Optimal Cooperative and Non-Cooperative Unconventional Monetary Policy Under Commitment

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I study optimal unconventional monetary policy under commitment in a two-country model. Financial intermediaries face occasionally binding financial constraints, which have two important implications. First, central banks should exit from their policy more slowly than the speed of deleveraging in financial sectors. This leads to positive balance sheets of central banks after a crisis. Second, policy commitment induces financial intermediaries to take too much risk in tranquil times, making the economy vulnerable to a financial crisis. On the international dimension, domestic policy entails larger positive spillovers to the foreign country due to financial integration. Cross-country policy cooperation benefits the global economy to the extent determined by the cost of interventions. In noncooperative equilibrium, interventions tend to be too strong in one country but too weak in the other. Finally, I find the simple rule that characterises the Ramsey policy responds to asset price gaps.
JEL: E59, F41, F42

I. Introduction

The recent financial crisis involved a significant disruption to financial intermediation, as evidenced by limited access to credit (e.g. Ivashina and Scharfstein, 2010) and high credit spreads. Such a disruption propagated internationally via integrated financial markets. To stabilize the financial system, fiscal and monetary authorities in major economies introduced the so-called unconventional monetary policy (UMP)[1]. This type of policy included the provision of large-scale liquidity and resulted in balance sheets of some central banks expanding 20 to 30 per cent of GDP. In the last ten years, various UMPs have been employed, a good summary of which

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[1] These policies may not be strictly monetary. For example, Kollmann et al. (2013) consider government support for banks as a fiscal policy. I use the terms unconventional policy and unconventional monetary policy interchangeably.

can be found in Borio and Zabai (2016). By employing UMP, policymakers hope to reduce long-term interest rates, boost lending, and stimulate real activity[2]. In the meantime, UMP may restore the functioning of financial markets on which the transmission mechanism of the conventional monetary policy depends (Altavilla, Canova and Ciccarelli, 2016). As a probably unintended consequence, domestic interventions in the markets of intentionally traded assets also affect financial conditions in foreign countries.

In this paper, I study optimal unconventional policy as large-scale liquidity provision under commitment (Ramsey policy) with and without cross-country policy cooperation. Three issues are addressed. First, how optimal policy responds to country-specific shocks, which may trigger a global financial crisis. Then, how central banks exit from their policy. Since UMP may not be permanent, the exit problem is particularly interesting a decade after the recent crisis. For instance, the Federal Reserve has been shrinking its balance sheet since September 2017. Bernanke (2017) argues from a policy communication perspective that this should be done in a passive and predictable way. In 2018, the European Central Bank has also announced its exit plan. Second, if policy cooperation matters and to what extent it matters. Third, how to conduct approximately the Ramsey policy using a simple feedback rule.

In answering these questions, I adopt a simple two-country model in which each country contains a stylized multinational banking sector similar to that in Gertler and Kiyotaki (2010). Banks face balance sheet constraints (financial constraints) derived from an agency problem between banks and their depositors. The constraints are slack in normal times but bind endogenously in periods of financial distress, which constitutes the systemic risk in this model. Given their high leverage, banks are vulnerable to both the shocks that negatively affect their asset value and the shocks that tighten the balance sheet constraints directly. When the constraints are binding, banks have difficulties of rolling over their short-term debts, which leads to a collapse in asset prices and investment. Though simple, this model allows two channels through which unconventional policy affects financial intermediation. First, there is a capital gain channel meaning that the provision of liquidity pushes up asset prices (reducing long-term interest rates). Banks holding these assets or substitutes enjoy an improved balance sheet condition. Second, there is an expectation channel meaning that the commitment to use unconventional policy when needed anchors expectations about future asset prices. This induces banks to take higher leverage and lend more in normal times. Furthermore, if the assets in question are traded internationally[3], the policy has large positive cross-country spillover

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[2]It is relatively well established that UMP reduces long-term interest rates. See, among many others, Gagnon et al. (2011) and Krishnamurthy and Vissing-Jorgensen (2011) for the Federal Reserve's QE, and Joyce et al. (2011) and Christensen and Rudebusch (2012) for the Bank of England's QE. However, UMP can have insignificant or unintended real effects through a bank lending channel, as shown by Chakraborty, Goldstein and MacKinlay (2017) and Acharya et al. (2017).

