Wealth in the utility, nonseparability and the New Keynesian model

Elliot Aurissergues

Abstract

This paper explores an alternative specification of the utility function and its theoretical implications for the New Keynesian model. I consider a model where wealth enters in the utility function. My contribution is to study the case of nonseparability between consumption and future wealth. I do not assume any functional form for the utility function. I consider the general case and derive linearized equations for consumption and labor supply. I show that nonseparability disentangles between the income effect on labor supply and the intertemporal substitution effect. I derive several implications for the dynamics of output, output gap, real wages and unemployment following monetary policy and demand shocks. Then, I estimate a medium scale DSGE model with bayesian methods and focus on the two key parameters introduced by (i) wealth in the utility function (ii) nonseparability between consumption and assets. I find large and positive values for both, providing some support for this specification of the utility function.

Elliot Aurissergues

PHD student, Paris School of Economics and University Paris Pantheon Sorbonne

elliot.aurissergues@gmail.com

1 Introduction

This paper explores an alternative specification of the utility function and its theoretical implications for the new keynesian model.

A recent strand of literature has introduced wealth in the utility function. Krishnamurthy and Vissing- Jorgensen (2012) use bonds in the utility function and infer demand functions to explain the behavior of the bond market. Michaillat and Saez (2014, 2018) explore consequences for the New Keynesian model, especially at the zero lower bound. Saez and Stantcheva (2017) look at implications for optimal capital taxation. Kumhof, Ranciere and Winant (2015) use it to match profiles of income distribution in a model of financial crisis. Michau (2017) introduces a similar specification in a model with downward nominal wage rigidity and zero lower bound on nominal interest rate. A related class of models comes from the overlapping generations literature. Yaari (1964) introduced the joy of giving specification to explain bequests. Under this specification, households care about the amount of wealth their children will inherit but do not care about the consumption of their children, a model refined by Abel and Warshawsky (1988).

My contribution is to study the case of nonseparability between consumption and future wealth. The utility function of a representative agent depends on three variables, consumption, leisure and next period wealth. Allowing for wealth in the utility function and nonseparability introduces two new parameters in the model. The first parameter, denoted κ, governs the discount rate in the linear Euler equation. The second parameter ν reflects the degree of complementarity between consumption and future wealth. A positive value of ν allows me to obtain a low elasticity of intertemporal substitution along a moderate income effect on labor supply whereas, under the standard specification, the former is the inverse of the latter. I call such complementarity between consumption and future wealth the intertemporal complementarity. Disentangling the income effect and the intertemporal substitution effect has important implications for the model. The elasticity of hours worked with respect to real wages becomes different from the elasticity with respect to the real interest rate, modifying the response of real wages and unemployment to a monetary policy shock and the response of output gap to a demand shock.

I extend the analysis to a medium scale DSGE model. I focus on implications for labor market variables, especially real wages and unemployment. Following Gali(2011), I introduce sticky wages and identify unemployment as the difference between the desired labor supply, given by the first order condition of a family behaving competitively, and the

effective labor demand. In the standard model, an expansionary monetary policy shock generates a relatively large response of real wages and a very large response of unemployment. This large response of unemployment is caused by the shift in labor demand but also by a large shift in labor supply. Introducing intertemporal complementarity allows me to reduce substantially this response of labor supply.

Then, I provide an estimation of parameters κ and ν. I estimate the medium scale model using Bayesian techniques. I find a large value for the parameter κ, suggesting a substantial discount rate in the Euler equation. I also find a large and positive value for the parameter ν, supporting intertemporal complementarity. This high value of ν is the consequence of the difference it creates between the income effect on labor supply and the intertemporal substitution effect. When the elasticity of intertemporal substitution is the inverse of the income effect, the estimation gives a low EIS and a large income effect, leading to implausible fluctuations in labor supply. A positive parameter ν allows for a low EIS and a moderate income effect providing a more plausible serie for labor supply. High values for ν and κ seem a robust result. I reestimate the model with several alternative specifications, like alternative prior for κ, habits consumption, a longer sample, and labor force participation instead of hours worked in observables. Outcomes are consistent with my baseline estimation.

Substituability between consumption and leisure time is an alternative way to separate the income effect and the intertemporal substitution effect. This has been extensively analyzed by Bilbiie (2009) to explain the response of consumption to fiscal policy shocks. Some degree of substituability seems plausible but the evolution of consumption at retirement provides an upper bound to it (See Kimball and Shapiro 2008 for some quantitative exercise). Compatibility with balanced growth is also a concern. It seems interesting to complement this approach by exploring an alternative specification focusing on the intertemporal choice.

Various explanations have been put forward to justify the inclusion of wealth in the utility function. Wealth can provide an important social status leading consumers to have a preference for it. Alternatively, it may capture several saving motivations. Under the standard specification, a representative agent only saves for consumption smoothing. Households also save to insure themselves against negative income shock, to increase their income at retirement or to hand their estate to their children. A model integrating explicitly all these motivations would be better. However, the cost in complexity would be very high. Wealth in the utility function may generate in some extent a similar

behavior for aggregate consumption and leisure whereas keeping the convenience of the representative agent framework.

Several pieces of literature have recently cast doubts on the standard model of intertemporal choice. The model implies that theoretical responses to expected monetary policy shocks are much larger than their empirical counterparts (Del Negro et al. 2013). A simple way to solve the forward guidance puzzle is to introduce a discount in the linear Euler equation (see McKay, Nakamura and Steinsson 2016 and Gabaix 2017). Such discount rate emerges immediately when wealth is in the utility function. It is interesting to note that intertemporal complementarity increases the discount in the linear Euler equation. Heterogeneous agents models suggest that a substantial part of the response of consumption to monetary policy could come from an indirect effect, through the increase of the income of "hand to mouth" households, and not from the direct effect through intertemporal substitution (Auclert 2017, Kaplan, Moll and Violante 2017).

The paper is organized as follows. The first section explores the household choice in a simple optimization problem in finite horizon with wealth in the utility. The second section extends the results to infinite horizon. The third gives several implications for the macroeconomic model using both a simple model to highlight intuition and a medium scale model to confirm these insights in a more "realistic" environment. I estimate the model in the fourth. I discuss some assumptions and implications in the fifth. I examine additional consequences for forward guidance in the sixth. The seventh section introduces time varying wealth in the model.

2   Intratemporal Household choice

When wealth enters into the utility function, households have two motives to accumulate it. First, because it increases the "income" of the next period. Second, because it increases their *current* utility. To get a better intuition of the household's behavior, it is useful to start by only considering this second motive. To do so, I consider an household which only cares about its current utility whose wealth is one of the arguments. I label this model as the "Wealth Targeting Model".

2.1 The household program

I consider the optimization problem of a consumer who does not care about future utility streams but whose current utility function accepts its future wealth as an argument. At period t, the objective function of the consumer is

$$U(C_t, L_t, A_{t+1}) \tag{1}$$

under the budget constraint

$$Q_t A_{t+1} + W_t L_t + C_t = A_t + W_t + \Pi_t \tag{2}$$

where C is the consumption, L is leisure time, W is real wage, and Π are profits distributed by firms. A is an asset which gives the right to receive one unit of consumption good at the next period. A does not provide utility through a continuation value but directly provides some utility, hence there is a positive demand for assets even if the optimization program of the household is purely static. Households still have a choice to make between current consumption and future assets.

To buy one unit of this asset, the consumer should pay a price Q. This price is the inverse of the interest factor.

$$Q_t = \frac{1}{1 + rr_t} \tag{3}$$

where rr is the real interest rate. Q_t is the price of future consumption goods.

For the moment, I identify wealth with safe bonds. The point is that consumers may buy in period t a "promise" on final good of period $t + 1$. The amount of this promise enters into the utility function. I consider alternative interpretations and some of their consequences in the section dedicated to the model with varying wealth. Until then, I will use "wealth" and "assets" as synonyms.

2.2 First order conditions

I now solve for first order conditions.

Proposition 1. *The utility function reaches its local maximum under the budget constraint if the following first order conditions are fulfilled*

$$U_C(C_t, L_t, A_{t+1}) = \Lambda_t \tag{4a}$$

$$U_L(C_t, L_t, A_{t+1}) = W_t \Lambda_t \tag{4b}$$

$$U_A(C_t, L_t, A_{t+1}) = Q_t \Lambda_t \tag{4c}$$

Where U_c (resp. U_L and U_A) is the first derivative of the utility function with respect to consumption(resp. leisure and wealth). Λ_t is the Lagrange multiplier.

The problem is a basic consumer choice problem. whose solution is straightforward. A proof is given in appendix **??**. Now, suppose that consumption C, wealth A an leisure L reach steady state values. I can linearize conditions from proposition **??** around the steady state. I defined c_t, a_t and l_t and λ_t as percentage deviation from their steady state value. More generally, small letters will denote percentage deviation from steady state or deviation from steady state.

Proposition 2. *A linear approximation of the system of equations defined in proposition **??** is*

$$\frac{U_{CC}C}{U_C}c_t + \frac{U_{CA}A}{U_C}a_{t+1} + \frac{U_{CL}L}{U_C}l_t = \lambda_t \tag{5a}$$

$$\frac{U_{CL}C}{U_L}c_t + \frac{U_{LA}A}{U_L}a_{t+1} + \frac{U_{LL}L}{U_L}l_t = w_t + \lambda_t \tag{5b}$$

$$\frac{U_{CA}C}{U_A}c_t + \frac{U_{AA}A}{U_A}a_{t+1} + \frac{U_{LA}L}{U_A}l_t = q_t + \lambda_t \tag{5c}$$

Terms U_C (resp U_L, U_A), and U_{CC} (and other similar terms), denote the steady state value of first and second order derivatives of the utility function.

Proof The linear approximation is a first order Taylor expansion of first order conditions defined in proposition **??**. Detailed computations are given in appendix **??** □

2.3 Separable preferences

Before considering the case of intertemporal complementarity, I show that, under separable preferences, first order conditions defined in proposition **??** are related to the first order conditions of the standard model.

Hypothesis 1. *Preferences are separable. Cross derivative of the utility function are equal to zero :*

$$U_{CL} = 0$$

$$U_{AL} = 0$$

$$U_{CA} = 0$$

To allow a proper comparison with the standard model, I combine first order conditions with several general equilibrium conditions. I assume that the supply of assets is fixed.

Thus, the deviation from steady state is equal to zero. I also introduce a relation between leisure time and hours worked.

Hypothesis 2. *the asset supply equation is given by*

$$A_{t+1} = \overline{A} \tag{6}$$

where A is a constant, hence the percentage deviation from steady state is

$$a_{t+1} = 0 \tag{7}$$

This assumption states that asset supply is not sensitive to the asset price and thus to the demand of assets by consumers. In the last section of the paper, I relax this assumption and study the model with a varying asset supply.

With assumptions **??** and **??**, The system of linear equations considered in proposition **??** becomes

$$\frac{U_{CC}C}{U_C}c_t = \lambda_t \tag{8a}$$

$$\frac{U_{LL}L}{U_L}l_t = w_t + \lambda_t \tag{8b}$$

$$q_t + \lambda_t = 0 \tag{8c}$$

To obtain more friendly equations, I substitute leisure with hours worked and the price of assets with real interest rate

Hours worked are given by

$$l_t = \eta n_t \tag{9}$$

where η is the ratio between the steady state working time and the steady state leisure time[1] .

The deviation from the steady state real interest rate rr_t is directly related with the percentage deviation from the steady state bond price q_t

$$rr_t = -q_t \tag{10}$$

It is convenient to make notations easier by introducing parameters $\sigma = \frac{U_{CC}C}{U_C}$ and $\theta = \frac{U_{LL}L}{U_L}$. I obtain two equations for labor supply and consumption.

---

[1]Often denoted $\frac{N}{1-N}$

6

Proposition 3. *Labor supply and consumption equation are*

$$\theta \eta n_t = w_t - \sigma c_t \tag{11}$$

$$\sigma c_t = -rr_t \tag{12}$$

Both equations are derived from the system **??**. Labor supply equation (**??**) is common with the standard model fof intertemporal choice. The difference lies in equation (**??**). Instead of having an equation for consumption growth, I have an equation for consumption *levels* with respect to interest rate. The parameter σ governs both the intertemporal substitution effect and the income effect on labor supply. This feature is shared with the standard model of intertemporal choice.

2.4 Intertemporal nonseparability

I now allow the cross derivative between wealth and consumption to be different from zero. Hypothesis **??** becomes

Hypothesis 3.

$$U_{CL} = 0$$

$$U_{AL} = 0$$

$$U_{CA} \neq 0$$

A positive cross derivative between consumption and wealth implies that assets and consumption are complements in the sense of Edgeworth, whereas a negative cross derivative means the two are substitutes.

I keep the separability assumption for leisure. $U_{AL} = 0$, $U_{CL} = 0$. This is a strong assumption but the goal is to keep a tractable model and to focus on intertemporal choice. My analysis roughly follows the analysis made by Bilbiie (2009) for the nonseparability between consumption and leisure.

Nonseparability between consumption and assets allows me to disentangle the consumption elasticity to interest rate from the income effect on labor supply.

Proposition 4. *The system from proposition* **??** *becomes*

$$\frac{U_{CC}C}{U_C}c_t = \lambda_t \tag{13a}$$

$$\frac{U_{LL}L}{U_L}l_t = w_t + \lambda_t \tag{13b}$$

$$\frac{U_{CA}C}{U_A}c_t = q_t + \lambda_t \tag{13c}$$

Let me define the parameter $\nu \equiv \frac{U_{CA}C}{U_A}$.

Corollary 1. *The intertemporal substitution effect is governed by $\sigma + \nu$ whereas the income effect is is governed by the parameter σ*

$$\theta \eta n_t = w_t - \sigma c_t \tag{14}$$

$$(\sigma + \nu)c_t = -rr_t \tag{15}$$

Intuition Some intuition may be given for this result. The sensitivity of leisure with respect to interest rate is equal to $\frac{\sigma}{\sigma+\nu}$. A large and positive ν is obtained if $U_{CA} > 0$ and thus complementarity between present consumption and future assets. A fall in real rates implies that the marginal utility of assets should rise relative to the marginal utility of consumption and relative to the marginal utility of leisure. With separable preferences, consumption increases and thus reduces the marginal utility of consumption. With non-separable preferences, if consumption and assets are complements, the rise in consumption decreases the marginal utility of consumption and increases the marginal utility of assets. Thus, a much lower rise in consumption may achieve the equality between the relative price of future consumption goods and the marginal rate of substitution between consumption and assets. A similar line of reasoning explains the smaller sensitivity of leisure to real interest rate. The increase in the marginal utility of assets implies a lower fall in the marginal utility of leisure and thus a lower rise in leisure.

Concavity and Noninferiority requirements Parameters σ and ν cannot be calibrated freely. They should respect concavity requirements for the utility function:

Proposition 5. *The utility function U is concave if*

$$U_{CC} \leq 0$$

$$U_{LL} \leq 0$$

$$U_{AA} \leq 0$$

$$U_{AA}U_{CC} - U_{CA}^2 \geq 0$$

The last condition implies that U_{CA} cannot be "too large" with respect to U_{CC} and thus ν should not be "too large" with respect to σ, except if U_{AA} is large enough. With our particular asset supply function, U_{AA} can be calibrated freely, allowing a low value for $\frac{\sigma}{\sigma+\nu}$.

