

# An Equilibrium Theory of Nominal Exchange Rates, Asset Flows and Country Portfolios

Marcus Hagedorn\*

University of Oslo and CEPR

This Version: November 4, 2018

## Abstract

This paper proposes an equilibrium theory of nominal exchange rates, asset flows and country portfolios which offers a new perspective to several issues in open economy macroeconomics. The nominal exchange rate and portfolio choices are jointly determined in equilibrium, which provides a new approach to overcome the indeterminacy results in Kareken and Wallace (1981). The determinants of the nominal exchange rate are the amount of assets issued by a country in its currency, the net foreign asset position, the nominal interest rate and productivity and show that changes in each of the determinants lead to depreciations or appreciations in line with empirical evidence. The novel theory also offers a different perspective on how international asset flows affect exchange rates, how a country can divorce itself from these flows and how a country can manage its exchange rate. The model also implies that a country with an exchange rate peg and free asset mobility faces a tetralemma and not a trilemma as it not only loses monetary but also fiscal policy independence. This suggest a new way to think about fiscal coordination in a monetary union as a response to within union asset flows.

---

\*University of Oslo, Department of Economics, Box 1095 Blindern, 0317 Oslo, Norway.  
Email: marcus.hagedorn@econ.uio.no

# 1 Introduction

How do fiscal and monetary policy, productivity shocks or a liquidity trap spill over to the rest of the world? What is the role of international asset flows in propagating regional policies and shocks to the rest of the world and how do they affect the nominal exchange rate? How does an increase in savings demand for US bonds affect asset flows, the exchange rate, the current account and the US economy? Can a country divorce itself from such global financial flows? And more generally, how can a country manage its exchange rate, for example engineer a depreciation? How independent is a country's policy in a monetary union with free capital mobility and is there a need for fiscal policy coordination among union members? Finally, how is the nominal exchange rate determined?

To answer these questions, this paper proposes a new theory of nominal exchange rates and country portfolios with three key ingredients:

1. Ricardian equivalence does not hold within each country.
2. Aggregate country risk is non-diversifiable.
3. Each country issues nominal government bonds denominated in its own currency.

The role of the latter assumption is clear. The model needs a nominal element for a meaningful discussion of nominal exchange rates. The model also needs at least two internationally traded bonds for meaningful portfolio choices. I use the simplest framework, a two-period, two-countries, overlapping generations (OLG) model without capital, to break Ricardian equivalence, the first assumption. Figure 1 illustrates how Portfolio choices, bond prices and the nominal exchange rate are then jointly determined in equilibrium. The starting point is that each country is exposed to some aggregate uncertainty, which cannot be diversified in international financial markets (assumption 2).<sup>1</sup> This uncertainty carries over to government bond prices which are risky in equilibrium. This renders old age consumption risky since the return on bonds bought when young is risky. The young generation therefore not only has to decide how much to save but in response to this old-age consumption risk, young households adjust their international bond portfolio decision leading to well-defined asset demands for each country's bonds.

---

<sup>1</sup>This assumption is generically satisfied but for knife-edge choices of technologies and preferences country returns can be collinear, see for example Kollmann (2006a).

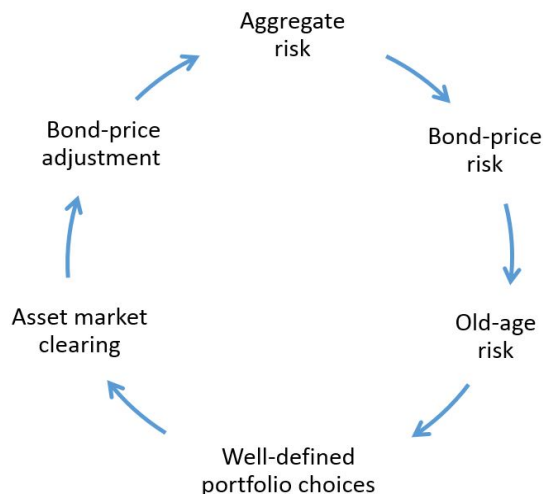


Figure 1: Portfolios and Nominal Exchange Rates as Equilibrium Outcomes

Bond market clearing then requires bond prices to adjust. This adjustment changes both the mean and the volatility of bond prices and thus changes the aggregate uncertainty households face and the circle starts over again. An equilibrium is reached if the portfolio choices given risky bond prices are consistent with the prices clearing the asset market.

A first main result is that this model leads to a new way to jointly determine both the nominal exchange rate and portfolios. To better understand the underlying mechanism and the implications for the questions motivating this research, it is instructive to recall the indeterminacy result by Kareken and Wallace (1981) (KW).<sup>2</sup> Consider two countries where monetary policy sets nominal interest rates. The uncovered interest rate parity condition then determines the expected change in the exchange rate only but leaves the level of the exchange rate indeterminate. An equivalent type of price level indeterminacy also arises in closed economies (Sargent and Wallace (1975)), but as pointed out in KW, the open economy frameworks adds another subtle type of indeterminacy.<sup>3</sup> The KW indeterminacy arises if assets are fully mobile across borders and households' portfolio choices and net foreign asset positions are indeterminate. Households are then indifferent for example between a portfolio with a strong home bias and one which is perfectly internationally diversified. At the aggregate level, this portfolio indeterminacy turns into an indeterminacy of the demand for the assets supplied by each country. Both a high and low demand for a country's assets

<sup>2</sup>Cavallo and Ghironi (2002) and Ghironi (2008) adopt an overlapping generations instead of a representative agent model (within a country) mainly to ensure stationarity. This assigns a role to the stock of real net foreign assets but does not deliver nominal exchange rate indeterminacy.

<sup>3</sup>In a closed economy, the Fisher equation determines a country's inflation rate - the expected change in the price level - but leaves the price level in each country indeterminate.

are equilibrium outcomes which are associated with different country price levels and thus exchange rates: the price level has to fall to absorb a high demand and has to increase if demand is low.

The solution in the textbook Mundell Fleming model is a normalization of the future expected exchange rate, in modern dynamic models it is fixing the long-run nominal exchange rate.<sup>4</sup> These assumptions on the long-run exchange rate anchor expectations with strong implications for agents' short-run and long-run behavior as well as for the full path of the exchange rate in the short-run and in the medium-run. Nominal rigidities then imply that this nominal indeterminacy turns into a real indeterminacy. Different nominal exchange rates correspond to different real exchange rates and thus to different levels of exports and imports as well as different levels of output and employment at home and abroad. The implications for output and employment therefore depend on the researcher's choices on the nominal exchange rate and are also likely to affect the answers to the questions which motivate this research in the first place. Fixing the long-run nominal exchange rate also limits the scope of policy. By assumption policy cannot affect the long-run nominal exchange rate which also limit its influence in the short- and medium-run.

This paper offers a different solution. Households are not indifferent between home and foreign bonds but instead use them to diversify aggregate risk. Determinacy of portfolio choices then carries over to exchange rates which have to adjust to clear bonds markets in all countries. The nominal exchange rate and portfolio choices are jointly determined with interesting interactions arising from the interplay of incomplete international markets and valuation effects. Suppose, that today the economy is hit by a shock. In response, households rebalance their portfolios, which in turn affects the exchange rate. The change in the nominal exchange rate induces valuation gains or losses on a country's international asset holdings.<sup>5</sup> These wealth gains or losses will again have effects on asset choices and thus

---

<sup>4</sup>Or one has to deviate from the consensus in monetary economics and central banks that monetary policy operates through setting nominal interest rates. The nominal exchange rate is determined if instead monetary policy sets money supply and in addition money is not freely mobile across countries so that agents cannot use any currency in every country without transaction costs. For a recent example where these assumptions lead to nominal exchange rate determinacy see Gabaix and Maggiori (2015). The focus of their paper is however quite different. These authors show that the intermediation of international capital flows leads to new (and more interesting and empirically relevant) determinants of exchange rates than the monetary textbook model, which they build on to obtain determinacy in the first place.

<sup>5</sup>For example, a country holding US dollar denominated bonds and appreciating vis-à-vis the US-dollar experiences a wealth loss. Several papers among them Lane and Milesi-Ferretti (2001, 2007), Tille (2003, 2008) Kollmann (2006b), Gourinchas and Rey (2007a,b), Devereux and Sutherland (2010), Pavlova and Rigobon (2012), Ghironi et al. (2015) have established the importance of such valuation effects. In particular the literature has documented that a large fraction of US foreign liabilities is denominated in US dollars whereas US foreign assets have a considerable non-dollar component.

again on the exchange rate. An equilibrium is reached if asset choices and exchange rates are mutually consistent. The literature has mainly focused on this valuation effect - how changes in exchange rates affect asset values - whereas this paper builds on these insights and adds the feedback from asset values to exchange rates, such that portfolio choices and exchange rates are jointly determined in equilibrium.

The benchmark model is on purpose simple to focus on the determination of the nominal exchange rate, portfolio choices and their interaction and to highlight the model mechanisms. There is only good so that the real exchange rate is equal to one, which makes it clear that movements in nominal exchange rates are not driven by movements in real exchange rates. One motivation to consider nominal exchange rate is however their strong co-movement with real exchange rates in the data and it is the latter rate that matters for trade decisions. Another implication of a constant real exchange rate is that the volatility of prices and nominal exchange rates is one-to-one related. To show that these are not essential elements of the theory proposed here, I extend the model along two dimensions. The extended model allows for both tradable and non-tradable goods implying that the real exchange rate is not constant. Prices are also sticky, implying a high correlation of nominal and real exchange rates and breaking the high correlation of prices and nominal exchange rates. At the same time nominal exchange rates and portfolios are determinate based on the same arguments used in the benchmark model. In Section 3, I provide a further generalization and show that the results derived in the simple OLG model carry over to a large class of incomplete markets (within countries) models with aggregate risk.<sup>6,7</sup>

I then use the theoretical models to shed light on what the answers to some of the motivating questions might be. The determinants of the nominal exchange rate are the amount of assets issued by a country in its currency, the net foreign asset position, the nominal interest rate and productivity. Issuing more government bonds leads to a depreciation. An increase in productivity and a tightening of monetary policy lead to an appreciation. An outflow of assets leads to a depreciation whereas an inflow of assets, say due to an increase in

---

<sup>6</sup>Clarida (1990), Willen (2004) and Mendoza et al. (2009) were the first among many other contributions to integrate the Bewley-Imrohoroglu-Huggett-Aiyagari incomplete markets model into an open economy model and show that this model class helps to understand global capital flows and trade imbalances. Here I use this same type of model and show that this model class, in addition to its well documented appealing quantitative predictions, provides an additional benefit over complete markets models: nominal exchange rate determinacy.

<sup>7</sup>Kollmann (2012) and Coeurdacier et al. (2011) use a different class of incomplete markets models - limited participation in asset markets - to address the Kollmann-Backus-Smith Consumption-Real Exchange rate anomaly. Corsetti et al. (2008) address the same anomaly in a model with internationally incomplete but nationally complete markets.

precautionary savings demand for US bonds by emerging countries, leads to an appreciation of the US exchange rate. The US can sterilize this latter effect on the exchange rate through acquiring foreign assets or just issuing government bonds. This suggests that a larger savings demand by the rest of the world (ROW) for US bonds can be accommodated without any effects on US prices or exchange rates, provided that the ROW's demand does not persistently increase at a faster rate than US GDP. If it does, then stabilizing the exchange rate will require an exploding US debt/gdp ratio, which is infeasible due to the limited US fiscal capacity. The US would then have to accept falling prices and an appreciation of its currency, a flexible exchange rate post Bretton Woods version of Triffin's dilemma. Or the ROW diverts its savings to other currencies, the Euro or the Yuan. The theory shows that various policies can be used to trigger a depreciation of a currency: Conduct an expansionary fiscal policy (increase debt), loosen monetary policy (lower nominal interest rates) or buy, without sterilizing, foreign assets.

The model also suggest that the classic policy trilemma in international economics - at most two out of the following three policies are simultaneously feasible: (i) unrestricted capital mobility; (ii) setting nominal interest rate independently (monetary policy independence); and (iii) a fixed exchange rate - turns into a tetralemma, as fixed exchange rates and free capital mobility not only imply the loss of monetary but also of fiscal policy independence. The argument is simple. Interest rate parity implies that monetary policy has to track foreign monetary policy to rule out anticipated changes in the exchange rate. Fiscal policy then has to ensure, for example through issuing more or less debt, that the exchange rate remains unchanged in response to unanticipated shocks. This restriction on fiscal policy is missing in the standard trilemma since there the level of the exchange rate is indeterminate and the focus is on the (anticipated) change only. Here, in contrast, monetary policy cannot stabilize the exchange rate on its own and fiscal policy has to step in when unanticipated shocks move the level of the exchange rate. The implication for monetary unions is that its members not only have to give up an independent monetary policy but de facto also an independent fiscal policy, at least if movements in the real exchange rate shall be avoided. This suggests a new perspective on the fiscal dimension of a monetary union: Fiscal policy coordination to jointly respond to asset and capital flows.

The rest of the paper is organized as follows. Section 2 develops the simple OLG model with a constant real exchange rate and flexible prices and explains the workings of this new theory and how it jointly determines exchange rates and asset choices. I extend the benchmark model in Section 2.5, where the real exchange rates is volatile and prices are

sticky. Section 3 extends the analysis to a large class of heterogeneous agents incomplete markets models with aggregate risk. Section 4 discusses implications for the questions which motivate this paper and a large literature and concern many policy makers and finally provides concluding remarks. Most derivations, proofs and the data description are delegated to the appendix.

## 2 Exchange Rates in a Simple OLG Model

In this section I illustrate the idea of exchange rate determination and how it is related to portfolio choices using a simple (partially linearized) OLG model. Households can invest in home and foreign nominal bonds, two assets which have different returns due to different stochastic prices across countries. To ensure a well-defined portfolio I also assume that investing in the foreign bonds is subject to a small transaction cost.<sup>8</sup> This results in a trade-off - foreign bonds feature better insurance properties than home bonds but are subject to a transaction cost which home bonds are not - and households in both countries are willing to hold positive amounts of bonds of both countries.

### 2.1 Open Economy OLG Model

The world economy consists of two countries, (H)ome and (F)oreign, where at each point of time  $t$  two generations, young and old, are alive. The state of the home country is  $s_t^H = s_t \in \mathcal{N}(0, \sigma^2)$  and independent over time. The state of the foreign country  $s_t^F = -s_t$  is perfectly negatively correlated with the home state, which in the symmetric benchmark below is equivalent to the assumption of no world risk. There is a single good such that the law of one price implies a real exchange rate equal to one. This assumption, which I relax in Section 2.5, establishes that movements in the nominal exchange rate are not driven by the real exchange rate. The nominal exchange rate is the home price of foreign currency such that an increase is a depreciation. I consider a cashless economy (Woodford (2003)) where monetary policy in each country  $H$  and  $F$  sets nominal interest rates  $i_H$  and  $i_F$  respectively. Fiscal policy sets nominal bonds  $B_H, B_F$  (denominated in their own currency), operates a social security system and sets taxes such that the steady-state government budget constraints hold in all states of the world.

---

<sup>8</sup>An alternative and maybe more appealing alternative would be to allow for a risk of default when investing abroad. This would however add some history-dependence to the model which is interesting but would render the model analytically non-tractable.

The after-tax real endowment of the young generation in countries H and F is

$$y_{H,s_t} = y_H(1 + \lambda_y^H s_t^H) = y_H(1 + \lambda_y s_t) \quad \text{and} \quad y_{F,s_t} = y_F(1 + \lambda_y^F s_t^F) = y_F(1 - \lambda_y s_t) \quad (1)$$

such that the period  $t$  budget constraint for young home households equals

$$c_{H,s_t}^y + A_{H,s_t}^H q_{s_t}^H + A_{H,s_t}^F q_{s_t}^F \leq y_H(1 + \lambda_y s_t) - T_{s_t}^H, \quad (2)$$

where household consumption is  $c_{H,s}^y$ ,  $A_{H,s}^H \geq 0$  are home and  $A_{H,s}^F \geq 0$  are foreign nominal government bond holdings of the home young generation, each dominated in their respective currencies,  $i^H$  is the home nominal interest rate and  $B_H$  is the supply of home government bonds. The real value of acquired bonds is  $A_{H,s}^H q_{H,s}$  and  $A_{H,s}^F q_{F,s}$ , where  $q_{H,s}$  ( $q_{F,s}$ ) is the inverse of the price level in country  $H$  ( $F$ ) in state  $s$ . Households have to pay taxes  $T_s^H$  which in are used to cover government's interest rate expenditures so that in equilibrium  $T_s^H = i^H B_H q_s^H$ .<sup>9</sup> The budget constraint for young foreign households equals

$$c_{F,s_t}^y + A_{F,s_t}^F q_{s_t}^F + A_{F,s_t}^H q_{s_t}^H \leq y_F(1 - \lambda_y s_t) - T_{s_t}^F, \quad (3)$$

where  $A_{F,s}^F \geq 0$  are foreign and  $A_{F,s}^H \geq 0$  are home bond holdings of the foreign young generation,  $c_{F,s}^y$  is consumption,  $i^F$  is the foreign nominal interest rate,  $B_F$  is the supply of foreign government bonds and  $T_s^F$  are taxes imposed on foreign households to cover the interest rate payments on foreign bonds,  $T_s^F = B_F i^F q_s^F$ . Home consumption when old,  $c_{H,s_{t+1},s_t}^o$ , equals

$$c_{H,s_{t+1},s_t}^o = y_{H,s_{t+1},s_t}^o + (1 + i^H) A_{H,s_t}^H q_{s_{t+1}}^H + (1 + i^F)(1 - \chi^F) A_{H,s_t}^F q_{s_{t+1}}^F, \quad (4)$$

where old age income,  $y_{H,s_{t+1},s_t}^o$ , is the sum social security benefits  $\kappa_1 s_t$  (linked to previous period's income state  $s_t$ ) and labor income  $y_H^o(1 + \lambda_o s_{t+1})$ .<sup>10</sup> The transaction cost for investing

<sup>9</sup>Tractability requires to eliminate any history-dependence so that young households' decision problem depends on the current state  $s_t$  only and not on previous states  $s_{t-1}$ . I therefore assume (implicitly) that social security contributions of the young generation are constant and that government spending is adjusted to balance the social security budget.