[3]Evidence for increasing global banking and financial integration can be found in Devereux and Yetman (2010), Perri and Quadrini (2011), Fillat, Garetto and Götz (2015), and Bank for International Settlements' international banking statistics.

effects through both channels.

My main findings are as follows. When the financial constraints are not binding, unconventional policy is not being used. But its existence increases investment, consumption, and output through the expectation channel. However, the economy takes much of the systemic risk and is pushed to the edge of a global financial crisis. When the economy falls into it, domestic and foreign unconventional policy responds asymmetrically. Domestic policy is more sensitive to domestic shocks. The degree of asymmetry depends on the nature of the shock, the cost of interventions, and banks' portfolios. Central banks should exit from their policy in accordance with banks deleveraging, the speed of which depends on a crowding-out effect of the policy. Moreover, the occasionally binding constraints (OBCs) give central banks a precautionary motivation to exit slowly, and central banks should maintain positive balance sheets even a few periods after the crisis. The policy's spillover effects leave some room for policy cooperation. The cooperation gain turns out to be an increasing function of the intervention cost. In noncooperative equilibrium, interventions are too strong in one country but too weak in the other. At last, in examining several simple rules, I find the rule that characterises the Ramsey policy responds to asset price gaps. However, this rule requires knowledge of asset prices that would be realised in a financially frictionless world.

This paper contributes to the literature that studies the normative perspective of unconventional policy in a multi-country environment. Importantly, I share with Bianchi (2016) the emphasis on the expectation channel in Ramsey equilibrium.[4] In his model, firms need to balance the desire to invest today with the risk of becoming financially constrained in the future. They have an incentive to borrow more, knowing that the more they borrow the larger transfer they can receive from the government in crises. A bailout policy faces the trade-off between the ex-ante overborrowing and the ex-post benefit of a faster recovery from a credit crunch. One important way this paper differ from Bianchi (2016) is that I drop his occasionally binding minimum dividend constraint on firms. This simplification makes it easier to study noncooperative policy. In Bianchi (2016)'s closed-economy model, this second OBC (in addition to the financial one) introduces an externality for labour demand such that the competitive equilibrium is constrained inefficient.[5] However, it is not at the centre of policy trade-offs.

Our two distinctive results, i.e. the expectation channel and the precautionary protection in post-crisis periods, depend on the nonlinearity originate from OBCs.[6] The emphasis on OBCs is in line with Del Negro, Hasegawa and Schorfheide (2016)

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[4]Through the expectation channel, I allow unconventional policy to affect how often financial constraints bind. This is also referred to as a risk-taking channel (effect) in the literature. Note that in the literature, the same name is also given to the policy's effect on how risky banks' portfolios are. See, for example, Tsiaras (2018) in the context of unconventional policy. See Pancost and Robatto (2017), Brunnermeier and Koby (2017), Coimbra and Rey (2017) among others in the context of monetary and macroprudential policy.

[5]The competitive equilibrium in our model is constrained efficient.

[6]To be sure, this paper focusing on occasionally frictional financial markets does not mean that this is the case in the real world. Data on credit spreads suggests that there are other types of frictions, which are abstracted from the model, haunt financial markets all the time. What the model tries to capture is the frictions that occasionally push the economy further into credit crunches.