Whereas not compulsory, it also seems reasonable to impose that assets and consumption are not inferior goods whose demand decreases when income rises. A positive value of ν (i.e. assets and consumption are complements) is however a sufficient condition for noninferiority.

Generalized nonseparable preferences In the previous paragraph, I focus on nonseparability between consumption and assets. It is useful to consider the general case with several forms of nonseparability. Notations are burdensome and I relegate computations to appendix **??**. The last equation of the appendix **??** gives the consumption equation when U_{CL}, U_{AL} and U_{CA} are different from zero. Combining complementarity between consumption and assets, complementarity between leisure and assets and substitutability between consumption and leisure reduces the consumption elasticity to real interest rate further without affecting the income effect on labor supply.

3   Intertemporal Household choice

The optimization problem I extend the analysis of the previous section in a more standard setup. Wealth still enters in the utility function but households care about future utility. They maximize

$$\sum_{T=t}^{+\infty} \beta^{T-t} E_t U(C_T, L_T, A_{T+1}) \tag{16}$$

Budget constraint is the same as in the previous section. It is important at this stage to note that I do not make any assumption about the value of the parameter β. In the standard model, $\frac{1}{\beta} - 1$ is equal to the steady state real interest rate. It necessarily implies a value of β close to one. Where wealth enters into the utility function, this equality no longer holds and β can be calibrated with more freedom as I show in proposition **??**. I now derive first order conditions.

Proposition 6. *First order conditions for the optimization problem are*

$$U_C(C_t, L_t, A_{t+1}) = \Lambda_t \tag{17a}$$

$$U_L(C_t, L_t, A_{t+1}) = w_t \Lambda_t \tag{17b}$$

$$U_A(C_t, L_t, A_{t+1}) + \beta E_t U_C(C_{t+1}, L_{t+1}, A_{t+2}) = Q_t \Lambda_t \tag{17c}$$

Proof The problem is close to the standard problem. The solution follows the same steps. See appendix **??** for details. $\square$

Compare to the intratemporal problem of the previous section, the only change is the forward looking term in the first order condition for wealth.

Steady state and linearization   Under the standard specification, the discount rate β is constrained to be the inverse of the interest factor. This restriction no longer holds with wealth in the utility function

Proposition 7. *At the steady state, there is a wedge between the discount rate and the inverse of the interest factor*

$$\beta = Q - \frac{U_A}{U_C} \tag{18}$$

Proof   It follows immediately from computing the steady state of the system of recursive equations given by the first order conditions above. Computations are given in appendix **??**.   □

I now linearize first order conditions around the steady state. I combine them with the asset supply equation (**??**). Asset supply is still fixed, implying $a_{t+1} = 0$. I keeps the nonseparability assumption of the previous section

Proposition 8. *Under hypothesis* **??**, *first order conditions for leisure and consumption becomes*

$$\frac{U_{LL}L}{U_L}l_t = w_t + \frac{U_{CC}C}{U_C}c_t$$

$$(1 - \frac{\beta}{Q})\frac{U_{CA}C}{U_A}c_t + \frac{\beta}{Q}\frac{U_{CC}C}{U_C}E_t c_{t+1} = q_t + \frac{U_{CC}C}{U_C}c_t$$

I denote $\kappa = 1 - \frac{\beta}{Q}$. Other notations are unchanged.

Proposition 9. *The system from proposition* **??** *can be rewritten*

$$\theta\eta n_t = w_t - \sigma c_t \tag{19a}$$

$$(\sigma + \kappa\nu)c_t = -rr_t + (1 - \kappa)\sigma E_t c_{t+1} \tag{19b}$$

Those equations are extremely close to those of the standard model but have two new parameters κ and ν. The term κ introduces a discount in the Euler equation for consumption. It reduces the elasticity of consumption with respect to **future** real interest rate values. As in the wealth targeting model, The term ν amplifies this discounting and as in the wealth targeting model, dampens the response of consumption to current real interest rate without modifying the income effect.

4 Implications for the New Keynesian model

I now study implications of intertemporal complementarity (IC thereafter) for the New Keynesian model. The more interesting property of IC is to relax the cross equation restriction between the income effect and the intertemporal substitution effect. For a given intertemporal substitution effect, I can obtain a lower income effect on labor supply. I show it has important consequences for responses to monetary policy shocks and demand shocks in the New Keynesian model (NK model thereafter). I use a very simple version of the NK model to derive those implications. I also verify in what extent they are still relevant in a medium scale model. The two models are presented in the first subsection. The second subsection is dedicated to monetary shocks and the third to demand shocks.

4.1 Framework

I display equations of the simple model in table **??**. There are four behavioral equations: the consumption equation, the labor supply equation, the monetary policy rule, and the Philips curve. Consumption is given by equation (**??**). It becomes the standard Euler equation if κ and ν are equal to zero. The monetary policy rule is unusual but simple. The nominal interest rate is equal to the expected inflation rate plus a disturbance[2]. The economic interpretation is that the central bank sets directly the real interest rate. It is not a realistic feature but aims at providing a better intuition by focusing on households' behavior. Indeed, whereas the New Keynesian Philips Curve is still there, it is no longer relevant for output and real variables in general. It only determines the path of inflation whose effects on real variables are neutralized by the response of the central bank.[3][4] Aggregate demand on the good market is equal to $\varphi c_t + d_t$. φ is the steady state consumption over output ratio. d_t is an exogenous shock directly expressed in terms of GDP percentage points. It encompasses all other components of aggregate demand including private investment and public consumption. The aggregate supply equation is the reduced form of the usual New Keynesian Philips Curve derived from Calvo Pricing. β_π

---

[2]Both the nominal interest rate and the expected inflation are in deviation from their steady state values

[3]This simple model can be viewed as an IS LM version of the New Keynesian model whereas the standard model with a Taylor rule would be as an AS-AD version

[4]This simple version does not grant determinacy. Our results are derived by assuming there are no sunspots. It is however easy to restore determinacy. For example, a monetary policy rule $r_t = \pi_{t+1} + \phi_y y_t$, ϕ_y being positive and possibly very small, would be sufficient.

is the coefficient associated with expected inflation and γ_p the coefficient associated with the marginal cost. On the labor market, I consider two variants of the model, a flexible wage variant in which the labor market is walrasian and a rigid wage variant in which the real wage does not depart from its steady state value and in which the difference between desired and effective hours worked is assimilated to the unemployment rate.

This very simple NK model is useful to provide some intuition. However, it is better to verify if results hold in a medium scale NK model. Equations are displayed in table **??**. I introduce a more conventional monetary policy rule along wage stickiness and wage and price indexation. I rely on simulations to compute impulse response functions. The calibration used to obtain these IRFs is displayed in table **??**. Frisch elasticity is equal to one. The consumption output ratio is set at 0.65, targeting the average value on US data between 1985 and 2007. Price and wage indexation parameters are both calibrated at 0.25 which is consistent with values found in estimated models. I set γ_p and γ_w at 0.1. The value of γ_p is high for reduced form estimation but is consistent with a yearly frequency for price changes. Coefficients for expected inflation β_π and β_w are set at 0.985.

Equation	Interpretation
$y_t = \alpha n_t$	Production Function
$y_t = \varphi c_t + d_t$	Market clearing on good market
$r_t - E_t \pi_{t+1} = rr_t$	Accounting equation
$\mu_t = w_t + n_t - y_t$	Marginal cost equation
$\pi_t = \gamma_p \mu_t + \beta_\pi E_t \pi_{t+1}$	Philips Curve
$r_t = E_t \pi_{t+1} + e_t$	Monetary policy rule
$n_t^s = \frac{1}{\theta \eta}(w_t - \sigma c_t)$	Desired Hours worked
$u_t = n_t - n_t^d$	Unemployment equation
$(\sigma + \kappa \nu)c_t = -rr_t + (1-\kappa)\sigma E_t c_{t+1}$	Consumption equation
$w_t = 0$	Rigid wage model
$u_t = 0$	Flexible wage model

Table 1: Simple Model

Identifying unemployment as the difference between desired and effective hours worked is a debatable assumption. Unemployment is an extensive margin phenomenon whereas the difference between desired and effective hours worked is an intensive margin phenomenon. However, Gali (2011) considers a model of indivisible labor in which house-

Equation	Interpretation
$y_t = \alpha n_t$	Production Function
$y_t = \varphi c_t + d_t$	Market clearing on good market
$r_t - \pi_{t+1} = rr_t$	Accounting equation
$\mu_t = w_t + n_t - y_t$	Marginal cost equation
$\pi_t = \frac{\beta_\pi}{1+\beta_\pi \tau_p} E_t \pi_{t+1} + \frac{\tau_p}{1+\beta_\pi \tau_p} \pi_{t-1} + \gamma_p \mu_t$	Philips Curve
$rg_t = \phi_\pi \pi_t + e_t$	Monetary policy rule
$r_t = \lambda r_{t-1} + (1-\lambda) rg_t$	Effective nominal interest rate
$n_t^s = \frac{1}{\theta \eta}(w_t - \sigma c_t)$	Desired Hours worked
$u_t = n_t - n_t^d$	Unemployment equation
$\pi_{w,t} = \frac{\beta_w}{1+\beta_w \tau_w} E_t \pi_{w,t+1} + \frac{\tau_w}{1+\beta_w \tau_w} \pi_{w,t-1} - \gamma_w u_t$	Wage Philips curve
$w_t = w_{t-1} + \pi_{w,t} - \pi_t$	Real wage equation
$(\sigma + \kappa \nu) c_t = -rr_t + (1-\kappa)\sigma E_t c_{t+1}$	Consumption equation

Table 2: Medium scale Model

Variable	value	Interpretation
α	0.68	Labor coefficient
ϕ_π	1.5	Inflation coefficient in MP rule
φ	0.65	consumption-output ratio
η	1	Hours-leisure ratio
θ	1	Inverse Frisch elasticity
β_π	0.985	Coefficient for the expected term in inflation equation
β_w	0.985	Coefficient for the expected term in wage equation
τ_p	0.25	Price indexation
τ_w	0.25	Wage indexation
γ_w	0.1	Wage Philips curve coefficient
γ_p	0.1	Philips curve coefficient
λ	0.7	Nominal rate persistence

Table 3: calibration

holds member differ by their labor disutility but have a common level of consumption. It shows that the reduced form for labor force participation is the same as the reduced form for desired hours worked in the classical model. In appendix **??**, I show that the Gali's

framework is compatible with a utility function whose wealth is one of the argument.

Wealth in the utility function has also some implications for the supply block. The derivation of the New Keynesian Philips curve for prices and wages is mostly unaffected but the relevant discount rate may be subject to some debate. Should firms (resp. "trade unions") discount profit streams (resp. utility streams) using the safe real interest rate or the pricing kernel $\beta \frac{U_c(C_{t+1})}{U_c(C_t)}$? If firms and trade unions maximize the utility of their shareholders (resp. members) and if only bonds enter in the utility function, the latter is the relevant one, leading to potentially large discount not only in the consumption equation but also in inflation and wage inflation equation. However, I want to focus on implications of WIU and IC on households' choice. Thus, I choose to allow for different discount rate in consumption, inflation and wage inflation equation. In the section dedicated to the estimation of the model, I check the robustness of my findings to this assumption.

4.2 The supply effect of the real interest rate

To better understand the effects of IC, it is useful to start by considering the standard model, when κ and ν are equal to 0. The same parameter governs the income effect on labor supply and the elasticity of intertemporal substitution. A first consequence of this cross equation restriction is that the real interest rate has the same impact on labor supply as the real wage growth. Indeed, consider consumption and desired hours worked equation from table **??** with κ and ν equal to zero

$$\sigma c_t = -rr_t + \sigma E_t c_{t+1}$$
$$- \theta \eta n_t^s = w_t - \sigma c_t$$

Combining the two equations allows me to derive an euler equation for hours.

$$\theta \eta (E_t n_{t+1}^s - n_t^s) = E_t w_{t+1} - w_t - rr_t \tag{20}$$

The elasticity of desired hours worked with respect to real wage growth is always equal to the elasticity with respect to the real interest rate. Real interest rate matters here because of the income effect on labor supply. But, it is important to keep in mind that the effect of the real interest rate on hours worked does not depend on the parameter σ which governs the income effect. A naive view of the problem would state that real interest rate affects labor supply because it affects consumption. Then, reducing the

sensitivity of consumption to real interest rate would lower the impact of the real interest rate. Equation **??** shows it is misleading. Real interest rate affects labor supply because it affects the *marginal utility of consumption* and under the standard specification, the real interest rate is equal to the growth rate of the marginal utility of consumption whatever the value of the parameter σ is.

Now, let me consider the case where κ and ν are positive. This supply effect of the real interest rate vanishes

Proposition 10. *If the utility function accepts wealth as an argument and if U_{CA} is positive, the elasticity of labor supply with respect to real interest rate is lower than the elasticity of labor supply to real wages.*

Indeed, deriving the labor supply equation for κ and ν different from zero and dropping expected terms for more clarity, I get

$$\theta \eta n_t^s = w_t + \frac{\sigma}{\sigma + \kappa\nu} rr_t \tag{21}$$

The elasticity of hours with respect to wages is given by $\frac{1}{\theta\eta}$ whereas the elasticity of hours with respect to real interest rate is $\frac{\sigma}{\sigma + \kappa\nu}\frac{1}{\theta\eta}$. If ν is positive, the elasticity with respect to real interest rate is lower. A more detailed proof is given in appendix **??**

4.3 Monetary shocks

4.3.1 Real wages in the flexible wage variant

The restriction on the hours equation has an important consequence for the relation between the real wage and the real interest rate in the flexible wage variant of the model.

I derive the New Keynesian labor demand conditional to a monetary policy shock. In the New Keynesian model, the production is determined by the demand in the short run. Firms collect orders and use production function to determine the amount of labor they need to satisfy these orders.[5]

---

[5]This New Keynesian labor demand is different from the neoclassical labor demand which in the NK model is more relevant in the long run. In the short run, the neoclassical labor demand is replaced by the marginal cost (or the inverse markup) equation and the Philips Curve which together with the monetary policy rule determine the real interest rate. In the simple model of this section, the real interest rate is directly set by the central bank and does not react to changes in inflation and thus changes in marginal cost. Because of this particular assumption, the supply block of the model is unimportant. The marginal cost equation may also act as a labor demand equation if the marginal cost is kept constant. For my

I first derive the labor demand in the standard model. I combine the consumption equation, the market clearing condition and the production function of table **??**. To make notations more friendly, I assume that the monetary shock is perfectly anticipated and that there is no demand shock. It allows me to drop the expectation operator and the term d_t.

$$\frac{\sigma\alpha}{\varphi}(n_{t+1} - n_t) = rr_t \tag{22}$$

I combine equation (**??**) which represents labor supply and equation (**??**) which represents labor demand. The equilibrium value of real wage growth appears and depends on real interest rate. I break down the real wage equation between a demand effect and a supply effect. The demand effect is defined as the change of real wages following a change in real interest rate when the labor supply curve is held constant. The supply effect is the change when the labor demand curve is held constant. I give a formal definition

Definition 1. *Consider the system of labor supply and demand*

$$\theta\eta(n^s_{t+1} - n^s_t) = w_{t+1} - w_t - rr_t$$
$$\frac{\sigma\alpha}{\varphi}(n_{t+1} - n_t) = rr_t$$

The demand effect is the response of real wages following a change in real interest rate when the labor supply curve does not shift (e.g in an "imaginary" world in which $\theta\eta(n^s_{t+1} - n^s_t) = w_{t+1} - w_t$). The supply effect is the difference between the total effect and the demand effect.

