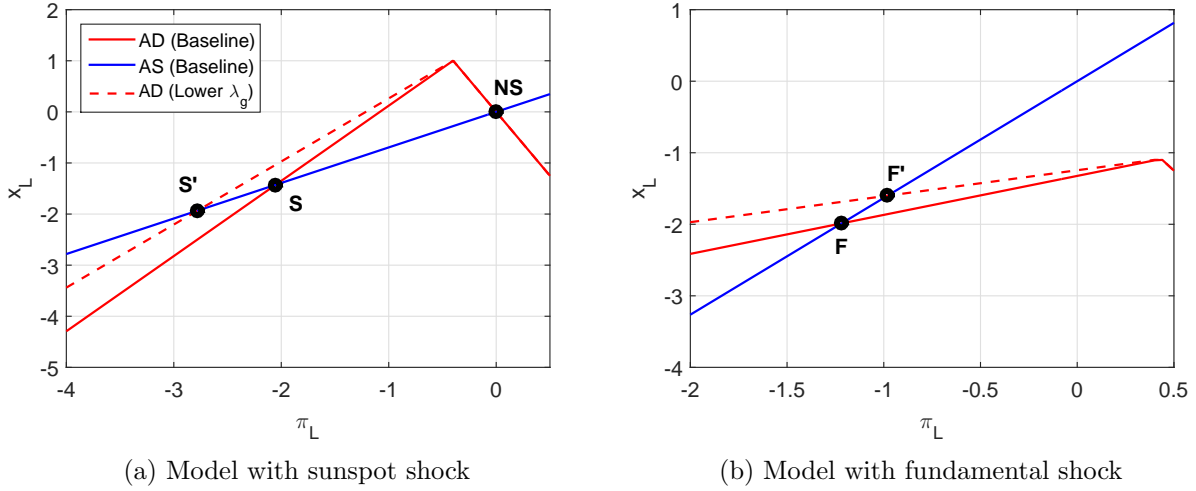
$$\text{AD-fundamental: } x_L = \min \left[ \frac{1}{\lambda_g + (1 - \Gamma)^2 \bar{\lambda}} \left( \frac{\sigma \lambda_g}{1 - p_L^f} r_L^n + \left( \frac{\sigma p_L^f \lambda_g}{1 - p_L^f} - (1 - \Gamma)^2 \kappa \right) \pi_L \right), -\frac{\kappa}{\bar{\lambda}} \pi_L \right] \quad (50)$$

$$\text{AS-fundamental: } x_L = \frac{1 - \beta p_L^f}{\kappa} \pi_L, \quad (51)$$

where again  $\pi_H$  and  $x_H$  have been set equal to zero.

Figure 11 depicts how the AD-AS curves are affected by a reduction in  $\lambda_g$ . The intersection point  $S$  in the left panel marks the sunspot equilibrium in the sunspot-shock model for the baseline calibration, and the intersection point  $NS$  marks the no-sunspot equilibrium. The intersection point  $F$  in the right panel, in turn, marks the fundamental equilibrium in the fundamental-shock model for the baseline calibration. In both models, the AD curve becomes flatter to the left of the kink when  $\lambda_g$  is lowered. Intuitively, when the policymaker adjusts government spending more aggressively to changes in inflation, aggregate demand, too, responds *ceteris paribus* more elastically to changes in inflation. In the model with the sunspot shock, the AD curve is steeper than the AS curve, and hence a flattening of the AD curve shifts the point at which the two curves intersect

Figure 11: The effect of reduction in  $\lambda_g$  on low-state aggregate demand and supply



Note: Solid lines:  $\lambda_g = \bar{\lambda}_g$ ; dashed lines:  $\lambda_g = \bar{\lambda}_g/10$ . In the left panel,  $S$  marks the sunspot equilibrium in the baseline,  $S'$  marks the sunspot equilibrium in case of a lower  $\lambda_g$  and  $NS$  marks the no-sunspot equilibrium. In the right panel,  $F$  marks the fundamental equilibrium in the baseline and  $F'$  marks the fundamental equilibrium in case of a lower  $\lambda_g$ . Inflation is expressed in annualized terms.

when the lower bound is binding to the south-west. In contrast, in the model with the fundamental shock, the AD curve is flatter than the AS curve, and hence a flattening of the AD curve shifts the point at which the two curves intersect to the north-east.

Propositions 12 and 13 together have a straightforward implication for the optimal value of  $\lambda_g$  in the sunspot equilibrium.

**Proposition 14** *Let  $\lambda_g^*$  denote the value of  $\lambda_g \in (0, \infty]$  that maximizes households' unconditional welfare  $EV_t$  where  $V_t$  is defined in equation (31). In the sunspot equilibrium,  $\lambda_g^* = \infty$ .*

It is easy to show that as  $\lambda_g \rightarrow \infty$ ,  $g_L \rightarrow 0$ . Intuitively, if it becomes infinitely costly for the policymaker to adjust government spending, she will not use it as a stabilization tool. This turns out to be the optimal configuration in the sunspot equilibrium. Put differently, introducing an additional policy tool in the form of government consumption reduces welfare in the sunspot equilibrium. Conditional on the existence of the sunspot equilibrium it is therefore optimal to make the use of the tool so expensive for the policymaker that she will refrain from using it.<sup>23</sup>

In the model with the fundamental shock, instead, fiscal activism can improve welfare. Specifically, Schmidt (2017) shows that  $\lambda_g^* < \bar{\lambda}_g$ .<sup>24</sup>

<sup>23</sup>Appendix D provides a numerical example of how  $\lambda_g$  affects allocations and welfare in the sunspot equilibrium.

<sup>24</sup>Schmidt (2017) provides a closed-form solution for the welfare-maximizing  $\lambda_g$  in case of  $p_H^f = 1$  and numerical results for  $p_H^f < 1$ .

### 6.3.1 Why is government spending raised in the low-confidence state?

If an expansionary fiscal policy in the low-confidence state moves the economy further away from target in both confidence states, why does the policymaker not simply refrain from raising government spending in the low-confidence state for any  $\lambda_g > 0$ ? To shed light on this question consider the following thought experiment. Suppose, for ease of exposition, that  $p_H = 1$  and  $\lambda_g \rightarrow \infty$ , i.e. the high-confidence state is an absorbing state and there is no systematic use of government spending for stabilization purposes in the low-confidence state. Consider some period  $T \geq 0$  where the economy is in the low-confidence state and the lower bound is binding. The private sector behavioral constraints for period  $T$  can be written as

$$x_L^T = (1 - \Gamma)g_L^T - p_L \frac{(1 - \beta p_L)\kappa^2 + (1 - \beta)(1 - \beta p_L + \beta(1 - p_H))\bar{\lambda} + \kappa\sigma(\kappa^2 + \bar{\lambda}(1 - \beta p_H))}{\kappa E} r^n + \sigma r^n$$

$$\pi_L^T = \kappa x_L^T - \beta p_L \frac{\kappa^2 + \bar{\lambda}(1 - \beta p_H)}{E} r^n,$$

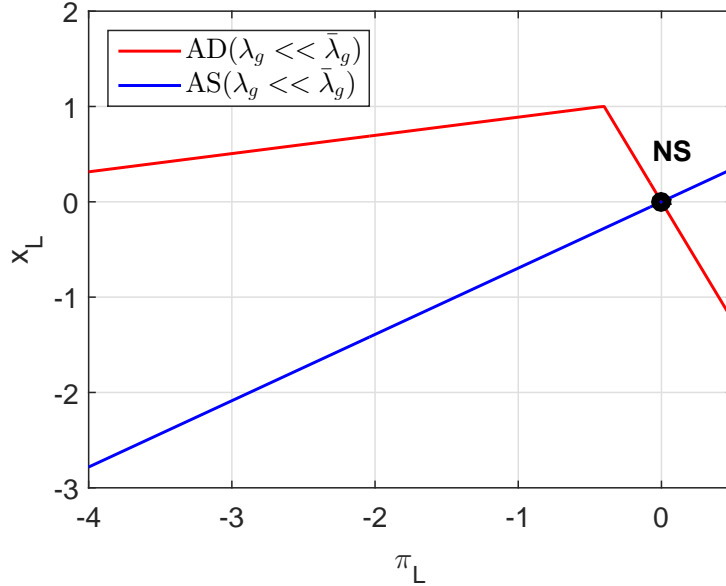
where  $\pi_L^T, x_L^T, g_L^T$  are the inflation rate, the modified output gap and government spending in period  $T$ . Now suppose that in period  $T$  there is an unexpected one-time increase in government spending. The marginal effect of this policy on the modified output gap and the inflation rate in period  $T$  is  $(\partial x_L^T / \partial g_L^T) = 1 - \Gamma > 0$  and  $(\partial \pi_L^T / \partial g_L^T) = \kappa(1 - \Gamma) > 0$ . In words, the unexpected and temporary government spending stimulus raises the modified output gap and inflation in the low-confidence state.

Hence, if expectations do not change, an increase in government spending is expansionary. However, if the discretionary policymaker uses government spending systematically, i.e. if  $\lambda_g < \infty$ , then agents anticipate government spending to be increased when the economy transitions from the high-confidence state to the low-confidence state, and to stay at this higher level for as long as the economy remains in the low-confidence state. We have already seen that this is detrimental for welfare. A discretionary policymaker who decides to raise government spending in the low-confidence state would thus like the private sector to expect the fiscal expansion to be temporary. However, when the economy is still in the low-confidence state in the next period, the discretionary policymaker has an incentive to renege on her promise. A policy announcement of a one-time fiscal stimulus is therefore not credible. In equilibrium, the policymaker keeps government spending high for as long as confidence is low and the forward-looking private sector internalizes this accordingly.

### 6.3.2 A policy paradox

Some of the results presented in this subsection seem to have conflicting implications for the design of fiscal policy in the model with the sunspot shock. Indeed, Propositions 11, 12 and 13 viewed together give rise to a paradox. According to Propositions 12 and 13, the appointment of a fiscally-activist policymaker ( $\lambda_g < \bar{\lambda}_g$ ) reduces welfare in the sunspot equilibrium. But according to Proposition 11, if the appointed policymaker is sufficiently activist—that is, if  $\lambda_g$  is small enough—the sunspot equilibrium ceases to exist.

Figure 12: Non-existence of the sunspot equilibrium in case of sufficient fiscal activism



Note:  $\lambda_g = 0.00012 \ll \bar{\lambda}_g$ . NS marks the no-sunspot equilibrium. Inflation is expressed in annualized terms.

Intuitively, when  $\lambda_g \rightarrow 0$ , the policymaker is willing to do “whatever it takes”—in terms of fiscal policy—to make sure that the weighted sum of inflation and the modified output gap are stabilized. Since the lower bound is not binding when this target criterion is met,  $\lambda_g \rightarrow 0$  rules out the sunspot equilibrium. In this case, the only stationary equilibrium in the model with the sunspot shock is the no-sunspot equilibrium where the shock does not affect agents’ behavior. In the no-sunspot equilibrium, all variables are at target in both confidence states. Figure 12 provides a graphical illustration. For a sufficiently low  $\lambda_g$  the AD curve to the left of the kink becomes flatter than the AS curve and there is only one intersection point left, which is the one associated with the no-sunspot equilibrium.

#### 6.4 Comparison with an exogenous government spending stimulus

In our analysis of fiscal policy, government spending is an endogenous variable set by an optimizing policymaker. A more common approach in the literature on fiscal policy in expectations-driven liquidity traps is to treat the fiscal policy instrument as an exogenous variable (e.g. Mertens and Ravn, 2014; Bilbiie, 2018). We therefore close the section with a brief comparison of these two approaches.

Suppose, government spending follows an exogenous process that is perfectly correlated with the sunspot shock, i.e.  $g_t = g_L$  if  $\xi_t = \xi_L$  and  $g_t = g_H$  if  $\xi_t = \xi_H$ , where  $g_L > g_H = 0$ . For this case, the definition of the sunspot equilibrium has to be slightly modified.

**Definition 5** *The sunspot equilibrium in the sunspot-shock model with **exogenous** fiscal policy is given by a vector  $\{x_H, \pi_H, i_H, x_L, \pi_L, i_L\}$  that solves the system of linear equations (33), (34), (36),*



(37), (38), (40), and satisfies the inequality constraints (41) and (42).