<sup>10</sup>Linking old age income to previous period's state adds some persistence which, as will become clear below, helps when quantifying the model but at the same time maintains independence of shocks across time. The modeling of income is irrelevant for the theoretical results though.



in the foreign country is  $\chi^F$ . Foreign consumption when old,  $c_{F,s_{t+1},s_t}^o$ , thus equals

$$c_{F,s_{t+1},s_t}^o = y_{F,s_{t+1},s_t}^o + (1 + i^F)A_{F,s_t}^F q_{s_{t+1}}^F + (1 + i^H)(1 - \chi^H)A_{F,s_t}^H q_{s_{t+1}}^H, \quad (5)$$

for old age income  $y_{F,s_{t+1},s_t}^o = y_F^o(1 - \lambda_o s_{t+1}) - \kappa_1 s_t$  and where now the transaction cost  $\chi^H$  is on home bonds (the foreign bonds for F investors) and F bonds feature no transaction costs. The young generation in period  $t$  derives utility

$$u(c_t^y) + E_t u(c_{t+1}^o), \quad (6)$$

where  $u(c) = -\exp(-\gamma c)$ .

I consider linear approximations of prices and portfolio choices implying that period  $t$  prices are linear in  $s = s_t$ ,

$$q_s^H = \bar{q}^H + \lambda_q^H s, \quad (7)$$

$$q_s^F = \bar{q}^F + \lambda_q^F s^F = \bar{q}^F - \lambda_q^F s, \quad (8)$$

and that both  $q_s^H$  and  $q_s^F$  are normally distributed. The exchange rate then equals

$$\epsilon_s = \frac{q_s^F}{q_s^H} = \frac{\bar{q}^F - \lambda_q^F s}{\bar{q}^H + \lambda_q^H s}. \quad (9)$$

The utility function therefore simplifies to

$$u(c_{H,s_t}^y) = e^{-\gamma \mu_{H,s_t} + \frac{(\gamma \sigma \Sigma_{H,s_t})^2}{2}}, \quad (10)$$

where

$$\mu_{H,s_t} = \kappa_1 s_t + y_H^o + (1 + i^H)A_{H,s_t}^H \bar{q}^H + (1 + i^F)(1 - \chi^F)A_{H,s_t}^F \bar{q}^F \quad (11)$$

is the mean and

$$\sigma \Sigma_{H,s_t} = \sigma [y_H^o \lambda_o + (1 + i^H)A_{H,s_t}^H \lambda_q^H + (1 + i^F)(1 - \chi^F)A_{H,s_t}^F \lambda_q^F] \quad (12)$$

is the standard deviation of old age home consumption so that old age consumption can be

written as

$$c_{H,s_{t+1},s_t}^o = \mu_{H,s_t} + \Sigma_{H,s_t} s_{t+1}.$$

Similarly, foreign young households expected utility equals

$$u(c_{F,s_t}^y) = e^{-\gamma\mu_{F,s_t} + \frac{(\gamma\sigma\Sigma_{F,s_t})^2}{2}}, \quad (13)$$

where

$$\mu_{F,s_t} = y_H^o - \kappa_1 s_t + (1+i^F)A_{F,s_t}^F \bar{q}^F + (1+i^H)(1-\chi^H)A_{F,s_t}^H \bar{q}^H, \quad (14)$$

$$\sigma\Sigma_{F,s_t} = \sigma[y_F^o \lambda_o + (1+i^F)A_{F,s_t}^F \lambda_q^F + (1+i^H)(1-\chi^H)A_{F,s_t}^H \lambda_q^H] \quad (15)$$

are the mean and the standard deviation of old-age foreign consumption.

## 2.2 Portfolio Choice and Exchange Rates

The first-order condition for home bonds acquired by home households,  $A_{H,s}^H$ , is then<sup>11</sup>

$$\begin{aligned} (\bar{q}^H + \lambda_q^H s_t) &= E[(e^{-\gamma(c_{H,s_{t+1},s_t}^o - c_{H,s_t}^y)})(1+i^H)(\bar{q}^H + \lambda_q^H s_{t+1})] \\ &= E[(e^{-\gamma(c_{H,s_{t+1},s_t}^o - c_{H,s_t}^y)})(1+i^H)\bar{q}^H + Cov[e^{-\gamma(c_{H,s_{t+1},s_t}^o - c_{H,s_t}^y)}, (1+i^H)\lambda_q^H s_{t+1}]] \\ &= \underbrace{[e^{-\gamma(\mu_{H,s_t} - c_{H,s_t}^y) + \frac{(\gamma\sigma\Sigma_{H,s_t})^2}{2}}]}_{E(SDF)} \underbrace{[(1+i^H)\bar{q}^H]}_{E(Payoff)} \underbrace{-(1+i^H)\lambda_q^H \gamma\sigma^2 \Sigma_{H,s_t}}_{Cov(SDF, Payoff)}, \end{aligned} \quad (16)$$

which delivers the standard decomposition into the expected stochastic discount factor,  $E(SDF)$ , the expected payoff,  $E(Payoff)$ , and a covariance term. Since prices and portfolios are linear, the first-order condition needs to be approximated. As will become clear below, a linear approximation with respect to  $s_t$  is sufficient:<sup>12</sup>

<sup>11</sup>Note that for a lognormal distribution  $X = exp(s)$  with mean 0 and variance  $\sigma^2$ ,

$$E[e^{c_0 + c_1 \log(X)} c_2 \log(X)] = e^{c_0 + \frac{(c_1 \sigma)^2}{2}} c_1 c_2 \sigma^2,$$

so that the covariance  $Cov(e^{-\gamma(c_{H,s_{t+1}}^o - c_{H,s_t}^y)}, s_{t+1})$  equals

$$Cov(e^{-\gamma(\mu_{H,s_t} + \Sigma_{H,s_t} s_{t+1})}, s_{t+1}) = e^{-\gamma\mu_{H,s_t} + \frac{(\gamma\Sigma_{H,s_t}\sigma)^2}{2}} \Sigma_{H,s_t} \sigma^2.$$

<sup>12</sup>For a variable  $x(s)$ ,  $\bar{x} = x(s=0)$  denotes the value of  $x$  at  $s=0$  and  $\hat{x}$  the deviation, so that  $x(s)$  is approximated as  $\bar{x} + \hat{x} \log(1+s) \approx \bar{x} + \hat{x}s$ .

Home Investors investing in Home Bonds  $A_{H,s_t}^H$ :

$$\underbrace{\bar{q}^H + \lambda_q^H s_t}_{\text{Price}} = \underbrace{[\bar{m}^H + s_t \hat{m}^H]}_{\text{E(SDF)}} \underbrace{(1 + i^H) \bar{q}^H}_{\text{E(Payoff)}} - \underbrace{\lambda_q^H \gamma \sigma^2 (1 + i^H) [(\bar{m}^H + s_t \hat{m}^H) \bar{\Sigma}_H + s_t \bar{m}^H \hat{\Sigma}_H]}_{\text{+ Cov( SDF, Payoff )}}, \quad (17)$$

where the expected stochastic discount factor (SDF) is approximated as

$$e^{-\gamma(\mu_{H,s_t} - c_{H,s_t}^y) + \frac{(\gamma \sigma \Sigma_{H,s_t})^2}{2}} \approx \bar{m}^H + \hat{m}^H s_t =: m_{s_t}^H, \quad (18)$$

$\Sigma_{H,s_t}$  and  $\Sigma_{F,s_t}$  are approximated as

$$\Sigma_{H,s_t} \approx \bar{\Sigma}_H + s_t \hat{\Sigma}_H \quad \text{and} \quad \Sigma_{F,s_t} \approx \bar{\Sigma}_F + s_t \hat{\Sigma}_F, \quad (19)$$

and the SDF at the point of approximation  $s = 0$  equals

$$\bar{m}^H = e^{\gamma(c_{H,s=0}^y - y_H^o) - \gamma \mu_{H,s=0} + \frac{(\gamma \sigma \Sigma_{H,s=0})^2}{2}}. \quad (20)$$

The covariance equals

$$- \left[ e^{-\gamma(\mu_{H,s_t} - c_{H,s_t}^y) + \frac{(\gamma \sigma \Sigma_{H,s_t})^2}{2}} \right] (1 + i^H) \lambda_q^H \gamma \sigma^2 \Sigma_{H,s_t} \quad (21)$$

and is approximated as

$$- \lambda_q^H \gamma \sigma^2 (1 + i^H) [(\bar{m}^H + s_t \hat{m}^H) \bar{\Sigma}_H + s_t \bar{m}^H \hat{\Sigma}_H]. \quad (22)$$

Note that future old age uncertainty is fully incorporated and the linearization is w.r.t. the state  $s_t$  which is known when young households take their portfolio decisions. Old age consumption depends on  $s_t$  though since first, social security payments are linked to young age income and second, the portfolio chosen when young depends on  $s_t$ . Also,  $\bar{m}^H + \hat{m}^H s_t$  is the expected SDF as it incorporates old age uncertainty but depends on the state of the world  $s_t$  when young. The approximation is valid only for  $s_t$  small enough which I assume to be the case. In particular  $s_t$  is small enough so that prices  $q^H$  and  $q^F$  are positive.

If households hold a diversified portfolio (which I show below to be true for small enough  $\chi$ ) the remaining first order conditions are

Home Investors: Foreign Bonds  $A_{H,s_t}^F$

$$(\bar{q}^F - \lambda_q^F s_t) = (1 + i^F)(1 - \chi)\{[\bar{m}^H + s_t \hat{m}^H] \bar{q}^F + \lambda_q^F \gamma \sigma^2 [(\bar{m}^H + s_t \hat{m}^H) \bar{\Sigma}_H + s_t \bar{m}^H \hat{\Sigma}_H]\} \quad (23)$$

Foreign Investors: Foreign Bonds  $A_{F,s_t}^F$

$$(\bar{q}^F - \lambda_q^F s_t) = (1 + i^F)\{[\bar{m}^F + s_t \hat{m}^F] \bar{q}^F - \lambda_q^F \gamma \sigma^2 [(\bar{m}^F + s_t \hat{m}^F) \bar{\Sigma}_F + s_t \bar{m}^F \hat{\Sigma}_F]\} \quad (24)$$

Foreign Investors: Home Bonds  $A_{F,s_t}^H$

$$(\bar{q}^H + \lambda_q^H s_t) = (1 + i^H)(1 - \chi)\{[\bar{m}^F + s_t \hat{m}^F] \bar{q}^H + \lambda_q^H \gamma \sigma^2 [(\bar{m}^F + s_t \hat{m}^F) \bar{\Sigma}_F + s_t \bar{m}^F \hat{\Sigma}_F]\}, \quad (25)$$

where the expected foreign stochastic discount factor is approximated as

$$e^{-\gamma(\mu_{F,s_t} - c_{F,s_t}^y) + \frac{(\gamma \sigma_{\Sigma_{F,s_t}})^2}{2}} \approx \bar{m}^F + s \hat{m}^F =: m_s^F \quad (26)$$

and at the point of approximation  $s = 0$  equals

$$\bar{m}^F = e^{\gamma(c_{F,s=0}^y - y_F^o) - \gamma \mu_{F,s=0} + \frac{(\gamma \sigma_{\Sigma_{F,s=0}})^2}{2}}. \quad (27)$$

Note that the SDFs  $m_s^H$  and  $m_s^F$  depend on the endogenous prices  $\bar{q}^H, \bar{q}^F, \lambda_q^H, \lambda_q^F$ , which renders the computation of the full equilibrium - prices and portfolio decisions jointly - a non-linear problem although all first-order conditions are linear in  $s$ . To obtain a trend-free exchange rate, monetary policy in both countries is assumed to be identical,  $i = i^H = i^F$ , but I allow for temporary monetary shocks below. Before computing the equilibrium, I will establish several properties of this new model to highlight its key mechanisms.

The home investor, i.e. the young generation in period  $t$ , portfolio choices are approximated around  $s_t = 0$  as

$$A_{H,s}^H \approx \bar{A}_H^H + s_t \hat{A}_H^H, \quad (28)$$

$$A_{H,s}^F \approx \bar{A}_H^F + s_t \hat{A}_H^F \quad (29)$$

and similarly for the foreign investor,

$$A_{F,s}^F \approx \bar{A}_F^F + s_t \hat{A}_F^F, \quad (30)$$

$$A_{F,s}^H \approx \bar{A}_F^H + s_t \hat{A}_F^H. \quad (31)$$

Using this notation allows me to write the zero- and first-order component of  $\Sigma_{H,s_t}$  as

$$\bar{\Sigma}_H = y_H^o \lambda_o + (1 + i^H) \bar{A}_H^H \lambda_q^H + (1 + i^F)(1 - \chi^F) \bar{A}_H^F \lambda_q^F, \quad (32)$$

$$\hat{\Sigma}_H = (1 + i^H) \hat{A}_H^H \bar{q}^H + (1 + i^F)(1 - \chi^F) \hat{A}_H^F \bar{q}^F, \quad (33)$$

and the zero- and first-order component of  $\Sigma_{F,s_t}$  as

$$\bar{\Sigma}_F = y_F^o \lambda_o + (1 + i^F) \bar{A}_F^F \lambda_q^F + (1 + i^H)(1 - \chi^H) \bar{A}_F^H \lambda_q^H, \quad (34)$$

$$\hat{\Sigma}_F = (1 + i^F) \hat{A}_F^F \bar{q}^F + (1 + i^H)(1 - \chi^H) \hat{A}_F^H \bar{q}^H. \quad (35)$$

First, I consider the zero-order component of the portfolio, the choices  $\bar{A}_H^H, \bar{A}_F^H, \bar{A}_F^F, \bar{A}_H^F$ , at the point of approximation  $s_t = 0$ . The standard first-order approximation approach where the non-stochastic steady state is used as the point of approximation cannot be used to compute the zero-order component (for example Devereux and Sutherland, 2011). A first-order approximation with respect to  $s_{t+1}$  would eliminate the covariance term and thus the risk, implying that every portfolio choice would be consistent with equilibrium. This problem is overcome here since no approximation (for  $s_{t+1}$ ) but instead the full nonlinear solution is considered, so that the four first-order conditions evaluated at  $s_t = 0$  are sufficient to solve for the four zero-order components  $\bar{A}_H^H, \bar{A}_F^H, \bar{A}_F^F, \bar{A}_H^F$ .<sup>13</sup>

**Result 1.** (*Portfolio Choice - zero order component*)

The zero-order components of the portfolio,  $\bar{A}_H^H, \bar{A}_F^H, \bar{A}_F^F, \bar{A}_H^F$  solve

$$\bar{q}^H = \bar{m}^H(1 + i^H)\bar{q}^H - \lambda_q^H \gamma \sigma^2 (1 + i^H) \bar{m}^H \bar{\Sigma}_H, \quad (36)$$

$$\bar{q}^F = \bar{m}^H(1 + i^F)(1 - \chi)\bar{q}^F + \lambda_q^F \gamma \sigma^2 (1 + i^F)(1 - \chi) \bar{m}^H \bar{\Sigma}_H, \quad (37)$$

$$\bar{q}^F = (1 + i^F) \bar{m}^F \bar{q}^F - \lambda_q^F \gamma \sigma^2 (1 + i^F) \bar{m}^F \bar{\Sigma}_F, \quad (38)$$

$$\bar{q}^H = (1 + i^H)(1 - \chi) \bar{m}^F \bar{q}^H + \lambda_q^H \gamma \sigma^2 (1 + i^H)(1 - \chi) \bar{m}^F \bar{\Sigma}_F, \quad (39)$$

where  $\bar{m}^H, \bar{m}^F, \bar{\Sigma}_H, \bar{\Sigma}_F$  are defined in (20), (27), (32) and (34).

The four first-order conditions are linear functions of  $s_t$ . Using this linear component is

---

<sup>13</sup>Devereux and Sutherland (2011) use second-order approximations of the same first-order conditions to determine the zero-order component. The assumptions on the utility function and the distribution allow me to fully incorporate risk about  $s_{t+1}$  in the second period without any need for approximation beyond linearly approximating prices.

sufficient to solve for the four first-order components  $\hat{A}_H^H, \hat{A}_F^H, \hat{A}_F^F, \hat{A}_H^F$ .

**Result 2.** (*Portfolio Choice - first order component*)

The first-order components of the portfolio,  $\hat{A}_H^H, \hat{A}_F^H, \hat{A}_F^F, \hat{A}_H^F$  solve

$$\lambda_q^H = \hat{m}^H(1+i^H)\bar{q}^H - \lambda_q^H\gamma\sigma^2(1+i^H)[\hat{m}^H\bar{\Sigma}_H + \bar{m}^H\hat{\Sigma}_H], \quad (40)$$

$$\lambda_q^F = (1+i^F)(1-\chi)\hat{m}^H\bar{q}^F + \lambda_q^F\gamma\sigma^2(1+i^F)(1-\chi)[\hat{m}^H\bar{\Sigma}_H + \bar{m}^H\hat{\Sigma}_H], \quad (41)$$

$$\lambda_q^F = (1+i^F)\hat{m}^F\bar{q}^F - \lambda_q^F\gamma\sigma^2(1+i^F)[\hat{m}^F\bar{\Sigma}_F + \bar{m}^F\hat{\Sigma}_F], \quad (42)$$

$$\lambda_q^H = (1+i^H)(1-\chi)\hat{m}^F\bar{q}^H + \lambda_q^H\gamma\sigma^2(1+i^H)(1-\chi)[\hat{m}^F\bar{\Sigma}_F + \bar{m}^F\hat{\Sigma}_F]. \quad (43)$$

where  $\hat{m}^H, \hat{m}^F, \hat{\Sigma}_H, \hat{\Sigma}_F$  are defined in (18), (26) and (33) and (35).