Real wages are given by

$$w_t = -\left(\underbrace{1}_{Supply\ effect} + \underbrace{\frac{\theta\eta\varphi}{\sigma\alpha}}_{Demand\ effect}\right)rr_t + w_{t+1} \tag{23}$$

Real interest rate affects the growth of real wages through the two channels. The demand effect represents the traditional keynesian channel. An expansionary monetary policy increases aggregate demand stimulating labor demand and thus real wages. In addition, a supply effect arises. A fall in real interest rate leads workers to substitute current leisure to future one and thus to reduce their labor supply, stimulating wage growth.

purpose, it seems more relevant to use the New Keynesian labor demand. Moreover, this "alternative" labor demand is difficult to interpret because the marginal cost is jointly determined with hours by firms and not an exogenous variable to their labor demand decision.

A counter intuitive result is that this supply effect on real wages *neither depends on the Frisch elasticity of the labor supply nor on the elasticity of intertemporal substitution.*

To better understand the intuition behind the response of real wages, I give a stylized representation of the labor market in figure **??**
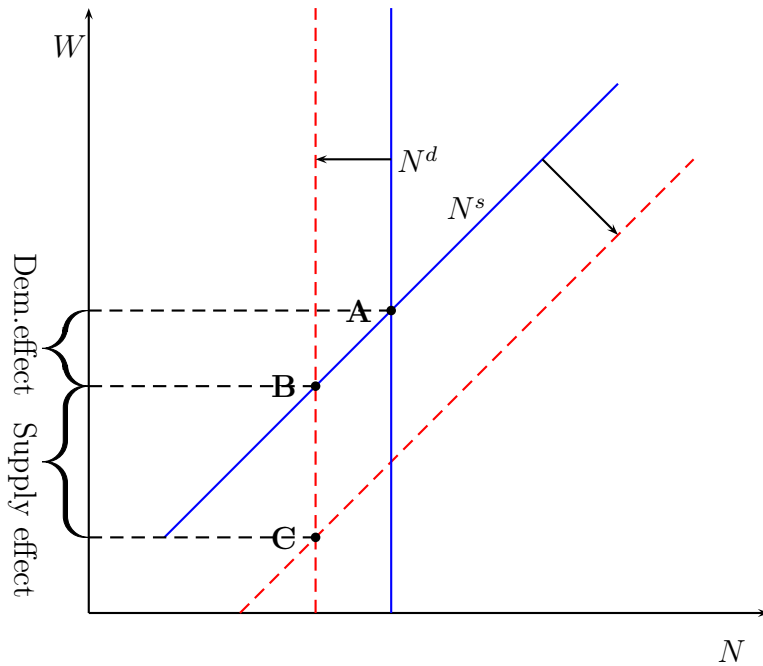

Figure 1: Real wages following a rise in real rate

In the short run, the labor demand is given by equation (**??**). It is vertical and does not depend on real wages. Labor supply increases with real wages for a given level of real interest rate. The point A is the initial equilibrium. A rise of real rate has two effects on the figure. It depresses aggregate demand, shifting the labor demand curve to the left, reducing equilibrium real wages. This is the standard keynesian channel. If only this channel is at play, the economy moves to the point B.

A second effect is the supply effect of the interest rate. The rise in the interest rate pushes the labor supply curve to the right, further lowering the equilibrium real wages, moving equilibrium to point C.

Consider now the case of intertemporal complementarity. Positive values for ν and κ alleviate the supply effect of the interest rate and thus lower the response of real wages to change in real interest rates.

Proposition 11. *The elasticity of equilibrium real wages with respect to real interest is decreasing with the value of the parameter ν governing intertemporal complementarity and*

with the value of the parameter κ which governs the discount rate in the euler equation.

Indeed, computing the equilibrium real wages on the labor market when κ and ν are different from zero gives

$$w_t = -\left(\underbrace{\frac{\sigma}{\sigma + \kappa\nu}}_{Supply\ effect} + \underbrace{\frac{\sigma}{\sigma + \kappa\nu}\frac{\theta\eta\varphi}{\sigma\alpha}}_{Demand\ effect}\right) rr_t + \frac{\sigma}{\sigma + \kappa\nu}(1 - \kappa)w_{t+1} \qquad (24)$$

Consider a temporary change in the real interest rate, the response of the real wage is lower when κ or σ are greater. Following a rise in real interest rate, High values for κ ans ν lower the response of leisure and thus the increase of labor supply. The rightward shift of labor supply is less important, limiting the fall in real wages. The demand effect is affected in a similar way. $\sigma + \kappa\nu$ governs the intertemporal effect of substitution. Higher values of κ and σ imply a lower impact of real interest rate on consumption and thus on aggregate demand.

What is interesting is that IC allows a lower supply effect for a given demand effect. The response of consumption to a certain path of real interest rate is mainly determined by the discount rate κ and the inverse of the consumption elasticity to real interest rate $\sigma + \kappa\nu$. Assuming both are given, a higher value of ν would lead to a lower value of σ lowering the supply effect whereas keeping the response of consumption to real interest rate roughly unchanged.

Proposition 12. *For a given value of $\sigma + \kappa\nu$ and a given value of κ, a larger value of ν (e.g a larger intertemporal complementarity) lowers the supply effect without affecting the demand effect.*

The proof immediately follows from the demand and supply effects highlighted in equation (**??**)

4.3.2 Unemployment in the rigid wage variant

The response of unemployment in the rigid wage model is very similar to the response of real wages in the flexible wage model. Shifts in labor demand and supply affects unemployment instead of real wages. Figure **??** provides some intuition. The equilibrium for labor and real wages is given by the intersection of the labor demand curve and the real wage curve whereas the difference between the labor demand and the labor supply for this real wage gives the unemployment rate. Unemployment is initially equal to zero.

Following a rise in real interest rate, labor demand shifts to the left and labor supply to the right. Real wages remain at the same level, causing a rise in unemployment, coming from both the demand and the supply effect.
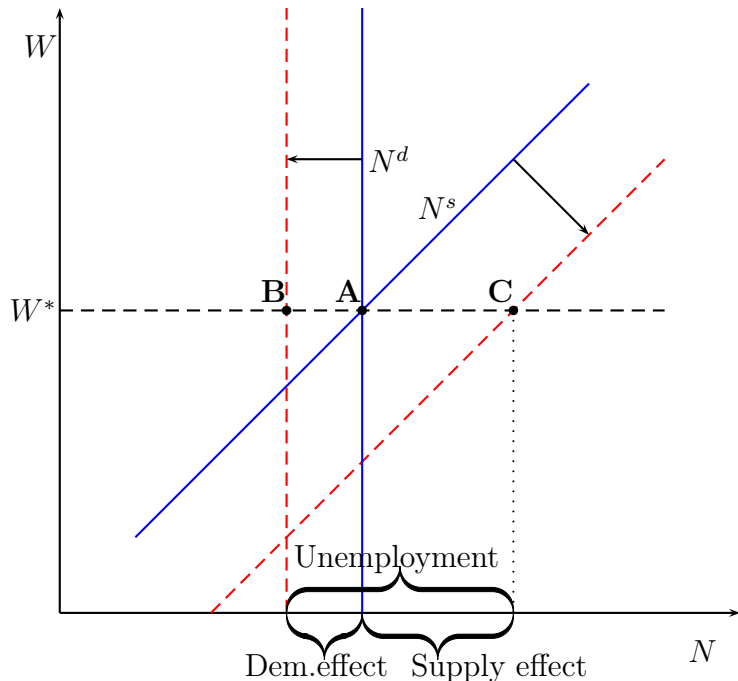

Figure 2: Unemployment following a rise in real rate

Computations confirm the graphical intuition. Unemployment is given by

$$u_t = -\left(\underbrace{\frac{\sigma}{\sigma + \kappa\nu}\frac{1}{\theta\eta}}_{Supply\ effect} + \underbrace{\frac{\sigma}{\sigma + \kappa\nu}\frac{\varphi}{\sigma\alpha}}_{Demand\ effect} \right) rr_t + \frac{\sigma}{\sigma + \kappa\nu}(1 - \kappa)u_{t+1} \tag{25}$$

4.3.3 The medium scale model

Are these insights still relevant in a medium scale model? Figure **??** represents impulse responses (IRFs thereafter) to a monetary policy shock for real wages, output, unemployment and expected real interest rate when κ and ν are equal to zero. The shock is an unexpected one percent decrease in the nominal rate in annual value. On impact, the real rate falls by 0.4 percent and output increases by the same amount. Real wages slightly underreacts on impact but displays a hump shaped response with a peak around 0.4 percent too. More striking is the response of unemployment. Unemployment falls by 1.2 points on impact, nearly three times the response of output. It suggests that the supply effect of monetary policy is still sizable in the medium scale model.
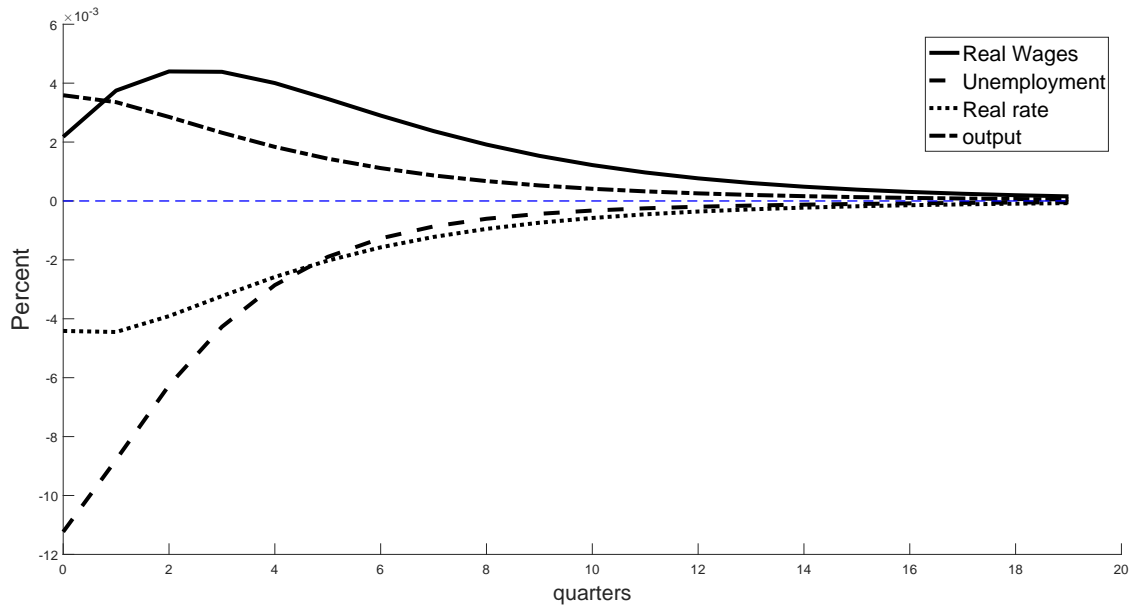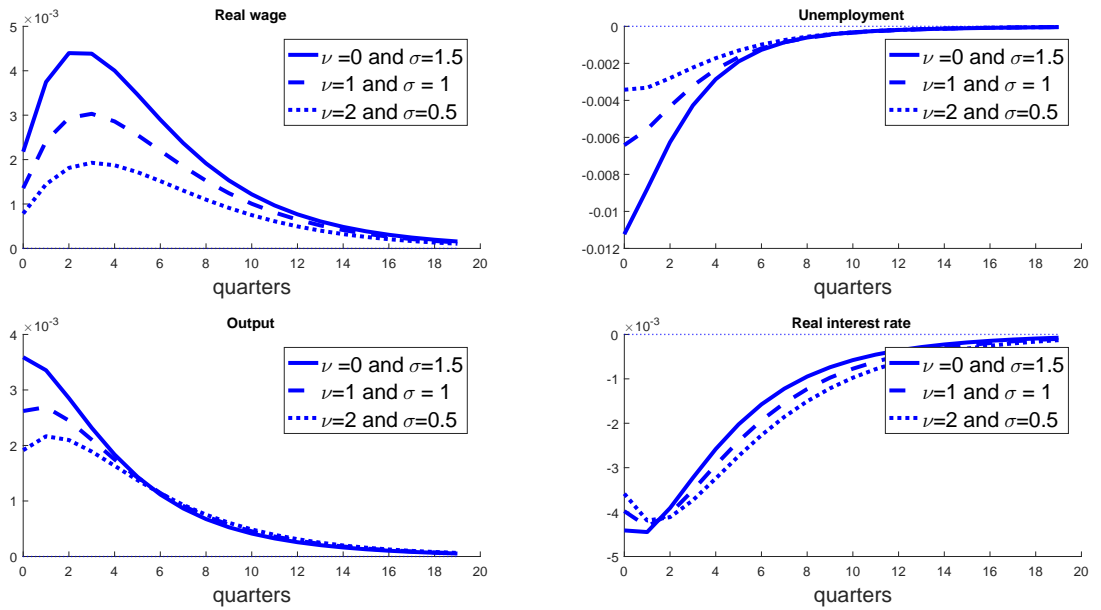
Figure 3: Responses to monetary policy shocks