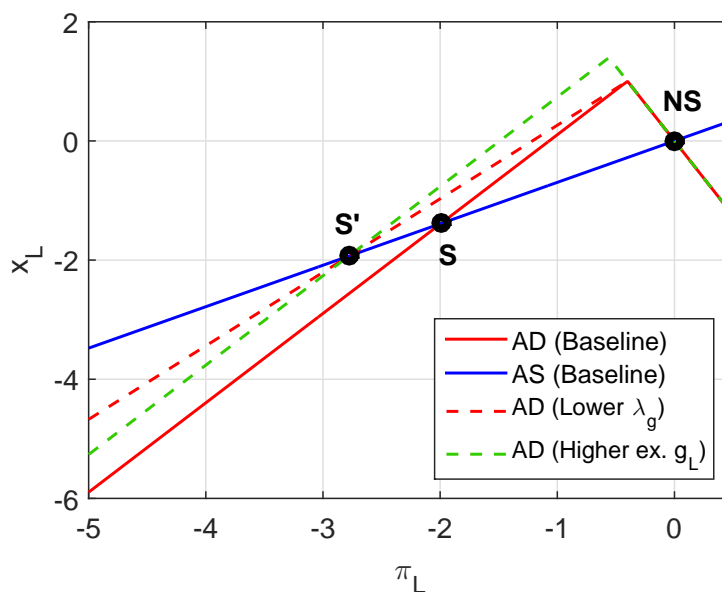
The low-confidence-state AD and AS curves in the sunspot-shock model with exogenous fiscal policy and an absorbing high-confidence state ( $p_H = 1$ ) are then given by

$$\mathbf{AD\text{-}sunspot\ g\text{-}ex:} \quad x_L = \min \left[ \left( \frac{\sigma}{1-p_L} r^n + (1-\Gamma)g_L \right) + \frac{\sigma p_L}{1-p_L} \pi_L, -\frac{\kappa}{\bar{\lambda}} \pi_L \right] \quad (52)$$

$$\mathbf{AS\text{-}sunspot\ g\text{-}ex:} \quad x_L = \frac{1-\beta p_L}{\kappa} \pi_L \quad (53)$$

Figure 13 compares the effects of a decline in  $\lambda_g$ —which in equilibrium results in an increase in  $g_L$ —on the AD-AS curves in the model with endogenous fiscal policy to those of an increase in  $g_L$  in the model with exogenous fiscal policy. For the baseline, it is assumed that  $\lambda_g = \infty$  for the model with endogenous fiscal policy and  $g_L = 0$  for the model with exogenous fiscal policy. Hence, in the baseline, the AD curve is the same whether fiscal policy is endogenous or exogenous. The sunspot equilibrium in the baseline is represented by the intersection of the AD curve (red solid line) with the AS curve (blue solid line), marked by point  $S$ . When considering an increase in low-state government spending in the model with exogenous fiscal policy, we calibrate the stimulus to be of the same size as the equilibrium increase in government spending that occurs in the model with endogenous fiscal policy in response to the reduction in  $\lambda_g$ .

Figure 13: Low-confidence state AD-AS curves: Endogenous vs exogenous fiscal policy



Note: Solid lines:  $\lambda_g = \infty$  (fiscal policy endogenous),  $g_L = 0$  (fiscal policy exogenous); red dashed line:  $\lambda_g = \bar{\lambda}_g/10$  (fiscal policy endogenous); green dashed line:  $g_L = 4$  (fiscal policy exogenous). Inflation is expressed in annualized terms.

As discussed before, in the model with endogenous fiscal policy a change in  $\lambda_g$  affects the slope of the AD curve to the left of the kink. A reduction in  $\lambda_g$  makes the AD curve flatter (red dashed

line). In the model with exogenous fiscal policy, a change in low-state government spending instead affects the intercept term in the AD curve and results in a level shift to the left of the kink. An increase in low-state government spending shifts the AD curve upwards (green dashed line). While the sunspot equilibria in the two models are observationally equivalent by construction (see point  $S'$ ), the two AD curves are not observationally equivalent.

## 7 Conclusion

Expectations-driven liquidity traps differ from fundamental-driven liquidity traps, both, in terms of their basic properties as well as in terms of their implications for the design of desirable monetary and fiscal stabilization policies. Moreover, policy design becomes more complicated when liquidity trap episodes are caused by changes in agents' confidence than when they are caused by changes in the economy's fundamentals.

The occurrence of occasional fundamental-driven liquidity trap events makes it desirable for society to assign a strict inflation-conservative objective function or an objective function with a strictly positive inflation target—high enough to generate positive inflation in the high state—to the central bank. No such clear-cut policy recommendations can be derived in case of expectations-driven liquidity trap events. The optimal inflation target may be negative or positive depending on the structural characteristics of the economy and the average persistence of the confidence states. Likewise, the optimal relative weight on inflation in the central bank's objective function may be smaller or larger than the weight that society puts on inflation stabilization. However, strict inflation conservatism or an inflation target that generates positive inflation in the high state are never optimal in the sunspot equilibrium.

Turning to fiscal policy, the use of optimal discretionary government spending is welfare-improving in the case of fundamental-driven liquidity traps and welfare-reducing in the case of expectations-driven liquidity traps. Nevertheless, it may be desirable to assign an explicit role to fiscal policy in the face of the latter too, for the appointment of a sufficiently fiscally-activist policymaker—i.e. one who puts a sufficiently small relative weight on stabilization of government expenditures—eliminates the sunspot equilibrium.

An obvious avenue for future work is the extension of the analysis to other policy frameworks that have featured prominently in the ongoing policy debate but have been omitted in this paper in case they are incompatible with a closed-form solution of the model.

## References

- ADAM, K. AND R. M. BILLI (2006): “Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates,” *Journal of Money, Credit and Banking*, 38, 1877–1905.
- (2007): “Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates,” *Journal of Monetary Economics*, 54, 728–752.
- ARMENTER, R. (2018): “The Perils of Nominal Targets,” *Review of Economic Studies*, 85, 50–86.
- ARUOBA, S. B., P. CUBA-BORDA, AND F. SCHORFHEIDE (2018): “Macroeconomic Dynamics Near the ZLB: A Tale of Two Countries,” *Review of Economic Studies*, 85, 87–118.
- BENHABIB, J., S. SCHMITT-GROHÉ, AND M. URIBE (2001): “The Perils of Taylor Rules,” *Journal of Economic Theory*, 96, 40–69.
- (2002): “Avoiding Liquidity Traps,” *Journal of Political Economy*, 110, 535–563.
- BILBIIE, F. O. (2018): “Neo-Fisherian Policies and Liquidity Traps,” CEPR Discussion Papers 13334, C.E.P.R. Discussion Papers.
- BILLI, R. M. (2017): “A Note On Nominal Gdp Targeting And The Zero Lower Bound,” *Macroeconomic Dynamics*, 21, 2138–2157.
- BOUAKEZ, H., M. GUILLARD, AND J. ROULLEAU-PASDELOUP (2016): “The Optimal Composition of Public Spending in a Deep Recession,” Cahiers de Recherches Economiques du Département d’Econométrie et d’Economie politique (DEEP) 16.09, Université de Lausanne, Faculté des HEC, DEEP.
- CLARIDA, R. H. (2019): “The Federal Reserve’s Review of Its Monetary Policy Strategy, Tools, and Communication Practices,” *2019 U.S. Monetary Policy Forum, sponsored by the Initiative on Global Markets at the University of Chicago Booth School of Business, New York, New York, February 22, 2019*.
- COIBION, O., Y. GORODNICHENKO, AND J. WIELAND (2012): “The Optimal Inflation Rate in New Keynesian Models: Should Central Banks Raise Their Inflation Targets in Light of the Zero Lower Bound?” *Review of Economic Studies*, 79, 1371–1406.
- COYLE, P. AND T. NAKATA (2018): “Optimal Inflation Target with Expectations-Driven Liquidity Traps,” Manuscript.
- EGGERTSSON, G. B. (2006): “The Deflation Bias and Committing to Being Irresponsible,” *Journal of Money, Credit and Banking*, 38, 283–321.
- EGGERTSSON, G. B. AND M. WOODFORD (2003): “The Zero Bound on Interest Rates and Optimal Monetary Policy,” *Brookings Papers on Economic Activity*, 34, 139–235.

- (2006): “Optimal Monetary and Fiscal Policy in a Liquidity Trap,” in *NBER International Seminar on Macroeconomics 2004*, National Bureau of Economic Research, Inc, NBER Chapters, 75–144.
- FEDERAL RESERVE BOARD OF GOVERNORS (2018): “Press release: Federal Reserve to review strategies, tools, and communication practices it uses to pursue its mandate of maximum employment and price stability, November 15, 2018,” available at <https://www.federalreserve.gov/newsevents/pressreleases/monetary20181115a.htm>.
- GALÍ, J. (2015): *Monetary Policy, Inflation, and the Business Cycle*, Princeton: Princeton University Press.
- HURSEY, T. AND A. L. WOLMAN (2010): “Monetary policy and global equilibria in a production economy,” *Economic Quarterly*, 317–337.
- JAROCIŃSKI, M. AND B. MAĆKOWIAK (2018): “Monetary-fiscal interactions and the euro area’s malaise,” *Journal of International Economics*, 112, 251 – 266.
- JUNG, T., Y. TERANISHI, AND T. WATANABE (2005): “Optimal Monetary Policy at the Zero-Interest-Rate Bound,” *Journal of Money, Credit and Banking*, 37, 813–835.
- MERTENS, K. AND M. O. RAVN (2014): “Fiscal Policy in an Expectations-Driven Liquidity Trap,” *The Review of Economic Studies*, 81, 1637–1667.
- MERTENS, T. M. AND J. C. WILLIAMS (2019): “Monetary Policy Frameworks and the Effective Lower Bound on Interest Rates,” Staff Reports 877, Federal Reserve Bank of New York.
- NAKATA, T. (2016): “Optimal Fiscal and Monetary Policy with Occasionally Binding Zero Bound Constraints,” *Journal of Economic Dynamics and Control*, 73, 220–240.
- (2018): “Reputation and Liquidity Traps,” *Review of Economic Dynamics*, 28, 252–268.
- NAKATA, T. AND S. SCHMIDT (2018): “Conservatism and Liquidity Traps,” *Journal of Monetary Economics*, forthcoming.
- (2019): “Gradualism and Liquidity Traps,” *Review of Economic Dynamics*, 31, 182 – 199.
- NAKOV, A. (2008): “Optimal and Simple Monetary Policy Rules with Zero Floor on the Nominal Interest Rate,” *International Journal of Central Banking*, 4, 73–127.
- ROULLEAU-PASDELOUP, J. (2019): “Optimal Monetary Policy, Determinacy and Policy Puzzles at the Effective Lower Bound,” Manuscript.
- SCHMIDT, S. (2013): “Optimal Monetary and Fiscal Policy with a Zero Bound on Nominal Interest Rates,” *Journal of Money, Credit and Banking*, 45, 1335–1350.

- (2016): “Lack of Confidence, the Zero Lower Bound, and the Virtue of Fiscal Rules,” *Journal of Economic Dynamics and Control*, 70, 36–53.
- (2017): “Fiscal Activism and the Zero Nominal Interest Rate Bound,” *Journal of Money, Credit and Banking*, 49, 695–732.
- SCHMITT-GROHÉ, S. AND M. URIBE (2014): “Liquidity Traps: An Interest-Rate-Based Exit Strategy,” *The Manchester School*, 82, 1–14.
- (2017): “Liquidity Traps and Jobless Recoveries,” *American Economic Journal: Macroeconomics*, 9, 165–204.
- SUGO, T. AND K. UEDA (2008): “Eliminating a Deflationary Trap through Superinertial Interest Rate Rules,” *Economics Letters*, 100, 119–122.
- WERNING, I. (2011): “Managing a Liquidity Trap: Monetary and Fiscal Policy,” NBER Working Papers 17344, National Bureau of Economic Research, Inc.
- WILKINS, C. A. (2018): “Choosing the Best Monetary Policy Framework for Canada,” *Remarks at the McGill University, Max Bell School of Public Policy, Montreal, Quebec, November 20, 2018*.
- WOODFORD, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton: Princeton University Press.

# Appendix

## A Sunspot equilibrium in the model without fiscal policy

### A.1 Proof of Proposition 1

To proof Proposition 1 on the necessary and sufficient conditions for existence of the sunspot equilibrium, it is useful to proceed in four steps. Each step is associated with an auxiliary proposition.