and Swarbrick, Holden and Levine (2017), who have shown that occasionally binding financial constraints help capture the sudden and discrete nature of financial crises and eliminate the financial acceleration mechanism during normal times. However, for reasons of tractability, much work in the literature focuses on log-linear dynamics around the steady state[7]. Furthermore, the literature has been mostly focusing on simple rules, which can be evaluated against our Ramsy policy. Dedola, Karadi and Lombardo (2013) is most closely related to this paper. They study the international dimension of public asset purchases in a two-country model where the financial constraints are always binding. In studying the policy in response to credit spreads, they find that the lack of policy cooperation reduces policy responses in both countries. This is not the case in the Ramsey equilibrium. My discussion on the exit strategy is linked to Foerster (2015) who also suggests slowly unwinding the central bank's balance sheet. Foerster (2015) lets policy respond to both credit spreads and it lagged self. I find that the main benefit of doing so is keeping expected spreads low at the cost of letting the spreads surge upon the shock. In equilibrium, this rule does not necessarily imply slow unwinding. He and Krishnamurthy (2013) compare three unconventional measures: borrowing subsidies, equity injections, and public asset purchases. Like them, I also find equity injections most efficient.

The rest of the paper is organized as follows. The next section presents the two-country model. After describing my quantitative method in section III, I report the main results of cooperative and noncooperative policy in section IV and V, respectively. I evaluate the performance of simple rules in section VI. The last section concludes.

II. The Model

The model mostly follows Dedola, Karadi and Lombardo (2013), i.e. a two-country real business cycle model of Backus, Kehoe and Kydland (1992) augmented by Gertler and Karadi (2011) style financial frictions. To focus on the channels outlined above, I assume the model is frictionless apart from the financial frictions and all markets are competitive. The world consists of two ex-ante symmetric countries, Home and Foreign. There is one final homogeneous good used for consumption, investment, and trade. In each country, production requires domestic labour and capital. Goods producers borrow from banks to finance their physical investment. Banks receive deposits from households in both countries and lend to goods producers in both countries.[8] The return on banks' lending is state-contingent. I use the term "non-financial sector" to refer to households and goods and capital producers, and the term "financial sector" to refer to banks. I now describe the Home economy. Foreign variables are denoted with "*". Lower case letters denote individual variables, and upper case letters denote aggregate variables.

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[7]Papers such as Del Negro et al. (2017) keep the assumption of always binding financial constraints but emphasise the nonlinearity of the Zero Lower Bound.

[8]I assume that the financial markets for both the banks' assets and liabilities are integrated across countries. As will be clearer shortly, the key assumption is the integration of the asset markets. Separating the liability markets would change the results mildly up to a miss-allocation of households' saving.

A. Households

There is a unit-continuum of infinitely lived households. Households consume final goods, supply labour, and save. The menu of assets available to them includes deposits in domestic banks $d_{h,t}$, deposits in foreign banks $d_{f,t}$, and domestic government bonds b_t. All these assets are risk-free one-period bonds denominated in final goods and carry a gross rate of return r_t or r_t^*. Households also own financial and non-financial firms.

Each household consists of workers and bankers who pool consumption risk perfectly. Workers are hired by goods producers and bring wages to the household. Bankers manage a bank and transfer profits to the household. It is convenient to assume that households do not save in their own banks. Complete consumption insurance allows me to work with a consolidated representative household. The household chooses consumption c_t, labour supply l_t, and end-of-period wealth to maximize its expected present discounted utility:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta_{t,t+j} \left[\frac{(c_{t+j} - hc_{t+j-1})^{1-\sigma}}{1-\sigma} - \chi \frac{l_{t+j}^{1+\varphi}}{1+\varphi} \right],$$

where $h \in [0,1)$ determines the degree of habit, $\sigma > 0$ is the coefficient of relative risk aversion, $\varphi > 0$ is the (inversed) Frisch elasticity of labor supply, $\chi \geq 0$ is the relative disutility weight on labour, and $\beta_{t,t+j} = \prod_{i=1}^{i=j} \beta_{t+i-1,t+i}$ is the subjective discount factor from $t+j$ to t. To induce stationarity of our model in which financial markets are incomplete, the discount factor depends on aggregate consumption relative to aggregate income, $\beta_{t,t+1} = \bar{\beta} + \psi_\beta \log \left(\frac{C_t}{Y_t} \right)$, following Kollmann (2016). Let Π_t be profits of firms and T_t a lump-sum tax, the household faces the budget constraint

$$c_t + d_{h,t} + b_t + d_{f,t} = w_t l_t + \Pi_t + (d_{h,t-1} + b_{t-1}) r_{t-1} + d_{f,t-1} r_{t-1}^* - T_t.$$