Figure 4: Responses for several calibration

Figure **??** represents the same IRFs in three different cases, corresponding to different values of σ and ν. κ is set at 0.5 and $\sigma + \kappa\nu$ is set at 1.5 in each case, hence parameters governing the demand effect are roughly unchanged. The main difference between the

three experiments is the income effect on labor supply. The solid line is the "standard case" with ν equal to 0 and σ equal to 1.5. The dotted line is the "IC case". It displays responses obtained with a significant level of intertemporal complementarity (ν equal to 2 and σ equal to 0.5). The dashed line is an intermediate case. The response of the real interest rate is quite similar across the different calibrations. The response of output slightly falls when ν increases (because a lower σ also lowers the discount rate in the Euler equation). Responses of real wages and unemployment are much more affected. They are roughly divided by three when the response of output is divided by 1.5. Indeed, the lower value of σ reduces the income effect on labor supply, dampening the response of labor supply to a change in real interest rate, and thus the response of real wages or the response of unemployment. In the meantime, the higher value of ν keeps the demand effect at a similar level across the three experiments. This interpretation is supported by figure **??**. It represents IRFs of labor demand and labor supply after a monetary policy shock in the standard case and in the IC case. In the standard case, the labor demand increases and the labor supply decreases, roughly by the same magnitude. The rise in unemployment is caused equally by the demand and the supply effect. In the IC model, the increase in unemployment is mainly caused by the increase in labor demand. The response of labor supply is ambiguous.
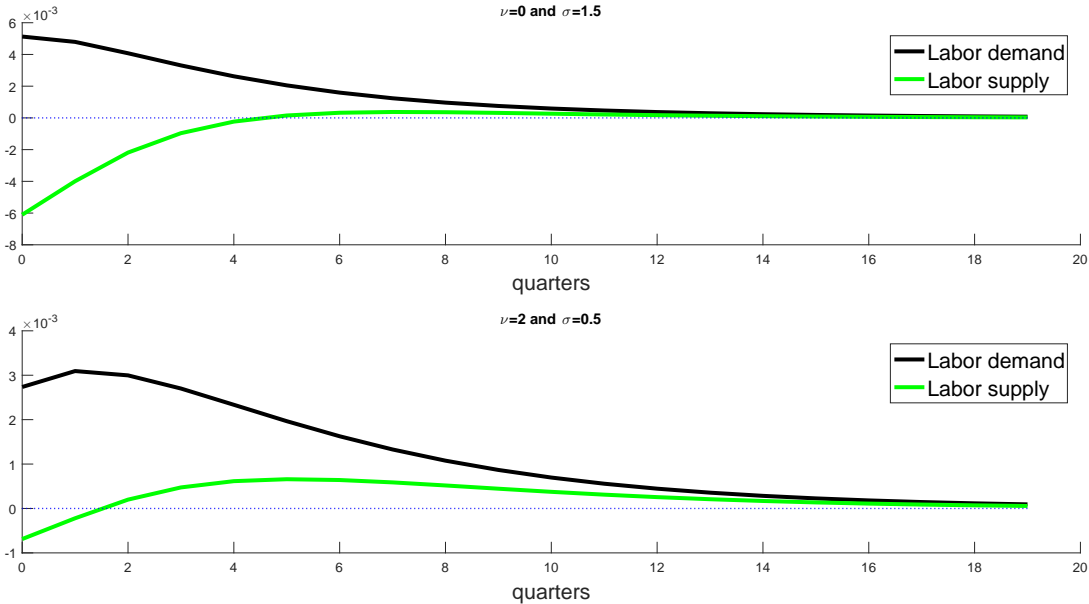

Figure 5: Labor supply and demand after a monetary policy shock

4.4 Demand Shock

Real wages and Unemployment after a demand shock In this section, I study the response of real wages and unemployment after a demand shock. There are several reasons to be interested in them. First, private investment and public consumption are probably important drivers of the business cycles. Second, in the conventional (keynesian) wisdom, effects of demand and monetary shocks on output, real wages and unemployment are similar. This conventional wisdom is only partially true in the NK model. A contractionary demand shock is equivalent to a contractionary monetary policy shock minus the supply effect.

Indeed, consider for example the response of real wages to a demand shock in the simple model[6].

$$w_t = -\frac{\theta\eta}{\alpha}(d_t - d_{t+1}) + w_{t+1} \tag{26}$$

An exogenous demand shock increases output, labor demand and thus real wages. In the meantime, real interest rate and thus consumption are unaffected keeping the labor supply curve unchanged. Obviously, it is an extreme result due to the very specific monetary policy rule of the simple model. In practice, real interest rates react to demand shocks.
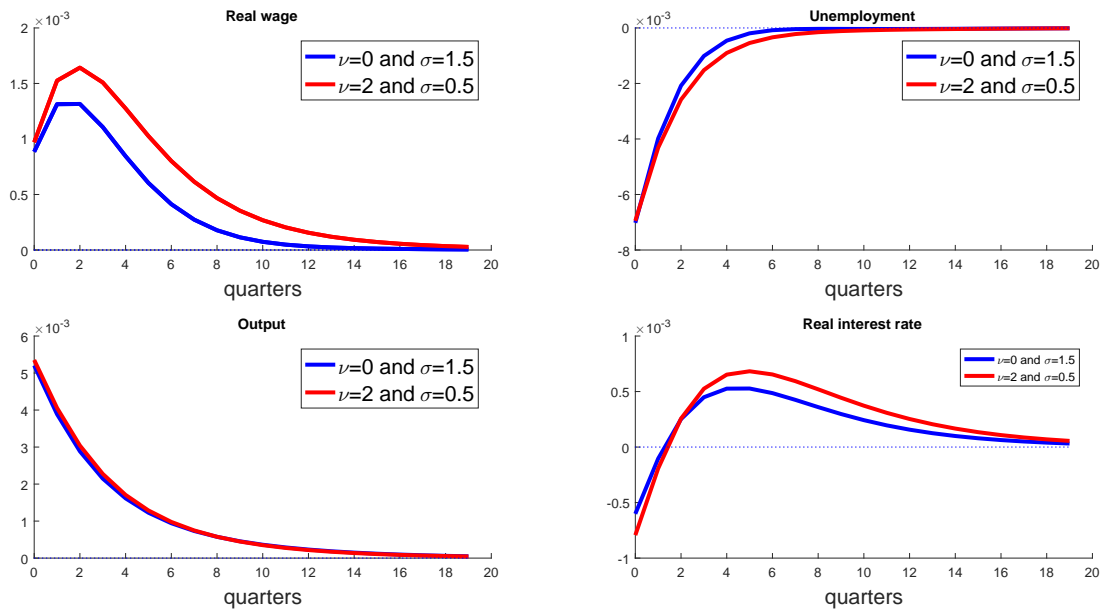

Figure 6: Response of Real wages to demand shocks

---

[6]As in the previous section, I assume that the demand shock is the only shock is perfectly anticipated , allowing me to drop the expectation term and the monetary shock.

Figure **??** shows that the result is actually quite robust in the medium scale model. I represent impulse responses of output, real rate, unemployment and real wages to a demand shock in the standard and in the IC case. IRFs are nearly unaffected by the different values of ν and σ.

Response of the output gap to demand shocks   Whereas ν and σ are unimportant for the response of output, real wages and unemployment, they actually matter for the response of output gap. Figure **??** displays the response of output, output gap, actual and natural interest rate following a demand shock in the standard case and in the IC case. In the standard model, the response of the output gap to a demand shock is nearly three times smaller than the response of output. The output gap has nearly disappeared after four quarters. This small response is not because the response of output is small but because the response of flexible price output is large. It is a consequence of the supply effect of real interest rate. Because of it, in the flexible price equilibrium, a small increase in the real interest rate may increase labor supply and decrease consumption enough to reestablish the equality between aggregate demand and the flexible price output. In other words, the response of the natural interest rate[7] to a demand shock is small, close to the the response of the real rate in the sticky price equilibrium. This one is initially negative because of nominal interest rate smoothing, but the effects of the smoothing vanished after four quarters and the real rate becomes only marginally different from the natural one. As a consequence, the response of output is close to the response of natural output and the output gap is small. In the model with IC, the response of the natural interest rate to a demand shock is much larger and farther away from the actual response of monetary policy. The response of output gap is larger and more persistent.

5   A Bayesian estimation of the model

In this section, I estimate parameter κ and ν. My approach is to perform a bayesian estimation of the medium scale model and to focus on parameters κ and ν.

The model I estimate is a variant of the model displayed in table **??**. I add five shocks: a productivity shock, a markup price shock, a markup wage shock, a labor disutility

---

[7]Here, the natural interest rate is the flexible price equilibrium real interest rate and not the steady state real interest rate.
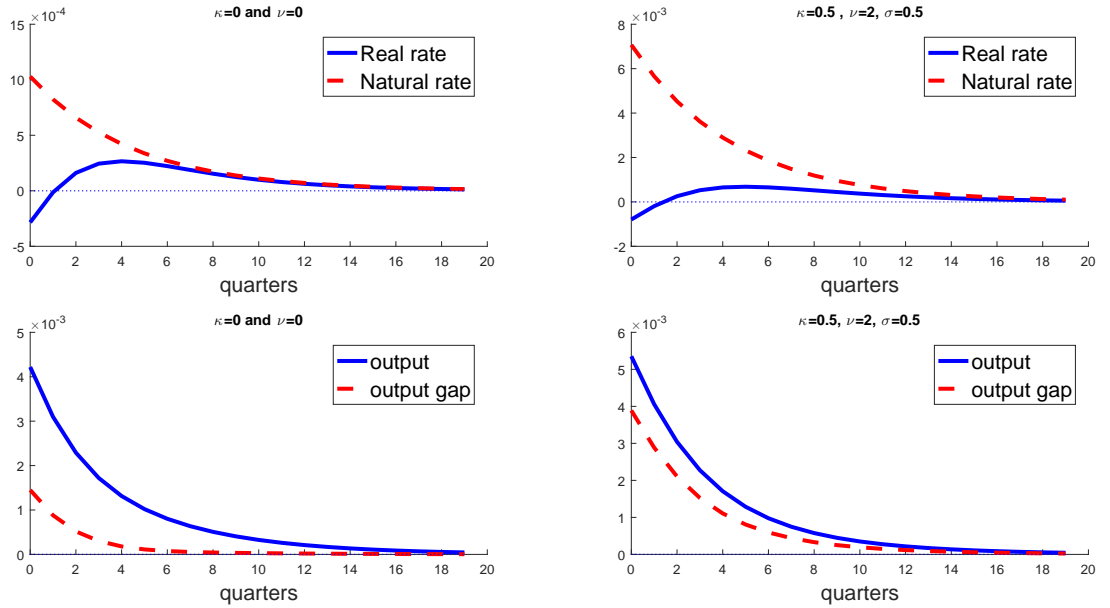
Figure 7: Response of output gap and output to demand shocks

shock and a discount factor shock. The complete specification can be consulted in table
??. Parameters γ_p and γ_w (e.g Philips Curve coefficients) are estimated directly. I use
quarterly data for seven macroeconomic variables : Real GDP, Real compensation per
Hour in Nonfarm business sector, GDP deflator, Hours worked by all persons in Nonfarm
business sector, Real Personal Consumption Expenditures, the effective FED funds rate,
and the Unemployment rate. Following Smets and Wouters(2007), observables are first
differences for the log of each of these variables, except for unemployment and FED funds
rate, which are simply detrended. The sample contains 91 data points from the 1985:2
to 2007:4 . This dataset is small but the risk of structural breaks would be higher over
a longer sample. Ours goes from the "great inflation" to the "great recession". A stable
relation between macroeconomic variables seems reasonable over that period. I perform
a robustness test with a longer sample.

5.1   Baseline estimation

Priors are given in table **??**. For usual parameters, they follow the literature. My prior on
κ is a beta distribution whose mean is 0.5 and whose variance is 0.2. This choice excludes
a value equal to zero. Thus, it is not possible to recover the standard model but it is still
possible to be very close from it. I also estimate the model with a more conservative prior
for κ. Results are described in the robustness subsection. The parameter ν is initially
supposed to follow a Gaussian distribution centered around a zero mean with a large

24

Variable	distribution	mean	std
κ	beta	0.5	0.2
ν	normal	0	1.5
σ	normal	1	1.5
θ	gamma	2	0.25
γ_p	normal	0.25	0.05
γ_w	normal	0.25	0.05
ϕ_π	normal	1.5	0.25
ϕ_y	normal	0.12	0.25
τ_p	beta	0.5	0.15
τ_w	beta	0.5	0.15

Table 4: Priors for parameters

standard deviation set at 1.5.

Posterior estimates are displayed in table ??. Figure ?? represents prior and posterior distributions of κ and ν. They show a substantial shift between the prior and the posterior. Both parameters are positive and large. The estimated discount rate κ is equal to 0.8 whereas the estimated value of ν is as high as 2. It means that the coefficient for the expected term in the consumption equation is roughly equal to 0.05. σ is close to 1 suggesting a still sizable income effect, whereas the inverse of the elasticity of consumption to real interest rate $\sigma + \kappa\nu$ is close to 3. Other parameters are in line with the literature, except for the marginal cost coefficient in the Philips curve which is close to zero.

It is interesting to compare these results with an estimation of the standard model. I set κ and ν equal to 0 and reestimate the model. Results are displayed in table ??. The main change is the mean estimate for σ. It is equal to 2.77 instead of 1.17. The estimate of σ in the standard model probably captures a small apparent response of consumption to changes in real interest rate. In my baseline estimation, this small response leads to a high value for the parameter ν whereas the value of σ is determined by the apparent income effect on labor supply. It is worth noting that the estimate of γ_p is also very low when estimating the standard model.

Variable	mean	mode	inf	sup
κ	0.797	0.839	0.644	0.961
ν	2.13	2.01	1.03	3.19
σ	1.17	1.11	0.627	1.69
θ	2.31	2.26	1.9	2.7
γ_p	0.0004	0.0005	-0.0006	0.0013
γ_w	0.264	0.284	0.178	0.374
ϕ_y	0.463	0.44	0.113	0.788
ϕ_π	1.47	1.46	1.07	1.86
τ_p	0.287	0.315	0.131	0.435
τ_w	0.126	0.0956	0.0378	0.21

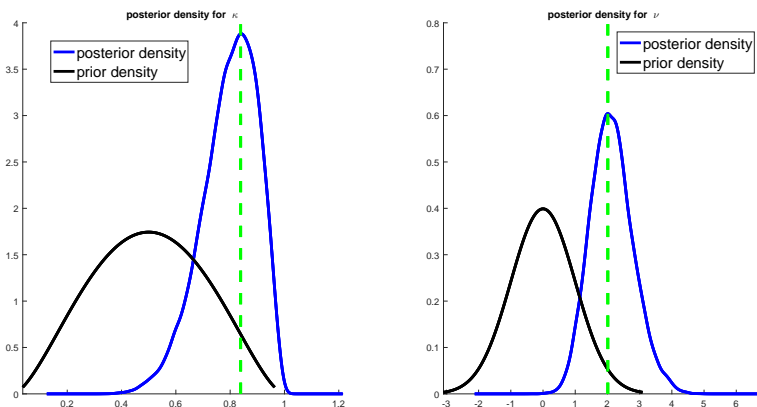Table 5: Posteriors for the baseline estimation

Figure 8: Posterior and prior distribution for κ and ν

5.2 Inspecting the mechanism

To better understand these results, it is useful to consider the figure **??**. This figure represents the observed labor supply, the "predicted" one and the labor disutility shock for the standard model and the model with wealth in the utility function and intertemporal complementarity. The "predicted" labor supply is the value predicted by the model, given observations of wages and consumption and without taking into account labor disutility shock. It is equal to $\frac{1}{\theta\eta}(w_t - \sigma c_t)$ where w_t and c_t are observed. For the standard model, the predicted labor supply seems very weakly correlated with the actual one and strongly negatively correlated with the shock. The labor disutility shock is large not because this is necessary to explain large changes in the *observed* labor supply but because this

Variable	mean	mode	inf	sup
σ	2.77	2.63	2.16	3.37
θ	2.58	2.51	2.14	2.98
γ_p	0.0042	0.0019	-0.0015	0.0109
γ_w	0.206	0.247	0.0513	0.318
ϕ_y	1.24	1.27	0.952	1.57
ϕ_π	1.75	1.66	1.23	2.3
τ_p	0.374	0.356	0.153	0.592
τ_w	0.138	0.104	0.0471	0.225

Table 6: Posteriors for the standard model estimation

explains a wide discrepancy between *predicted* changes in labor supply and *observed* ones. In other words, with the standard specification and the estimated value for σ, the model predicts large changes in labor supply because of observed changes in consumption and wages. These changes are not observed in data and the model has to create a large labor disutility shock in order to explain that the observed labor supply does not change.