Let

$$A := -\beta\lambda(1 - p_H), \quad (\text{A.1})$$

$$B := \kappa^2 + \lambda(1 - \beta p_H), \quad (\text{A.2})$$

$$C := \frac{(1 - p_L)}{\sigma\kappa}(1 - \beta p_L + \beta(1 - p_H)) - p_L, \quad (\text{A.3})$$

$$D := -\frac{(1 - p_L)}{\sigma\kappa}(1 - \beta p_L + \beta(1 - p_H)) - (1 - p_L) = -1 - C, \quad (\text{A.4})$$

and

$$E := AD - BC. \quad (\text{A.5})$$

**Proposition A.1** *There exists a vector  $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$  that solves the system of linear equations (8)–(13).*

**Proof:** Rearranging the system of equations (8)–(13) and eliminating  $y_H$  and  $y_L$ , we obtain two unknowns for  $\pi_H$  and  $\pi_L$  in two equations

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \begin{bmatrix} \kappa^2\pi^* \\ r^n \end{bmatrix}. \quad (\text{A.6})$$

For what follows, it is useful to show that  $E = 0$  is generically inconsistent with existence of the sunspot equilibrium. Since  $B > 0$ , we can always write  $\pi_H = \kappa^2/B\pi^* - A/B\pi_L$ . Plugging this into  $C\pi_L + D\pi_H = r^n$  and multiplying both sides by  $B$ , we get  $D\kappa^2\pi^* - E\pi_L = Br^n$ . Since the right-hand side of this equation is strictly positive,  $E = 0$  is inconsistent with the existence of the sunspot equilibrium for generic  $\pi^*$ .

Hence, we can invert the matrix on the left-hand-side of (A.6)

$$\begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \frac{1}{AD - BC} \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} \kappa^2\pi^* \\ r^n \end{bmatrix}. \quad (\text{A.7})$$

Thus,

$$\pi_H = -\frac{C\kappa^2}{E}\pi^* + \frac{A}{E}r^n \quad (\text{A.8})$$

and

$$\pi_L = \frac{D\kappa^2}{E}\pi^* - \frac{B}{E}r^n. \quad (\text{A.9})$$

From the Phillips curves in both states, we obtain

$$y_H = \frac{\kappa(\beta(1-p_H) - (1-\beta)C)}{E}\pi^* + \frac{\beta\kappa(1-p_H)}{E}r^n \quad (\text{A.10})$$

and

$$y_L = \frac{\kappa(\beta p_L - 1 - (1-\beta)C)}{E}\pi^* - \frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L + \beta(1-p_H))\lambda}{\kappa E}r^n. \quad (\text{A.11})$$

**Proposition A.2** *Suppose equations (8)–(13) are satisfied. Then  $\lambda y_L + (\kappa\pi_L - \pi^*) < 0$  if and only if (i)  $E > 0$  and  $\pi^* > -\frac{\kappa^2 + \lambda(1-\beta)}{\kappa^2}r^n$  or (ii)  $E < 0$  and  $\pi^* < -\frac{\kappa^2 + \lambda(1-\beta)}{\kappa^2}r^n$ .*

**Proof:** Using (A.9) and (A.11), we have

$$\lambda y_L + \kappa(\pi_L - \pi^*) = -\frac{\kappa^2 + \lambda(1-\beta p_L + \beta(1-p_H))}{E}\kappa\left(\pi^* + \frac{\kappa^2 + \lambda(1-\beta)}{\kappa^2}r^n\right). \quad (\text{A.12})$$

Notice that  $(\kappa^2 + \lambda(1-\beta p_L + \beta(1-p_H)))\kappa > 0$ , and  $\frac{\kappa^2 + \lambda(1-\beta)}{\kappa^2}r^n > 0$ . Thus, if  $E > 0$  and  $\pi^* > -\frac{\kappa^2 + \lambda(1-\beta)}{\kappa^2}r^n$ , then  $\lambda y_L + \kappa(\pi_L - \pi^*) < 0$ . Similarly, if  $E < 0$  and  $\pi^* < -\frac{\kappa^2 + \lambda(1-\beta)}{\kappa^2}r^n$ , then  $\lambda y_L + \kappa(\pi_L - \pi^*) < 0$ .

**Proposition A.3** *Suppose equations (8)–(13) are satisfied,  $E > 0$  and  $\pi^* > -\frac{\kappa^2 + \lambda(1-\beta)}{\kappa^2}r^n$ . Then  $i_H > 0$  if and only if  $p_L - (1-p_H) - \frac{1-p_L+1-p_H}{\kappa\sigma}(1-\beta p_L + \beta(1-p_H)) > 0$ .*

**Proof:**  $i_H$  is given by

$$\begin{aligned} i_H &= \frac{1-p_H}{\sigma}(y_L - y_H) + p_H\pi_H + (1-p_H)\pi_L + r^n \\ &= \frac{\left(p_L - (1-p_H) - \frac{1-p_L+1-p_H}{\kappa\sigma}(1-\beta p_L + \beta(1-p_H))\right)\kappa^2}{E}\left(\pi^* + \frac{\kappa^2 + \lambda(1-\beta)}{\kappa^2}r^n\right), \end{aligned} \quad (\text{A.13})$$

where in the second row we made use of (A.8)–(A.11).

**Proposition A.4** *Suppose equations (8)–(13) are satisfied,  $E < 0$  and  $\pi^* < -\frac{\kappa^2 + \lambda(1-\beta)}{\kappa^2}r^n$ . Then  $i_H < 0$ .*

**Proof:** First, substitute equations (A.1), (A.2), and (A.4) into equation (A.5) to obtain

$$E = \beta\lambda(1 - p_H) - (\kappa^2 + \lambda(1 - \beta))C. \quad (\text{A.14})$$

Hence,  $E < 0$  implies  $C > 0$ .

**Corollary A.1**  $C < 0$  implies  $E > 0$ .

Next, note that

$$p_L - (1 - p_H) - \frac{1 - p_L + 1 - p_H}{\kappa\sigma} (1 - \beta p_L + \beta(1 - p_H)) = -C - (1 - p_H) \frac{1 - \beta p_L + \beta(1 - p_H) + \kappa\sigma}{\kappa\sigma}.$$

Hence,  $C > 0$  implies  $p_L - (1 - p_H) - \frac{1 - p_L + 1 - p_H}{\kappa\sigma} (1 - \beta p_L + \beta(1 - p_H)) < 0$ .

**Corollary A.2**  $p_L - (1 - p_H) - \frac{1 - p_L + 1 - p_H}{\kappa\sigma} (1 - \beta p_L + \beta(1 - p_H)) > 0$  implies  $C < 0$ .

From equation (A.13), it follows that  $p_L - (1 - p_H) - \frac{1 - p_L + 1 - p_H}{\kappa\sigma} (1 - \beta p_L + \beta(1 - p_H)) < 0$ ,  $E < 0$  and  $\pi^* < -\frac{\kappa^2 + \lambda(1 - \beta)}{\kappa^2} r^n$  imply  $i_H < 0$ .

We are now ready to proof Proposition 1. For notational convenience, define

$$\Omega(p_L, p_H, \kappa, \sigma, \beta) \equiv p_L - (1 - p_H) - \frac{1 - p_L + 1 - p_H}{\kappa\sigma} (1 - \beta p_L + \beta(1 - p_H)). \quad (\text{A.15})$$

**Proof of “if” part:** Suppose that  $\Omega(\cdot) > 0$  and  $\pi^* > -\frac{\kappa^2 + \lambda(1 - \beta)}{\kappa^2} r^n$ . According to Proposition A.1 there exists a vector  $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$  that solves equations (8)–(13). According to Corollary A.2,  $\Omega(\cdot) > 0$  implies  $C < 0$ . According to Corollary A.1,  $C < 0$  implies  $E > 0$ . According to Proposition A.2,  $E > 0$  and  $\pi^* > -\frac{\kappa^2 + \lambda(1 - \beta)}{\kappa^2} r^n$  imply  $\lambda y_L + \kappa(\pi_L - \pi^*) < 0$ . According to Proposition A.3, given  $E > 0$  and  $\pi^* > -\frac{\kappa^2 + \lambda(1 - \beta)}{\kappa^2} r^n$ ,  $\Omega(\cdot) > 0$  implies  $i_H > 0$ .

**Proof of “only if” part:** Suppose that the vector  $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$  solves (8)–(13), and satisfies  $\lambda y_L + \kappa(\pi_L - \pi^*) < 0$  and  $i_H > 0$ . According to Proposition A.2,  $\lambda y_L + \kappa(\pi_L - \pi^*) < 0$  implies that either (i)  $E > 0$  and  $\pi^* > -\frac{\kappa^2 + \lambda(1 - \beta)}{\kappa^2} r^n$  or (ii)  $E < 0$  and  $\pi^* < -\frac{\kappa^2 + \lambda(1 - \beta)}{\kappa^2} r^n$ . According to Proposition A.4, (ii) is inconsistent with  $i_H > 0$ . Hence,  $E > 0$  and  $\pi^* > -\frac{\kappa^2 + \lambda(1 - \beta)}{\kappa^2} r^n$ . According to Proposition A.3, given  $E > 0$  and  $\pi^* > -\frac{\kappa^2 + \lambda(1 - \beta)}{\kappa^2} r^n$ ,  $i_H > 0$  implies  $\Omega(\cdot) > 0$ .



## A.2 Proof of Proposition 2

The allocations and prices in the sunspot equilibrium are given by

$$\begin{aligned}\pi_L &= -\frac{(C+1)\kappa^2}{E}\pi^* - \frac{\kappa^2 + \lambda(1-\beta p_H)}{E}r^n \\ y_L &= \frac{\kappa(\beta p_L - 1 - (1-\beta)C)}{E}\pi^* - \frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L + \beta(1-p_H))\lambda}{\kappa E}r^n \\ \pi_H &= -\frac{C\kappa^2}{E}\pi^* - \frac{\beta\lambda(1-p_H)}{E}r^n \\ y_H &= \frac{\kappa(\beta(1-p_H) - (1-\beta)C)}{E}\pi^* + \frac{\beta\kappa(1-p_H)}{E}r^n\end{aligned}$$

Assuming  $\pi^* = 0$  and  $\lambda > 0$ , it holds

$$\begin{aligned}\pi_L &= -\frac{\kappa^2 + \lambda(1-\beta p_H)}{E}r^n < 0 \\ y_L &= -\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L + \beta(1-p_H))\lambda}{\kappa E}r^n < 0 \\ i_L &= 0 \\ \pi_H &= -\frac{\beta\lambda(1-p_H)}{E}r^n < 0 \\ y_H &= \frac{\beta\kappa(1-p_H)}{E}r^n > 0 \\ i_H &= \left(1 - \frac{(1-p_H)}{E} \left( \frac{(\kappa^2 + \lambda(1-\beta))(1-\beta p_L + \beta(1-p_H))}{\kappa\sigma} + \kappa^2 + \lambda \right)\right) r^n > 0\end{aligned}$$

## A.3 Proof of Proposition 3

Assuming  $\pi^* = 0$ , it holds

$$\begin{aligned}\frac{\partial \pi_L}{\partial p_L} &= \frac{(\kappa^2 + \lambda(1-\beta p_H))(\kappa^2 + \lambda(1-\beta)) [1 + (\kappa\sigma)^{-1}(1 + \beta - 2\beta p_L + \beta(1-p_H))]}{E^2} r^n > 0 \\ \frac{\partial y_L}{\partial p_L} &= \frac{\beta E + [(1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L + \beta(1-p_H))\lambda] [1 + (\kappa\sigma)^{-1}(1 + \beta - 2\beta p_L + \beta(1-p_H))]}{\kappa E^2} \\ &\quad \times (\kappa^2 + \lambda(1-\beta)) r^n > 0,\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \pi_H}{\partial p_L} &= \frac{\beta\lambda(1-p_H)(\kappa^2 + \lambda(1-\beta)) [1 + (\kappa\sigma)^{-1}(1 + \beta - 2\beta p_L + \beta(1-p_H))]}{E^2} r^n \geq 0 \\ \frac{\partial y_H}{\partial p_L} &= -\frac{\beta\kappa(1-p_H)(\kappa^2 + \lambda(1-\beta)) [1 + (\kappa\sigma)^{-1}(1 + \beta - 2\beta p_L + \beta(1-p_H))]}{E^2} r^n \leq 0\end{aligned}$$

## A.4 Proof of Proposition 4

Assuming  $\pi^* = 0$ , it holds

$$\begin{aligned}\frac{\partial \pi_L}{\partial p_H} &= \frac{(1-p_L)\beta(\kappa^2 + \lambda(1-\beta))}{\kappa\sigma E^2} [\kappa^2 - (\kappa\sigma + \beta(1-p_L))\lambda] r^n \\ \frac{\partial y_L}{\partial p_H} &= \frac{(1-p_L)\beta(\kappa^2 + \lambda(1-\beta))}{\kappa E^2} \left[ \frac{\kappa^2(1-\beta p_L)}{\kappa\sigma} - \lambda \right] r^n,\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \pi_H}{\partial p_H} &= \frac{\beta\lambda(\kappa^2 + \lambda(1-\beta))}{E^2} \left[ p_L - \frac{1-p_L}{\kappa\sigma}(1-\beta p_L) \right] r^n \\ \frac{\partial y_H}{\partial p_H} &= -\frac{\beta\kappa(\kappa^2 + \lambda(1-\beta))}{E^2} \left[ p_L - \frac{1-p_L}{\kappa\sigma}(1-\beta p_L) \right] r^n\end{aligned}$$

Note that the second term in square brackets on the right-hand side of the above equations equals  $\Omega(p_L, 1, \kappa, \sigma, \beta)$ . Furthermore, note that  $\frac{\partial \Omega(\cdot)}{\partial p_H} = 1 + \frac{1-\beta p_L + \beta(1-\beta p_H)}{\kappa\sigma} + \beta \left( \frac{1-p_L+1-p_H}{\kappa\sigma} \right) > 0$ . Hence,  $\Omega(p_L, 1, \kappa, \sigma, \beta) > 0$  is a necessary condition for equilibrium existence for any  $p_H \geq 0$ . Consequently, the second term in square brackets is strictly positive, and it holds  $\frac{\partial \pi_H}{\partial p_H} > 0$  and  $\frac{\partial y_H}{\partial p_H} < 0$ .