The two previous results establish the partial equilibrium result, which maps prices into portfolio choices. Before moving to the equilibrium results, I derive properties of these portfolio choices. First, I show that portfolios are diversified if  $\kappa_1 \neq 1$ :

**Result 3** (Diversified Portfolios). *Home and foreign investors hold positive amounts of each asset:*

$$0 < A_H^H, A_F^H < B_H; \quad 0 < A_F^F, A_H^F < B_F.$$

The logic behind this result is quite simple and has two steps. First, autarky where each country only holds its own bonds is not an equilibrium. The output uncertainty carries over to prices such that bonds are risky even in autarky. That is where  $\kappa_1 \neq 1$  is needed since otherwise  $\hat{m}^H = 0$  and prices would be independent of  $s$ . To reduce their risk exposure, home investors hold fewer home bonds (negative correlation with autarky-SDF) and more foreign bonds (positive correlation with autarky-SDF) than in autarky. Second, investors face a mean-variance trade-off such that they hold positive amounts of both bonds. In a world without uncertainty and thus constant exchange rates, a positive transaction cost implies that investing in the home bond is the return dominant strategy. However, in the presence of uncertainty the home bond is a risky asset as indicated by the negative covariance with the SDF whereas the foreign bond has a positive covariance such that moving away from a fully home biased portfolio reduces risk. This diversification strategy enhances utility if either the transaction cost  $\chi$  is not too high or the price of the foreign bond relative to the home bond is low enough, which is the case if  $|s|$  is large enough.

A symmetric world, that is both countries are identical including except that the states are perfectly negatively correlated,  $s^F = -s^H$ , allows for a precise characterization of the portfolios as a function of prices. A symmetric world allows to drop country super. and subscripts,  $B = B_F = B_H$ ,  $\bar{q} = \bar{q}^H = \bar{q}^F$ ,  $\lambda_q = \lambda_q^H = \lambda_q^F$ ,  $\bar{m} = \bar{m}^H = \bar{m}^F$ ,  $\hat{m} = \hat{m}^H = \hat{m}^F$ .

**Result 4.** (*Portfolio Choices*) *In a symmetric world*

### Home Portfolio Choice

$$\bar{A}_H^H = B \frac{1 - \chi}{2 - \chi} + \frac{y^o \lambda_o}{(\chi - 2) \lambda_q^H (1 + i)} + \frac{\bar{q}^H \chi}{(\chi - 2)^2 (1 + i) \gamma \sigma^2 (\lambda_q^H)^2} \quad (44)$$

$$\hat{A}_H^H = B \frac{4\chi - 4}{\gamma \chi (1 + i) \lambda_q^H \sigma^2 (\chi - 2)^2} \quad (45)$$

$$\bar{A}_H^F = B - \bar{A}_H^H \quad (46)$$

$$\hat{A}_H^F = \hat{A}_H^H \quad (47)$$

### Foreign Portfolio Choice

$$\bar{A}_F^F = B - \bar{A}_H^F \quad (48)$$

$$\hat{A}_F^F = B - \hat{A}_H^F \quad (49)$$

$$\bar{A}_F^H = B - \bar{A}_H^H \quad (50)$$

$$\hat{A}_F^H = B - \hat{A}_H^H \quad (51)$$

However, the diversified portfolio does not eliminate all the risk but balances the higher mean return (from home bonds) and the lower variance (from foreign bonds). The linearized model allows to solve for the volatility of the portfolio payoff  $\Sigma_s \neq 0$ , for the SDF which is not constant and not equal to  $1/(1 + i)$  and for the exchange rate which is not constant either.

**Result 5.** (*Presence of Risk*)

### Portfolio Volatility:

$$\bar{\Sigma} + s \hat{\Sigma} = \frac{\bar{q} \chi}{(2 - \chi) \lambda_q \gamma \sigma^2} - s \frac{4(1 - \chi)}{\gamma \sigma^2 (2 - \chi)^2} \neq 0 \quad (52)$$

**SDF:**

$$\bar{m}^H + s\hat{m}^H = \frac{1}{1+i} \frac{2-\chi}{2(1-\chi)} - s \frac{\lambda_q \chi}{2\bar{q}(1-\chi)(1+i)} \neq \frac{1}{1+i} \quad (53)$$

**Echange Rate Volatility:**

$$Var[\bar{\epsilon} + s\hat{\epsilon}] = \sigma^2 \left(2 \frac{\lambda_q}{\bar{q}}\right)^2 > 0 \quad (54)$$

The nominal risk renders all assets risky so that the SDF at  $s = 0$

$$\bar{m}^H = \frac{1}{1+i} \frac{2-\chi}{2(1-\chi)} > \frac{1}{1+i} \quad (55)$$

is larger than  $1/(1+i)$ , echoing the well-known result that the interest rate is lower in incomplete market models with precautionary savings than if markets were complete.

The source of the state-contingency in the portfolio is risk in the exchange rate. To reduce this risk households also invest in foreign bonds such that at the margin the marginal gain from lower risk is balanced with the transaction costs. Is an equilibrium without exchange rate risk possible? No, since then home bonds would return dominate the foreign bonds and portfolios would be fully home-biased. But Result 3 shows that in this case prices are not constant in equilibrium contradicting the assumption that there is no risk in exchange rates, ruling out a fully home-biased portfolio as an equilibrium outcome.

The results so far are partial equilibrium results describing the mapping from prices to portfolio choices but do not impose asset market clearing yet. A stationary equilibrium are state-contingent portfolio choices and prices for the two countries such that optimal portfolio choices are as characterized in Results 1 and 2 and asset markets clear for all states  $s$ :

Asset market clearing

Home bond market:

$$\underbrace{(\bar{A}_H^H + s\hat{A}_H^H)}_{\text{Home demand}} + \underbrace{(\bar{A}_F^H + s\hat{A}_F^H)}_{\text{Foreign demand}} = \underbrace{B_H}_{\text{Supply}} \quad (56)$$

Foreign bond market:

$$\underbrace{(\bar{A}_H^F + s\hat{A}_H^F)}_{\text{Home demand}} + \underbrace{(\bar{A}_F^F + s\hat{A}_F^F)}_{\text{Foreign demand}} = \underbrace{B_F}_{\text{Supply}} \quad (57)$$



The equations characterizing the equilibrium are linear in  $s$  but are not linear in prices  $\bar{q}$  and  $\lambda_q$  since  $\bar{m}^H, \bar{m}^F, \bar{\Sigma}_H, \bar{\Sigma}_F$  do not depend on  $s$  but are nonlinear in  $\bar{q}$  and  $\lambda_q$ . I therefore resort to a numerical analysis to compute the equilibrium prices, to illustrate the workings of the equilibrium and to consider non-symmetric worlds. One case of asymmetry,  $B_H \neq B_F$ , can be handled theoretically though since flexible prices imply some form of nominal neutrality. Let  $\bar{q}^H, \lambda_q^H, \bar{q}^F, \lambda_q^F$  be the equilibrium prices in the same world but now with the same amount of bonds in both countries,  $B = \frac{B_F + B_H}{2}$ . Then the prices in the asymmetric bond case are  $\bar{q}^H \frac{(B_F + B_H)/2}{B_H}, \lambda_q^H \frac{(B_F + B_H)/2}{B_H}, \bar{q}^F \frac{(B_F + B_H)/2}{B_F}, \lambda_q^F \frac{(B_F + B_H)/2}{B_F}$ . The portfolio holdings stay unchanged in real terms, as for example  $\bar{A}_H^H$  home nominal bond holdings in the symmetric world turns into  $\bar{A}_H^H \frac{B_H}{(B_F + B_H)/2}$  in the asymmetric bond case.

## 2.3 Limit portfolio and exchange rate

In all model parametrizations in the previous Section, both the exchange rate and the portfolio choice are jointly determined in equilibrium. The volatility in exchange rates implies that households hold diversified portfolios. And well-defined portfolio choices imply a determinate exchange rate. The determinacy result of the equilibrium exchange rate thus has two parts, one that is well understood and one that is new and the main contribution of this paper. The well understood part is the mapping from exchange rates to asset and portfolio choices. This is standard finance theory. The new part is the mapping from assets to the exchange rate which together with the portfolio choices determines the exchange rate.

To zoom in on this mapping from portfolios to exchange rates, I consider the limit economy when both the uncertainty and the default risk vanish,  $\sigma^2 \rightarrow 0, \chi \rightarrow 0$ . The previous analysis shows that I obtain a well defined equilibrium portfolio choice and an exchange rate for every combination of positive  $\sigma$  and  $\chi$ , implying well-defined limits

$$\lim_{\sigma, \chi \rightarrow 0} \epsilon_{s=0}(\sigma, \chi) = \epsilon \quad (58)$$

$$\lim_{\sigma, \chi \rightarrow 0} \bar{q}^H(\sigma, \chi) \bar{A}_H^H(\sigma, \chi) = S_H^H \quad \lim_{\sigma, \chi \rightarrow 0} \bar{q}^F \bar{A}_H^F(\sigma, \chi) = S_H^F \quad (59)$$

$$\lim_{\sigma, \chi \rightarrow 0} \bar{q}^F \bar{A}_F^F(\sigma, \chi) = S_F^F \quad \lim_{\sigma, \chi \rightarrow 0} \bar{q}^H \bar{A}_F^H(\sigma, \chi) = S_F^H \quad (60)$$

The demand of home households for home real bonds converges to  $S_H^H$  and for foreign real bonds to  $S_H^F$ . The demand of foreign households for foreign real bonds converges to  $S_F^F$  and for home real bonds to  $S_F^H$ . The limit real asset demand in the home and the foreign country are  $S_H = S_H^H + S_H^F$  and  $S_F = S_F^F + S_F^H$  respectively. Similar to Judd and Guu

(2001), considering the limit of vanishing uncertainty delivers the zero-order component of the portfolio as well as of prices, which can be precisely characterized:

**Result 6.** (*Limit of Vanishing Uncertainty*)

*In a symmetric world but allowing  $B_F \neq B_H$  the limits are:*

### Limit Nominal Portfolios

$$\lim_{\sigma, \chi \rightarrow 0} \bar{A}_H^H(\sigma, \chi) = \lim_{\sigma, \chi \rightarrow 0} \bar{A}_F^H(\sigma, \chi) = \frac{B_H}{2}, \quad (61)$$

$$\lim_{\sigma, \chi \rightarrow 0} \bar{A}_H^F(\sigma, \chi) = \lim_{\sigma, \chi \rightarrow 0} \bar{A}_F^F(\sigma, \chi) = \frac{B_F}{2}. \quad (62)$$

### Limit Prices and Exchange Rate

$$\lim_{\sigma, \chi \rightarrow 0} \bar{q}^H(\sigma, \chi) = \frac{\gamma(y - y^o) - \ln(1/(1+i))}{2\gamma B_H(1+i)}, \quad (63)$$

$$\lim_{\sigma, \chi \rightarrow 0} \bar{q}^F(\sigma, \chi) = \frac{\gamma(y - y^o) - \ln(1/(1+i))}{2\gamma B_F(1+i)}, \quad (64)$$

$$\lim_{\sigma, \chi \rightarrow 0} \epsilon(\sigma, \chi) = \frac{B_H}{B_F}. \quad (65)$$

### Limit Real Portfolios

$$S_H^H = S_F^H = \frac{\gamma(y - y^o) - \ln(1/(1+i))}{4\gamma(1+i)}, \quad (66)$$

$$S_H^F = S_F^F = \frac{\gamma(y - y^o) - \ln(1/(1+i))}{4\gamma(1+i)}. \quad (67)$$

## 2.4 Numerical Analysis

Prices are assumed to be linear in  $s$  and at the same time have to be positive. I therefore restrict the analysis to  $\pm 2$  standard deviations. Output  $y_H = 100$  and  $y_F = 100$ , nominal bond supplies  $B_H = 10$  and  $B_F = 10$  and the standard deviation  $\sigma = 1$  is normalized to one. A one standard deviation in  $s$  changes output by  $\lambda_y = \lambda_y^H = \lambda_y^F = 0.4$  (percent). The simple model has some features which are irrelevant for exchange rate and portfolio determinacy but allow me to ensure that the model is well behaved in spite of linear prices. For example, to increase the correlation of young and old income I assumed that social security is linked to

previous income, implying that the strength of intertemporal substitution is not too state-dependent. I set old age non-asset income for home households  $\kappa_0 y_H(1 + \lambda_y^H s) + \kappa_1 s_{-1} = 12.2 + 0.5\lambda_y s + \lambda_y s_{-1}$  and for foreign households  $\kappa_0 y_F(1 - \lambda_y s) + \kappa_1 s_{-1} = 12.2 - 0.5\lambda_y s - \lambda_y s_{-1}$ . The nominal interest  $i = 0.7$  corresponding to about 2 percent per each year of the 30-year model period. The transaction cost  $\chi = \chi^H = \chi^F = 0.89$  to match the home bias bond measure in Coeurdacier and Rey (2013),  $1 - \frac{\bar{A}_H^F}{B_F/(B_H+B_F)} = 1 - 2\bar{A}_H^F = 0.75$ . The risk aversion parameter  $\gamma = 5$  for the old generation (implying a relative risk aversion of about 3) and I choose for numerical reasons a different risk aversion for the young generation of 1. Using this parametrization, the unique solution for prices is

$$\bar{q}^H + \lambda_q^H s = 1 + 0.12s, \quad (68)$$

$$\bar{q}^F - \lambda_q^F s = 1 - 0.12s, \quad (69)$$

with associated portfolio choices

$$\bar{A}_H^H + s\hat{A}_H^H = (0.875 - 0.061s)B_H, \quad (70)$$

$$\bar{A}_H^F + s\hat{A}_H^F = (0.125 - 0.061s)B_F, \quad (71)$$

$$\bar{A}_F^F + s\hat{A}_F^F = (0.875 + 0.061s)B_F, \quad (72)$$

$$\bar{A}_F^H + s\hat{A}_F^H = (0.125 + 0.061s)B_H. \quad (73)$$

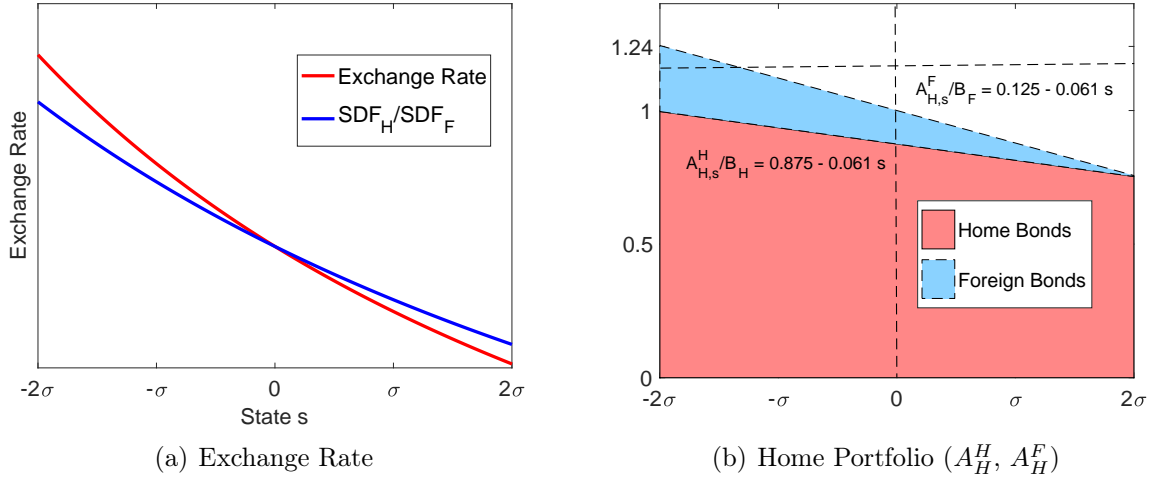
Panel a) of Figure 2 shows the exchange rate

$$\epsilon_s = \frac{q_s^F}{q_s^H} = \frac{\bar{q}^F - \lambda_q^F s}{\bar{q}^H + \lambda_q^H s} = \frac{1 - 0.12s}{1 + 0.12s} \quad (74)$$

together with the ratio of home and foreign SDFs,

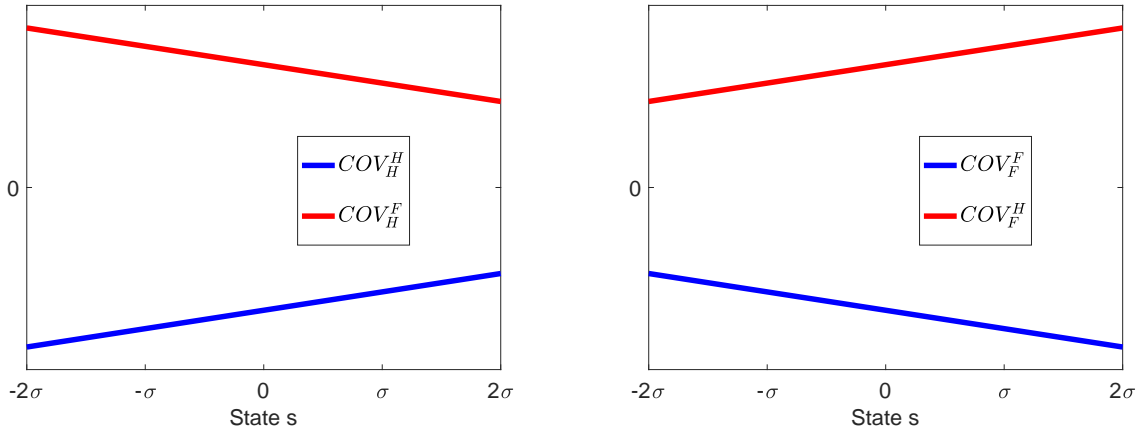
$$\frac{\bar{m}^H + s\hat{m}^H}{\bar{m}^F + s\hat{m}^F} = \frac{4.587 - 0.441s}{4.587 + 0.441s}. \quad (75)$$

Figure 2 shows that a varying risk-premium - the covariance term - implies that the exchange rate is more volatile than the SDF. The slope of the exchange rate in  $s$  around  $s = 0$  equals  $-0.24$  whereas the slope of the ratio of SDFs is  $-0.19$ . At the same time and again due to varying risk-premia the portfolio choice varies quite a bit with the state  $s$  as Panel b) of Figure 2 shows: An increase in  $s$  decreases home's holding of both home and foreign bonds.