With intertemporal complementarity, the main change for the predicted labor supply is a much lower estimated value for σ. The serie seems more reasonable. The predicted value still seems unable to explain the actual one but does not induce an artificially large labor disutility shock. Data points to moderate values for σ. But, such values are not compatible with the observed response of consumption to real interest rate. Introducing the parameter ν allows to combine a moderate income effect and a small elasticity of consumption with respect to real interest rate.

5.3 Robustness

I now perform several robustness exercises. Posterior mean and confidence interval of κ, ν and σ are displayed in table **??** for all these exercises.

In the first one, I assume a different prior for κ. κ still follows a beta distribution but whose mean is 0.2 and whose standard deviation is 0.1 . Results show that the posterior estimate of κ is lower than in the baseline estimation but still very high around 0.55.

In a second one, I introduce some form of habits consumption. Indeed, not adding a lag in the consumption equation may introduce some bias. I choose external habits.
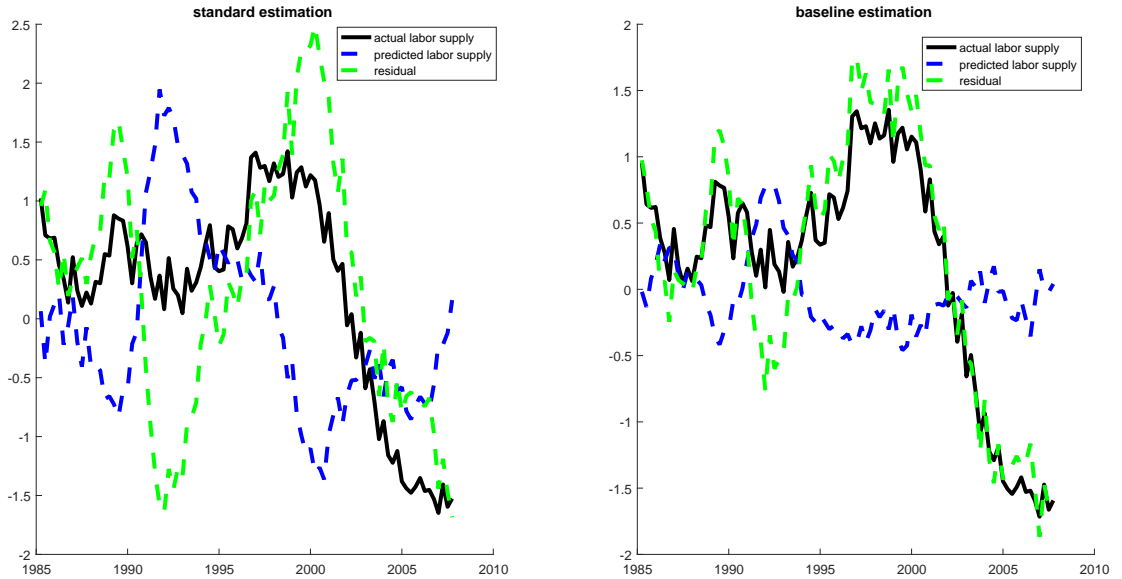
Figure 9: Predicted and actual labor supply

Utility is provided by $c_t - hC_{t-1}$ where c_t is individual consumption and C_{t-1} aggregate consumption. h is calibrated at 0.7. Posterior mean of κ and ν are slightly lower than in the baseline estimation. The more affected parameter is σ whose posterior mean collapses.

In the third one, I estimate the model again by using a longer sample from 1954:3 to 2007:4. The estimate κ is still at 0.8 whereas ν is larger than in the baseline estimation.

In a fourth "experiment", I replace hours worked by civilian labor force participation among the observables. Results for ν and κ are roughly equivalent. Interestingly the value of σ is lower than in the baseline estimation, around 0.4, suggesting that income effect on labor supply is lower with labor force participation being the measure of labor supply.

In the fifth exercise, the discount rate in price and wage Philips curve is directly related to households' discount rate. Results seems unaffected. I also perform an estimation in which β_π and β_w are estimated. It leads to very low values for both of them (0.13 and 0.27 respectively) but also to a more reasonable value for γ_p, close to 0.08 compatible with a yearly frequency for price change.

The value of γ_p is very small in my baseline estimation (and in the standard one). Eventually, I estimate the model with γ_p and γ_w calibrated at 0.025. Whereas I obtain a larger estimate for σ, I still find large and positive values for κ and ν.

	Post. mean for κ	Post. mean for ν	Post. mean for σ
Alter. prior for κ	0.556 [0.4,0.7]	2.607 [1.56,3.8]	1.14 [0.64,1.7]
Habits consumption	0.652 [0.41,0.91]	1.67 [0.75,2.5]	0.196 [0.03,0.35]
Long sample	0.86 [0.76,0.98]	3.542 [2.37,4.51]	0.75 [0.4, 1.1]
Labor force part. as observable	0.82 [0.69,0.96]	2.80 [1.41,4.07]	0.43 [0.09, 0.81]
High disc. rate for NKPC	0.79 [0.60,0.94]	2.11 [1.11,2.94]	1.12 [0.65, 1.52]
γ_p and γ_w calibrated	0.7252 [0.53,0.93]	1.47 [0.4, 2.6]	1.49 [0.92,2.01]

Table 7: Posterior mean of key parameters for different variants

5.4 Quantitative implications of the estimated model

In this paragraph, I look at IRFs generated by the estimated model. Figure **??** represents responses of output, expected real interest rate, unemployment and real wages. The fall in unemployment is in line with the rise of output suggesting that changes in unemployment are mostly explained by the demand side. The magnitude of the response of output is relatively low as compared to the response of the real rate. However, it should be kept in mind that this is only the response of consumption. Residential investment is a part of the exogenous demand shock d_t and thus is not sensitive to the real interest rate in the model, whereas in reality it represents a substantial part of the response of aggregate demand. The response of real wages is quite large, reflecting the high value of γ_w (e.g frequent price changes). Figure **??** displays response of labor demand and supply. Interestingly, labor supply increases following the expansionary monetary policy shock. In the calibrated exercise of section 3, labor supply was decreasing in the standard model and flat with intertemporal complementarity. I conjecture that this positive response of labor supply is related to the relatively large response of real wages. The fall in real rates pushes the labor supply curve to the left but labor supply also moves along the labor supply curve with the rise in real wages. Figure **??** shows the response of output and output gap following a demand shock. The two responses are quite close. It is worth noting that the demand shock generates a substantial rise in the natural interest rate.
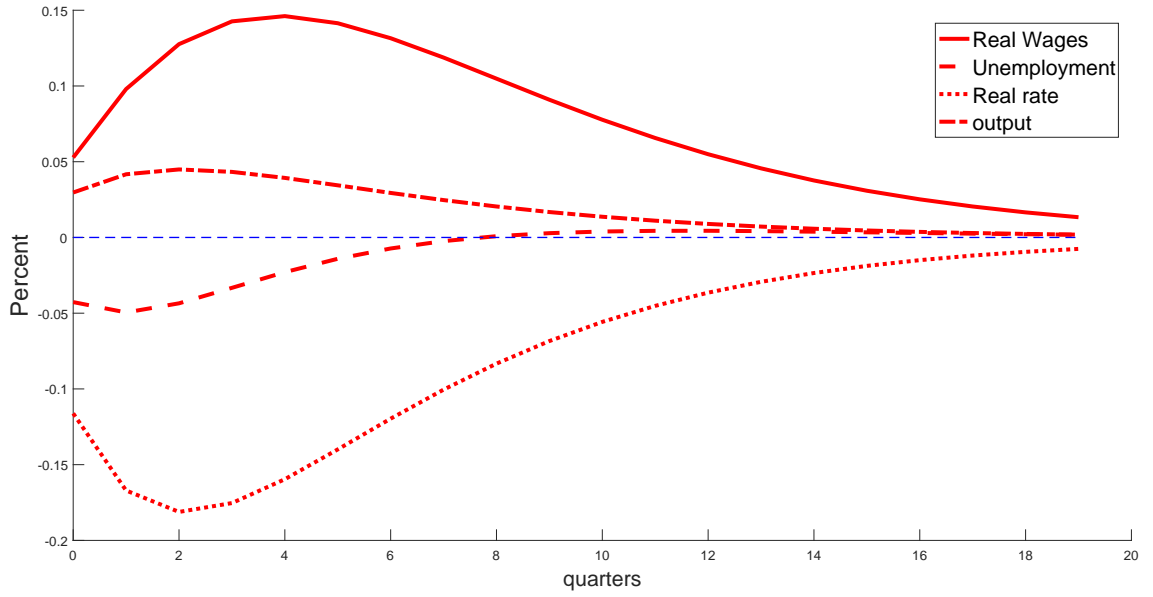
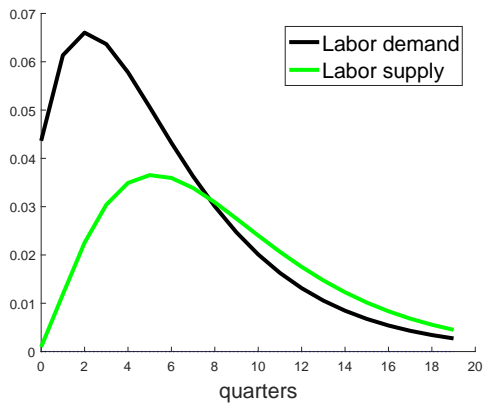Figure 10: Impulse responses to Monetary Policy shocks

Figure 11: Responses of labor supply and demand to Monetary Policy Shocks

6   Discussion

In this section, I discuss various pieces of literature which may support the specification proposed in this paper.

Labor market response to Monetary Policy Shocks   Wealth in the utility function and intertemporal complementarity have important implications for the behavior of labor market variables following a monetary policy shocks. A substantial literature has dealt with the issue.

Intertemporal complementarity could help to explain the cyclical behavior of labor force participation. Pieces of evidence from VAR models collected by Christiano, Trabandt
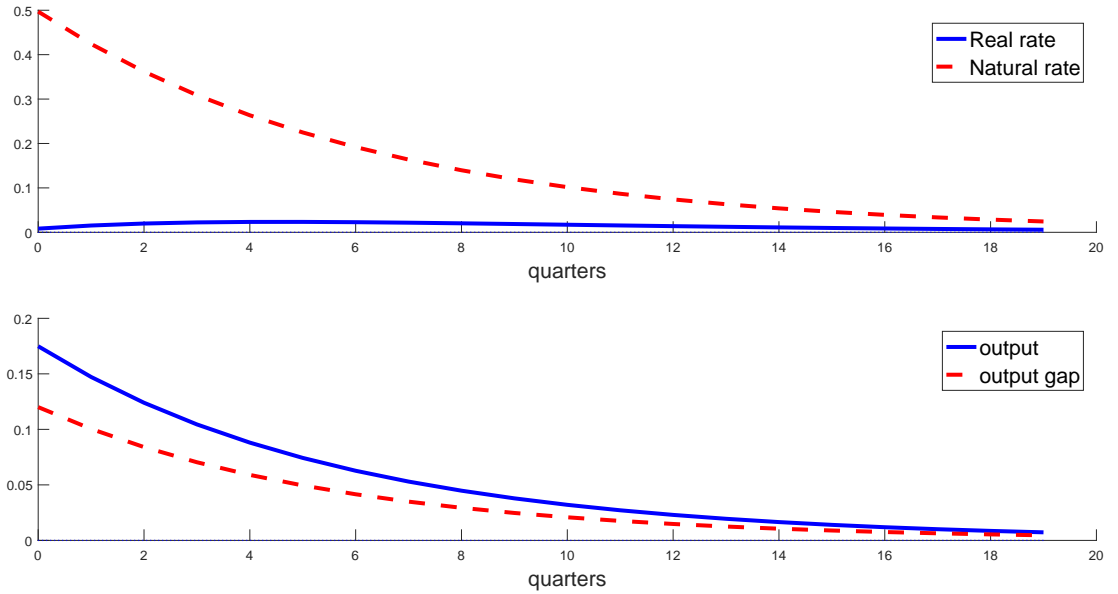
Figure 12: Response of output gap to demand shocks

and Walentin (2010) suggest that labor force is mildly procyclical conditionally to a monetary policy shock. By contrast, labor supply, if identified in the way proposed by Gali (2011), responds positively to a contractionary monetary policy shock in the standard model because of the income effect on labor supply. Reducing the income effect is necessary to reconcile the model and the data. WIU and IC are a possibility to do so.

Simple New Keynesian model also fails to match the response of real wages to monetary policy shocks. Sims and Zha (1998) finds that a very persistent increase of nominal rate by 0.4 percent in annual value have not a significant impact on average real wages when looking at US postwar data. Christiano, Eichenbaum and Evans (1997) find relatively similar results. A less persistent 0.7 percent increase in nominal interest rate rises real wages by 0.1 percent. Both papers focus on US data, but estimations across countries do not support a strongly procyclical response of real wages. Peersman and Smets (2001) finds that response of real wages is small in most countries of the euro area. Normandin (2006) finds a similar result for United Kingdom and Canada. These last two papers also find a counter-cyclical response of real wages for several countries. Wage rigidity provides a simple explanation for this mild procyclicality of real wages conditional to monetary policy shocks. However, the degree of wage stickiness needed to match data could be lower with an alternative specification of the utility function disentangling between the income effect on labor supply and the elasticity of intertemporal substitution.

Empirical evidence on euler equation A large empirical literature has dealt with the Euler equation. Hall(1988) finds no evidence of intertemporal substitution, a result confirmed for example by Yogo (2004). Another disappointing result was the negative correlation found between the FED funds rate and the real rate implied by consumption growth, found by Canzoneri et al. (2007) for several widely used consumption models. More positive results came from several papers by Attanasio and Weber (1993, 1995, 2010). Using microeconomic data on individual consumption and introducing controls for demographics and labor supply, they find a larger elasticity of intertemporal substitution. Results from these various papers are conflicting but pieces of evidence accumulated by the literature do not clearly endorse the standard model.

More importantly, such tests do not really allow the econometrician to choose between the standard model and a model with a discounted Euler equation, or even between the standard model and our wealth targeting model. Indeed, an equation giving consumption levels as a decreasing function of real interest rate also implies a positive correlation between consumption growth and real interest rate if changes in real interest rate are positively auto correlated. Conversely, under the same condition, an equation giving consumption growth as an increasing linear function of real interest rate implies a negative correlation between real interest rate and consumption levels.