## A.5 Proof of Proposition 5

Assuming  $\pi^* = 0$ , it holds

$$\begin{aligned}\frac{\partial \pi_L}{\partial \kappa} &= -\frac{2\beta\kappa\lambda(1-p_H)(1+C) - (\kappa^2 + \lambda(1-\beta + \beta(1-p_H))) (\kappa^2 + \lambda(1-\beta)) \frac{1-p_L}{\kappa\sigma} \frac{1-\beta p_L + \beta(1-p_H)}{\kappa}}{E^2} r^n \\ \frac{\partial y_L}{\partial \kappa} &= -\frac{1}{(\kappa E)^2} \left[ \beta\lambda\kappa^2(1-p_L)(1-p_H) + 2\beta\lambda\kappa^2(1-\beta)(1-p_H)C - (\kappa^2 + \lambda(1-\beta))^2 (1-\beta p_L)p_L \right. \\ &\quad \left. - \lambda^2(1-\beta)^2\beta(1-p_H)p_L - (1-\beta p_L + \beta(1-p_H))(1-\beta)\beta\lambda(1-p_H) \right] r^n.\end{aligned}$$

For  $p_H = 1$ , we get

$$\frac{\partial \pi_L}{\partial \kappa} = \frac{(\kappa^2 + \lambda(1-\beta))^2 (1-p_L)(1-\beta p_L)}{(\kappa E)^2 \sigma} r^n > 0 \quad (\text{A.16})$$

$$\frac{\partial y_L}{\partial \kappa} = \frac{(\kappa^2 + \lambda(1-\beta))^2 (1-\beta p_L)p_L}{(\kappa E)^2} r^n > 0. \quad (\text{A.17})$$

## A.6 Proof of Proposition 6

Suppose  $\pi^* = 0$  and  $p_H < 1$ . It holds

$$\begin{aligned}
\frac{\partial \pi_L}{\lambda} &= \frac{\beta \kappa^2 (1 - p_H)(1 - p_L) \kappa \sigma + (1 - \beta p_L + \beta(1 - p_H))}{E^2 \kappa \sigma} r^n > 0 \\
\frac{\partial y_L}{\partial \lambda} &= \frac{\beta \kappa (1 - p_H)(1 - p_L) \kappa \sigma + (1 - \beta)(1 - \beta p_L + \beta(1 - p_H))}{E^2 \kappa \sigma} r^n > 0 \\
\frac{\partial \pi_H}{\partial \lambda} &= - \frac{\beta \kappa^2 (1 - p_H)}{E^2} \left[ \Omega(p_L, p_H, \kappa, \sigma, \beta) + (1 - p_H) \frac{\kappa \sigma + (1 - \beta p_L + \beta(1 - p_H))}{\kappa \sigma} \right] r^n < 0 \\
\frac{\partial y_H}{\partial \lambda} &= - \frac{\beta \kappa (1 - p_H)}{E^2} \left[ (1 - \beta) \Omega(p_L, p_H, \kappa, \sigma, \beta) + (1 - p_H) \frac{\kappa \sigma + (1 - \beta)(1 - \beta p_L + \beta(1 - p_H))}{\kappa \sigma} \right] r^n < 0
\end{aligned}$$

## A.7 Proof of Proposition 7

Note first that

$$EV = - \frac{1}{1 - \beta} \frac{1}{2} \left[ \frac{1 - p_L}{1 - p_L + 1 - p_H} (\pi_H^2 + \bar{\lambda} y_H^2) + \frac{1 - p_H}{1 - p_L + 1 - p_H} (\pi_L^2 + \bar{\lambda} y_L^2) \right], \quad (\text{A.18})$$

where  $V$  is defined in equation (3).

Assuming  $\pi^* = 0$ , the partial derivative of  $EV$  with respect to  $\lambda$  is

$$\begin{aligned}
\frac{\partial EV}{\partial \lambda} &= \frac{\beta ((1 - p_H) r^n)^2}{(1 - \beta)(1 - p_L + 1 - p_H) E^3} \left\{ \left[ \beta \kappa^2 (1 - p_L) C + \kappa^2 (1 - \beta p_H)(C + 1) \right. \right. \\
&\quad \left. \left. + \bar{\lambda} (1 - \beta)(1 - \beta p_L + \beta(1 - p_H)) ((1 - \beta)(C + 1) + \beta(1 - p_L)) \right] \lambda \right. \\
&\quad \left. + \beta \kappa^2 [(1 - p_L)(1 - p_H) \beta \bar{\lambda} - (1 - p_L)(1 - \beta) C \bar{\lambda}] + \kappa^4 (C + 1) \right. \\
&\quad \left. + \bar{\lambda} (1 - \beta p_L) \kappa^2 ((1 - \beta)(C + 1) + \beta(1 - p_L)) \right\}.
\end{aligned}$$

Note that since  $(C + 1) > 0$  and  $C < 0$ , all terms in curly brackets are positive except for the very first one,  $\beta \kappa^2 (1 - p_L) C < 0$ . Also note that since in the sunspot equilibrium  $E > 0$ , the term in front of the curly brackets is positive for any  $\lambda \geq 0$ . Since the only negative term in curly brackets is multiplied by  $\lambda$ ,  $\frac{\partial EV}{\partial \lambda} |_{\lambda=0} > 0$ , and therefore  $\lambda^* > 0$ .

Furthermore, if

$$\kappa^2 \beta (1 - p_L) C + \kappa^2 (1 - \beta p_H)(C + 1) + \bar{\lambda} (1 - \beta) (1 - \beta p_L + \beta(1 - p_H)) ((C + 1)(1 - \beta) + \beta(1 - p_L)) \geq 0,$$

then  $\frac{\partial EV}{\partial \lambda} > 0$  for all  $\lambda \geq 0$ . Hence, in this case no interior solution for  $\lambda^*$  exists and  $\lambda^* = \infty$ .

If instead

$$\kappa^2\beta(1-p_L)C + \kappa^2(1-\beta p_H)(C+1) + \bar{\lambda}(1-\beta)(1-\beta p_L + \beta(1-p_H))((C+1)(1-\beta) + \beta(1-p_L)) < 0,$$

then

$$\lambda^* = -\frac{\beta\kappa^2[(1-p_L)(1-p_H)\beta\bar{\lambda} - (1-p_L)(1-\beta)C\bar{\lambda}] + \kappa^4(C+1) + \bar{\lambda}(1-\beta p_L)\kappa^2((1-\beta)(C+1) + \beta(1-p_L))}{\kappa^2\beta(1-p_L)C + \kappa^2(1-\beta p_H)(C+1) + \bar{\lambda}(1-\beta)(1-\beta p_L + \beta(1-p_H))((C+1)(1-\beta) + \beta(1-p_L))}$$

[TO BE ADDED: SECOND ORDER CONDITION]

In this case,  $\lambda^* > \bar{\lambda}$  if

$$(\beta\kappa)^2(1-p_L)\bar{\lambda}\underbrace{(C+1-p_H)}_{<0} + \kappa^2(\kappa^2 + (1-\beta p_H)\bar{\lambda})(C+1) + (\kappa^2(1-\beta p_L) + (1-\beta)\bar{\lambda}(1-\beta p_L + \beta(1-p_H)))(\beta(1-p_L) + (1-\beta)(C+1))\bar{\lambda} > 0$$

## A.8 Proof of Proposition 8

Keeping in mind that  $-1 < C < 0$  in the sunspot equilibrium, it holds,

$$\frac{\partial\pi_L}{\partial\pi^*} = -\frac{C+1}{E}\kappa^2 < 0 \quad (\text{A.19})$$

$$\frac{\partial y_L}{\partial\pi^*} = -\frac{\beta(1-p_L) + (1-\beta)(C+1)}{E}\kappa < 0, \quad (\text{A.20})$$

and

$$\frac{\partial\pi_H}{\partial\pi^*} = -\frac{C}{E}\kappa^2 > 0 \quad (\text{A.21})$$

$$\frac{\partial y_H}{\partial\pi^*} = \frac{\beta(1-p_H) - (1-\beta)C}{E}\kappa > 0. \quad (\text{A.22})$$

## A.9 Proof of Lemma 1

If  $\pi^0$  exists, it holds  $-\frac{C\kappa^2}{E}\pi^0 - \frac{\beta\lambda(1-p_H)}{E}r^n = 0$ . Solving for  $\pi^0$ , one obtains

$$\pi^0 = -\frac{\beta\lambda(1-p_H)}{C\kappa^2}r^n, \quad (\text{A.23})$$

where  $C < 0$ , and hence  $\pi^0 > 0$ .

## A.10 Proof of Proposition 9

The partial derivative of EV with respect to  $\pi^*$  is

$$\begin{aligned} \frac{\partial EV}{\partial \pi^*} = & - \frac{1}{(1-\beta)(1-p_L+1-p_H)E^2} \left\{ \left[ (\kappa^2 + \bar{\lambda}(1-\beta)^2) ((1-p_H)(C+1)^2 + (1-p_L)C^2) \right. \right. \\ & + \bar{\lambda}\beta(1-p_H)(1-p_L)(1-\beta p_L + 1 - \beta p_H) \left. \right] \kappa^2 \pi^* + \left[ \bar{\lambda}(\kappa^2 + \bar{\lambda}(1-\beta)) (1-\beta p_L + \beta(1-p_H)) \right. \\ & (\beta(1-p_L) + (1-\beta)(C+1)) + (\kappa^2 + \bar{\lambda}(1-\beta + \beta^2(1-p_L+1-p_H))) \kappa^2(C+1) \\ & \left. \left. - (\beta\kappa)^2 \bar{\lambda}(1-p_L) \right] (1-p_H)r^n \right\}. \end{aligned}$$

Note that all terms in the square brackets which are multiplied by  $\pi^*$  are positive. In the square brackets which are multiplied by  $r^n$  all terms are positive except for the last one,  $-(\beta\kappa)^2 \bar{\lambda}(1-p_L) < 0$ .

The first-order necessary condition for the welfare-maximizing inflation target is  $\frac{\partial EV}{\partial \pi^*} = 0$ . Solving for  $\pi^*$ , one obtains

$$\pi^{**} = - \frac{1-p_H}{\kappa^2} \frac{\bar{\lambda}(\kappa^2 + \bar{\lambda}(1-\beta)) (1-\beta p_L + \beta(1-p_H)) (\beta(1-p_L) + (1-\beta)(C+1)) + (\kappa^2 + \bar{\lambda}(1-\beta + \beta^2(1-p_L+1-p_H))) \kappa^2(C+1) - (\beta\kappa)^2 \bar{\lambda}(1-p_L)}{(\kappa^2 + \bar{\lambda}(1-\beta)^2) ((1-p_H)(C+1)^2 + (1-p_L)C^2) + \bar{\lambda}\beta(1-p_H)(1-p_L)(1-\beta p_L + 1 - \beta p_H)} r^n$$

[TO BE ADDED: SECOND ORDER CONDITION]

Note that  $\pi^{**} > \frac{\kappa^2 + \bar{\lambda}(1-\beta)}{\kappa^2} r^n$  whenever existence condition (22) is satisfied. Specifically,  $\pi^{**} > \frac{\kappa^2 + \bar{\lambda}(1-\beta)}{\kappa^2} r^n$  if and only if

$$\begin{aligned} & (\kappa^2 + \bar{\lambda}(1-\beta)) \{ (\kappa^2 + \bar{\lambda}(1-\beta)^2) C [(1-p_L+1-p_H)C+1-p_H] \} \\ & > [ (\kappa^2 + \bar{\lambda}(1-\beta)) \bar{\lambda}\beta(1-\beta)(1-p_H) + (\beta\kappa)^2 \bar{\lambda}(1-p_H) ] [(1-p_L+1-p_H)C+1-p_H], \end{aligned}$$

where  $(1-p_L+1-p_H)C+1-p_H = -(1-p_L)\Omega(p_L, p_H, \kappa, \sigma, \beta) < 0$ . Hence, the left-hand side of the inequality is positive and the right-hand side is negative, so that the inequality is satisfied.