(a) Exchange Rate

(b) Home Portfolio ( $A_H^H, A_H^F$ )



(c) Home investor Covariance: Home bonds( $COV_H^H$ ); Foreign Bonds( $COV_H^F$ ); (d) Foreign investor Covariance: Foreign bonds( $COV_F^F$ ); Home Bonds( $COV_F^H$ )

Figure 2: Portfolio Choices and Nominal Exchange Rates

To better understand how the movements in exchange rates and portfolios are connected, consider an appreciation of the home currency in response to an increase in  $s$ . This appreciation renders investment into home bonds less attractive from an expected return perspective for both home and foreign investors such that both countries want to sell home bonds and buy foreign bonds. Market clearing, however, rules out that all agents are sellers and requires that the risk properties of home bonds improve and make them a more attractive investment.

For home investors home bonds are risky - the payoff is positively correlated with consumption and the covariance term is negative - and selling home bonds reduces their risk. For foreign investors on the other hand home bonds are risk-reducing - the payoff is negatively correlated with consumption and the covariance term is positive - and selling home bonds

reduces their risk. As a result foreign investors buy home bonds and home investors sell them. As Panel c) and d) of Figure 2 show this trade reduces the risk-premium - increases the covariance terms  $COV_H^H$  and  $COV_F^H$  - for both countries.<sup>14</sup> For the home country since they reduce their exposure to the risky investment and for the foreign country since they increase holding risk-reducing investments, such that risk premium of home bonds falls for both countries and makes buying them more attractive for both countries.

For foreign bonds the situation is symmetric taking into account that the foreign country depreciates, which raises the expected return for home and foreign investors and renders both countries buyers. To ensure market clearing, the risk premium for foreign bond has to increase to reduce the demand. For home investors foreign bonds are risk reducing so selling them increases the risk. For foreign investors foreign bonds are risky, so buying them increases the risk. As Panel c) and d) of Figure 2 show this trade increases the risk-premium - decreases the covariance terms  $Cov_H^F$  and  $Cov_F^F$  - for both countries.

Figure 1 illustrates how Portfolio choices, bond prices and the nominal exchange rate are jointly determined in equilibrium. The starting point is that each country is exposed to some aggregate uncertainty, for example output fluctuations as above. This uncertainty carries over to bond prices which are risky in equilibrium (Results 3 and 5). Therefore old age consumption is risky as some of it is financed from risky payoff of bonds bought when young. In response to this risky old-age consumption, young households adjust their portfolio decision leading to well-defined asset demands for each country's bonds. Asset market clearing then requires bond prices to adjust. This adjustment changes both the mean and the volatility of bond prices and thus changes the aggregate uncertainty households face and the circle starts over again. An equilibrium is reached if the portfolio choices given risky bond prices are consistent with the prices clearing the asset market. As illustrated above, expected returns and risk premia have to adjust jointly to ensure market clearing.

The model so far emphasized productivity movements as the driving force. I now switch the focus and consider variation in monetary policy, modeled as exogenous fluctuations in

---

<sup>14</sup>Recall that

$$\begin{aligned}
COV_H^H &= -\lambda_q^H \gamma \sigma^2 (1 + i^H) [(\bar{m}^H + s_t \hat{m}^H) \bar{\Sigma}_H + s_t \bar{m}^H \hat{\Sigma}_H], \\
COV_H^F &= +\lambda_q^F \gamma \sigma^2 (1 + i^F) [(\bar{m}^H + s_t \hat{m}^H) \bar{\Sigma}_H + s_t \bar{m}^H \hat{\Sigma}_H], \\
COV_F^F &= -\lambda_q^F \gamma \sigma^2 (1 + i^F) [(\bar{m}^F + s_t \hat{m}^F) \bar{\Sigma}_F + s_t \bar{m}^F \hat{\Sigma}_F], \\
COV_F^H &= +\lambda_q^H \gamma \sigma^2 (1 + i^H) [(\bar{m}^F + s_t \hat{m}^F) \bar{\Sigma}_F + s_t \bar{m}^F \hat{\Sigma}_F].
\end{aligned}$$

nominal interest rates,

$$i_s^H = \bar{i} + \lambda_i s \quad (76)$$

$$i_s^F = \bar{i} - \lambda_i s \quad (77)$$

where I maintain for simplicity the assumption that  $s^F = -s^H$ . I shut down all other sources of variation to focus on the effects of monetary policy which requires to adjust the parametrization.<sup>15</sup>

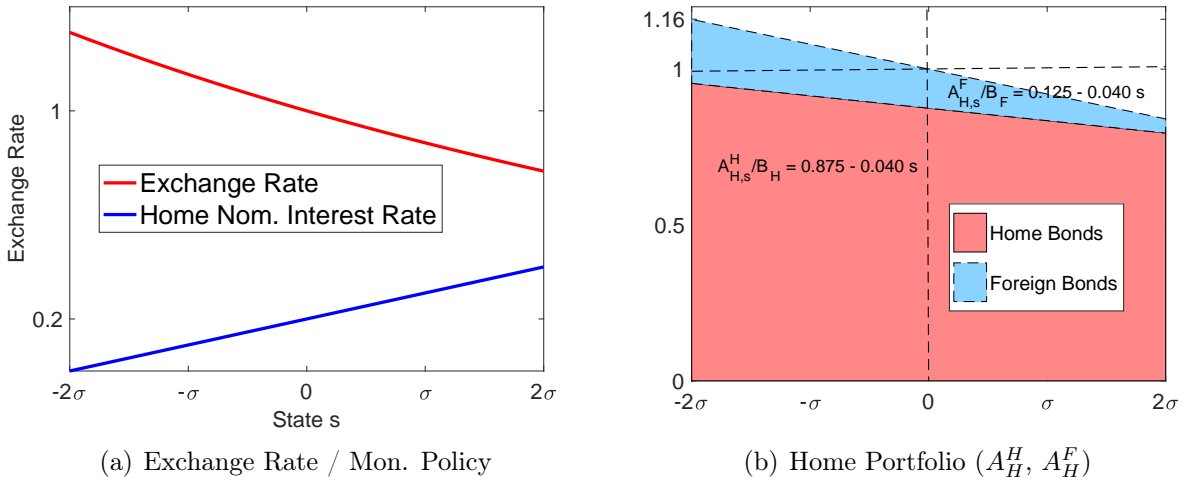


Figure 3: Portfolio Choices and Nominal Exchange Rates: Monetary Policy

A tightening of home monetary policy leads to an appreciation (see panel a) of Figure 3) and the home portfolio is adjusted towards foreign bonds in response to an increase in the nominal interest rate as panel b) of Figure 3) illustrates. Asset market clearing then requires that foreign investors buy more home bonds when the return  $i^H$  increases. The associated price of home bonds is  $q_s^H = 1 + 0.066s$  and of foreign bonds is  $q_s^F = 1 - 0.066s$ . To better understand the underlying mechanism consider an increase  $i^H = 0.3$  and the associated decrease  $i^F = 0.1$ , that is  $\hat{s} = 1$ . As a result the price for home bonds,  $q_s^H$ , increases and the price for foreign bonds,  $q_s^F$ , falls. For home investors the nominal return from home bonds is  $(1+i^H)$  and increases by 0.1 and the expected return from foreign bonds is  $(1+i^F)(1-\chi)\frac{E_t\epsilon_{t+1}}{\epsilon_t}$  and increases by 0.035, suggesting a violation of the interest rate parity condition which can be exploited through investing in home bonds. Similarly for the foreign country the expected return from home bonds is  $(1+i^H)(1-\chi)\frac{E_t\epsilon_t}{\epsilon_{t+1}}$  and falls by 0.06 and the expected return

<sup>15</sup>The changed parameter values are  $\chi = \chi_H = \chi_F = 0.3388$ ,  $\lambda_y = \kappa_1 = 0$ ,  $\kappa_0 = 0.07$ ,  $\lambda_i = 0.1$  and  $\bar{i} = 0.2$ .

from foreign bonds is  $(1 + i^F)$  and decreases by 0.1 so that home bonds return dominate foreign bonds for investors in both countries. These considerations ignore the different risk properties of home and foreign bonds. To ensure market clearing home bonds' risk premium has to increase whereas the risk premium of foreign bonds has to fall. The higher interest rate on home bonds increases the variance of their payoff which is proportional to the interest rate whereas the lower interest rate on foreign bonds decreases the variance of their payoff. This increase in the risk premium of home bonds and the decrease in the risk premium of foreign bonds is larger than necessary to clear the market. Home investors therefore acquire fewer home bonds since this decreases the risk premium for home bonds. For home investors, the necessary market clearing adjustment in the risk premium is thus the sum of an increase induced by an increase in the interest rate and the decrease to a reduction in home bond holdings. For foreign investors, the adjustment in the risk premium is the sum of an increase induced by an increase in the interest rate and the decrease to an increase in home bond holdings. The increase in the excess return of home over foreign bonds is thus compensated by an increase in the the riskiness of home relative to foreign bonds.

## 2.5 Implications for Real Exchange Rates

To focus on the determination of the nominal exchange rate, portfolio choices and their interaction, the analysis so far assumed a single good implying that the real exchange rate is equal to one. This assumption also establishes that the movements in the nominal exchange rate are not caused by movements in the real exchange rate.

One motivation to consider nominal exchanges rate is their strong co-movement with real exchange rates in the data and it is the latter rate that matters for trade decisions. One purpose of this Section is to show that the theory does not rely on a constant real exchange rate and that instead the real exchange rate inherits the volatility of the nominal exchange rate and that both rates are highly correlated. I therefore extend the model by allowing for non-tradable goods in both countries and assume prices to be sticky in the non-tradable sector. I show first, that the results on nominal exchange rates from the previous sections carry over to the richer model in this Section and second, that the real exchange rate is volatile and strongly correlated with its nominal counterpart. The movements in the nominal exchange rate now carry over to the real exchange rate since prices are sticky.

Another implication of a constant real exchange rate is that the volatility of prices and nominal exchange rates is one-to-one related. Building on this model one might think that

the theory relies on equally volatile price levels and nominal exchange rates, whereas in the data the latter one is more volatile. A second objective of this Section is to show that adding price stickiness overcomes this tight relationship. The volatility of the price index is dampened while at the same time the volatility of the nominal exchange rate is unaffected.

### 2.5.1 A Model with Non-Tradables

Young home households purchase tradable consumption goods  $c_{H,s}^{y,T}$  at price  $p_{H,s}^T$  and non-tradable goods  $c_{H,s}^{y,N}$  at price  $p_{H,s}^N$  where both prices are in home currency. Old households only consume tradables  $c_{H,s}^{o,T}$ . The young home generation has after-tax real tradable good endowment  $y_{H,s} = y_H(1 + \lambda_y s)$ , provides labor  $l_{H,s}$  to the non-tradable sector at a wage  $w_{H,s}$  and receives dividends  $d_{H,s}$  so that the budget constraint for young home households equals, after dividing by the price for tradables,

$$c_{H,s}^{y,T} + A_{H,q_s^H}^H + A_{H,q_s^F}^F \epsilon q_s^H + p_{H,s}^N q_s^H c_{H,s}^{y,N} \leq y_H(1 + \lambda_y s) + p_{H,s}^N q_s^H (w_{H,s} l_{H,s} + d_{H,s}). \quad (78)$$

The prices  $q_s^H$  and  $q_s^F$  are now the inverse of the prices of tradables  $p_{H,s}^T$  and  $p_{F,s}^T$  and not the inverse of the price level as in Section 2 above. Prices are again assumed to be linear in  $s$ ,

$$q_s^H = \bar{q}^H + \lambda_q^H s, \quad (79)$$

$$q_s^F = \bar{q}^F + \lambda_q^F s^F = \bar{q}^F - \lambda_q^F s. \quad (80)$$

The non-tradable sector is subject to price adjustment costs and I assume them to be infinite, an assumption that is easy to relax as I explain below. Prices in the non-tradable sector are therefore constant

$$\bar{p}_H^N = p_{H,s}^N, \quad \bar{p}_F^N = p_{F,s}^N. \quad (81)$$

Firms in the non-tradable have to satisfy all demand at these constant prices such that the market clears. Households' expenditures on non-tradables then equals their labor and dividend income from this sector,

$$\bar{p}_H^N q_s^H c_{H,s}^{y,N} = \bar{p}_H^N q_s^H (w_{H,s} l_{H,s} + d_{H,s}), \quad (82)$$

$$\bar{p}_F^N q_s^F c_{F,s}^{y,N} = \bar{p}_F^N q_s^F (w_{F,s} l_{F,s} + d_{F,s}). \quad (83)$$



Tradables goods are identical across countries so that the nominal exchange rate is the ratio of the prices of tradables,

$$\epsilon_s = \frac{q_s^F}{q_s^H} = \frac{\bar{q}^F - \lambda_q^F s}{\bar{q}^H + \lambda_q^H s}. \quad (84)$$

The real value of acquired home bonds in terms of tradable goods is  $A_H^H q_{H,s}$  and of foreign bonds is  $A_H^F \epsilon q_s^H = A_H^F q_{F,s}$ , so that young home and foreign household budget constraints simplify to

$$c_{H,s}^{y,T} + A_H^H q_s^H + A_H^F q_s^F \leq y_H(1 + \lambda_y s), \quad (85)$$

$$c_{F,s}^{y,T} + A_F^F q_s^F + A_F^H q_s^H \leq y_F(1 - \lambda_y s). \quad (86)$$

Consumption of the home old generation equals as before

$$c_{H,s}^{o,T} = \kappa_0 y_H(1 + \lambda_y^H s) + \kappa_1 s_{-1} + (1 + i^H) A_H^H q_s^H + (1 + i^F)(1 - \chi^F) A_H^F q_s^F, \quad (87)$$

where their non-asset income,  $\kappa_0 y_H(1 + \lambda_y^H s) + \kappa_1 s_{-1}$ , is in tradables. Similarly foreign old generation consumption is

$$c_{F,s}^{o,T} = \kappa_0 y_F(1 - \lambda_y^F s) - \kappa_1 s_{-1} + (1 + i^F) A_F^F q_s^F + (1 + i^H)(1 - \chi^H) A_F^H q_s^H. \quad (88)$$

The utility function is as before,

$$u(c_t^y) + E_t u(c_{t+1}^{o,T}), \quad (89)$$

where  $u(c) = -exp(-\gamma c)$  with the only difference that now the young generation consumption  $c_t^y$  is an aggregation of tradable and non-tradable consumption,

$$c_t^y = (c_t^{T,y})^\alpha (c_t^{N,y})^{1-\alpha}. \quad (90)$$

Young household optimization yields

$$c_s^{N,y} = c_s^{T,y} \frac{1-\alpha}{\alpha} \frac{1}{\bar{p}^N (\bar{q}^H + \lambda_q^H s)} \quad (91)$$

and thus

$$c_s^y = (c_s^{T,y})^\alpha (c_s^{N,y})^{1-\alpha} = c_{t,s}^{T,y} \left( \frac{1-\alpha}{\alpha} \frac{1}{\bar{p}^N (\bar{q}^H + \lambda_q^H s)} \right)^{1-\alpha}. \quad (92)$$

The previous analysis applies again and the expected stochastic discount factor (SDF) equals

$$e^{\gamma(c_s^y - y_H^y) - \gamma\mu_{H,s} + \frac{(\gamma\sigma_{H,s})^2}{2}} \approx \bar{m}^H + \hat{m}^H s, \quad (93)$$

but where now  $c_s^y$  is a consumption aggregator of tradable and non-tradable consumption.

The price indexes in the home and foreign country are (omitting multiplicative constants)

$$P_s^H = (q_s^H)^{-\alpha} (\bar{p}^H)^{1-\alpha}, \quad (94)$$

$$P_s^F = (q_s^F)^{-\alpha} (\bar{p}^F)^{1-\alpha}. \quad (95)$$

The real exchange rate equals, given the nominal exchange rate  $\epsilon_s$ ,

$$rer_s = \frac{\epsilon_s P_s^F}{P_s^H}, \quad (96)$$

which simplifies using  $\bar{p}^H = \bar{p}^F$  and  $\epsilon_s = q_s^F / q_s^H$ ,

$$rer_s = \epsilon_s \left( \frac{q_s^H}{q_s^F} \right)^\alpha \left( \frac{\bar{p}^H}{\bar{p}^F} \right)^{1-\alpha} = \left( \frac{q_s^F \bar{p}^H}{q_s^H \bar{p}^F} \right)^{1-\alpha} = \left( \frac{q_s^F}{q_s^H} \right)^{1-\alpha} = (\epsilon_s)^{1-\alpha}. \quad (97)$$

## 2.5.2 Results

We thus obtain a simple relationship between the nominal and the real exchange rate, where the log real exchange rate is the nominal exchange rate scaled by  $1-\alpha$ . The same linearization of the FOC for  $A_H^H, A_H^F, A_F^F, A_F^H$ , (17), (23), (24), (25), applies here. The only difference is that the linearized SDFs  $\bar{m}^H + \hat{m}^H s$  and  $\bar{m}^F + \hat{m}^F s$  could have a different mean and variance since the underlying consumption process of young households does. Vice versa if the SDF is the same in the two models, then the same set of equations describe equilibrium prices  $q_s$  and the nominal exchange rate  $\epsilon_s$ , implying the identical solution for  $\epsilon_s$  in both models.