Indeed, consider a consumption equation in level like in the wealth targeting model. $c_t = -\varphi rr_{t+1}$. Consumption growth becomes $c_{t+1} - c_t = -\varphi(rr_{t+2} - rr_{t+1})$, implying $cov(c_{t+1} - c_t, rr_{t+1}) = \varphi(1 - \rho_r)$ where ρ_r is the autocorrelation coefficient of real interest rate.

Consider now a consumption equation in growth $c_{t+1} - c_t = \sigma rr_{t+1}$. Correlation between consumption levels and real interest rate is equal to $cov(c_t, rr_{t+1}) = cov(c_t, \frac{c_{t+1}-c_t}{\sigma}) = \frac{\rho_c - 1}{\sigma}$ where ρ_c denotes consumption autocorrelation

Most macroeconomic shocks are positively auto correlated and persistence coefficients are often large. As a consequence, both consumption and real interest rate are very persistent at business cycles frequencies. Thus, a model with discounted Euler equation and the standard model have similar predictions for the sign of the two correlations.

Income effect on labor supply and low elasticity of intertemporal substitution Intertemporal complementarity disentangles between the income effect on labor supply and the intertemporal substitution effect.

A moderate but non negligible income effect is supported by survey directly asking to

participants their labor supply response after an exogenous change in income (typically a lottery prize) (Kimball and Shapiro 2008). This result was confirmed by Cesarini et al. (2015). A moderate but not very small income effect is also supported the relative stability of hours worked in the long run despite large changes in real wages.

The small elasticity of consumption with respect to change in real interest rate also comes from a variety of observations. Direct estimations of the elasticity of intertemporal substitution have usually found a very low value for the EIS, sometimes close to zero (Hall 88, Yogo 2004). These small values are compatible with VAR evidence. The response of consumption to a positive monetary policy shocks is small given the response of the real interest rate (Bernanke and Gertler 95). Moreover, a large fraction of this response could be generated by indirect effects of monetary policy rather than by the direct effect on intertemporal substitution (Auclert 2017, Kaplan, Moll and Violante 2017).

In the standard model, under separable preferences, these two facts are hard to reconcile. The parameter governing the income effect of labor supply is also the inverse of the intertemporal elasticity of substitution. A small EIS implies a very large income effect.

With intertemporal complementarity, the income effect is related to the parameter σ and $\sigma + \kappa\nu$ is the equivalent of the EIS. The complementarity between consumption and assets, measured by ν allows the elasticity of consumption to interest rate to be reduced substantially whereas keeping a moderate value for the income effect.

Precautionnary saving and adjustment cost in consumption   Another argument can be drawn in favor of complementarity between current consumption and future wealth. I show in appendix **??** that it naturally arises when habits consumption and precautionary savings are combined. Consider an agent living two periods. He works and consumes in period one. In period 2, He does not work but uses assets accumulated in period one to consume. The utility at period 2 is affected by habits. It is not given by period two consumption but by the difference between the consumption of period two and a fraction of the consumption of period one. It is possible to rewrite the decision problem of period one by replacing the period two utility function by an indirect function depending on assets and period one consumption. The cross derivative of this indirect utility function is positive, indicating complementarity.

7   Implications for the forward guidance puzzle

A related issue is the forward guidance puzzle. Michaillat and Saez (2018) study conse-
quences of the wealth in the utility function for forward guidance at the zero lower bound.
They keep a separable form for the utility function. In this section, I show that intertem-
poral complementarity may also help to solve the puzzle. I give a formal characterization
of forward guidance by computing the response of output and inflation to an expected
monetary policy shock in the New Keynesian model. Whereas it does not encompass
all forms of forward guidance, the experiment clearly shows the overreaction of output.
Then, I show analytically that intertemporal complementarity dampens the response.

A formal characterization   The response to an expected shock on interest rate de-
pends on the duration between the announcement and the realization of the shock but
also depends on the contemporary reaction of monetary policy with respect to inflation
and output gap. I choose the lower computational burden. I compute output and in-
flation multipliers with respect to an expected shock on nominal interest rate with two
additional assumptions. First, the expected shock occurs in period $t + 1$. Second, the
nominal interest rate in period t is kept constant by the central bank and do not react
either to inflation or output gap. The underlying idea is that multipliers for other forward
guidance shocks are linked to multipliers for this simple case.

The multipliers for output and inflation in response to such expected shock are denoted
by $\mathcal{M}_y$ and $\mathcal{M}_\pi$. I compute them relatively to multipliers associated with a contempora-
neous monetary policy shock. I denote these multipliers Ψ_y and Ψ_π. I use a baseline New
Keynesian model (see details in appendix **??**)

$$
\mathcal{M}_y = \left(1 + \frac{\psi(\theta\eta + \sigma\alpha + 1 - \alpha)}{\sigma(1 - \beta_\pi\rho)\alpha} \right) \Psi_y \geq \Psi_y
$$
$$
\mathcal{M}_\pi = \Psi_\pi \left(\beta_\pi + \frac{\psi}{\alpha\sigma}(\theta\eta + \sigma\alpha + 1 - \alpha) \right)
$$

Current output overreacts. Its response to the future shock is always superior to the
response to a current shock. Inflation is very likely to overreact as well. The response of
inflation is superior to β_π which is usually close to 1.

Forward guidance and the supply effect of interest rate   It is worth noting that
the supply effect of interest rate enhances a forward guidance shock. Indeed, the term

$\sigma\alpha$ at the numerator of the expression of $\mathcal{M}_y$ is a consequence of the supply effect. In a imaginary world in which labor supply would only depend on wages and not on consumption whereas the Euler equation for consumption remains unchanged, the expression for $\mathcal{M}_y$ would be $\left(1 + \frac{\psi(\theta\eta+1-\alpha)}{\sigma(1-\beta\rho)\alpha}\right)\Psi_y$. There is still a substantial overreaction but lower than in the standard model.

Forward guidance and intertemporal complementarity   I now compute the response to a forward guidance shock in the WIU model. Consumption is given by equation (**??**). Computing the output multiplier leads to

$$\mathcal{M}_y = \left(\frac{(1-\kappa)\sigma}{\varphi} + \frac{\psi(\theta\eta + \sigma\alpha + 1 - \alpha)}{\varphi(1-\beta_\pi\rho)\alpha}\right)\Psi_y$$

where $\varphi \equiv \sigma + \kappa\nu$. Note that $\frac{(1-\kappa)\sigma}{\varphi}$ is simply the discount rate in the linear euler equation. The multiplier with respect to expected shocks is not always superior to the multiplier with respect to current monetary policy shock. There is no longer systematic overreaction of current output. A high value of φ dampens the response to forward guidance announcements not only by increasing the discount in the euler equation but also by diminishing the effects of expected inflation.

8   The model with varying wealth

Until then, I have identified wealth with safe assets and I have supposed that the supply of bonds is not varying. In this section, I allow for a varying supply and for different interpretations. It may alter significantly the response of leisure and consumption in the wealth targeting model. I conjecture that similar issues may arise in the wealth in the utility model. I assume separable preferences to make computations easier.

The system of equations defined in proposition **??** may be rewritten

$$-\sigma c_t = -\gamma a_{t+1} - q_t \tag{27a}$$

$$-\theta l_t = w_t - q_t - \gamma a_{t+1} \tag{27b}$$

where parameters σ and θ are usual and γ is defined by

$$\gamma = -\frac{U_{AA}A}{U_A}$$

In the first order conditions above, a represents the amount of wealth *desired* by households for a given real wage and a given real interest rate. At this stage, they should not be viewed as equilibrium values even if obviously those two equations holds at equilibrium.

How wealth reacts to changes in interest rate deeply modifies the response of leisure and consumption. If wealth is an increasing function of q_t, the two responses are enhanced whereas they are dampened if wealth is a decreasing function of q_t.

I now give three examples of wealth which gives different outcomes for monetary policy shocks.

A broad definition of wealth   The first example is the closest to the standard model. The wealth is defined as the sum of the financial wealth and the labor wealth

$$A_t = F_t + \Omega_t$$

Where, F_t is the financial wealth and Ω_t is the labor wealth defined by the recursive equation

$$\Omega_t = W_t + Q_t \Omega_{t+1}$$

If the previous equation is iterated forward, the labor wealth is the discounted sum of future real wages

$$\Omega_t = \sum_{T=0}^{+\infty} [\prod_{k=0}^{T} Q_{t+k}] W_{t+T}$$

It is easy to see this model is very close to the standard model in many respect. Indeed, the budget constraint is nothing else than the usual intertemporal budget constraint. $\Omega_{t+1} + F_{t+1}$ is the income at period $t+1$ and thus is equal to the discounted sum of consumption and leisure spending. The intertemporal first order condition relates current consumption C_t with future wealth $\Omega_{t+1} + A_{t+1}$. In period $t+1$, consumption C_{t+1} will be a function of this wealth. Thus, you recover an equation linking current consumption with future ones and real interest rate.

Predictions with respect to monetary policy effects are also close. Assuming that supply of financial assets is null and linearizing of labor wealth gives

$$a_{t+1} = \omega_{t+1}$$
$$\omega_t = (1 - \beta)w_t + \beta(\omega_{t+1} + q_t)$$

A persistent fall in real interest rate will increase interest factors q and the future wealth ω_{t+1}. It will cause a rise in consumption and in leisure, implying a strong response of real wage to a current shock and strong responses to forward guidance.

Fixed public debt   The second example leads to a very different conclusion. I assume that wealth is only financial and take the form of public debt. Responses to monetary policy are affected by fiscal policy.

I assume that the government has a very simple fiscal policy rule. Public debt, denoted B_t is fixed, equal to $\overline{B}$. Market clearing for public debt implies

$$Q_t A_{t+1} = \overline{B}$$

The linearized equation is simply

$$a_{t+1} = -q_t$$

Consumption and leisure may be expressed with respect to real wages and interest rate

$$\sigma c_t = (1 - \gamma) q_t$$
$$\theta l_t = (1 - \gamma) q_t - w_t$$

If $\gamma > 1$, both leisure and consumption becomes an increasing function of real interest rate. Obviously, under such parameters, effect of current and future monetary policy shocks on output are reversed.

A two agent framework   The third example shows how preferences heterogeneity may affect response to monetary policy. I consider a model with two types of agents. Agents differ by the elasticity of their wealth to interest rate for a given marginal utility of consumption and by their Frisch elasticity of leisure. The first type is called "debtors" and the second type "creditors". Behavioral equations are more complicated, but aggregation and linearization remain straightforward (see appendix **??**). Combining them leads to a three equation system for leisure, consumption and financial assets of creditors (also equal to the financial liabilities of debtors). Asset distribution affects both leisure and consumption and makes their response to change in real interest rates ambiguous.

The asset distribution equation also introduces an endogenous amplification mechanism in the model. A shift in creditors' assets in period t will affect labor supply in period $t + 1$. Following an expansionary monetary policy shocks, creditors lower their savings and increase their labor supply on impact. At the next period, they have fewer assets and thus reduce their labor supply and their consumption through income effect.

9   Conclusion

In this paper, I refined wealth in the utility function model by considering a more general specification of the utility function. I show that complementarity between consumption and future wealth may have important implications for the New Keynesian model. It disentangles the income effect on labor supply and the intertemporal substitution effect, allowing for a lower response of labor supply to monetary policy shocks and a larger response of output gap to a demand shock. Then, I estimate the model using bayesian methods. I find a large value for both ν which governs the intertemporal complementarity and κ which governs the discount factor in the Euler equation. These findings probably reflects the ability of the model to provide a more plausible time serie for labor supply than the standard model. They are robust to several alternative specifications.

References

[1] **Abel. A.B. and Warshawsky. M.** 1988. "Specification of the Joy of Giving: Insights from Altruism", *The Review of Economics and Statistics*, 70(1), 145-149.

[2] **Attanasio. O. and Davis. S.J.** 1996. "Relative Wage Movements and the Distribution of Consumption", *Journal of Political Economy*, 104, 1227-1262.

[3] **Auclert. A.** 2017. "Monetary policy and the redistribution channel", *The Review of Economics and Statistics*, mimeo.

[4] **Barakchian. S.M. and Crowe. C.** 2013. "Monetary policy matters : Evidence from new shocks data",*Journal of Monetary Economics*, 60, 950-966.

[5] **Bernanke. B., and Gertler. M.** 1995. "Inside the black box: The credit channel of Monetary Policty Transmission", *NBER working papers*, 5146.

[6] **Bernanke. B., and Mihov. I.** 1998. "Measuring Monetary Policy",*The Quarterly Journal of Economics*, 113, 869-902.

[7] **Bilbiie. F.** 2011. "Nonseparable Preferences, Frisch Labor Supply, and the Consumption Multiplier of Government Spending: One Solution to a Fiscal Policy Puzzle", *Journal of Money, Credit and Banking*, 43, 221-251.

[8] **Bilbiie. F.** 2009. "Nonseparable Preferences, Fiscal Policy Puzzles, and Inferior Goods", *Journal of Money, Credit and Banking*, 41, 443-450.

[9] **Bilbiie. F.** 2017. "The New Keynesian Cross: Understanding Monetary Policy with Hand-to-Mouth Households", mimeo.

[10] **Camerer. C., Babcock. L., Loewenstein.G., Thaler. R.** 1997. "Labor supply of New York city cabdrivers : one day at a time", *Quarterly Journal of Economics*, 407-441.

[11] **Campbell. J. and Mankiw. G.** 1990. "Permanent Income, Current Income, and Consumption" , *Journal of Business Economics and Statistics*, 8, 265-279.

[12] **Canzoneri. M. B., Cumby. R. E. and Diba. B. T.** 2007. "Euler equations and money market interest rates: A challenge for monetary policy models", *Journal of Monetary Economics*, 79(1), 1863-1881.

[13] **Cesarini. D., Lindqvist. E., Notowidigdo. M. and Ostling. R.** 2017. "The Effect of Wealth on Individual and Household Labor Supply: Evidence from Swedish Lotteries", mimeo.

[14] **Challe. E. , Matheron. J. , Ragot. X. and Rubio Ramirez. J.** 2017. "Precautionary saving and aggregate demand", *Quantitative Economics*, 8, 435-478.

[15] **Christiano. L.J. , Eichenbaum. M. and Evans. C.L.** 1997. "Sticky Price and Limited Participation Models of Money: A Comparison", *European Economic Review*, 41, 1201-1249.

[16] **Christiano. L.J. , Trabandt. M. and Walentin. K.** 2010. "Involuntary Unemployment and the Business Cycles", NBER working papers.

[17] **Christiano. L.J. , Eichenbaum. M. and Evans. C.L.** 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", *Journal of Political Economy*, 113(1), 1-45.

[18] **Del Negro. M., Giannoni. M., and Patterson, C.** 2013. "The Forward Guidance Puzzle", Federal Reserve Bank of New York Staff Report 574.

[19] **Fahri. E. and Werning. I.** 2017. "Monetary Policy, Bounded Rationality and Incomplete Markets", mimeo.

[20] **Frisch. R.** 1959. "A Complete Scheme for Computing All Direct and Cross Demand Elasticities in a Model with Many Sectors", *Econometrica*, 177-196.