Next, we show that  $\pi^{**} < \pi^0$ . This requires

$$-\frac{\beta\bar{\lambda}}{C} > - \frac{\bar{\lambda}(\kappa^2 + \bar{\lambda}(1-\beta)) (1-\beta p_L + \beta(1-p_H)) (\beta(1-p_L) + (1-\beta)(C+1)) + (\kappa^2 + \bar{\lambda}(1-\beta + \beta^2(1-p_L+1-p_H))) \kappa^2(C+1) - (\beta\kappa)^2 \bar{\lambda}(1-p_L)}{(\kappa^2 + \bar{\lambda}(1-\beta)^2) ((1-p_H)(C+1)^2 + (1-p_L)C^2) + \bar{\lambda}\beta(1-p_H)(1-p_L)(1-\beta p_L + 1 - \beta p_H)},$$

which can be rewritten as

$$\begin{aligned} & \beta\bar{\lambda}\kappa^2(1-p_L)(1-\beta)C^2 + \beta\bar{\lambda}^2(1-\beta)^2(1-p_L)C^2 + \beta\bar{\lambda}(\kappa^2 + \bar{\lambda}(1-\beta)^2) (1-p_H)(C+1)^2 + (\beta\bar{\lambda})^2(1-p_L)(1-p_H)(1-\beta p_L + 1 - \beta p_H) \\ & > (\beta\kappa)^2 \bar{\lambda}(1-p_L)(1-p_H)C + \kappa^2 (\kappa^2 + \bar{\lambda}(1-\beta p_H)) C + [\kappa^2(1-\beta p_L) + \bar{\lambda}(1-\beta)(1-\beta p_L + 1 - \beta p_H)] [\beta(1-p_L) + (1-\beta)(C+1)] \bar{\lambda}C. \end{aligned}$$

Note that all terms on the left-hand side of the inequality sign are strictly positive and all terms on the right-hand side are strictly negative. This completes the proof.

## A.11 Proof of Proposition 10

Let  $X_{S|\lambda=\bar{\lambda},\pi^*=\hat{\pi}^*}$  denote the outcome of variable  $X \in \{\pi, y\}$  in state  $S \in \{H, L\}$  of the sunspot equilibrium when  $\lambda = \bar{\lambda}$  and  $\pi^* = \hat{\pi}^*$ , and  $X_{S|\lambda=\hat{\lambda},\pi^*=0}$  when  $\lambda = \hat{\lambda}$  and  $\pi^* = 0$ . We need to show that  $X_{S|\lambda=\bar{\lambda},\pi^*=\hat{\pi}^*} = X_{S|\lambda=\hat{\lambda},\pi^*=0}$  for all  $X \times S$  and any  $\hat{\lambda} \geq 0$ .

High-state inflation:

$$\begin{aligned} \pi_{H|\lambda=\bar{\lambda},\pi^*=\hat{\pi}^*} &= - \frac{C\kappa^2}{[\beta\bar{\lambda}(1-p_H) - (\kappa^2 + \bar{\lambda}(1-\beta))C]} \frac{\beta(1-p_H)(\bar{\lambda} - \hat{\lambda})}{[\beta\hat{\lambda}(1-p_H) - (\kappa^2 + \hat{\lambda}(1-\beta))C]} r^n \\ &\quad - \frac{\beta\bar{\lambda}(1-p_H)}{\beta\bar{\lambda}(1-p_H) - (\kappa^2 + \bar{\lambda}(1-\beta))C} r^n \\ &= - \frac{\beta\hat{\lambda}(1-p_H)}{\beta\hat{\lambda}(1-p_H) - (\kappa^2 + \hat{\lambda}(1-\beta))C} r^n \\ &= \pi_{H|\lambda=\hat{\lambda},\pi^*=0} \end{aligned}$$

High-state output:

$$\begin{aligned} y_{H|\lambda=\bar{\lambda},\pi^*=\hat{\pi}^*} &= \frac{\kappa(\beta(1-p_H) - (1-\beta)C)}{[\beta\bar{\lambda}(1-p_H) - (\kappa^2 + \bar{\lambda}(1-\beta))C]} \frac{\beta(1-p_H)(\bar{\lambda} - \hat{\lambda})}{[\beta\hat{\lambda}(1-p_H) - (\kappa^2 + \hat{\lambda}(1-\beta))C]} r^n \\ &\quad + \frac{\beta\kappa(1-p_H)}{\beta\bar{\lambda}(1-p_H) - (\kappa^2 + \bar{\lambda}(1-\beta))C} r^n \\ &= \frac{\beta\kappa(1-p_H)}{\beta\hat{\lambda}(1-p_H) - (\kappa^2 + \hat{\lambda}(1-\beta))C} r^n \\ &= y_{H|\lambda=\hat{\lambda},\pi^*=0} \end{aligned}$$

Low-state inflation:

$$\begin{aligned} \pi_{L|\lambda=\bar{\lambda},\pi^*=\hat{\pi}^*} &= \frac{D\kappa^2}{[\beta\bar{\lambda}(1-p_H) - (\kappa^2 + \bar{\lambda}(1-\beta))C]} \frac{\beta(1-p_H)(\bar{\lambda} - \hat{\lambda})}{[\beta\hat{\lambda}(1-p_H) - (\kappa^2 + \hat{\lambda}(1-\beta))C]} r^n \\ &\quad - \frac{\kappa^2 + \bar{\lambda}(1-\beta p_H)}{\beta\bar{\lambda}(1-p_H) - (\kappa^2 + \bar{\lambda}(1-\beta))C} r^n \\ &= - \frac{\kappa^2 + \hat{\lambda}(1-\beta p_H)}{\beta\hat{\lambda}(1-p_H) - (\kappa^2 + \hat{\lambda}(1-\beta))C} r^n \\ &= \pi_{L|\lambda=\hat{\lambda},\pi^*=0} \end{aligned}$$

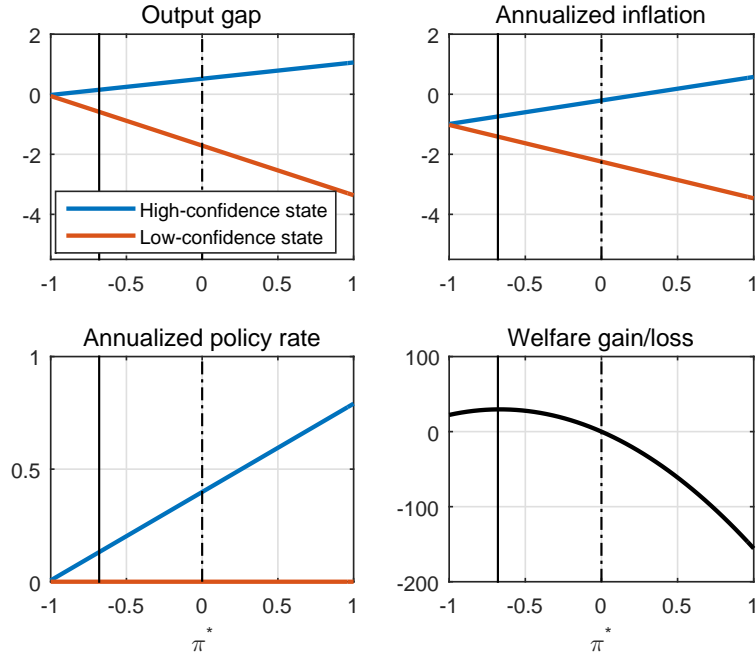
Low-state output:

$$\begin{aligned}
 y_{L|\lambda=\bar{\lambda},\pi^*=\hat{\pi}^*} &= \frac{\kappa(\beta p_L - 1 - (1 - \beta)C)}{[\beta\bar{\lambda}(1 - p_H) - (\kappa^2 + \bar{\lambda}(1 - \beta))C]} \frac{\beta(1 - p_H)(\bar{\lambda} - \hat{\lambda})}{[\beta\hat{\lambda}(1 - p_H) - (\kappa^2 + \hat{\lambda}(1 - \beta))C]} r^n \\
 &\quad - \frac{\kappa^2(1 - \beta p_L) + \bar{\lambda}(1 - \beta)(1 - \beta p_L + \beta(1 - p_H))}{\beta\bar{\lambda}(1 - p_H) - (\kappa^2 + \bar{\lambda}(1 - \beta))C} r^n \\
 &= - \frac{\kappa^2(1 - \beta p_L) + \hat{\lambda}(1 - \beta)(1 - \beta p_L + \beta(1 - p_H))}{\beta\hat{\lambda}(1 - p_H) - (\kappa^2 + \hat{\lambda}(1 - \beta))C} r^n \\
 &= y_{L|\lambda=\hat{\lambda},\pi^*=0}
 \end{aligned}$$

## A.12 Numerical example

This subsection provides a numerical example of how allocations and welfare depend on the central bank's inflation target  $\pi^*$ . The parameterisation follows Table 1. In addition,  $p_L = 0.9375$  and  $p_H = 0.98$ . In this particular example, the optimal inflation target is negative.

Figure 14: Allocations and welfare as a function of  $\pi^*$



Note: Dash-dotted vertical lines indicate the case where the central bank has the same objective function as society as a whole, i.e.  $\pi^* = 0$ . Solid vertical lines indicate the welfare-maximizing inflation target. The welfare gain/loss is expressed relative to the welfare level achieved when the inflation target is zero (in percent).

## B Fundamental equilibrium in the model without fiscal policy

Here we formally derive those results for the fundamental equilibrium in the model with the two-state natural real rate shock that are not provided in Nakata and Schmidt (2018).

### B.1 Allocations and prices

For the sake of completeness, we restate the policy functions associated with the fundamental equilibrium when  $\pi^* = 0$

$$\pi_L = -\frac{\kappa^2 + \lambda(1 - \beta p_H^f)}{E^f} r_L^n < 0 \quad (\text{B.1})$$

$$y_L = -\frac{(1 - \beta p_L^f)\kappa^2 + (1 - \beta)(1 - \beta p_L^f + \beta(1 - p_H^f))\lambda}{\kappa E^f} r_L^n < 0 \quad (\text{B.2})$$

$$i_L = 0 \quad (\text{B.3})$$

$$\pi_H = -\frac{\beta\lambda(1 - p_H^f)}{E^f} r_L^n < 0 \quad (\text{B.4})$$

$$y_H = \frac{\beta\kappa(1 - p_H^f)}{E^f} r_L^n > 0 \quad (\text{B.5})$$

$$i_H = r^n - \frac{(1 - p_H^f)}{E^f} \left( \frac{(\kappa^2 + \lambda(1 - \beta))(1 - \beta p_L^f + \beta(1 - p_H^f))}{\kappa\sigma} + \kappa^2 + \lambda \right) r_L^n > 0, \quad (\text{B.6})$$

where  $E^f = \beta\lambda(1 - p_H^f) - (\kappa^2 + \lambda(1 - \beta)) \left[ \frac{1 - p_L^f}{\kappa\sigma} (1 - \beta p_L^f + \beta(1 - p_H^f)) - p_L^f \right] < 0$  in the fundamental equilibrium (see Nakata and Schmidt, 2018).