A simple reparametrization ensures the same SDF in the model with non-tradables as in the model with a fixed real exchange rate. I set the two new parameters,  $\alpha = 0.5$ , and the constant price of non-tradables equal to the steady-state price of tradables,  $\bar{p}^H = \bar{p}^F = 1/\bar{q}^H = 1/\bar{q}^F$ . Old-age income is increased by 0.00134 and the standard deviation

of young-age endowment of tradables from 0.02 to 0.158, so that  $y_{H,s} = 1 + 0.158s$  and  $y_{H,s} = 1 - 0.158s$ . Although the volatility of the tradable endowment is much higher now, the standard deviation of aggregate consumption is unchanged and equal to 0.022. A higher endowment in tradables leads to a fall in its price (an increase in  $q_s$ ). This drop in the relative price  $\bar{p}/q_s$  implies a demand shift from non-tradables to tradables, which leaves aggregate consumption to a large degree unaffected. Figure 4 shows the result. The log real exchange rate is perfectly correlated with the nominal exchange rate but less volatile.

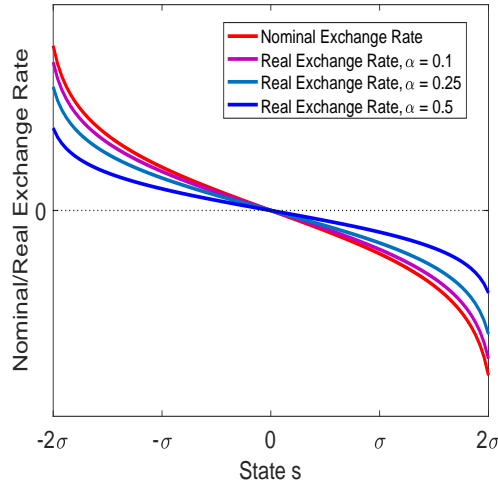


Figure 4: Log Nominal and Real Exchange Rates for different degrees of aggregate price rigidity,  $\alpha = 0.5$  (benchmark),  $\alpha = 0.25$  (medium rigidity),  $\alpha = 0.1$  (high rigidity).

The volatility of the real exchange rate is inversely related to the rigidity of prices, which is parameterized through  $\alpha$ . A lower value for  $\alpha$  implies a more rigid aggregate price level since the share of the constant price non-tradables increases. Figure 4 also shows the result when the price rigidity is increased to  $\alpha = 0.25$  and  $\alpha = 0.1$ . Not surprisingly, the volatility of real exchange rate increases when moving from  $\alpha = 0.5$  to  $\alpha = 0.25$  and 0.1. For each value of  $\alpha$ , I reparameterize the model to obtain the same SDF as in the benchmark, implying that the volatility of the nominal exchange rate is the same across all degrees of price rigidities. The volatility of the price index changes though, establishing that it is unrelated to the nominal exchange rate volatility.

Several conclusions can be drawn from this analysis. First, the theory of nominal exchange rate determination does not rely on a constant real exchange rate. Instead, this assumption just serves to show that the nominal exchange rate movements are not caused by real exchange rate movements. Second, the nominal and the real exchange rate are highly

correlated. Third, the volatility of the price index is unrelated to the volatility of the nominal exchange rate and inversely related to the volatility of the real exchange rate. In the extreme case when prices are almost fully sticky,  $\alpha$  close to 0, the aggregate price is basically constant while the nominal and the real exchange rate become almost equally volatile.

### 3 Exchange Rates and Portfolios: General Case

In all model parametrizations in the previous Section, both the exchange rate and the portfolio choice are jointly determined in equilibrium. The volatility in exchange rates implies that households hold diversified portfolios. And well-defined portfolio choices imply a determinate exchange rate. The determinacy result of the equilibrium exchange rate thus has two parts, one that is well understood and one that is new and the main contribution of this paper. The well understood part is the mapping from exchange rates to asset and portfolio choices. This is standard finance theory. The new part is the mapping from assets to the exchange rate which together with the portfolio choices determines the exchange rate. The remainder of the paper is devoted to this new part.

To zoom in on this mapping from portfolios to exchange rates I consider the limit economy when both the uncertainty and the default risk vanish,  $\sigma^2 \rightarrow 0, \chi \rightarrow 0$ . The previous analysis shows that I obtain a well defined equilibrium portfolio choice and an exchange rate for every combination of positive  $\sigma$  and  $\chi$ , implying well-defined limits

$$\lim_{\sigma, \chi \rightarrow 0} \epsilon_{s=0}(\sigma, \chi) = \epsilon \quad (98)$$

$$\lim_{\sigma, \chi \rightarrow 0} \bar{q}^H(\sigma, \chi) \bar{A}_H^H(\sigma, \chi) = S_H^H \quad \lim_{\sigma, \chi \rightarrow 0} \bar{q}^F \bar{A}_H^F(\sigma, \chi) = S_H^F \quad (99)$$

$$\lim_{\sigma, \chi \rightarrow 0} \bar{q}^F \bar{A}_F^F(\sigma, \chi) = S_F^F \quad \lim_{\sigma, \chi \rightarrow 0} \bar{q}^H \bar{A}_F^H(\sigma, \chi) = S_F^H \quad (100)$$

The demand of home households for home real bonds converges to  $S_H^H$  and for foreign real bonds to  $S_H^F$ . The demand of foreign households for foreign real bonds converges to  $S_F^F$  and for home real bonds to  $S_F^H$ . The limit real asset demand in the home and the foreign country are  $S_H = S_H^H + S_H^F$  and  $S_F = S_F^F + S_F^H$  respectively. Similar to Judd and Guu (2001), considering the limit of vanishing uncertainty delivers the zero-order component of the portfolio.

Using this limiting argument I argue in this Section that exchange rate determinacy and the previous section's arguments extend beyond the OLG economy and indeed hold more generally. Specifically, I argue that the assumption of incomplete asset markets within each

country can replace the OLG assumption. I first show that incomplete markets models deliver determinacy before I turn to explaining the role of my assumptions. In this Section I only consider steady states. Once this step is accomplished, the uncovered interest rate parity condition and vanishing uncertainty imply that determinacy outside the steady state follows from determinacy of the steady state. Outside steady states one can use domestic investors' Euler equations (in nominal terms)

$$1 = E_t \left( m_{t+1}^H \frac{\epsilon_{t+1}}{\epsilon_t} (1 + i_{t+1}^F) (1 - \chi) \right) \quad (101)$$

$$1 = E_t \left( m_{t+1}^H (1 + i_{t+1}^H) \right), \quad (102)$$

where  $m^H$  is the domestic stochastic discount factor and the uncovered (risky) interest parity condition

$$0 = E_t \left[ m_{t+1}^H \left( (1 + i_{t+1}^H) - \frac{\epsilon_{t+1}}{\epsilon_t} (1 + i_{t+1}^F) (1 - \chi) \right) \right]. \quad (103)$$

Starting from the long-run and determinate steady state exchange rate distribution  $\tilde{\epsilon}_{ss}$  and using the equilibrium Euler equation for foreign bonds,

$$\epsilon_t = E_t \left( m_{t+1}^H (1 + i_{t+1}^F) (1 - \chi) \epsilon_{t+1} \right), \quad (104)$$

to iterate backwards yields for the period  $t$  exchange rate

$$\epsilon_t = E_t \left( \prod_{s=t+1}^{\infty} m_s^H (1 + i_{F,s}) (1 - \chi) \tilde{\epsilon}_{ss} \right). \quad (105)$$

This implies that the determinacy of the exchange rate at time  $t$  (outside steady states) follows from the determinacy of the steady state exchange rate  $\tilde{\epsilon}_{ss}$ . I will therefore focus on the latter.<sup>16</sup>

In the zero uncertainty limit, where all agents have perfect foresight, the long-run ex-

---

<sup>16</sup>In the conceivable case that both countries invest in their own bonds only (for example because of high transaction costs), the price levels in the two countries and thus the exchange rate are equilibrium outcomes consistent with this full home bias. Starting from a distribution of long-run and determinate steady state prices  $\tilde{q}^H$  and  $\tilde{q}^F$ , recognizing that now the SDFs depend on prices  $q^H$ ,  $m_t^H(q_t^H, q_{t+1}^H)$ , and  $q^F$ ,  $m_t^F(q_t^F, q_{t+1}^F)$ , one can again use the consumption Euler equations to iterate backwards to obtain the current prices  $q_t^H$  and  $q_t^F$  and thus the current exchange rate  $\epsilon_t = q_t^F / q_t^H$ .

change rate  $\epsilon_{ss}$  is a number and the the period  $t$  exchange rate is

$$\epsilon_t = \left( \prod_{s=t+1}^{\infty} \frac{1 + i_{F,s}}{1 + i_{H,s}} \right) \epsilon_{ss}. \quad (106)$$

Note that  $\epsilon_{ss}$  is a number, the well defined unique limit of vanishing uncertainty, whereas  $\tilde{\epsilon}_{ss}$  is a distribution in the full model with uncertainty. Similarly, the period  $t$  exchange rate is a number which just depends on monetary policies and the long-run exchange rate  $\epsilon_{ss}$ .

### 3.1 The Graphical Approach

I start with the the simple OLG model of the previous Section and use a graphical representation to highlight the determination of exchange rates. First, the asset market clearing condition for home real bonds is rewritten, using the notation introduced in (98), as<sup>17</sup>

$$\frac{B_H}{P_H} = S_H^H + S_F^H, \quad (107)$$

where  $S_H^H + S_F^H$  is the sum of the home and the foreign country demand for home real bonds. For foreign bonds the market clearing condition is

$$\frac{B_F}{P_F} = S_F^F + S_H^F. \quad (108)$$

where  $S_F^F + S_H^F$  is the sum of the home and the foreign country demand for foreign real bonds. As above, the equilibrium steady-state price levels  $P_H (= 1/q_H)$  and  $P_F (= 1/q_F)$  and thus the exchange rate  $\epsilon = P_H/P_F$  are characterized as the solution to these two asset market clearing conditions (107) and (108).

Some simple algebra yields an equivalent but empirically more applicable characterization of prices and the exchange rate in terms of each countries observed asset holdings. Observe first that by definition nominal net foreign asset holdings by the home country,  $NFA_H$  (denominated in the home currency), satisfy

$$\frac{NFA_H}{P_H} = S_H^F - S_F^H, \quad (109)$$

---

<sup>17</sup>For the ease of exposition I assume that both  $B_H$  and  $B_F$  are constant. It is straightforward to relax this since in a steady-state the real value of bonds is constant and  $\frac{B_{H,t}}{P_{H,t}} = \frac{B_H(1+\pi_H)^t}{P_H(1+\pi_H)^t} = \frac{B_H}{P_H}$  and  $\frac{B_{F,t}}{P_{F,t}} = \frac{B_F(1+\pi_F)^t}{P_F(1+\pi_F)^t} = \frac{B_F}{P_F}$ .

and by the foreign country,  $NFA_F$  (denominated in the foreign currency), satisfy

$$\frac{NFA_F}{P_F} = S_F^H - S_H^F = -\frac{NFA_H}{P_H}. \quad (110)$$

Using this in (107) and (108) and rearranging yields:

$$\frac{B_H + NFA_H}{\mathbf{P}_H} = S_H^H + S_H^F = S_H, \quad (111)$$

$$\frac{B_F + NFA_F}{\mathbf{P}_F} = S_F^F + S_F^H = S_F, \quad (112)$$

which defines a mapping from assets to prices and exchange rates. The advantage of this characterization is that it is stated in terms of empirically observable assets  $B_H, NFA_H, B_F, NFA_F$  and depends only on a countries total savings  $S_H, S_F$  but not on the portfolio decisions  $S_H^H, S_H^F, S_F^F, S_F^H$  separately.

The latter characterization also allows to use the Metzler diagram for a graphical illustration. Figure 5 shows how prices and the exchange rate are derived. The left and right panels of Figure 5 report the home and foreign savings curves  $S_H$  and  $S_F$  as a function of the world real interest rate  $1 + r$ . On the horizontal axis they also show the real value of home assets,  $B_H/P_H + NFA_H/P_H$ , and the real value of foreign assets,  $B_F/P_F + NFA_F/P_F = B_F/P_F - NFA_H/P_H$ , where I used that  $NFA_F/P_F = -NFA_H/P_H$ . The right panel tells us that the price level  $P_H$  can be determined as clearing the home market,

$$B_H/\mathbf{P}_H + NFA_H/\mathbf{P}_H = S_H(1 + r, \dots), \quad (113)$$

which then pins down the real value of net foreign assets,  $NFA_F/P_F = -NFA_H/P_H$ . Using this in the left panel pins down the price level  $P_F$  from asset market clearing in the foreign country,

$$B_F/\mathbf{P}_F - NFA_H/P_H = S_F(1 + r, \dots). \quad (114)$$

Therefore the exchange rate  $\epsilon_t = P_{H,t}/P_{F,t}$  is determinate and solves

$$\epsilon_t = \frac{B_{H,t} + NFA_{H,t}}{S_H(1 + r, \dots)} \frac{S_F(1 + r, \dots)}{B_{F,t} - NFA_{H,t}/\epsilon_t}. \quad (115)$$

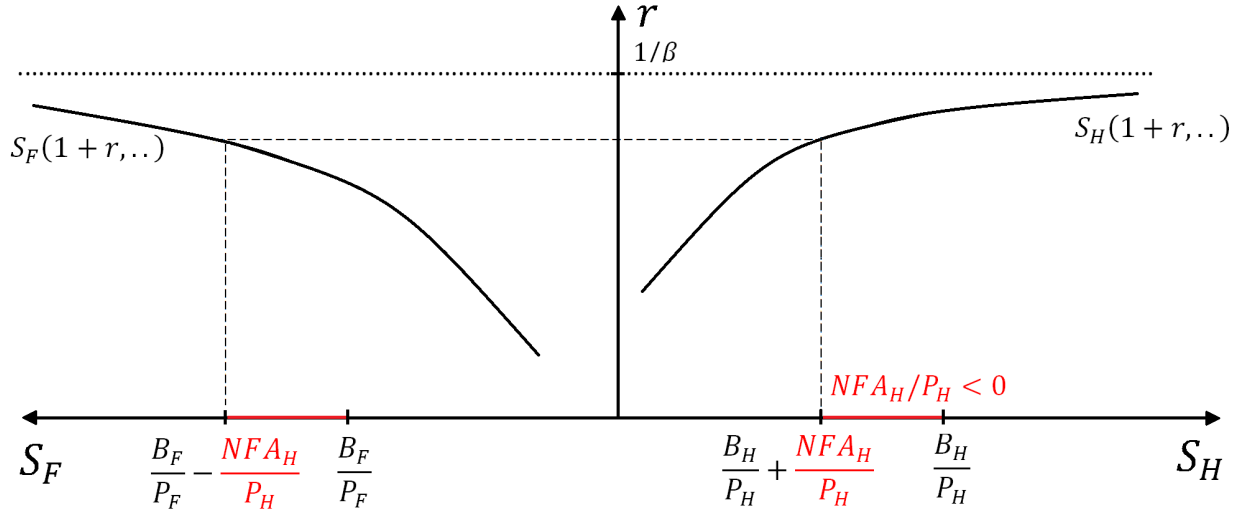


Figure 5: Exchange Rate Determination in Metzler Diagram

Note that the determinacy result does not depend on specific properties of an OLG model but as I will argue now extends to a wide class of models with three properties:

1. Market incompleteness: asset markets within each country are incomplete.

↔ Well-defined aggregate savings within each country.

2. Non-diversifiable Aggregate Risk

↔ Well-defined international portfolios for each country.

3. Nominal assets

↔ Assigns a role for nominal prices.

The necessity of the later property - assets are (partially) nominal - is clear. If assets were fully price-indexed, then there would be no role for prices since the whole economy would be specified in real terms only. It is however sufficient that assets are partially nominal, i.e. a fraction less than 100% could be indexed.

The role of the other two assumptions - market incompleteness and aggregate risk - is more subtle. To understand this, it is useful to first consider a frictionless world without aggregate risk and where markets are complete. In such a world indeterminacies of the Sargent and Wallace (1975) (SW) and the Kareken and Wallace (1981) (KW) type arise. The steady state nominal interest rates  $i_H$  and  $i_F$  just determine the expected change of the



nominal exchange rate,  $E_t \frac{\epsilon_{t+1}}{\epsilon_t}$ , but not the levels  $\epsilon_t$  and  $\epsilon_{t+1}$ . The uncovered interest rate parity condition,

$$1 + i_H = (1 + i_F) E_t \frac{\epsilon_{t+1}}{\epsilon_t}, \quad (116)$$

if satisfied for a pair  $(\epsilon_t, \epsilon_{t+1})$ , is also satisfied for any multiple  $(\lambda\epsilon_t, \lambda\epsilon_{t+1})$  for all  $\lambda > 0$ . This is the analog for exchange rates of the price level indeterminacy pointed out by SW. Accordingly, the derivation illustrated in Figure 5 does not apply anymore. With complete markets the steady-state savings curve is degenerate and becomes a horizontal line at the steady-state real interest rate  $1/\beta$  (for a discount factor  $\beta$ ). As Figure 6 illustrates, asset market clearing in both countries is consistent with a continuum of prices, e.g.  $P_H^1, P_H^2, P_H^3$  for the home country and  $P_F^1, P_F^2, P_F^3$  for the foreign country, and hence with a continuum of exchange rates  $\epsilon = P_H/P_F$ .