The first-order conditions are standard:

$$w_t = \frac{\chi l_t^\varphi}{(c_t - hc_{t-1})^{-\sigma} - \beta_{t,t+1} h (c_{t+1} - hc_t)^{-\sigma}},$$

(1) $$\mathbb{E}_t \left[\Xi_{t,t+1} r_t \right] = 1,$$

(2) $$\mathbb{E}_t \left[\Xi_{t,t+1} r_t^* \right] = 1,$$

where $\Xi_{t,t+1}$ is the stochastic discount factor. (1) and (2) imply that the risk-free rates are equalized across countries.

B. Non-financial firms

There are two types of non-financial firms: capital producers and goods producers.

GOODS PRODUCERS. — Goods producers have a standard Cobb-Douglas technology $y_t = A_t \left(\xi_t k_{t-1}\right)^\alpha l_t^{1-\alpha}$ where α is the capital share, A_t is total factor productivity, and k_t is the capital stock at the end of period t. Let δ be the depreciation rate of capital and ξ_t the exogenous quality of capital, a goods producer acquires additional capital $i_t = k_t - (1-\delta)\xi_t k_{t-1}$ at the price q_t. To finance its physical investment, the goods producer borrows from banks by issuing securities

$$(3) \qquad\qquad q_{t,t}^s \left(s_t - s_{t-1}\right) = i_t q_t,$$

where s_t is the number of securities issued at the end of period t, and $q_{T,t}^s$ is the period T price of securities issued in period t. Each unit of the securities is a state-contingent claim to the future returns from one unit of investment: z_{t+1}, $(1-\delta)\xi_{t+1}z_{t+2}$, $(1-\delta)^2\xi_{t+1}\xi_{t+2}z_{t+3}$, ... with z_t denoting gross profits per unit of capital.

The goods producers solve

$$\max_{\{l_{t+j}, k_{t+j}, s_{t+j}\}_{j=0}^\infty} \mathbb{E}_t \sum_{j=0}^\infty \Xi_{t,t+j}$$
$$\times \left[y_{t+j} - w_{t+j} l_{t+j} - i_{t+j} q_{t+j} + q_{t+j,t+j}^s \left(s_{t+j} - s_{t+j-1}\right) - z_{t+j} s_{t+j-1} \right],$$

subject to (3), the production function, and the capital accumulation equation. Let the multiplier associated with (3) be λ_t^y, the first-order conditions with respect to labour, capital, and securities are

$$w_t = (1-\alpha)\frac{y_t}{l_t},$$

$$(4) \qquad q_t \left(1 + \lambda_t^y\right) = \mathbb{E}_t \Xi_{t,t+1}\left[\frac{\partial y_{t+1}}{\partial k_t} + (1-\delta)\xi_{t+1}q_{t+1}\left(1 + \lambda_{t+1}^y\right)\right],$$

$$(5) \qquad q_t^s \left(1 + \lambda_t^y\right) = \mathbb{E}_t \Xi_{t,t+1}\left[z_{t+1} + q_{t+1}^s \left(1 + \lambda_{t+1}^y\right)\right].$$

(3) or $\lambda_t^y \neq 0$ is an important assumption. Otherwise, firms can borrow directly from households by paying a negative dividend, which makes the banking sector trivial. Using (4) and (5), it is easy to show that the time T price of securities issued at time t is $q_{T,t}^s = q_T \left(1-\delta\right)^T \prod_{j=1}^T \xi_{t+j}$ and $s_t = k_t$, given $s_0 = k_0$. I define the return of holding securities for one period as