### B.2 Effect of a marginal increase in $p_L^f$

Assuming  $\pi^* = 0$ , it holds

$$\begin{aligned} \frac{\partial \pi_L}{\partial p_L^f} &= \frac{(\kappa^2 + \lambda(1 - \beta p_H^f)) (\kappa^2 + \lambda(1 - \beta)) \left[ 1 + (\kappa\sigma)^{-1} (1 + \beta - 2\beta p_L^f + \beta(1 - p_H^f)) \right]}{(E^f)^2} r_L^n < 0 \\ \frac{\partial y_L}{\partial p_L^f} &= \left[ \beta\lambda(1 - p_H^f) + (\kappa^2 + \lambda(1 - \beta)) \left( 1 + \frac{\beta(1 - \beta p_L^f)(1 - p_L^f) + (1 - \beta p_L^f + \beta(1 - p_H^f))(1 - \beta)}{\kappa\sigma} \right) \right. \\ &\quad \left. + (1 - \beta)\beta\lambda(1 - p_H^f) \frac{1 + \beta - 2\beta p_L^f + \beta(1 - p_H^f)}{\kappa\sigma} \right] \frac{\kappa^2 + \lambda(1 - \beta)}{(E^f)^2} r_L^n < 0 \end{aligned}$$



and

$$\frac{\partial \pi_H}{\partial p_L^f} = \frac{\beta \lambda (1 - p_H^f) (\kappa^2 + \lambda (1 - \beta)) \left[ 1 + (\kappa \sigma)^{-1} \left( 1 + \beta - 2\beta p_L^f + \beta (1 - p_H^f) \right) \right]}{(E^f)^2} r_L^n \leq 0$$

$$\frac{\partial y_H}{\partial p_L^f} = - \frac{\beta \kappa (1 - p_H^f) (\kappa^2 + \lambda (1 - \beta)) \left[ 1 + (\kappa \sigma)^{-1} \left( 1 + \beta - 2\beta p_L^f + \beta (1 - p_H^f) \right) \right]}{(E^f)^2} r_L^n \geq 0$$

### B.3 Effect of a marginal increase in $p_H^f$

Assuming  $\pi^* = 0$ , it holds

$$\frac{\partial \pi_L}{\partial p_H^f} = \frac{(1 - p_L^f) \beta (\kappa^2 + \lambda (1 - \beta))}{\kappa \sigma (E^f)^2} \left[ \kappa^2 - \left( \kappa \sigma + \beta (1 - p_L^f) \right) \lambda \right] r_L^n$$

$$\frac{\partial y_L}{\partial p_H^f} = \frac{(1 - p_L^f) \beta (\kappa^2 + \lambda (1 - \beta))}{\kappa (E^f)^2} \left[ \frac{\kappa^2 (1 - \beta p_L^f)}{\kappa \sigma} - \lambda \right] r_L^n$$

$$\frac{\partial \pi_H}{\partial p_H^f} = \frac{\beta \lambda (\kappa^2 + \lambda (1 - \beta))}{(E^f)^2} \left[ p_L^f - \frac{1 - p_L^f}{\kappa \sigma} (1 - \beta p_L^f) \right] r_L^n$$

$$\frac{\partial y_H}{\partial p_H^f} = - \frac{\beta \kappa (\kappa^2 + \lambda (1 - \beta))}{(E^f)^2} \left[ p_L^f - \frac{1 - p_L^f}{\kappa \sigma} (1 - \beta p_L^f) \right] r_L^n$$

### B.4 Effect of a marginal increase in $\kappa$ on low-state allocations/prices

Assuming  $\pi^* = 0$ , it holds

$$\frac{\partial \pi_L}{\partial \kappa} = - \frac{2\beta \kappa \lambda (1 - p_H^f) (1 + C^f) - \left( \kappa^2 + \lambda (1 - \beta + \beta (1 - p_H^f)) \right) (\kappa^2 + \lambda (1 - \beta)) \frac{1 - p_L^f}{\kappa \sigma} \frac{1 - \beta p_L^f + \beta (1 - p_H^f)}{\kappa}}{(E^f)^2} r_L^n \quad (\text{B.7})$$

$$\frac{\partial y_L}{\partial \kappa} = - \frac{1}{(\kappa (E^f))^2} \left[ \beta \lambda \kappa^2 (1 - p_L^f) (1 - p_H^f) + 2\beta \lambda \kappa^2 (1 - \beta) (1 - p_H^f) C^f - (\kappa^2 + \lambda (1 - \beta))^2 (1 - \beta p_L^f) p_L^f \right. \\ \left. - \lambda^2 (1 - \beta)^2 \beta (1 - p_H^f) p_L^f - (1 - \beta p_L^f + \beta (1 - p_H^f)) (1 - \beta) \beta \lambda (1 - p_H^f) \right] r_L^n \quad (\text{B.8})$$

where  $C^f = \frac{1 - p_L^f}{\kappa \sigma} \left( 1 - \beta p_L^f + \beta (1 - p_H^f) \right) - p_L^f > 0$  in the fundamental equilibrium (see Nakata and Schmidt, 2018).

For  $p_H^f = 1$ , we get

$$\frac{\partial \pi_L}{\partial \kappa} = \frac{(\kappa^2 + \lambda (1 - \beta))^2 (1 - p_L^f) (1 - \beta p_L^f)}{(\kappa (E^f))^2 \sigma} r_L^n > 0 \quad (\text{B.9})$$

$$\frac{\partial y_L}{\partial \kappa} = \frac{(\kappa^2 + \lambda (1 - \beta))^2 (1 - \beta p_L^f) p_L^f}{(\kappa (E^f))^2} r_L^n > 0 \quad (\text{B.10})$$

## C Policy problem in the model with fiscal policy

At the beginning of time, society delegates monetary and fiscal policy to a discretionary policymaker. The objective function of the policymaker is given by

$$V_t^{MF} = -\frac{1}{2} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j (\pi_{t+j}^2 + \bar{\lambda} x_{t+j}^2 + \lambda_g g_{t+j}^2), \quad (\text{C.1})$$

where for  $\lambda_g = \bar{\lambda}_g$ , the policymaker's objective function coincides with society's objective function.

The optimization problem of a generic policymaker acting under discretion is as follows. Each period  $t$ , she chooses the inflation rate, the modified output gap, government spending, and the nominal interest rate to maximize its objective function (C.1) subject to the behavioral constraints of the private sector and the lower bound constraint, with the policy functions at time  $t+1$  taken as given. Since the model features no endogenous state variable, the central bank solves a sequence of static optimization problems

$$\max_{\pi_t, x_t, g_t, i_t} -\frac{1}{2} (\pi_t^2 + \bar{\lambda} x_t^2 + \lambda_g g_t^2) \quad (\text{C.2})$$

subject to

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} \quad (\text{C.3})$$

$$x_t = \mathbb{E}_t x_{t+1} + (1 - \Gamma)(g_t - g_{t+1}) - \sigma (i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \quad (\text{C.4})$$

$$i_t \geq 0 \quad (\text{C.5})$$

The consolidated first order conditions are

$$(\kappa \pi_t + \bar{\lambda} x_t) i_t = 0 \quad (\text{C.6})$$

$$\kappa \pi_t + \bar{\lambda} x_t \leq 0 \quad (\text{C.7})$$

$$i_t \geq 0 \quad (\text{C.8})$$

$$\lambda_g g_t + (1 - \Gamma)(\kappa \pi_t + \bar{\lambda} x_t) = 0 \quad (\text{C.9})$$

together with the private sector behavioral constraints.

## D Sunspot equilibrium in the model with fiscal policy

### D.1 Proof of Proposition 11

To proof Proposition 11 on the necessary and sufficient condition for existence of the sunspot equilibrium, it is useful to proceed in three steps. Each step is associated with an auxiliary proposition.

Let

$$\tilde{C} := \lambda_g C + (\kappa^2 + \bar{\lambda}(1 - \beta p_L)) \frac{(1 - \Gamma)^2}{\kappa \sigma} (1 - p_L), \quad (\text{D.1})$$

$$\tilde{D} := \lambda_g D - \beta \bar{\lambda} \frac{(1 - \Gamma)^2}{\kappa \sigma} (1 - p_L)^2, \quad (\text{D.2})$$

and

$$\begin{aligned} \tilde{E} &:= A\tilde{D} - B\tilde{C} \\ &= \lambda_g E - \frac{(1 - \Gamma)^2 (1 - p_L)}{\kappa \sigma} (\kappa^2 + \bar{\lambda}(1 - \beta)) [\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H))], \end{aligned} \quad (\text{D.3})$$

where  $A, B, C, D$  and  $E$  are defined in (A.1)–(A.5).

**Proposition D.1** *There exists a vector  $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$  that solves the system of linear equations (33)–(40).*

**Proof:** Rearranging the system of equations (33)–(40) and eliminating  $x_H, i_H, g_H, x_L, i_L$  and  $g_L$ , we obtain two unknowns for  $\pi_H$  and  $\pi_L$  in two equations

$$\begin{bmatrix} A & B \\ \tilde{C} & \tilde{D} \end{bmatrix} \begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda_g r^n \end{bmatrix}. \quad (\text{D.4})$$

For what follows, it is useful to show that  $\tilde{E} = 0$  is inconsistent with existence of the sunspot equilibrium. Since  $B > 0$ , we can always write  $\pi_H = -A/B\pi_L$ . Plugging this into  $\tilde{C}\pi_L + \tilde{D}\pi_H = \lambda_g r^n$  and multiplying both sides by  $B$ , we get  $-\tilde{E}\pi_L = B\lambda_g r^n$ . Since the right-hand side of this equation is strictly positive for  $\lambda_g > 0$ ,  $\tilde{E} = 0$  is inconsistent with the existence of the sunspot equilibrium. Hence, we can invert the matrix on the left-hand-side of (D.4)

$$\begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \frac{1}{A\tilde{D} - B\tilde{C}} \begin{bmatrix} \tilde{D} & -B \\ -\tilde{C} & A \end{bmatrix} \begin{bmatrix} 0 \\ \lambda_g r^n \end{bmatrix}. \quad (\text{D.5})$$

Thus,

$$\pi_H = \frac{A}{\tilde{E}} \lambda_g r^n \quad (\text{D.6})$$

and

$$\pi_L = \frac{-B}{\tilde{E}} \lambda_g r^n. \quad (\text{D.7})$$

From the Phillips curves in both states, we obtain

$$x_H = \frac{\beta \kappa (1 - p_H)}{\tilde{E}} \lambda_g r^n \quad (\text{D.8})$$

and

$$x_L = -\frac{(1 - \beta p_L)\kappa^2 + (1 - \beta)(1 - \beta p_L + \beta(1 - p_H))\bar{\lambda}}{\kappa\tilde{E}}\lambda_g r^n. \quad (\text{D.9})$$

Using the target criterion for fiscal policy in the low-confidence state (39), we obtain

$$g_L = \frac{(1 - \Gamma)(\kappa^2 + \bar{\lambda}(1 - \beta))(\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H)))}{\kappa\tilde{E}}r^n. \quad (\text{D.10})$$

Using the consumption Euler equation in the high-confidence state (33), we obtain

$$i_H = \left[ 1 - \frac{1 - p_H}{\tilde{E}} \left( \lambda_g \left( \kappa^2 + \bar{\lambda} + (\kappa^2 + \bar{\lambda}(1 - \beta)) \frac{1 - \beta p_L + \beta(1 - p_H)}{\kappa\sigma} \right) + \frac{(1 - \Gamma)^2}{\kappa\sigma} (\kappa^2 + \bar{\lambda}(1 - \beta)) (\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H))) \right) \right] r^n. \quad (\text{D.11})$$

Finally, from equations (35) and (40), we have  $g_H = 0$ , and  $i_L = 0$ .

**Proposition D.2** *Suppose equations (33)–(40) are satisfied. Then  $\bar{\lambda}x_L + \kappa\pi_L < 0$  if and only if  $\tilde{E} > 0$ .*

**Proof:** Using (D.7) and (D.9), we have

$$\bar{\lambda}x_L + \kappa\pi_L = -\frac{(\kappa^2 + \bar{\lambda}(1 - \beta))(\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H)))}{\kappa\tilde{E}}\lambda_g r^n \quad (\text{D.12})$$

Notice that  $\lambda_g r^n > 0$  and  $(\kappa^2 + \bar{\lambda}(1 - \beta))(\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H))) > 0$ . Thus, if  $\bar{\lambda}x_L + \kappa\pi_L < 0$ , then  $\tilde{E} > 0$ . Similarly, if  $\tilde{E} > 0$ , then  $\bar{\lambda}x_L + \kappa\pi_L < 0$ .