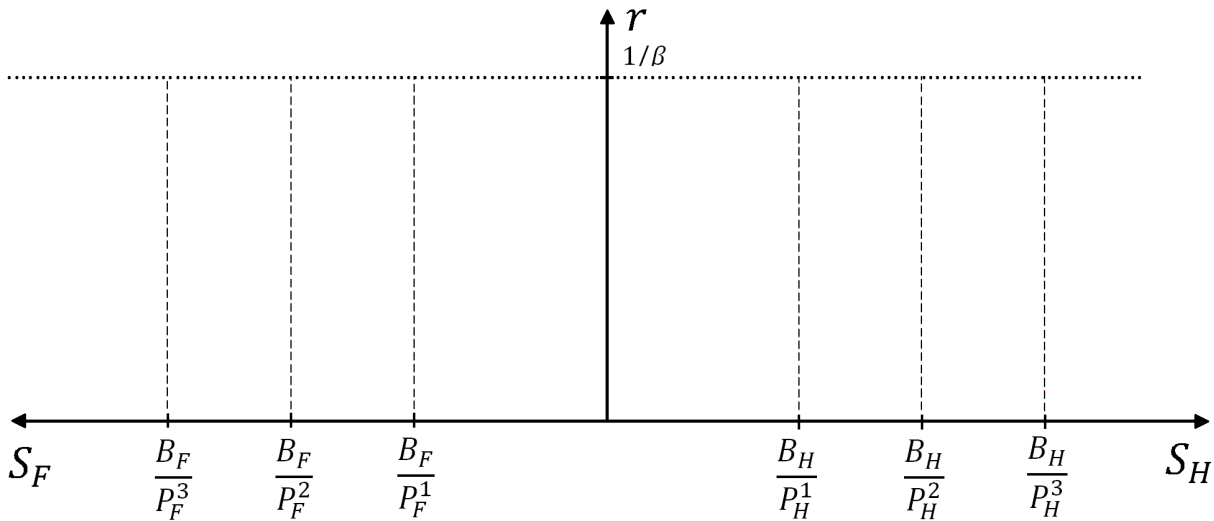


Figure 6: Complete Markets: Exchange Rate Indeterminacy of Sargent and Wallace (1975) type

What incomplete markets contribute are well defined steady-state aggregate savings function  $S_H$  and  $S_F$  as explained above. While adding incomplete markets overcomes the SW indeterminacy it still does not deliver determinacy as now the KW type indeterminacy kicks in. Since bonds are freely mobile across borders and there are no transactions costs, the

world asset market clears when

$$S_H + S_F = \frac{B_H}{P_H} + \frac{B_F}{P_F} = \frac{B_H}{P_H} + \epsilon \frac{B_F}{P_H}, \quad (117)$$

which, for every exchange rate  $\epsilon > 0$ , has a different solution  $P_H$ .<sup>18</sup> However, all of these different exchange rates and price levels are associated with different net foreign asset positions,

$$\frac{NFA_H}{P_H} = S_H - \frac{B_H}{P_H}. \quad (119)$$

For example one can choose price levels  $P_H^-$  and  $P_F^-$  such that the world asset markets clear

$$S_H + S_F = \frac{B_H}{P_H^-} + \frac{B_F}{P_F^-}, \quad (120)$$

and that the associated net foreign asset positions are

$$NFA_H^- = P_H^- S_H - B_H < 0 \quad (121)$$

$$NFA_F^- = P_F^- S_F - B_F > 0 \quad (122)$$

and the exchange rate equals  $\epsilon^- = P_H^-/P_F^-$ . Similarly one can pick world asset market clearing prices  $P_H^0$  and  $P_F^0$  such that

$$NFA_H^0 = P_H^0 S_H - B_H = 0 \quad (123)$$

$$NFA_F^0 = P_F^0 S_F - B_F = 0 \quad (124)$$

---

<sup>18</sup>If mobility was restricted, as an extreme example if each country can only hold its own bonds, then the exchange rate would be determined. This mobility restriction implies separate asset market clearing conditions for each country  $H$  and  $F$ ,

$$S_H = \frac{B_H}{P_H} \quad \text{and} \quad S_F = \frac{B_F}{P_F} \quad (118)$$

which determine price levels  $P_H$  and  $P_F$  and thus the nominal exchange rate  $\epsilon = P_H/P_F$ . However, in this case  $NFA \equiv 0$  prevents a meaningful discussion of cross-border asset flows.

That part of the literature which assumes that monetary policy sets money supplies instead of interest rates makes similar assumptions and typically restricts the usage of a country's currency to this country (The assumption is that households derive utility only from holding their own currency). This full home bias for cash, in contrast to bonds, seems to reflect reality well for many developed countries.

or prices  $P_H^+$  and  $P_F^+$  such that

$$NFA_H^+ = P_H^+ S_H - B_H > 0 \quad (125)$$

$$NFA_F^+ = P_F^+ S_F - B_F < 0 \quad (126)$$

and again world asset markets clear. All these choices are equilibrium outcomes but are associated with different exchange rates  $\epsilon^- = P_H^-/P_F^- < \epsilon^0 = P_H^0/P_F^0 < \epsilon^+ = P_H^+/P_F^+$ , different prices  $P_H^- < P_H^0 < P_H^+$  and  $P_F^- > P_F^0 > P_F^+$  and different NFAs.<sup>19</sup>

This is where assumption 2 (aggregate risk) becomes relevant. Aggregate country risk delivers well defined portfolio choices how to split a country's savings between home and foreign bonds. This adds NFAs to the list of equilibrium objects and eliminates it as a free parameter. In particular, total assets  $A_H = B_H + NFA_H$  is an outcome of agents diversification of aggregate risk. Figure 5 then illustrates the mapping from  $A_H = B_H + NFA_H$  to  $P_H$  and of  $A_F = B_F + NFA_F$  into prices  $P_H$  and  $P_F$  and the exchange rate  $\epsilon = P_H/P_F$ .

## 3.2 The Determinants of the Exchange Rate

The graphical analysis is also informative on some of the determinants of the exchange rate. The exchange rate moves either because assets  $A_H$  or  $A_F$  change or because the savings curves  $S_H$  or  $S_F$  shift. Total assets  $A_H = B_H + NFA_H$  in turn can change either because bond supply  $B_H$  changes or because net foreign assets  $NFA_H$  change. All comparative static exercises that follow use the graphical analysis and are comparisons across steady-states and describe partial effects. General equilibrium effects are not taken into account in the Metzler diagram but are incorporated when I report the corresponding results of the same experiments using the simple OLG model.

### 3.2.1 Government Debt and Exchange Rates

The Metzler diagram can be used to understand how an increase in home supplied assets to  $B'_H > B_H$  affects the exchange rate. This leads to a depreciation or an appreciation depending on whether the home country or the foreign country absorbs those assets. Panel

---

<sup>19</sup>Different NFAs position mean different wealth transfers across regions. For example  $NFA_H^+ < 0$  means that the home country transfers interest rate payments to the foreign country. These wealth transfers change countries' asset demands and are taken into account below but are omitted here as they are irrelevant for the indeterminacy argument. The world asset market clearing condition is still one equation in two unknowns.

b) of Figure 7 shows the case when the increase in  $B_H$  is fully absorbed by the home country and Panel a) is the steady state before the policy change. Note that here I start from a scenario with  $NFA = 0$ , which eliminates valuation gains or losses on initial NFAs from exchange rate movements and allows me to focus on the effects of an expansion of home government debt. A comparison of panel a) and b) shows that total home assets  $A_H = B_H + NFA_H$  increase, the nominal  $NFA_H = 0$  does not change since home absorbs the increase in  $B_H$ , and the home price level  $P_H$  increases to match the real savings of the home country, such that total assets equal  $B'_H$ . Note that this increase of the price to  $P'_H$  does not affect the real value of assets held by the foreign country since  $NFA_H = 0$  and thus does not require a change in the foreign price level. As a result the exchange rate  $\epsilon = P_H/P_F$  depreciates (increases). Panel c) of Figure 7 shows the case when the increase in  $B_H$  is fully absorbed by the foreign country which leads to a fall in  $NFA_H$  and an increase in  $A_F$ . To clear the market for savings by foreigners, the price level  $P_F$  has to increase such that  $A_F/P_F$  matches real savings. Since the increase in  $B_H$  is fully absorbed abroad,  $A_H = B_H + NFA_H$  is unchanged and so is the price level  $P_H$ . As a result the exchange rate  $\epsilon = P_H/P_F$  appreciates (decreases).

In the OLG economy in Section 2 for example a 10% increase in foreign bonds is fully absorbed by the foreign country and the foreign price level increases by 10% whereas the home price level remains unchanged. This is because the home country holds no foreign bonds - neither before nor after the expansion - and therefore does not experience any valuation losses due to the increase in the foreign price level. A 10% increase in home bonds is absorbed by both the home and foreign country, such that the portfolio weights are unchanged and each countries holding of home nominal bonds increases by 10%. As a result the home price level increases by 10% keeping the real value of home bonds for both countries constant. However, there are valuation effects in this example since the foreign country holds home bonds. The real value of foreign held home bonds does not change though since the 10% valuation loss due to the 10% depreciation is exactly offset by the 10% increase in nominal holdings. The real value of foreign bonds and the foreign price level are unaffected by these considerations and are thus unchanged.

These neutrality results are a combination of familiar monetary textbook results and of unrestricted savings and portfolio decisions by households, echoing the results in Backus and Kehoe (1989). Other type of policy interventions break this neutrality, but require a different fiscal policy. For example, the foreign country can acquire home bonds by levying taxes on the current young generation and then adjust government consumption to balance

the government budget. Using this fiscal policy, panel c) of Figure 7 then describes the price movements if the foreign country buys all newly issued home bonds using this type of fiscal policy.<sup>20</sup> I will discuss such policy interventions and exchange rate management strategies in more detail below. Prior to that I consider how changes in output in both countries affects the exchange rate.

### 3.2.2 Output and Exchange Rates

Higher output in the foreign country, a permanent change since this is a comparison across steady states, increases asset demand in this country, shifting the  $S^F$  curve in Figure 8 outwards. To restore equilibrium in the asset market for foreign bonds, the foreign price level falls and the real value of foreign bonds has to increase. Since the home country does not hold any foreign bonds, the real value of NFAs is unaffected and the equilibrium does not require further price or portfolio adjustments. As a result, the foreign output expansion leads to an appreciation of the foreign currency or equivalently to a depreciation of the home currency.

The consequences of an increase in home output are more complicated (and more interesting) since the foreign country experiences valuation gains at the expense of the home country. To understand this I proceed stepwise and first recognize that higher home output increases asset demand in the home country, shifting the  $S^H$  curve in panel a) of Figure 9 outwards, similarly to the shift in Figure 8. Asset market clearing again requires the price level to fall and the real value of home bonds to increase. However, that is not an equilibrium now since the fall in home prices to  $P'_H$  increases the real value of NFAs, which is a wealth transfer from the home to the foreign country. These valuation gains and losses dampen asset demand in the home country and stimulate it in the foreign country, as the two shifting demand curves in Panel b) of Figure 9 show. Since the value of real assets held by the foreign country increases more than their asset demand, the foreign price level has to increase to  $P''_F$  and the home price decreases less to  $P''_H > P'_H$ . As a result, the home output expansion leads to an appreciation of the home currency or equivalently to a depreciation of the foreign currency.

Using again the simple OLG model of Section 2, I find, consistent with these general arguments, that a 1% permanent increase in foreign output leads to a 1.6% depreciation of

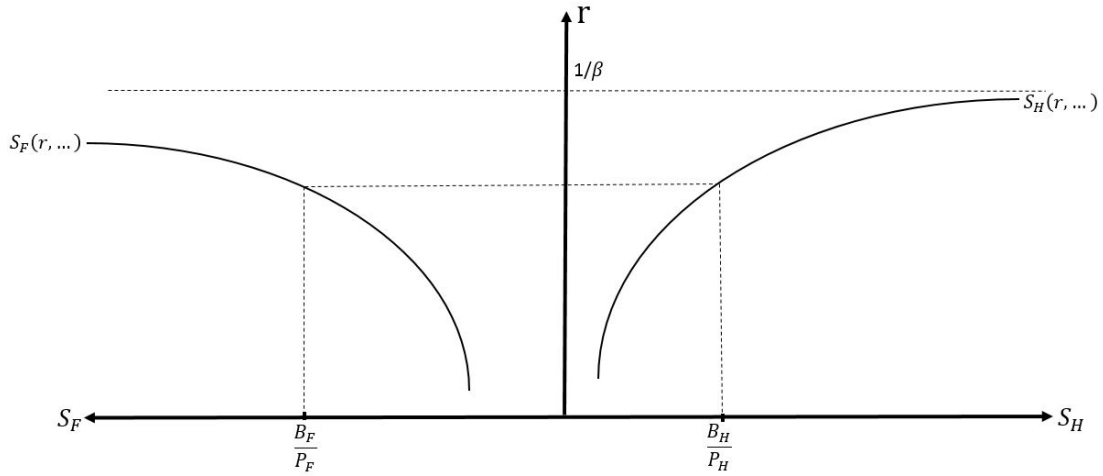
---

<sup>20</sup>This policy leads to the same price movements since  $P_F = A_F/S_F$  and in panel c) of Figure 7 the price  $P_F$  increases because  $A_F$  increases and the policy intervention implies that  $P_F$  increases because  $S_F$  falls due to higher taxes and households holdings of safe liquid assets  $A_F$  is unaffected by the government purchase.

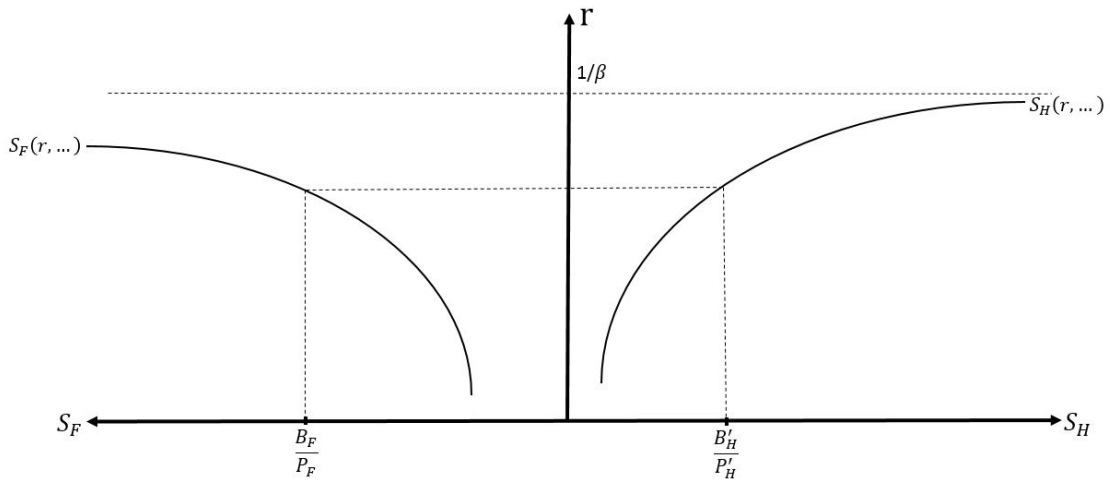
the home currency and a 1% increase in home output leads to a 7.3% appreciation. Both permanent shifts and the temporary shocks to output considered before move the exchange rate in the same direction - output increases lead to an appreciation - as  $\lambda_q^H > 0$  and  $\lambda_q^F > 0$  in the OLG model.

### 3.2.3 Income Risk and Exchange Rates

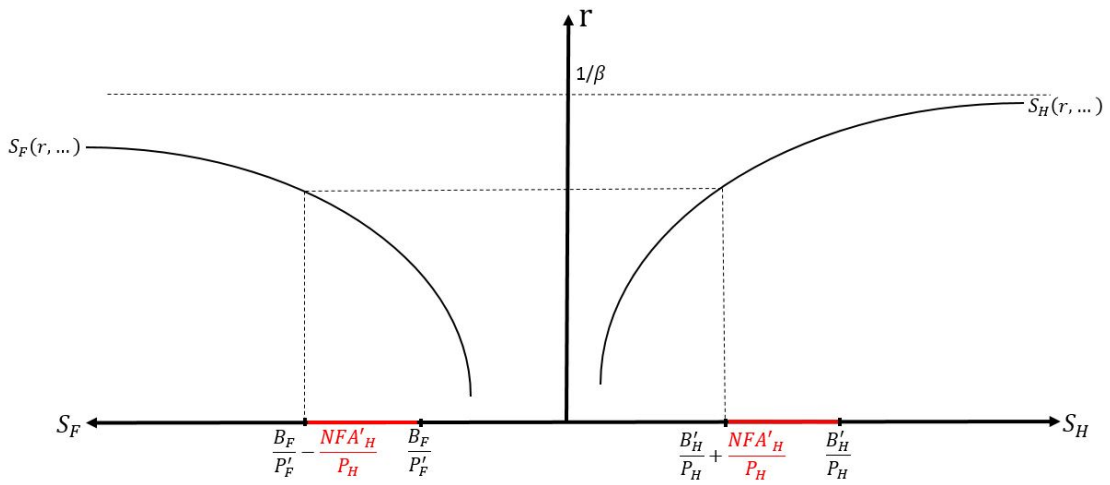
A country's NFA position is the result of households' equilibrium responses to the risks they face. The experiments so far kept the risk constant and as a result households' portfolios basically did not change and a country's portfolio moved only if fiscal policy intervenes in international bond markets. If however the risks households face change, households will re-optimize their portfolios and not only how much they save. For example, increasing the standard deviation of old age income by 0.01 in the OLG model, leads to a 4.9% increase in foreign holdings of home bonds and market clearing implies a 4.9% decrease in home holdings of home bonds. Figure 10 illustrates the implications of this portfolio rebalancing for the exchange rate. The decrease in home's NFA position has two main effects. First, it decreases the total amount of assets held by home households to  $B_H + NFA'_H$  and accordingly increases the total asset holdings of foreign households as indicated on the horizontal axis in panel a) of Figure 10. Second, it decreases home households' wealth, through higher interest rate payments on NFAs, shifting the asset demand curve inwards. Symmetrically, foreign households experience a positive wealth effect, shifting their asset demand curve outwards, and their total asset holdings increase. Since the shift in asset demand is due to the shift in interest payments, it is quantitatively small relative to the change in total assets. Asset market clearing therefore requires that the foreign price level increases and that the home price level falls implying an appreciation of the home currency. Indeed, foreign prices increase and home prices fall in the OLG model such that the exchange rate appreciates by 18.8%.