[21] **Gabaix. X.** 2014. "A Sparsity-Based Model of Bounded Rationality", *Quarterly Journal of Economics*, 1661-1710.

[22] **Gabaix. X.** 2017. "A Behavioral New Keynesian Model", mimeo.

[23] **Gali. J.** 2008. *Monetary Policy, Inflation, and the Business Cycle - An Introduction to the New Keynesian Framework*, Princeton University Press.

[24] **Gali. J.** 2011. *Unemployment Fluctuations and Stabilization Policies*, MIT Press.

[25] **Gali. J.** 2015. "Hysteresis and the European Unemployment Revisited", NBER Working Papers.

[26] **Gali. J., Smets. F., Wouters. R.** 2011. "Unemployment in an Estimated New Keynesian Model", NBER Working Papers.

[27] **Gertler. M. and Karadi. P.** 2015. "Monetary Policy Surprises, Credit Costs, and Economic Activity", *American Economic Journal: Macroeconomics*, 7(1), 44–76.

[28] **Gourinchas. P.O. and Parker. J.** 2002. "Consumption over the Life Cycle", *Econometrica*, 70(1), 47–89.

[29] **Greenwood. M., Hercowitz. S. and Huffman. G.** 1994. "Investment, capacity utilization, and the real business cycle", *American Economic Review*, 78, 402-417.

[30] **Hall. R.** 1988. "Intertempral Substituability in Consumption", *Journal of Political Economy*, 96, 339-357.

[31] **Kaplan. G., Moll. B. and Violante. G** 2018. "Monetary Policy According to HANK", *American Economic Review*, 108(3), 697-743.

[32] **Kimball. M. and Shapiro. M.** 2008. "Labor Supply: Are the Income and Substitution Effects Both Large or Both Small?", NBER Working Papers.

[33] **Krishnamurthy. A. and Vissing Jorgensen. A.** 2012. "The Aggregate Demand for Treasury Debt", *Journal of Political Economy*, 120(2), 233-267.

[34] **Kumhof. M. , Ranciere. R. and Winant. P.** 2015. "Inequality, leverage and crisis", *American Economic Review*, 105(3), 1217-1245.

[35] **Lusardi. A.** 2008. "Household Saving Behavior: The Role Of Financial Literacy, Information, And Financial Education Programs", NBER working papers, 13824.

[36] **Michaillat. P. and Saez. E.** 2014. "An Economical Business-Cycle Model", NBER working papers.

[37] **Michaillat. P. and Saez. E.** 2018. "Wealth in the Utility function and the New Keynesian model", mimeo.

[38] **Michau. J.B.** 2017. "Secular Stagnation: Theory and Remedies", mimeo.

[39] **McKay. A., Nakamura. E., and Steinsson. J.** 2016. "The Power of Forward Guidance Revisited",*American Economic Review*, 106(10), 3133–3158.

[40] **McKay. A., Nakamura. E., and Steinsson. J.** 2016. "The Discounted Euler Equation: A note", mimeo.

[41] **Normandin. M.** 2006. "The Effects of Monetary-Policy Shocks on Real Wages: A Multi-Country Investigation", mimeo.

[42] **Peersman. G. and Smets. F.** 2001. "The monetary transmission mechanism in the euro area : more evidence from VAR analysis", ECB working papers.

[43] **Romer. C.D., and Romer. D.H.** 1989. " Does Monetary Policy Matter? A New Test in the Spirit of Friedman and Schwartz," in Olivier J. Blanchard and Stanley Fischer, eds., NBER macroeconomics annual 1989. Cambridge, MA: MIT Press, 121–70.

[44] **Romer. C.D., and Romer. D.H.** 2004. "A New Measure of Monetary Shocks: Derivation and Implications",*American Economic Review*, 94(4), 1055-1084.

[45] **Saez. E. and Stantcheva. S.** 2016. "Simpler Theory of Optimal Capital Taxation", NBER working papers.

[46] **Sims. C. and Zha. T.** 1998. "Does Monetary Policy Generate Recessions?", FRB Atlanta Working Papers.

[47] **Smets. F., Wouters. R.** 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach", *American Economic Review*, 97(3), 586-606.

[48] **Uhlig. H.** 2005. "What are the effects of monetary policy on output? Results from an agnostic identification procedure",*Journal of Monetary Economics*, 52, 381-419.

[49] **Woodford. M.** 2005. *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University press.

[50] **Woodford. M.** 2010. "Simple Analytics of the Government Expenditure Multiplier", NBER Working Papers.

[51] **Yaari. M.** 1964. "On the consumer's lifetime allocation process",*International Economic Review*, 304-317.

[52] **Yogo. M.** 2004. "Estimating the Elasticity of Intertemporal Substitution when instruments are weak",*The Review of Economics and Statistics*, 797-810.

A Proofs

A.1 Proof of proposition ??

Consider the lagrangean

$$U(C_t, L_t, A_{t+1}) + \Lambda_t \left(W_t + A_t - \Pi_t - W_t L_t - C_t - Q_t A_{t+1} \right)$$

First order condition are

$$U_C(C_t, L_t, A_{t+1}) - \Lambda_t = 0 \tag{28a}$$

$$U_L(C_t, L_t, A_{t+1}) - W_t \Lambda_t = 0 \tag{28b}$$

$$U_A(C_t, L_t, A_{t+1}) - Q_t \Lambda_t = 0 \tag{28c}$$

A.2 Proof of proposition ??

A first order Taylor expansion around the steady state of $U_C(C_t, L_t, A_{t+1})$ gives

$$U_c(C_t, L_t, A_{t+1}) = U_c + U_{cc}(C_t - C) + U_{ca}(A_{t+1} - A) + U_{cl}(L_t - L) + \epsilon$$

$$\frac{U_c(C_t, L_t, A_{t+1}) - U_c}{U_C} = \frac{U_{cc}C}{U_c} \frac{(C_t - C)}{C} + \frac{U_{ca}A}{U_c} \frac{(A_{t+1} - A)}{A} + \frac{U_{cl}L}{U_c} \frac{(L_t - L)}{L}$$

$$u_{c,t} = \frac{U_{CC}C}{U_C} c_t + \frac{U_{CA}A}{U_C} a_{t+1} + \frac{U_{CL}L}{U_C} l_t$$

Where lowercase letters denotes log linear values. For example $c_t = \frac{C_t - C}{C}$ and Letters without t index denotes steady state value.

The linearization of the two other equations use the same method.

A.3 Proof of proposition ??

The proposition rewrites the system of linear equations (??) with notations introduced in the paper and eliminate the Lagrange multiplier λ_t.

A.4 Proof of proposition ??

The proposition rewrites the system of linear equations from proposition ?? with assumption ?? and ??

A.5 Proof of corollary ??

The proposition rewrites the system of linear equations from proposition ?? with notations introduced in the paper and eliminate the Lagrange multiplier λ_t.

A.6 Proof of proposition ??

The hessian matrix of the utility function

$$
\begin{pmatrix}
U_{ll} & U_{cl} & U_{al} \\
U_{cl} & U_{cc} & U_{ca} \\
U_{al} & U_{ca} & U_{aa}
\end{pmatrix}
$$

Using assumption ??, it becomes

$$
\begin{pmatrix}
U_{ll} & 0 & 0 \\
0 & U_{cc} & U_{ca} \\
0 & U_{ca} & U_{aa}
\end{pmatrix}
$$

The utility function is concave if the hessian matrix is semidefinite negative, thus if all eigenvalues of the hessian are negative. The eigenvalues are given by U_{ll} and eigenvalues of the matrix

$$
\begin{pmatrix}
U_{cc} & U_{ca} \\
U_{al} & U_{aa}
\end{pmatrix}
$$

Eigenvalues of this matrix are negative if the trace is negative and the determinant positive. Sufficient conditions are

$$
U_{cc} \leq 0
$$
$$
U_{aa} \leq 0
$$
$$
U_{aa}U_{cc} - U_{ca}^2 \geq 0
$$

A.7 Proof of proposition ??

The recursive formulation for the optimization problem is

$$
V(A_t) = Max_{A_{t+1}, L_t, C_t} U(C_t, L_t, A_{t+1}) + \beta E_t V(A_{t+1})
$$

The lagrangean is

$$U(C_t, L_t, A_{t+1}) + E_t V(A_{t+1}) + \Lambda_t \left(W_t + A_t - \Pi_t - W_t L_t - C_t - Q_t A_{t+1} \right)$$

First order condition are

$$U_C(C_t, L_t, A_{t+1}) - \Lambda_t = 0$$

$$U_L(C_t, L_t, A_{t+1}) - W_t \Lambda_t = 0$$

$$U_A(C_t, L_t, A_{t+1}) + \beta E_t \frac{\partial V}{\partial A_{t+1}} - Q_t \Lambda_t = 0$$

Using envelope theorem, the derivative of the value function is

$$\frac{\partial V}{\partial A_{t+1}} = \Lambda_{t+1} = U_C(C_{t+1}, L_{t+1}, A_{t+2})$$

Leading to equations highlighted in the proposition.

A.8   Proof of proposition ??

Consider the modified euler condition of the optimization problem

$$U_A(C_t, L_t, A_{t+1}) + \beta U_C(C_{t+1}, L_{t+1}, A_{t+2}) - Q_t U_C(C_t, L_t, A_{t+1}) = 0$$

Consider now this equation at the steady state

$$U_A + \beta U_C - Q U_C = 0 \Rightarrow \beta = Q - \frac{U_A}{U_c}$$

A.9   Proof of proposition ??

The linearization method is the same as in the proof of the proposition **??**. The log linear equation for leisure is the same.

I now compute the log linear Euler equation. I denote the log linear approximation of x as $\mathcal{M}x$), except for variables A, C, Q and L whose log linear approximations are denoted in lowercase.

$$\mathcal{M}\left(U_A(C_t, L_t, A_{t+1}) + \beta E_t U_C(C_{t+1}, L_{t+1}, A_{t+2})\right) = \mathcal{M}\left(Q_t U_C(C_t, L_t, A_{t+1})\right)$$

By standard properties of log linear first order approximations, I have

$$\frac{U_A}{U_A + \beta U_c} \mathcal{M}\left(U_A(C_t, L_t, A_t)\right) + \frac{\beta U_c}{U_A + \beta U_c} \mathcal{M}\left(E_t U_C(C_{t+1}, L_{t+1}, A_{t+2})\right) = q_t + \mathcal{M}\left(U_C(C_t, L_t, A_{t+1})\right)$$

$$(29)$$

I use the steady state condition $U_A + \beta U_C = Q U_c$ to obtain

$$(1 - \frac{\beta}{Q})\mathcal{M}\left(U_A(C_t, L_t, A_t)\right) + \frac{\beta}{Q}\mathcal{M}\left(E_t U_C(C_{t+1}, L_{t+1}, A_{t+2})\right) = q_t + \mathcal{M}\left(U_C(C_t, L_t, A_{t+1})\right)$$

Using standard properties for log linear approximations of expectations, first order taylor expansion of U_c and U_A and the hypothesis $a_{t+1} = 0$, I obtain

$$(1 - \frac{\beta}{Q})\frac{U_{CA}C}{U_A}c_t + \frac{\beta}{Q}\frac{U_{CC}C}{U_C}E_t c_{t+1} = q_t + \frac{U_{CC}C}{U_C}c_t$$

A.10   Proof of proposition ??

The proposition rewrites the system of equations from proposition **??** with notations introduced in the paper.

A.11   Proof of proposition ??

Combination of equations (**??**) and (**??**) gives

$$\theta\eta n_t = w_t + \frac{\sigma}{\sigma + \kappa\nu}rr_t + \frac{(1-\kappa)\sigma}{\sigma + \kappa\nu}(\theta\eta n_{t+1} - w_{t+1})$$

$$\theta\eta(n_t - \frac{(1-\kappa)\sigma}{\sigma + \kappa\nu}n_{t+1}) = \left(w_t - \frac{(1-\kappa)\sigma}{\sigma + \kappa\nu}w_{t+1}\right) + \frac{\sigma}{\sigma + \kappa\nu}rr_t$$

Consider a non persistent and exogenous change in real interest rate. Forward looking terms are no longer relevant. The equation becomes

$$\theta\eta n_t = w_t + \frac{\sigma}{\sigma + \kappa\nu}rr_t$$

If $U_{CA} > 0$, then $\nu > 0$ and $\frac{\sigma}{\sigma + \kappa\nu} < 1$. The elasticity of hours with respect to real interest rate is lower than the elasticity with respect to real wages.

A.12   Proof of proposition ??

The new keynesian labor demand becomes

$$(\sigma + \kappa\nu)\alpha n_t = -rr_t + (1-\kappa)\sigma\alpha n_{t+1} \tag{30}$$

From it, I deduce

$$(n_t - \frac{(1-\kappa)\sigma}{\sigma + \kappa\nu}n_{t+1}) = -\frac{1}{\alpha(\sigma + \kappa\nu)}rr_t \tag{31}$$

I use the equation giving labor supply as a function of real wages and real interest rate

$$\theta\eta(n_t - \frac{(1-\kappa)\sigma}{\sigma+\kappa\nu}n_{t+1}) = \left(w_t - \frac{(1-\kappa)\sigma}{\sigma+\kappa\nu}w_{t+1}\right) + \frac{\sigma}{\sigma+\kappa\nu}rr_t$$

And replace hours by real interest rate using the relation derived from the labor demand

$$-\frac{\theta\eta}{\alpha(\sigma+\kappa\nu)}rr_t = \left(w_t - \frac{(1-\kappa)\sigma}{\sigma+\kappa\nu}w_{t+1}\right) + \frac{\sigma}{\sigma+\kappa\nu}rr_t \qquad (32)$$

rearranging gives

$$w_t = -\left(\underbrace{\frac{\sigma}{\sigma+\kappa\nu}}_{Supply\ effect} + \underbrace{\frac{\theta\eta}{(\sigma+\kappa\nu)\alpha}}_{Demand\ effect}\right)rr_t + \frac{(1-\kappa)\sigma}{\sigma+\kappa\nu}w_{t+1} \qquad (33)$$

B   Wealth in the utility function and indivisible labor

In this section, I show how the framework developed by Gali(2011) may be adapted to accommodate wealth in the utility function and nonseparability between wealth and consumption.

The household is composed of a continuum of members indexed by j. Each member may work either one or zero unit of time. The labor is indivisible. Members also differ by the disutility associated to then work time. The jth household suffer a disutility equal to j^θ. There is perfect risk sharing across household's members for consumption and assets. The head of the household chooses household's consumption assets and working household's members. He maximizes

$$U(C_t, A_{t+1}) + \int_0^{N_t} j^\theta dj$$
$$\Rightarrow U(C_t, A_{t+1}) + \frac{N_t^{1+\theta}}{1+\theta}$$

Utility is separable between consumption and labor force participation but nonseparable between assets and consumption like in the intertemporal complementarity model.