**Proposition D.3** *Suppose equations (33)–(40) are satisfied and  $\tilde{E} > 0$ . Then  $i_H > 0$  if and only if  $\lambda_g\Omega(p_L, p_H, \kappa, \sigma, \beta) - (1 - \Gamma)^2 \frac{1 - p_L + 1 - p_H}{\kappa\sigma} [\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H))] > 0$ , where  $\Omega(\cdot)$  is defined in (A.15).*

**Proof:** First, notice that  $i_H$  is given by

$$\begin{aligned} i_H &= \frac{1 - p_H}{\sigma} (x_L - x_H + (1 - \Gamma)(g_H - g_L)) + p_H\pi_H + (1 - p_H)\pi_L + r^n \\ &= \frac{(\kappa^2 + \bar{\lambda}(1 - \beta))r^n}{\tilde{E}} \left[ \lambda_g\Omega(p_L, p_H, \kappa, \sigma, \beta) - (1 - \Gamma)^2 \frac{1 - p_L + 1 - p_H}{\kappa\sigma} (\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H))) \right], \end{aligned} \quad (\text{D.13})$$

where in the second row we made use of (D.6)–(D.10). Notice also that  $\frac{(\kappa^2 + \bar{\lambda}(1-\beta))r^n}{\tilde{E}} > 0$ . Thus, if  $\lambda_g \Omega(p_L, p_H, \kappa, \sigma, \beta) - (1-\Gamma)^2 \frac{1-p_L+1-p_H}{\kappa\sigma} [\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))]$   $> 0$  then  $i_H > 0$ . Similarly, if  $i_H > 0$  then  $\lambda_g \Omega(p_L, p_H, \kappa, \sigma, \beta) - (1-\Gamma)^2 \frac{1-p_L+1-p_H}{\kappa\sigma} [\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))]$   $> 0$ .

We are now ready to proof Proposition 11. For notational convenience, define

$$\tilde{\Omega}(p_L, p_H, \kappa, \sigma, \beta, \Gamma, \lambda_g) = \lambda_g \Omega(p_L, p_H, \kappa, \sigma, \beta) - (1-\Gamma)^2 \frac{1-p_L+1-p_H}{\kappa\sigma} [\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))] \quad (\text{D.14})$$

**Proof of “if” part:** Suppose that  $\tilde{\Omega}(\cdot) > 0$ . According to Proposition D.1 there exists a vector  $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$  that solves equations (33)–(40). Notice that

$$\begin{aligned} (\kappa^2 + \bar{\lambda}(1-\beta))\tilde{\Omega}(\cdot) = & \tilde{E} - (1-p_H) \left[ \lambda_g \left( \kappa^2 + \bar{\lambda} + (\kappa^2 + \bar{\lambda}(1-\beta)) \frac{1-\beta p_L + \beta(1-p_H)}{\kappa\sigma} \right) \right. \\ & \left. + \frac{(1-\Gamma)^2}{\kappa\sigma} (\kappa^2 + \lambda(1-\beta)) (\kappa^2 + \lambda(1-\beta p_L + \beta(1-p_H))) \right]. \end{aligned}$$

Hence,  $\tilde{\Omega}(\cdot) > 0$  implies  $\tilde{E} > 0$ . According to Proposition D.2,  $\tilde{E} > 0$  implies  $\bar{\lambda}x_L + \kappa\pi_L < 0$ . According to Proposition D.3, given  $\tilde{E} > 0$ ,  $\tilde{\Omega}(\cdot) > 0$  implies  $i_H > 0$ .

**Proof of “only if” part:** Suppose that the vector  $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$  solves (33)–(40), and satisfies  $\bar{\lambda}x_L + \kappa\pi_L < 0$  and  $i_H > 0$ . According to Proposition D.2,  $\bar{\lambda}x_L + \kappa\pi_L < 0$  implies  $\tilde{E} > 0$ . According to Proposition D.3,  $\tilde{E} > 0$  and  $i_H > 0$  imply  $\tilde{\Omega}(\cdot) > 0$ .

## D.2 Proof of Proposition 12

In the sunspot equilibrium, allocations and prices are given by

$$\pi_L = -\frac{\kappa^2 + \bar{\lambda}(1 - \beta p_H)}{\tilde{E}} \lambda_g r^n < 0 \quad (\text{D.15})$$

$$x_L = -\frac{(1 - \beta p_L)\kappa^2 + (1 - \beta)(1 - \beta p_L + \beta(1 - p_H))\bar{\lambda}}{\kappa \tilde{E}} \lambda_g r^n < 0 \quad (\text{D.16})$$

$$i_L = 0 \quad (\text{D.17})$$

$$g_L = \frac{(1 - \Gamma)(\kappa^2 + \bar{\lambda}(1 - \beta))(\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H)))}{\kappa \tilde{E}} r^n > 0 \quad (\text{D.18})$$

$$\pi_H = -\frac{\beta \bar{\lambda}(1 - p_H)}{\tilde{E}} \lambda_g r^n < 0 \quad (\text{D.19})$$

$$x_H = \frac{\beta \kappa(1 - p_H)}{\tilde{E}} \lambda_g r^n > 0 \quad (\text{D.20})$$

$$i_H = \left[ 1 - \frac{1 - p_H}{\tilde{E}} \left( \lambda_g \left( \kappa^2 + \bar{\lambda} + (\kappa^2 + \bar{\lambda}(1 - \beta)) \frac{1 - \beta p_L + \beta(1 - p_H)}{\kappa \sigma} \right) + \frac{(1 - \Gamma)^2}{\kappa \sigma} (\kappa^2 + \bar{\lambda}(1 - \beta)) (\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H))) \right) \right] r^n > 0 \quad (\text{D.21})$$

$$g_H = 0, \quad (\text{D.22})$$

where  $\tilde{E} > 0$  is defined in equation (D.3).

### D.3 Proof of Proposition 13

In the sunspot equilibrium, it holds

$$\frac{\partial \pi_L}{\partial \lambda_g} = \frac{(\kappa^2 + \bar{\lambda}(1 - \beta p_H))(1 - \Gamma)^2 (\kappa \sigma)^{-1} (1 - p_L) (\kappa^2 + \bar{\lambda}(1 - \beta)) [\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H))]}{\tilde{E}^2} r^n > 0$$

$$\begin{aligned} \frac{\partial x_L}{\partial \lambda_g} &= [\kappa^2(1 - \beta p_L) + \bar{\lambda}(1 - \beta)(1 - \beta p_L + \beta(1 - p_H))] \\ &\times \frac{(1 - \Gamma)^2 (\kappa \sigma)^{-1} (1 - p_L) (\kappa^2 + \bar{\lambda}(1 - \beta)) [\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H))]}{\kappa \tilde{E}^2} r^n > 0 \end{aligned}$$

$$\frac{\partial g_L}{\partial \lambda_g} = -\frac{(1 - \Gamma)(\kappa^2 + \bar{\lambda}(1 - \beta))(\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H)))}{\kappa \tilde{E}^2} E r^n < 0$$

and

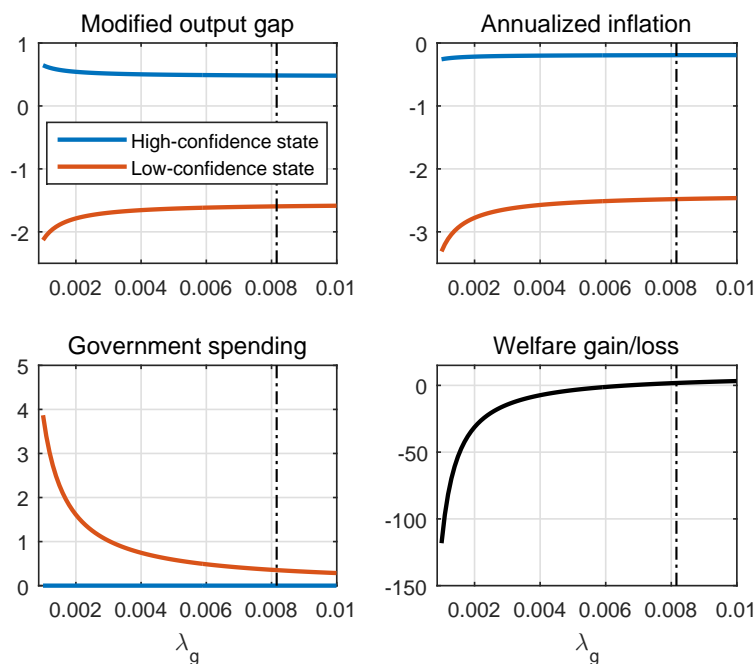
$$\frac{\partial \pi_H}{\partial \lambda_g} = \frac{\beta \bar{\lambda}(1 - p_H)(1 - \Gamma)^2 (\kappa \sigma)^{-1} (1 - p_L) (\kappa^2 + \bar{\lambda}(1 - \beta)) [\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H))]}{\tilde{E}^2} r^n > 0$$

$$\frac{\partial x_H}{\partial \lambda_g} = -\frac{\beta \kappa(1 - p_H)(1 - \Gamma)^2 (\kappa \sigma)^{-1} (1 - p_L) (\kappa^2 + \bar{\lambda}(1 - \beta)) [\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H))]}{\tilde{E}^2} r^n < 0$$

## D.4 Numerical example

This subsection provides a numerical example of how allocations and welfare depend on the relative weight that the policymaker's objective function puts on government spending stabilization  $\lambda_g$ . The parameterisation follows Table 1 except that we account for a non-zero steady-state government spending to output ratio of 0.2, which implies that the inverse of the elasticity of the marginal utility of private consumption with respect to output  $\sigma$  becomes 0.4. The inverse of the elasticity of the marginal utility of public consumption with respect to output  $\nu$  is set to 0.1, as in Section 6. This implies  $\bar{\lambda}_g = 0.0082$ . In addition,  $p_L = 0.9375$  and  $p_H = 0.98$ .

Figure 15: Allocations and welfare as a function of  $\lambda_g$



Note: Dash-dotted vertical lines indicate the case where the central bank has the same objective function as society as a whole, i.e.  $\lambda_g = \bar{\lambda}_g$ . The welfare gain/loss is expressed relative to the welfare level achieved when  $\lambda_g = \bar{\lambda}_g$  (in percent).

## E Fundamental equilibrium in the model with fiscal policy

### E.1 Existence of the fundamental equilibrium

**Proposition 15** *The fundamental equilibrium in the model with government consumption and a two-state natural real rate shock exists if and only if*

$$\begin{aligned} \tilde{E}^f < (1 - p_H^f) \frac{r_L^n}{r^n} \left[ \lambda_g \left( \kappa^2 + \bar{\lambda} + (\kappa^2 + \bar{\lambda}(1 - \beta)) \frac{1 - \beta p_L^f + \beta(1 - p_H^f)}{\kappa\sigma} \right) \right. \\ \left. + \frac{(1 - \Gamma)^2}{\kappa\sigma} (\kappa^2 + \bar{\lambda}(1 - \beta)) (\kappa^2 + \bar{\lambda}(1 - \beta p_L^f + \beta(1 - p_H^f))) \right] \end{aligned} \quad (\text{E.1})$$

where  $\tilde{E}^f \equiv \lambda_g E^f - \frac{(1 - \Gamma)^2 (1 - p_L^f)}{\kappa\sigma} (\kappa^2 + \bar{\lambda}(1 - \beta)) [\kappa^2 + \bar{\lambda}(1 - \beta p_L^f + \beta(1 - p_H^f))]$ .

To proof Proposition 15, we proceed again in three steps. Each step is associated with an auxiliary proposition.