(a) Pre Expansion



(b) Home Country Absorption of  $B'_H > B_H$



(c) Foreign Country Absorption of  $B'_H > B_H$

Figure 7: Home Asset Supply  $B_H$  and Nominal Exchange Rates

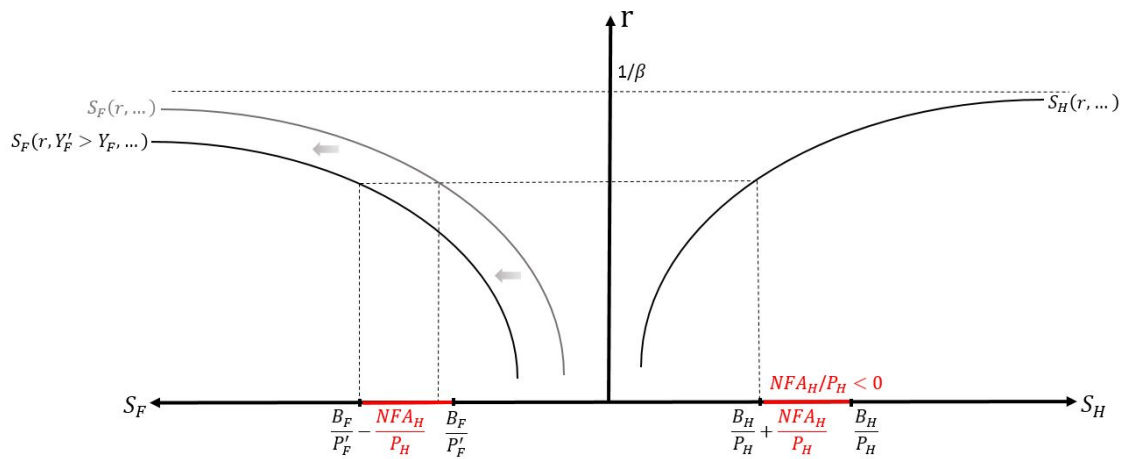


Figure 8: Output Expansion in Foreign Country

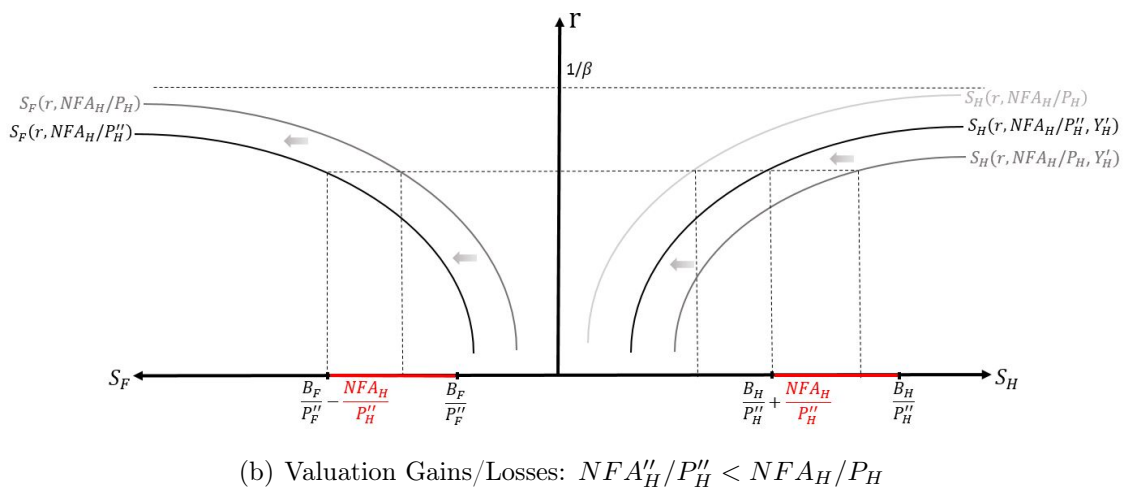
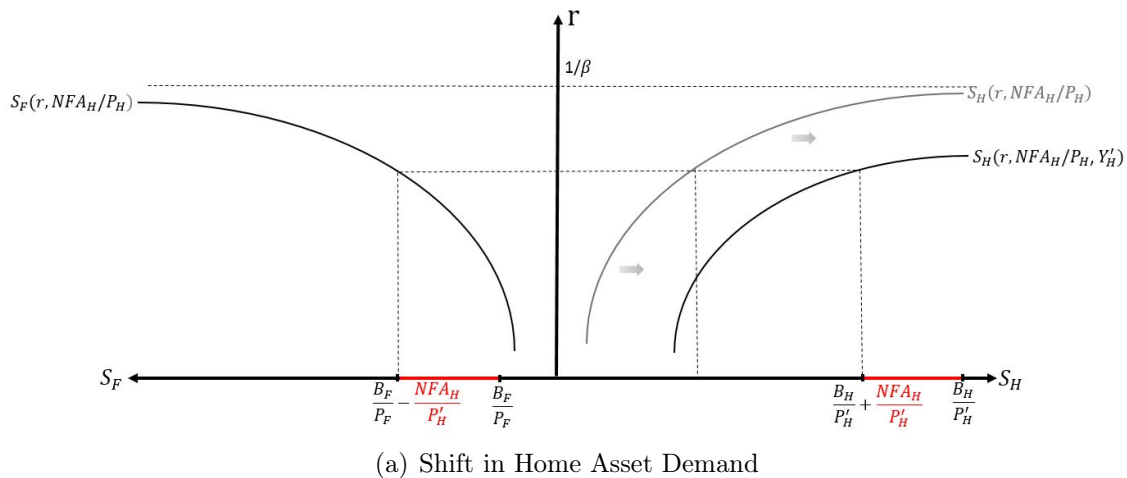
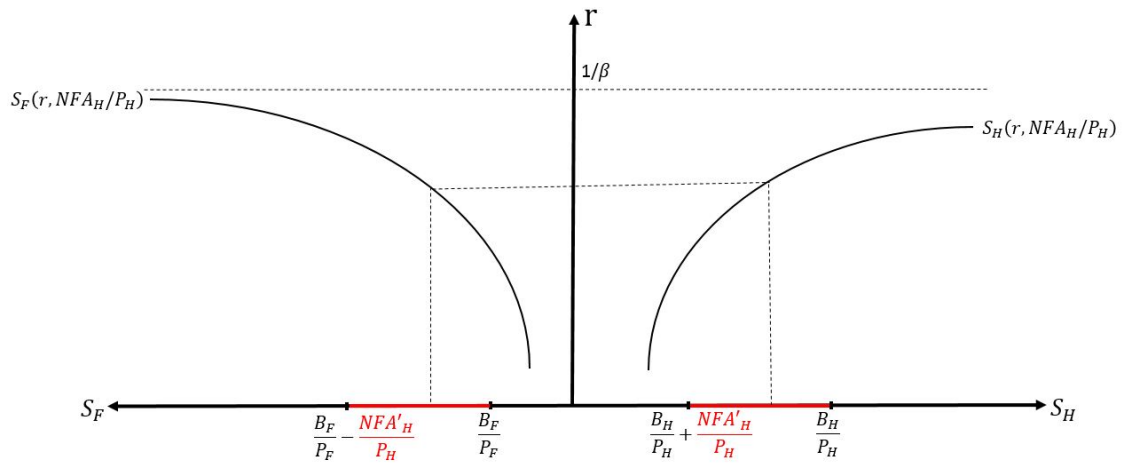
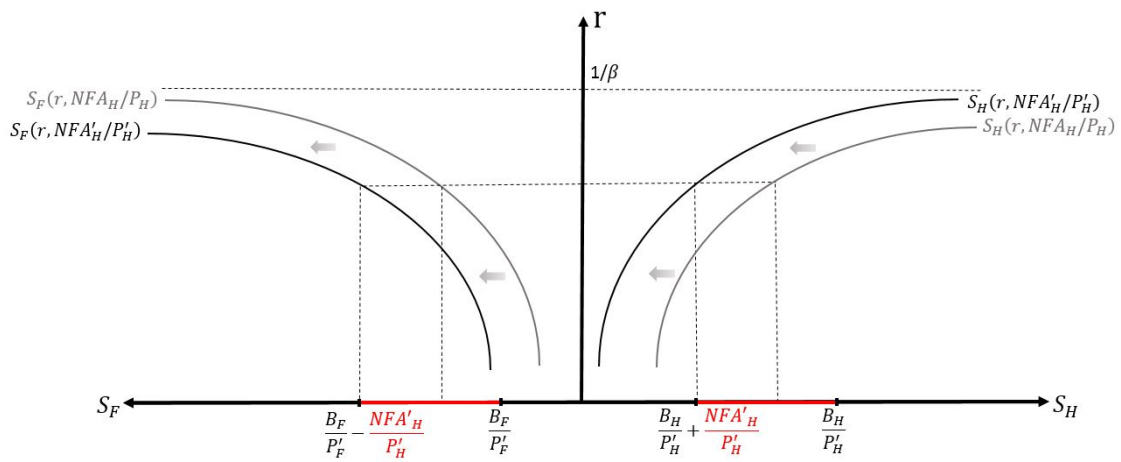


Figure 9: Output Expansion in Home Country





(a) Shift in Total Assets



(b) Wealth Effects: Shift in Asset Demand

Figure 10: Portfolio Adjustment

## 4 Implications and Concluding Remarks

A key element of the previous analysis is that both the exchange rate and portfolios are equilibrium objects which means that they only change if at least one of its potentially many determinants changes, implying that measuring exogenous shocks to the exchange rate in the data is likely to measure a combination of changes of its determinants. This paper provides a theory where both the nominal exchange rate and portfolio choices are jointly determinate and in addition is informative on its determinants.

In this Section I discuss several implications of this theory for several questions: How does a sudden asset outflow affect the exchange rate? How does an increase in savings demand in the rest of the world affect asset flows and the exchange rate? Can a country divorce itself from such global financial flows? And more generally, how can a country manage its exchange rate, for example engineer a depreciation? Finally, I argue that exchange rate determinacy transforms the open macroeconomics policy trilemma into a tetralemma: A country with a fixed exchange rate and free capital mobility loses both monetary and fiscal policy independence. In a monetary union, this tetralemma requires fiscal policy coordination.<sup>21</sup>

### Exchange rates and Asset outflows/inflows

How does a sudden asset outflow affect the exchange rate? The model can speak to this question although a final answer presumably requires to distinguish between bonds and capital what this paper does not. Although asset holdings are also endogenous in the model, it is still instructive to assume that the rest of the world (ROW) pulls out assets, that is I consider first a thought experiment where NFAs move exogenously, for example because of an intervention by a foreign central bank. What this means is that the ROW sells assets to the home country such that assets held by the home investors,  $A_H$ , increase. Foreign investors asset holdings  $A_F$  are unaffected. What matters for the price and exchange rate determination is the ratio of total nominal assets held by investors of a country relative to

---

<sup>21</sup>The tetralemma argues that both fiscal and monetary policy lose independence if exchange rates are fixed but does not restrict policies if exchange rates are freely floating. Rey (2015) considers this latter floating exchange rate scenario and argues that monetary policy is not independent even in a world with floating exchange rates and free capital mobility invalidating the trilemma and leading to a dilemma. At first sight the tetralemma and the dilemma might look contradicting each other but they are actually consistent since the first considers policy for fixed exchange rates and the latter for flexible exchange rates.

their real asset demand, as the simplified version of equation (115) shows

$$\epsilon = \frac{A_H S_F}{S_H A_F}, \quad (127)$$

where  $A_H = B_H + NFA_H$ , denominated in home currency, and  $A_F = B_F - NFA_H/\epsilon$ , denominated in foreign currency. The increase in home held assets (an increase in  $NFA_H$ ) and their unchanged asset demand implies an increase in the home price level and thus a depreciation. Vice versa, an asset inflow to the home country, that is the ROW buys home assets (a reduction of  $NFA_H$ ), leads to an appreciation of the home currency. An asset market based intuition can be grasped from the Metzler diagram, introduced above. Indeed, the same experiment of an exogenous change in  $NFA_H$  was considered in Section 3.2.3 using Figure 10 and I explained why a decrease in  $NFA_H$  leads to an appreciation of the home currency. Note that this result holds only if the total number of assets increases as well. This is where the thought experiment of exogeneity in asset flows is relevant as it rules out rebalancing the portfolio or changes in the amount saved.

These implications are consistent with the basic Mundell Fleming model as well as more modern extensions of it. Both in this paper and in Mundell Fleming, it is important to remember the absence of capital when assessing the empirical validity of model predictions. There is however a key difference between my model and Mundell Fleming, a difference which motivates this paper. In this paper the exchange rate is determined as clearing the world asset and goods market whereas in the textbook Mundell Fleming model one has to fix expected future exchange rates to some arbitrary value.

An endogenous driver of international asset flows is changes in the demand for liquidity induced by an increase in risk. If in response the ROW increases their precautionary savings through accumulating more home bonds, the model predicts an appreciation of the home currency. In the OLG model this is the case, as an increase of old age income uncertainty by 0.01 leads to an 18.8% appreciation of the home country (a 23.1% depreciation of the foreign country) and an 4.9% percentage point increase in home assets held abroad. The model therefore suggests that higher precautionary savings for example (and realistically) in developing countries and larger world risk is disproportionately absorbed by the US - maybe due to the depth of US financial markets or the US dollar being the leading reserve currency - and the model predicts that these capital flows lead to an appreciation of the US dollar.

Before I turn to discuss how policy can respond to changes in financial flows to or from abroad and consider a more general exchange rate management in my framework, I briefly

relate to an older literature prominent in the 1970s and 1980s - portfolio balance models - which was also partially motivated by the aim to better understand how policy can affect the exchange rate.

### Portfolio Balance Models and Exchange Rates

Portfolio Balance Models start from “postulated”, that is not micro-founded, asset demand functions which in equilibrium must adjust to equal the available supplies of these assets (see Branson and Henderson (1985) for a survey). This older literature and my paper take similar approaches to open economy macroeconomics. In both approaches there are asset demand functions, that is mappings from prices to portfolio choices. And in both approaches prices are determined to ensure asset market clearing for each asset. This paper’s approach however overcomes a main criticism brought up against portfolio balance models, which is particularly relevant for discussing policy interventions. Since asset demand is only postulated and not micro-founded, the Lucas (1976) critique applies.

For example, Backus and Kehoe (1989) show that for some class of sterilized foreign exchange interventions, micro-founded models imply the irrelevance of this policy option, that is equilibrium prices and quantities are unaffected. It is effective only if combined with changes in monetary and/or fiscal policy. This paper shares many of these conclusions but differs in one important aspect. Micro-founded models where monetary policy controls the nominal interest rate and not money supply suffer from the indeterminacy problems pointed out in Sargent and Wallace (1975) and Kareken and Wallace (1981). This implies, as I will explain below, that, in contrast to Backus and Kehoe (1989), increasing the supply of market traded bonds of one country while at the same time decreasing the supply of the other country by the same amount has an effect on the nominal exchange rate. Real quantities, including real bond holdings/and portfolios, are however unaffected in both papers. The difference to Backus and Kehoe (1989) which explains this is that here monetary policy sets the nominal interest and not money supply and that the supply of bonds and not of the amount of money matters for the price level. Changing bond supplies therefore changes the price level.

The portfolio approach to open economy macroeconomics responded and provided models with micro-founded asset and portfolio choices. Whereas many assumed an exogenous process for prices, others built on Lucas (1978) and Cox et al. (1985a,b) and derived asset prices in general equilibrium. These are however complete market models which as I argued above do not overcome the indeterminacy problem since Ricardian equivalence holds and the steady-state aggregate demand for government bonds is degenerated. Determining

prices and the exchange rate then requires to deviate from the consensus view that monetary policy sets nominal interest rates and instead assume that central banks set money supply.

Divorcing from global financial flows As explained above, an inflow of assets into the economy leads to an appreciation whereas an outflow leads to a depreciation. A policy maker who is concerned about appreciations and would like to avoid them, has to deal with the inflow of assets which caused the appreciation. The model framework in this paper suggests which policy measures are effective in neutralizing the asset inflow and thus the associated appreciation. Which policy measure is effective depends however on the reason of the asset flows and the appreciation. If it is due to an intervention by a foreign central bank, simple sterilizations can be used. In case of a mere central bank balance sheet operation - the foreign central bank buys assets from the home central bank - the home central bank can just undo the trade. If the foreign central bank buys from the private sector, a different type of measure is necessary since the supply of traded bonds shrinks. The home central bank can either sell the same amount of home bonds to the private investors that the foreign central bank bought. Or if the the foreign central bank bought  $x\%$  of traded home assets, the home central bank can buy  $x\%$  of traded foreign assets. In both cases the exchange rate, the real net foreign asset position and the real amount of assets held by home and ROW investors are unchanged, but in the first case prices are also unchanged whereas in the latter case the price level falls by  $x\%$ . In this case the policy response is easy since the foreign central bank changes the relative supply of home and foreign bonds and the home central bank just reestablishes the old level. Things get more complicated if the asset flows and the exchange rate movements are triggered by changing fundamentals and not by policy. Now, the exchange rate moves not because the relative supply of assets changes but as a result of investors' optimal response to the change in fundamentals. The central bank cannot simply undo the asset flows and moreover has to take into account that investors respond to any policy change. A simpler strategy for the home country, if the objective is to only stabilize the exchange rate, is then to issue more government debt to match the increase in demand for this asset. Whereas issuing the right amount of debt can fix the exchange rate, the net foreign asset position changes. In the example above where an increase of old age income uncertainty by 0.01 leads to an 18.8% appreciation of the home country (a 23.1% depreciation of the foreign country) and an 4.9% percentage point increase in home assets held abroad, stabilizing the exchange rate for the home country requires to increase their bond supply by 23.1%. For the foreign country it would require to contract their bond supply by 18.8%. Another possibility to stabilize the exchange rate would be if both countries coor-

dinate and for example the home country increases bond supply by 10.4% and the foreign country decreases it by 10.4%. All these interventions undo the exchange rate movement but cannot undo the portfolio adjustment. The foreign country always holds 54.9% of home assets, 4.9% percentage points more than before the change in fundamentals.<sup>22</sup>

This reasoning suggest that a larger savings demand by the ROW for US bonds can be accommodated without any effects on US prices or exchange rates. However, if the ROW's savings demand permanently increases at a faster rate than US output, the US debt/gdp ratio would eventually explode. Since the US fiscal capacity is bounded and the default probability on US bonds would become non-negligible at such high debt levels and render US bonds not safe anymore, this debt-issuing policy would not be feasible. The US would have to accept (permanently) falling prices and a (permanent) appreciation of its currency, a flexible exchange post Bretton Woods version of Triffin's dilemma. Or the ROW diverts its savings to other currencies - the Euros or the Yuan - provided those are considered safe.