C   Bayesian estimation

First, I display the equations of the estimated model

I now display posteriors for the different estimation

Equation	Interpretation
$y_t = \alpha n_t + a_t$	Production Function
$y_t = \varphi c_t + d_t$	Market clearing on good market
$r_t - \pi_{t+1} = rr_t$	Accounting equation
$\mu_t = w_t + n_t - y_t$	Marginal cost equation
$\pi_t = \frac{\beta_\pi}{1+\beta_\pi \tau_p} E_t \pi_{t+1} + \frac{\tau_p}{1+\beta_\pi \tau_p} \pi_{t-1} + \gamma_p \mu_t + ep_t$	Philips Curve
$rg_t = \phi_\pi \pi_t + \phi_y(y_t - \tilde{y}_t) + e_t$	Monetary policy rule
$r_t = \lambda r_{t-1} + (1-\lambda)rg_t$	Effective nominal interest rate
$\theta \eta n_t^d = w_t - \sigma c_t$	Labor supply
$u_t = n_t^d - n_t$	Unemployment
$\pi_{w,t} = \frac{\beta_w}{1+\beta_w \tau_w} E_t \pi_{w,t+1} + \frac{\tau_w}{1+\beta_w \tau_w} \pi_{w,t-1} + \gamma_w u_t + ew_t$	Wage Philips curve
$w_t = w_{t-1} + \pi_{w,t} - \pi_t$	Real wage equation
$(\sigma + \kappa\nu)c_t = -rr_t + (1-\kappa)\sigma E_t c_{t+1} + ec_t$	Consumption equation

Table 8: Estimated model

Variable	distribution	mean	std
κ	beta	0.5	0.2
ν	normal	0	1.5
σ	normal	1	1.5
θ	gamma	2	0.25
γ_p	normal	0.25	0.05
γ_w	normal	0.25	0.05
ϕ_π	normal	1.5	0.25
ϕ_y	normal	0.12	0.25
h	beta	0.5	0.15
τ_p	beta	0.5	0.15
τ_w	beta	0.5	0.15

Table 9: Priors for parameters

Variable	distribution	mean	std
ρ_d	beta	0.5	0.2
ρ_e	beta	0.5	0.2
ρ_a	beta	0.5	0.2
ρ_{ec}	beta	0.5	0.2
ρ_{ep}	beta	0.5	0.2
ρ_{ew}	beta	0.5	0.2
σ_d	inv gamma	0.4	0.5
σ_e	inv gamma	0.4	0.5
σ_a	inv gamma	0.4	0.5
σ_{ec}	inv gamma	0.4	0.5
σ_{ep}	inv gamma	0.4	0.5
σ_{ew}	inv gamma	0.4	0.5

Table 10: Priors for standard deviation and persistence

Variable	mean	mode	inf	sup
σ	2.77	2.63	2.16	3.37
θ	2.58	2.51	2.14	2.98
γ_p	0.0042	0.0019	-0.0015	0.0109
γ_w	0.206	0.247	0.0513	0.318
ϕ_y	1.24	1.27	0.952	1.57
ϕ_π	1.75	1.66	1.23	2.3
τ_p	0.374	0.356	0.153	0.592
τ_w	0.138	0.104	0.0471	0.225

Table 11: Posteriors for the standard model estimation

Variable	mean	mode	inf	sup
κ	0.797	0.839	0.644	0.961
ν	2.13	2.01	1.03	3.19
σ	1.17	1.11	0.627	1.69
θ	2.31	2.26	1.9	2.7
γ_p	0.0004	0.0005	-0.0006	0.0013
γ_w	0.264	0.284	0.178	0.374
ϕ_y	0.463	0.44	0.113	0.788
ϕ_π	1.47	1.46	1.07	1.86
τ_p	0.287	0.315	0.131	0.435
τ_w	0.126	0.0956	0.0378	0.21
ρ_a	0.925	0.924	0.884	0.969
ρ_d	0.849	0.853	0.782	0.917
ρ_{ep}	0.101	0.0663	0.014	0.181
ρ_e	0.65	0.651	0.521	0.778
ρ_{en}	0.958	0.969	0.93	0.986
ρ_{ec}	0.94	0.953	0.907	0.973
ρ_{ew}	0.752	0.811	0.644	0.913
σ_d	0.516	0.506	0.454	0.577
σ_a	0.202	0.199	0.177	0.226
σ_e	0.12	0.118	0.105	0.134
σ_{ep}	0.0696	0.0678	0.059	0.0789
σ_{ec}	0.209	0.207	0.179	0.238
σ_{en}	0.288	0.281	0.248	0.325
σ_{ew}	0.221	0.206	0.176	0.267

Table 12: Posteriors for the baseline estimation

D   Generalized nonseparable preferences in the model

In this section, I consider the case where all form of nonseparabilities are possible.

Reconsider the model with all forms of nonseparabilities.

$$\frac{U_{CC}C}{U_C}c_t + \frac{U_{CA}A}{U_C}a_{t+1} + \frac{U_{CL}L}{U_C}l_t = \lambda_t$$

$$\frac{U_{CL}C}{U_L}c_t + \frac{U_{LA}A}{U_L}a_{t+1} + \frac{U_{LL}L}{U_L}l_t = w_t + \lambda_t$$

$$\frac{U_{CA}C}{U_A}c_t + \frac{U_{AA}A}{U_A}a_{t+1} + \frac{U_{LA}L}{U_A}l_t = q_t + \lambda_t$$

I adopt more friendly notations. All elasticities are denoted by δ plus a subscript corresponding to the second derivative. Thus, I have $\frac{U_{CC}C}{U_C} = \delta_{cc}, \frac{U_{AA}A}{U_A} = \delta_{aa}$ etc. For cross derivative, I have $\delta_{la} = \frac{U_{LA}L}{U_A}$, $\delta_{ca} = \frac{U_{CA}C}{U_A}$ and $\delta_{cl} = \frac{U_{CL}C}{U_L}$. Moreover, I denote $m_{cl} = \frac{WL}{C}$, $m_{ac} = \frac{QA}{C}$ and $m_{al} = \frac{QA}{WL}$.

The system becomes

$$\delta_{cc}c_t + \delta_{ca}m_{ca}a_{t+1} + \delta_{cl}m_{cl}l_t = \lambda_t$$

$$\delta_{cl}c_t + \delta_{la}m_{la}a_{t+1} + \delta_{ll}l_t = w_t + \lambda_t$$

$$\delta_{ca}c_t + \delta_{aa}a_{t+1} + \delta_{al}l_t = q_t + \lambda_t$$

If $a_{t+1} = 0$, we get

$$\delta_{cc}c_t + \delta_{cl}m_{cl}l_t = \lambda_t$$

$$\delta_{cl}c_t + \delta_{ll}l_t = w_t + \lambda_t$$

$$\delta_{ca}c_t + \delta_{al}l_t = q_t + \lambda_t$$

eliminating λ_t

$$-(\delta_{cl}m_{cl} - \delta_{ll})l_t = w_t - (\delta_{cl} - \delta_{cc})c_t$$

$$(\delta_{ca} - \delta_{cc})c_t + (\delta_{al} - \delta_{cl}m_{cl})l_t = q_t$$

The consumption equation can be written with respect to wages and interest factors

$$\left(\delta_{ca} - \delta_{cc} + \frac{(\delta_{al} - \delta_{cl}m_{cl})(\delta_{cl} - \delta_{cc})}{\delta_{cl}m_{cl} - \delta_{ll}}\right)c_t = q_t + \frac{\delta_{al} - \delta_{cl}m_{cl}}{\delta_{cl}m_{cl} - \delta_{ll}}w_t$$

E   Adjustment cost for consumption in an overlapping generations model

Consider an household living two periods, receiving wages only in the first period. The utility at the second period features habit consumption

$$\max_{c_y, c_o} u(c_y) + u(c_o - hc_y)$$

$$w.r.t \quad w = c_y + a$$

$$w.r.t \quad a = c_o$$

Where a are assets accumulated at the first period. c_y and c_o are consumption levels the agent is young and old. h is between 0 and 1. $u'(.) > 0$, $u''(.) < 0$

The objective function can also be written under the form

$$u(c^y) + v(a, c^y)$$

where $v(a, c^y) = u(a - hc_y)$

I compute first order and cross derivative of v

$$\frac{dv}{da} = u'(a - hc_y)$$

The cross derivative with respect to consumption of the young is

$$\frac{d^2v}{dadc_y} = -hu''(a - hc_y) > 0 \tag{34}$$

The cross derivative is positive

F   Forward guidance computations

F.1   Forward guidance in the standard model

I consider a standard linear New Keynesian model.

$$\sigma(c_{t+1} - c_t) = r_t - \pi_{t+1}$$

$$-\theta l_t = w_t - \sigma c_t$$

$$l_t = -\eta n_t$$

$$\mu_t = w_t + n_t - y_t$$

$$y_t = \alpha n_t$$

$$\pi_t = \psi \mu_t + \beta_\pi \pi_{t+1}$$

$$r_t = \phi_\pi \pi_t + \epsilon_t$$

$$y_t = c_t$$

Where c is consumption, r is nominal interest rate, π is inflation, l is leisure, n is hours worked, y is output, w is real wage, μ is the marginal cost (the inverse of the markup), and ϵ is a monetary policy shock.

First, I compute multipliers for a contemporaneous monetary policy shock. They are denoted $\psi_{(.)}$. the subscript denotes the variable of interest.

$$\Psi_w = \frac{\theta\eta + \sigma\alpha}{\alpha}\psi_y$$

$$\Psi_\mu = \frac{\theta\eta + \sigma\alpha + 1 - \alpha}{\alpha}\psi_y$$

$$\Psi_\pi = \frac{\psi}{1 - \beta_\pi\rho}\frac{\theta\eta + \sigma\alpha + 1 - \alpha}{\alpha}\Psi_y$$

$$\Psi_y = \frac{(1 - \beta_\pi\rho)\alpha}{(1 - \beta_\pi\rho)\alpha\sigma(\rho - 1) - (\phi_\pi - \rho)\psi(\theta\eta + \sigma\alpha + 1 - \alpha)}$$

The first multiplier is obtained by combining labor supply equation, production function and market clearing condition. The second comes from the combination of the markup equation, the previous result for wages and the production function. The markup multiplier immediately gives the multiplier for inflation.

I now compute multipliers for a shock occurring in $t + 1$ under the assumption that nominal interest rate in t is fixed. I denote these multipliers $\mathcal{M}_{(.)}$. I derive the system relating current multiplier for output and inflation to future ones.

$$\sigma\mathcal{M}_y = \sigma\Psi_y + \Psi_\pi$$

$$\mathcal{M}_\pi = \psi\frac{\theta\eta + \sigma\alpha + 1 - \alpha}{\alpha}\mathcal{M}_y + \beta_\pi\Psi_\pi$$

Solving the system leads to

$$\mathcal{M}_y = \left(1 + \frac{\psi(\theta\eta + \sigma\alpha + 1 - \alpha)}{\sigma(1 - \beta_\pi\rho)\alpha}\right)\Psi_y \geq \Psi_y \tag{35a}$$

$$\mathcal{M}_\pi = \Psi_\pi\left(\beta_\pi + \frac{\psi}{\alpha\sigma}(\theta\eta + \sigma\alpha + 1 - \alpha)\right) \tag{35b}$$

F.2 Forward guidance with wealth in the utility

The equation for consumption is now

$$(\sigma + \kappa\nu)c_t = -r_t + \pi_{t+1} + (1 - \kappa)\sigma c_{t+1} \tag{36}$$

Ψ_w, Ψ_μ and Ψ_π remains unchanged.

The output multiplier associated with a contemporaneous monetary policy shock is

$$\Psi_y = \frac{(1 - \beta_\pi\rho)\alpha}{(1 - \beta_\pi\rho)\alpha(\sigma + \kappa\nu - (1 - \kappa)\sigma\rho) - (\phi_\pi - \rho)\psi(\theta\eta + \sigma\alpha + 1 - \alpha)}$$

Computing multiplier for a shock occuring in $t + 1$ gives

$$(\sigma + \kappa\nu)\mathcal{M}_y = (1 - \kappa)\sigma\Psi_y + \Psi_\pi$$

$$\mathcal{M}_\pi = \psi\frac{\theta\eta + \sigma\alpha + 1 - \alpha}{\alpha}\mathcal{M}_y + \beta_\pi\Psi_\pi$$

For the output multiplier, the solution is now

$$\mathcal{M}_y = \left(\frac{(1 - \kappa)\sigma}{\sigma + \kappa\nu} + \frac{\psi(\theta\eta + \sigma\alpha + 1 - \alpha)}{(\sigma + \kappa\nu)(1 - \beta_\pi\rho)\alpha}\right)\Psi_y \tag{37}$$

G   Model with creditors and debtors

Compare to the standard model, the labor supply and the consumption equation are replaced by first order conditions for both agents, and aggregation equation . In their linear version, it gives

$$-\sigma c_t^d = -\gamma^d a_{t+1}^d - q_t$$

$$-\theta_d l_t^d = w_t - q_t - \gamma a_{t+1}^d$$

$$-\sigma c_t^c = -\gamma^c a_{t+1}^c - q_t$$

$$-\theta_c l_t^c = w_t - q_t - \gamma a_{t+1}^c$$

$$\lambda_c c_t^c + (1 - \lambda_c) c_t^d = c_t$$

$$\lambda_l l_t^c + (1 - \lambda_c) l_t^d = l_t$$

$$a_{t+1}^d = -a_{t+1}^c$$

Where λ_c and λ_l are the share of debtors for respectively consumption and leisure at steady state. Parameters γ, θ and σ have the same meaning as in the paper but are specific to each type of household and are indexed by subscripts d or c. To solve the problem, I need the budget constraint of one of the two agents. The budget constraint of debtors is

$$W_t + A_t^d = W_t L_t + C_t + Q_t A_{t+1}^d$$

I denote $m = \frac{C}{W+A+\Pi}$ the average propensity to consume at steady state and $v = \frac{WL}{W+A}$ its equivalent for leisure.

The linearized budget constraint gives

$$vw_t + vl_t + mc_t^d + (1 - m - v)(q_t + a_{t+1}^d) = \frac{1 - m - v}{q} a_t^d + \frac{m + v}{q} w_t \qquad (38)$$

I can now derive an equation for debtor asset law of motion by combining the budget constraint with first order conditions for debtors

$$\left(1 - m - v + \frac{v\gamma_d}{\theta_d}\right) a_{t+1}^d = w_t \left(\frac{m + v}{q} - v + \frac{v}{\theta_d}\right) + a_t^d \left(\frac{1 - m - v}{q}\right) - q_t \left(1 - m - v + \frac{v}{\theta_d} + \frac{m}{\sigma}\right) \qquad (39)$$

Equations for consumption and aggregate leisure are

$$-\sigma c_t = -q_t + [\lambda_c \gamma_c - \gamma_d(1 - \lambda_c)] a_{t+1}^d$$

$$-(\theta_d(1 - \lambda_l) + \theta_c \lambda_l) l_t = w_t - q_t + (\lambda_l \gamma_c - \gamma_d(1 - \lambda_l)) a_{t+1}^d$$