$$r_{k,t+1} = \frac{z_{t+1} + (1-\delta)\,\xi_{t+1}q_{t+1}}{q_t},$$

where z_t is obtained from the zero-profit condition

$$z_t = \frac{y_t - w_t l_t}{k_{t-1}} = \alpha \frac{y_t}{k_{t-1}}.$$

CAPITAL PRODUCERS. — Given the demand for new capital i_t and the market price q_t, a capital producer maximises its expected discounted profits:

$$\max_{\{i_{t+j}\}_{j=0}^{\infty}} \mathbb{E}_t \sum_{j=0}^{\infty} \Xi_{t,t+j} \left[q_{t+j}i_{t+j} - f\left(k_{t+j-1}, i_{t+j}\right) \right],$$

where the cost function is given by

$$f\left(\cdot\right) = i_t + \frac{\eta}{2} \left(\frac{i_t}{\delta k_{t-1}} - 1 \right)^2 \delta k_{t-1},$$

and $\eta \geq 0$. The first-order condition pins down the market price of new capital

$$(6) \qquad\qquad q_t = 1 + \eta \left(\frac{i_t}{\delta k_{t-1}} - 1 \right).$$

C.   Banks

A bank is a financial intermediary, which engages in maturity and liquidity transformation. It receives deposits amounting to $d_{h,t}$ and $d_{h,t}^*$ from domestic and foreign households, respectively. It purchases $s_{h,t}$ and $s_{f,t}$ units of securities from domestic and foreign goods producers. The bank's balance sheet is hence

$$(7) \qquad\qquad \omega_t \equiv q_t s_{h,t} + q_t^* s_{f,t} = d_{h,t} + d_{h,t}^* + n_t,$$

where ω_t is the total assets, n_t is the bank's net worth at the beginning of period t given by

$$(8) \qquad n_t \equiv q_{t-1}s_{h,t-1}r_{k,t} + q_{t-1}^* s_{f,t-1}r_{k,t}^* - d_{h,t-1}r_{t-1} - d_{h,t-1}^* r_{t-1}^*.$$

The bank's leverage is defined as

$$\phi_t = \frac{\omega_t}{n_t}.$$

As in Gertler and Karadi (2011), banks shut down with probability $r_{n,t}$ at the end of each period, upon which banks distribute their net worth to households. The notation $r_{n,t}$ follows the suggestion of Swarbrick, Holden and Levine (2017) that the probability of shutting down can be interpreted as an exogenous dividend rate. Then, bankers become workers. In the meantime, a similar number of workers from the same household randomly become new bankers. New bankers receive "start-up" funds from their household as a proportion ϖ of the total assets owned by the incumbent and the central bank.[9] The probability of shutting down could be stochastic. It plays two roles. First, an infinitely lived bank will sooner or later accumulate enough net worth to finance its investment without borrowing from households. In this case, the financial constraint detailed shortly will never bind. Second, it ensures that banks are always "less patient" than households, so funds always flow from households to banks.

The bank chooses its lending and borrowing $(s_{h,t},\ s_{f,t},\ d_{h,t},\ d_{h,t}^*)$ to maximize the expected present value of net worth upon closure

$$
\begin{aligned}
V_t\left(n_t\right) & = \max \mathbb{E}_t \sum_{j=0}^{\infty} r_{n,t+j,t+j}\left(1-r_{n,t,t+j-1}\right)\Xi_{t,t+j+1}\left(n_{t+1+j}\right) \\
& = \max \mathbb{E}_t \Xi_{t,t+1}\left[r_{n,t,t}n_{t+1}+\left(1-r_{n,t,t}\right)V_{t+1}\left(n_{t+1}\right)\right] \\
& = \nu_{n,t}n_t,
\end{aligned}
$$