Managing the Exchange rate The theory is explicit about what policy can do, which instruments it can use and how to use them to induce changes in the exchange rate. These policy experiments are well defined since the exchange rate is an endogenous variable at all horizons (in the short-run, medium-run and long-run) without any exogenously imposed restrictions. If policy aims for a change in the exchange rate, it needs to change the amount of debt (the fiscal policy channel) or the amount of foreign assets (FX channel) or interest rates (monetary policy channel). A desired depreciation requires to either conduct an expansionary fiscal policy (increase debt), o buy foreign assets or to loosen monetary policy (lower nominal interest rates), which all stimulate home demand relative to foreign demand and lead to a depreciation. Vice versa an appreciation requires to either conduct a contractionary fiscal policy (decrease debt), to tighten monetary policy (increase nominal interest rates) or to sell foreign assets, which all depress home demand relative to foreign demand and lead to an appreciation. All three channels were shown to work in the simple OLG model and the first two channels were also illustrated using the Metzler diagram (Figures ?? and 7).

Tetralemma and Monetary Unions The classic policy trilemma in international economics is that at most two out of the following three policies are simultaneously feasible: (i) unrestricted capital mobility; (ii) setting nominal interest rate independently (monetary policy

---

<sup>22</sup>Home investors buy 45.1% and foreign investors buy 54.9% of the home bond supply increase by 23.1%. Home nominal asset holdings  $A_H$  increase by 23.1% with an unchanged real value. Foreign nominal asset holdings  $A_F$  are unchanged and so is their real value. Nominal  $A_F$  is unchanged since it is denominated in foreign currency and the 23.1% increase in home asset holdings is exactly offset by the 23.1% depreciation, so that  $NFA_F = -NFA_H/\epsilon$  is constant. As a result  $A_H/A_F$  increases by 23.1%, the depreciation rate in (127).

independence); and (iii) a fixed exchange rate. The underlying logic is quite simple. Free asset flows imply that the uncovered interest rate parity holds such that a fixed exchange rate regime requires to set the domestic nominal interest rate equal to the ROW nominal interest rate.

The interest rate parity condition when the future exchange rate  $\epsilon_{t+1}$  is known at time  $t$ ,

$$\epsilon_t = \frac{1 + i_{t+1}^F}{1 + i_{t+1}^H} \epsilon_{t+1}, \quad (128)$$

show this logic. If the exchange rate is constant,  $\frac{\epsilon_{t+1}}{\epsilon_t} = 1$ , the interest rate parity condition simplifies further and implies that  $(1 + i^H) = (1 + i^F)$ .

However, giving up an independent monetary policy is necessary but not sufficient to stabilize the level of the exchange rate. The reason is that the above logic neglects two aspects. First, the above argument only shows that the exchange rate is constant,  $\epsilon_t = \epsilon_{t+1}$  if  $i_{t+1}^F = i_{t+1}^H$ , and thus equal to the long-run exchange rate  $\epsilon_{ss}$ . It leaves out what the level of the exchange rate is. A complete argument would require to fix for example  $\epsilon_{ss}$  at some constant level such that all previous exchange rates are then equal to this constant as well if  $i_s^F = i_s^H$  for all  $s$ . As a result the exchange rate is fixed through monetary policy only. However, this argument is based on an unjustified fixing of  $\epsilon_{ss}$  to some constant. To see that this is unjustified, consider again an unanticipated permanent increase in output in one country. This will lead to an appreciation of the long-run exchange rate  $\epsilon_{ss}$  and thus using the same interest rate parity condition to an appreciation of the same magnitude of the exchange rate in period  $t$ . The exchange rate is not constant although at the same time  $i_s^F = i_s^H$  for all  $s$  and the interest rate parity condition holds, simply because  $\epsilon$  shifts by the same amount in all periods.

The above argument also neglects the presence of risk, for example output shocks hitting the economy. These shocks move the current exchange rate  $\epsilon_t$  without necessarily moving  $\epsilon_{t+1}$  by the same magnitude. The OLG model provides an example. The nominal interest rate in both the home and the foreign countries are constant and equal but as Figure 2 shows the exchange rate is quite volatile and responds to unanticipated output shocks. The above interest rate parity condition logic does not hold in the presence of risks as a covariance term needs to be added. For example, a period  $t$  positive output shock leads to a fall of  $\epsilon_t$  (a period  $t$  appreciation) and to an expected increase in period  $t + 1$ ,  $E_t \epsilon_{t+1} > \epsilon_t$ . Using (128) would suggest that the home investor should invest more in the foreign bond and less in the home bond. As panel (b) of Figure 2 shows, this is what happens. But whereas the model

without risk suggests to short sell home bonds, the model with risk takes into account that the portfolio shift towards more foreign bonds changes the risk properties of future old age consumption. Home bonds become less risky, the covariance increases (panel c), and foreign bonds become more risky, the covariance decreases (panel d). As a result, the portfolio weight of foreign bonds increases but to a number less than one, and the portfolio weight of home bonds falls but not below zero. The expected return of the portfolio increases since  $E_t \epsilon_{t+1} > \epsilon_t$ , but so does its riskiness.

This suggest that a country faces a tetralemma. Unrestricted capital mobility and a fixed exchange rate imply that a country loses both monetary and fiscal policy independence, or more generally loses its ability to manage aggregate domestic demand. An exchange rate peg requires fiscal policy to absorb shocks hitting the economy to stabilize the exchange rate while home monetary policy perfectly tracks foreign monetary policy. In a world with cooperation between the two countries, each country uses fiscal policy to eliminate the impact of shocks on households in their country. For example in response to a positive output shock, taxes are increased to fully absorb the output increase, and vice versa in response to a negative output shock, taxes are decreased to undo the output drop. The tax revenue in- and decreases can be thought of as being one-to-one used for government spending from which households derive no utility (or the utility is separable in private and government consumption). This coordinated policy fixes the exchange rate. If on the other hand foreign policy does not respond, domestic policy can by itself fix the exchange rate but has to operate at a larger scale. Home policy has to set taxes such that home households' income changes by the same amount as does foreign households' income. For example if the home country experiences an output increase and the foreign country an output decrease, then taxes have to be increased such that home income falls as much as does foreign income. Although the price levels in both countries are not constant, home fiscal policy replicating the foreign country implies that the home price level now moves one-to-one with the foreign price level such that the exchange rate is constant.

The implications for a monetary union, where capital can freely move and the nominal exchange rate is fixed, are quite unpleasant. Not only do union-member countries have to give up monetary policy but they also lose an independent fiscal policy. Not implementing the fiscal policy necessary to stabilize the counterfactual nominal exchange rate will in a monetary union lead to a change in the real exchange rate. For example a capital inflow, say into Spain, would require a contractionary fiscal policy in Spain or an expansionary fiscal policy in the rest of the Euro area. If instead Spanish fiscal policy is unchanged or even becomes



expansionary, this inevitably leads to a real appreciation with the likely effects on exports, imports and output. This suggests a new perspective on the fiscal dimension of a monetary union: Fiscal policy coordination to respond to the capital flows which cause exchange rate movements or more precisely would have caused changes of the nominal exchange rate if it was flexible.

## Concluding Remarks

This paper proposes a new equilibrium theory where nominal and real exchange rates and international portfolio choices are jointly determined. I use the model to discuss what the answers to several questions motivating a large literature in open economy macroeconomics might be. How does a sudden asset outflow affect the exchange rate? How does an increase in savings demand in the rest of the world affect asset flows and the exchange rate? Can a country divorce itself from such global financial flows? And more generally, how can a country manage its exchange rate, for example engineer a depreciation? Finally, I argue that exchange rate determinacy transforms the open macroeconomics policy trilemma into a tetralemma: A country with a fixed exchange rate and free capital mobility loses both monetary and fiscal policy independence. In a monetary union, this tetralemma requires fiscal policy coordination.

A full answer to these questions certainly requires to move to a quantitative analysis and enrich the model. For example, adding capital, although irrelevant for determinacy, allows to obtain a full picture of a country's capital account which is in particular relevant for the US, the "Venture Capitalist of the World", which can be roughly described as issuing debt liabilities and investing in physical capital (equity and direct investment) abroad (Gourinchas and Rey (2007b,a)). Although these models are much richer than the simple one in this paper, it is important to notice that the same mechanism to determine the exchange rate is operating in the simple model and in more richer models. It is the mechanism proposed in this paper which enables the researcher to quantitatively and simultaneously account for the observed fall in US interest rates, the flow of capital and assets in and out of the US, the large current account US deficit and the evolution of exchange rates within a coherent equilibrium model. The mechanism also allows to consider different theories of "global imbalances" within a consistent framework. One theory put forth in Caballero et al. (2008) is that different regions of the world differ in their capacity to generate financial assets from real investments. Another explanation focuses on exchange rates and argues that emerging countries, mainly in

Asia, have undervalued exchange rates, impose capital controls and accumulate reserve asset claims on the US (Dooley et al. (2003, 2014)). A joint assessment of these theories requires a model with a determinate equilibrium exchange rate; this is what this paper provides. This paper also enables to study spillovers of foreign fiscal and monetary policy as well as of foreign shocks and a foreign liquidity trap on the home macroeconomy. A key aspect when studying such policy or shock spillovers is the potential absorbing role of exchange rate adjustments, which requires a theory how the exchange rate is determined.

## Bibliography

- BACKUS, D. K. AND P. J. KEHOE (1989): “On the denomination of government debt: A critique of the portfolio balance approach,” *Journal of Monetary Economics*, 23, 359 – 376.
- BRANSON, W. H. AND D. W. HENDERSON (1985): “Chapter 15 The specification and influence of asset markets,” Elsevier, vol. 2 of *Handbook of International Economics*, 749 – 805.
- CABALLERO, R. J., E. FARHI, AND P.-O. GOURINCHAS (2008): “An Equilibrium Model of "Global Imbalances" and Low Interest Rates,” *American Economic Review*, 98, 358–93.
- CAVALLO, M. AND F. GHIRONI (2002): “Net foreign assets and the exchange rate: Redux revived,” *Journal of Monetary Economics*, 49, 1057 – 1097.
- CLARIDA, R. H. (1990): “International Lending and Borrowing in a Stochastic, Stationary Equilibrium,” *International Economic Review*, 31, 543–558.
- COEURDACIER, N., T. OCAKTAN, AND H. REY (2011): “Limited Participation and International Risk Sharing,” mimeo.
- COEURDACIER, N. AND H. REY (2013): “Home Bias in Open Economy Financial Macroeconomics,” *Journal of Economic Literature*, 51, 63–115.
- CORSETTI, G., L. DEDOLA, AND S. LEDUC (2008): “International Risk Sharing and the Transmission of Productivity Shocks,” *The Review of Economic Studies*, 75, 443–473.
- COX, J. C., J. E. INGERSOLL, AND S. A. ROSS (1985a): “An Intertemporal General Equilibrium Model of Asset Prices,” *Econometrica*, 53, 363–384.

- (1985b): “A Theory of the Term Structure of Interest Rates,” *Econometrica*, 53, 385–407.
- DEVEREUX, M. B. AND A. SUTHERLAND (2010): “Valuation effects and the dynamics of net external assets,” *Journal of International Economics*, 80, 129 – 143, special Issue: {JIE} Special Issue on International Macro-Finance.
- (2011): “Country Portfolios in Open Economy Macro-Models,” *Journal of the European Economic Association*, 9, 337–369.
- DOOLEY, M. P., D. FOLKERTS-LANDAU, AND P. GARBER (2003): “An Essay on the Revived Bretton Woods System,” Working Paper 9971, National Bureau of Economic Research.
- DOOLEY, M. P., D. FOLKERTS-LANDAU, AND P. M. GARBER (2014): “The Revived Bretton Woods System’s First Decade,” Working Paper 20454, National Bureau of Economic Research.
- GABAIX, X. AND M. MAGGIORI (2015): “International Liquidity and Exchange Rate Dynamics,” *Quarterly Journal of Economics*, 130, 1369–1420.
- GHIRONI, F. (2008): “The role of net foreign assets in a New Keynesian small open economy model,” *Journal of Economic Dynamics and Control*, 32, 1780 – 1811.
- GHIRONI, F., J. LEE, AND A. REBUCCI (2015): “The valuation channel of external adjustment,” *Journal of International Money and Finance*, 57, 86 – 114.
- GOURINCHAS, P.-O. AND H. REY (2007a): *From World Banker to World Venture Capitalist: U.S. External Adjustment and the Exorbitant Privilege*, University of Chicago Press, 11–66.
- (2007b): “International Financial Adjustment,” *Journal of Political Economy*, 115, 665–703.
- JUDD, K. L. AND S.-M. GUU (2001): “Asymptotic methods for asset market equilibrium analysis,” *Economic Theory*, 18, 127–157.
- KAREKEN, J. AND N. WALLACE (1981): “On the Indeterminacy of Equilibrium Exchange Rates,” *The Quarterly Journal of Economics*, 96, 207–222.

- KOLLMANN, R. (2006a): “A Dynamic Equilibrium Model of International Portfolio Holdings: Comment,” *Econometrica*, 74, 269–273.
- (2006b): “International Portfolio Equilibrium and the Current Account,” Working paper, London, Centre for Economic Policy Research.
- (2012): “Limited asset market participation and the consumption-real exchange rate anomaly,” *The Canadian Journal of Economics / Revue canadienne d’Economie*, 45, 566–584.
- LANE, P. R. AND G. M. MILESI-FERRETTI (2001): “The external wealth of nations: measures of foreign assets and liabilities for industrial and developing countries,” *Journal of International Economics*, 55, 263 – 294.
- (2007): “The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970-2004,” *Journal of International Economics*, 73, 223 – 250.
- LUCAS, R. E. (1976): “Econometric policy evaluation: A critique,” *Carnegie-Rochester Conference Series on Public Policy*, 1, 19 – 46.
- LUCAS, R. J. (1978): “Asset Prices in an Exchange Economy,” *Econometrica*, 46, 1426–1445.
- MENDOZA, E. G., V. QUADRINI, AND J.-V. RIOS-RULL (2009): “Financial Integration, Financial Development, and Global Imbalances,” *Journal of Political Economy*, 117, 371–416.
- PAVLOVA, A. AND R. RIGOBON (2012): “Equilibrium Portfolios and External Adjustment Under Incomplete Markets,” Ifa working paper.
- REY, H. (2015): “Dilemma not trilemma: the global financial cycle and monetary policy independence,” Tech. rep., National Bureau of Economic Research.
- SARGENT, T. J. AND N. WALLACE (1975): “Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule,” *Journal of Political Economy*, 83, 241–254.
- TILLE, C. (2003): “The Impact of Exchange Rate Movements on U.S. Foreign Debt,” *Federal Reserve Bank of New York Current Issues in Economics and Finance* 9, 9.

——— (2008): “Financial integration and the wealth effect of exchange rate fluctuations,” *Journal of International Economics*, 75, 283 – 294.

WILLEN, P. S. (2004): “Incomplete Markets and Trade,” Research department working papers no. 04-8, Federal Reserve Bank of Boston.

WOODFORD, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton: Princeton University Press.

## APPENDIX

### A.1 Derivations

#### Derivation of Result 3 [Diversified Portfolio]

Result 3 claims that home and foreign investors hold positive amounts of each asset:

$$0 < A_H^H, A_F^H < B_H; \quad 0 < A_F^F, A_H^F < B_F.$$

Suppose not and that instead investors follow an autarky strategy and hold their own bonds only,  $A_H^H = B_H, A_H^F = 0$  and  $A_F^F = B_F, A_F^H = 0$  and that  $\lambda_q^H = \lambda_q^F = 0$ . The first-order condition for home investors in home bonds would then be

$$\bar{q}^H = \bar{m}^H + s\hat{m}^H(1 + i^H)\bar{q}^H,$$

which is a contradiction since  $\kappa_1 \neq 1$  implies  $\hat{m}^H \neq 0$  and thus this condition cannot hold for all  $s$ . Thus  $\lambda_q^H = \lambda_q^F \neq 0$  and it is beneficial for home investors to also buy foreign bonds for  $|s|$  large enough where the price of home bonds,  $\bar{q}^H - \lambda_q^H s$ , is sufficiently higher than the price of foreign bonds,  $\bar{q}^F - \lambda_q^F s$ .

#### Derivation of Result 5 [Presence of Risk]: Asymmetric Case

##### Portfolio Volatility:

$$\bar{\Sigma}_H + s\hat{\Sigma}_H = \frac{\bar{q}^H \bar{q}^F \chi(1 + i)}{(1 - \chi)\bar{q}^H + \bar{q}^F} - s \frac{4\lambda_q(1 - \chi)(1 + i)}{\lambda_q \gamma \sigma^2 (2 - \chi)^2} \neq 0 \quad (\text{A1})$$

##### SDF:

$$\bar{m}^H + s\hat{m}^H = \frac{1}{1 + i} \frac{2 - \chi}{2(1 - \chi)} - s \frac{\lambda_q \chi}{2} \bar{q}(1 - \chi)(1 + i) \neq \frac{1}{1 + i} \quad (\text{A2})$$

**Exchange Rate Volatility:**

$$Var[\bar{\epsilon} + s\hat{\epsilon}] = \sigma^2 \left(2\frac{\lambda_q}{\bar{q}}\right)^2 > 0 \quad (A3)$$