**Proposition E.1** *There exists a vector  $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$  that solves the system of linear equations (35), (36), (39), (40), and (43)–(46).*

**Proof:** Let

$$A^f := -\beta \bar{\lambda} (1 - p_H^f), \quad (\text{E.2})$$

$$B^f := \kappa^2 + \bar{\lambda} (1 - \beta p_H^f), \quad (\text{E.3})$$

$$C^f := \frac{(1 - p_L^f)}{\sigma\kappa} (1 - \beta p_L^f + \beta(1 - p_H^f)) - p_L^f, \quad (\text{E.4})$$

$$D^f := -\frac{(1 - p_L^f)}{\sigma\kappa} (1 - \beta p_L^f + \beta(1 - p_H^f)) - (1 - p_L^f) = -1 - C^f, \quad (\text{E.5})$$

and

$$E^f := A^f D^f - B^f C^f. \quad (\text{E.6})$$

Rearranging the system of equations and eliminating  $x_H, i_H, g_H, x_L, i_L$  and  $g_L$ , we obtain two unknowns for  $\pi_H$  and  $\pi_L$  in two equations

$$\begin{bmatrix} A^f & B^f \\ \tilde{C}^f & \tilde{D}^f \end{bmatrix} \begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda_g r_L^n \end{bmatrix}, \quad (\text{E.7})$$

where

$$\tilde{C}^f := \lambda_g C^f + \left( \kappa^2 + \bar{\lambda} (1 - \beta p_L^f) \right) \frac{(1 - \Gamma)^2}{\kappa\sigma} (1 - p_L^f), \quad (\text{E.8})$$

$$\tilde{D}^f := \lambda_g D^f - \beta \bar{\lambda} \frac{(1 - \Gamma)^2}{\kappa\sigma} (1 - p_L^f)^2. \quad (\text{E.9})$$



Define  $\tilde{E}^f := A^f \tilde{D}^f - B^f \tilde{C}^f$ . For what follows, it is useful to show that  $\tilde{E}^f = 0$  is inconsistent with existence of the fundamental equilibrium. Since  $B > 0$ , we can always write  $\pi_H = -A^f/B^f \pi_L$ . Plugging this into  $\tilde{C}^f \pi_L + \tilde{D}^f \pi_H = \lambda_g r_L^n$  and multiplying both sides by  $B^f$ , we get  $-\tilde{E}^f \pi_L = B^f \lambda_g r_L^n$ . Since the right-hand side of this equation is strictly negative for  $\lambda_g > 0$ ,  $\tilde{E}^f = 0$  is inconsistent with the existence of the fundamental equilibrium. Hence, we can invert the matrix on the left-hand-side of (E.7)

$$\begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \frac{1}{A^f \tilde{D}^f - B^f \tilde{C}^f} \begin{bmatrix} \tilde{D}^f & -B^f \\ -\tilde{C}^f & A^f \end{bmatrix} \begin{bmatrix} 0 \\ \lambda_g r_L^n \end{bmatrix}. \quad (\text{E.10})$$

Thus,

$$\pi_H = \frac{A^f}{\tilde{E}^f} \lambda_g r_L^n \quad (\text{E.11})$$

and

$$\pi_L = \frac{-B^f}{\tilde{E}^f} \lambda_g r_L^n. \quad (\text{E.12})$$

From the Phillips curves in both states, we obtain

$$x_H = \frac{\beta \kappa (1 - p_H^f)}{\tilde{E}^f} \lambda_g r_L^n \quad (\text{E.13})$$

and

$$x_L = -\frac{(1 - \beta p_L^f) \kappa^2 + (1 - \beta)(1 - \beta p_L^f + \beta(1 - p_H^f)) \bar{\lambda}}{\kappa \tilde{E}^f} \lambda_g r_L^n. \quad (\text{E.14})$$

Using the target criterion for fiscal policy in the low-confidence state (39), we obtain

$$g_L = \frac{(1 - \Gamma) (\kappa^2 + \bar{\lambda}(1 - \beta)) (\kappa^2 + \bar{\lambda}(1 - \beta p_L^f + \beta(1 - p_H^f)))}{\kappa \tilde{E}^f} r_L^n. \quad (\text{E.15})$$

Using the consumption Euler equation in the high-confidence state (43), we obtain

$$\begin{aligned} i_H = r^n - \frac{1 - p_H^f}{\tilde{E}^f} & \left( \lambda_g \left( \kappa^2 + \bar{\lambda} + (\kappa^2 + \bar{\lambda}(1 - \beta)) \frac{1 - \beta p_L^f + \beta(1 - p_H^f)}{\kappa \sigma} \right) \right. \\ & \left. + \frac{(1 - \Gamma)^2}{\kappa \sigma} (\kappa^2 + \bar{\lambda}(1 - \beta)) (\kappa^2 + \bar{\lambda}(1 - \beta p_L^f + \beta(1 - p_H^f))) \right) r_L^n. \end{aligned} \quad (\text{E.16})$$

Finally, from equations (35) and (40), we have  $g_H = 0$ , and  $i_L = 0$ .

**Proposition E.2** *Suppose equations (35), (36), (39), (40), and (43)–(46) are satisfied. Then  $\bar{\lambda} x_L + \kappa \pi_L < 0$  if and only if  $\tilde{E}^f < 0$ .*

**Proof:** Using (E.12) and (E.14), we have

$$\bar{\lambda}x_L + \kappa\pi_L = -\frac{(\kappa^2 + \bar{\lambda}(1 - \beta)) \left( \kappa^2 + \bar{\lambda}(1 - \beta p_L^f + \beta(1 - p_H^f)) \right)}{\kappa \tilde{E}^f} \lambda_g r_L^n \quad (\text{E.17})$$

Notice that  $\lambda_g r_L^n < 0$  and  $(\kappa^2 + \bar{\lambda}(1 - \beta)) \left( \kappa^2 + \bar{\lambda}(1 - \beta p_L^f + \beta(1 - p_H^f)) \right) > 0$ . Thus, if  $\bar{\lambda}x_L + \kappa\pi_L < 0$ , then  $\tilde{E}^f < 0$ . Similarly, if  $\tilde{E}^f < 0$ , then  $\bar{\lambda}x_L + \kappa\pi_L < 0$ .

**Proposition E.3** *Suppose equations (35), (36), (39), (40), and (43)–(46) are satisfied and  $\tilde{E}^f < 0$ . Then  $i_H > 0$  if and only if  $\tilde{E}^f < \underline{\tilde{E}}^f$ ,*

where  $\underline{\tilde{E}}^f \equiv (1 - p_H^f) \frac{r_L^n}{r^n} \left[ \lambda_g \left( \kappa^2 + \bar{\lambda} + (\kappa^2 + \bar{\lambda}(1 - \beta)) \frac{1 - \beta p_L^f + \beta(1 - p_H^f)}{\kappa\sigma} \right) + \frac{(1 - \Gamma)^2}{\kappa\sigma} (\kappa^2 + \bar{\lambda}(1 - \beta)) \left( \kappa^2 + \bar{\lambda}(1 - \beta p_L^f + \beta(1 - p_H^f)) \right) \right]$

**Proof:** First, notice that  $i_H$  is given by

$$\begin{aligned} i_H &= \frac{1 - p_H^f}{\sigma} (x_L - x_H + (1 - \Gamma)(g_H - g_L)) + p_H^f \pi_H + (1 - p_H^f) \pi_L + r^n \\ &= r^n - \frac{1 - p_H^f}{\tilde{E}^f} \left[ \lambda_g \left( \kappa^2 + \bar{\lambda} + (\kappa^2 + \bar{\lambda}(1 - \beta)) \frac{1 - \beta p_L^f + \beta(1 - p_H^f)}{\kappa\sigma} \right) \right. \\ &\quad \left. + \frac{(1 - \Gamma)^2}{\kappa\sigma} (\kappa^2 + \bar{\lambda}(1 - \beta)) \left( \kappa^2 + \bar{\lambda}(1 - \beta p_L^f + \beta(1 - p_H^f)) \right) \right] r_L^n, \end{aligned} \quad (\text{E.18})$$

The term in square brackets is strictly positive,  $r^n > 0$ ,  $r_L^n < 0$  and  $\tilde{E}^f < 0$ . Thus, if  $\tilde{E}^f < \underline{\tilde{E}}^f$  then  $i_H > 0$ . Similarly, if  $i_H > 0$  then  $\tilde{E}^f < \underline{\tilde{E}}^f$ .

We are now ready to proof Proposition 15.

**Proof of “if” part:** Suppose that  $\tilde{E}^f < \underline{\tilde{E}}^f$ . According to Proposition E.1 there exists a vector  $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$  that solves equations (35), (36), (39), (40), and (43)–(46). Notice that  $\tilde{E}^f < 0$ . Hence,  $\tilde{E}^f < \underline{\tilde{E}}^f$  implies  $\tilde{E}^f < 0$ . According to Proposition E.2,  $\tilde{E}^f < 0$  implies  $\bar{\lambda}x_L + \kappa\pi_L < 0$ . According to Proposition E.3, given  $\tilde{E}^f < 0$ ,  $\tilde{E}^f < \underline{\tilde{E}}^f$  implies  $i_H > 0$ .

**Proof of “only if” part:** Suppose that the vector  $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$  solves (35), (36), (39), (40), (43)–(46), and satisfies  $\bar{\lambda}x_L + \kappa\pi_L < 0$  and  $i_H > 0$ . According to Proposition E.2,  $\bar{\lambda}x_L + \kappa\pi_L < 0$  implies  $\tilde{E}^f < 0$ . According to Proposition E.3,  $\tilde{E}^f < 0$  and  $i_H > 0$  imply  $\tilde{E}^f < \underline{\tilde{E}}^f$ .

## E.2 Allocations and prices

In the fundamental equilibrium, allocations and prices are given by:

$$\pi_L = -\frac{\kappa^2 + \bar{\lambda}(1 - \beta p_H^f)}{\tilde{E}^f} \lambda_g r_L^n < 0 \quad (\text{E.19})$$

$$x_L = -\frac{(1 - \beta p_L^f) \kappa^2 + (1 - \beta)(1 - \beta p_L^f + \beta(1 - p_H^f)) \bar{\lambda}}{\kappa \tilde{E}^f} \lambda_g r_L^n < 0 \quad (\text{E.20})$$

$$i_L = 0 \quad (\text{E.21})$$

$$g_L = \frac{(1 - \Gamma) (\kappa^2 + \bar{\lambda}(1 - \beta)) \left( \kappa^2 + \bar{\lambda}(1 - \beta p_L^f + \beta(1 - p_H^f)) \right)}{\kappa \tilde{E}^f} r_L^n > 0 \quad (\text{E.22})$$

$$\pi_H = -\frac{\beta \bar{\lambda}(1 - p_H^f)}{\tilde{E}^f} \lambda_g r_L^n < 0 \quad (\text{E.23})$$

$$x_H = \frac{\beta \kappa(1 - p_H^f)}{\tilde{E}^f} \lambda_g r_L^n > 0 \quad (\text{E.24})$$

$$i_H = r^n - \frac{1 - p_H^f}{\tilde{E}^f} \left( \lambda_g \left( \kappa^2 + \bar{\lambda} + (\kappa^2 + \bar{\lambda}(1 - \beta)) \frac{1 - \beta p_L^f + \beta(1 - p_H^f)}{\kappa \sigma} \right) \right. \\ \left. + \frac{(1 - \Gamma)^2}{\kappa \sigma} (\kappa^2 + \bar{\lambda}(1 - \beta)) \left( \kappa^2 + \bar{\lambda}(1 - \beta p_L^f + \beta(1 - p_H^f)) \right) \right) r_L^n > 0 \quad (\text{E.25})$$

$$g_H = 0 \quad (\text{E.26})$$

### E.3 Effects of a marginal change in $\lambda_g$

The partial derivatives of the policy functions with respect to  $\lambda_g$  are

$$\frac{\partial \pi_L}{\partial \lambda_g} = \frac{(\kappa^2 + \bar{\lambda}(1 - \beta p_H^f))(1 - \Gamma)^2 (\kappa \sigma)^{-1} (1 - p_L^f) (\kappa^2 + \bar{\lambda}(1 - \beta)) \left[ \kappa^2 + \bar{\lambda}(1 - \beta p_L^f + \beta(1 - p_H^f)) \right]}{(\tilde{E}^f)^2} r_L^n < 0$$

$$\frac{\partial x_L}{\partial \lambda_g} = \left[ \kappa^2(1 - \beta p_L^f) + \bar{\lambda}(1 - \beta)(1 - \beta p_L^f + \beta(1 - p_H^f)) \right] \\ \times \frac{(1 - \Gamma)^2 (\kappa \sigma)^{-1} (1 - p_L^f) (\kappa^2 + \bar{\lambda}(1 - \beta)) \left[ \kappa^2 + \bar{\lambda}(1 - \beta p_L^f + \beta(1 - p_H^f)) \right]}{\kappa (\tilde{E}^f)^2} r_L^n < 0$$

$$\frac{\partial g_L}{\partial \lambda_g} = -\frac{(1 - \Gamma) (\kappa^2 + \bar{\lambda}(1 - \beta)) \left( \kappa^2 + \bar{\lambda}(1 - \beta p_L^f + \beta(1 - p_H^f)) \right)}{\kappa (\tilde{E}^f)^2} E^f r_L^n,$$

and

$$\frac{\partial \pi_H}{\partial \lambda_g} = \frac{\beta \bar{\lambda} (1 - p_H^f) (1 - \Gamma)^2 (\kappa \sigma)^{-1} (1 - p_L^f) (\kappa^2 + \bar{\lambda} (1 - \beta)) \left[ \kappa^2 + \bar{\lambda} (1 - \beta p_L^f + \beta (1 - p_H^f)) \right]}{(\tilde{E}^f)^2} r_L^n < 0$$

$$\frac{\partial x_H}{\partial \lambda_g} = - \frac{\beta \kappa (1 - p_H^f) (1 - \Gamma)^2 (\kappa \sigma)^{-1} (1 - p_L) (\kappa^2 + \bar{\lambda} (1 - \beta)) \left[ \kappa^2 + \bar{\lambda} (1 - \beta p_L^f + \beta (1 - p_H^f)) \right]}{(\tilde{E}^f)^2} r_L^n > 0$$