(9)

where the third equality follows the conjecture that the value function is linear in net worth, $\nu_{n,t}=V_t'\left(n_t\right)$ is a time-varying coefficient to be solved, and $\left(1-r_{n,i,j}\right)$ is the probability that the bank operates until the end of period j conditional on the bank operating at the beginning of period i. The bank faces an incentive constraint (the financial constraint)

$$
(10) \qquad\qquad OBC_t \equiv \nu_{n,t}n_t - \theta_t\omega_t \geq 0
$$

where OBC_t measures the distance of the constraint from its bound, shocks to $\theta_t \in [0,1]$ tightens the constraint directly and is referred to as financial shocks (Perri and Quadrini, 2011; Dedola and Lombardo, 2012; Del Negro et al., 2017). Rearrange (10), it is easy to see that the constraint is an upper bound on leverage, $\frac{\nu_{n,t}}{\theta_t} \geq \phi_t$. The intuition behind this constraint is as follows. Banks are able to declare bankruptcy and exit from the market. Should this happens, the banker diverts to his or her family a proportion θ_t of the total assets. Creditors can reclaim only the remaining. Therefore, creditors are willing to lend to a bank only if the bank has no incentive to default, i.e. (10) not being violated.

For convenience, the lending decisions can be written in terms of the total assets ω_t and the portfolio $\alpha_{p,t}=\frac{q_t^* s_{f,t}}{\omega_t}$. Let the multiplier associated with (10) be $\lambda_t \geq 0$, the necessary conditions of the maximisation include $OBC_t\lambda_t = 0$ and the first-order

[9]This is to ensure that the start-up funds are not affected by the central bank's purchase of assets.

conditions with respect to ω_t and $\alpha_{p,t}$:

$$
(11) \qquad
\begin{aligned}
\mathbb{E}_t \Xi_{t,t+1} \left(r_{n,t,t} + (1 - r_{n,t,t}) \nu_{n,t+1} \right) \left(r_{k,t+1} - r_t \right) &\equiv \nu_{\omega,t} \\
&= \frac{\lambda_t}{1 + \lambda_t} \theta_t
\end{aligned},
$$

$$
(12) \qquad
\begin{aligned}
\mathbb{E}_t \Xi_{t,t+1} \left(r_{n,t,t} + (1 - r_{n,t,t}) \nu_{n,t+1} \right) \left(r_{k,t+1} - r^*_{k,t+1} \right) &\equiv \nu_{\alpha_p,t} \\
&= 0
\end{aligned},
$$

where (11) follows the fact that $\nu_{\alpha_p,t} = 0$ for all t thanks to market integration. The unknown coefficient $\nu_{n,t}$ can be solved using the first-order conditions and the financial constraint:

$$
(13) \qquad \nu_{n,t} = \nu_t \left(\frac{\nu_{\omega,t}}{\theta_t - \nu_{\omega,t}} + 1 \right),
$$

where $\nu_t \equiv \mathbb{E}_t \Xi_{t,t+1} \left(r_{n,t,t} + (1 - r_{n,t,t}) \nu_{n,t+1} \right) r_t$ is defined similarly to $\nu_{\omega,t}$, $\nu_{\alpha_p,t}$. (13) verifies the earlier conjecture that the value function is linear. Given $\nu_{n,t+1} \geq 1$, the term $\Xi_{t,t+1} \left(r_{n,t,t} + (1 - r_{n,t,t}) \nu_{n,t+1} \right)$ suggests that banks are generally less patient than households.

It is worth discussing how the financial constraint affects banks' behaviours. To begin, use (7), (8), and (9) to write the value function as

$$
\nu_{n,t} n_t = \nu_{\omega,t} \omega_t - \nu_{\alpha_p,t} \omega_t \alpha_{p,t} + \nu_t n_t,
$$

where $\nu_{n,t}$, $\nu_{\omega,t}$, $\nu_{\alpha_p,t}$, ν_t can be conveniently interpreted as the expected marginal value of net worth, total assets, portfolio, and deposits, respectively. Consider a benchmark case in which the financial constraint never binds. We Immediately have $\lambda_t = \nu_{\omega,t} = 0$ meaning that having one unit of investment does not raise banks' value at the margin. In addition, (13) becomes $\nu_{n,t} = \nu_t = 1$ meaning that external and internal funds are equally valued.

Next, consider that the constraint binds due to a positive shock on θ_t. Because the current period net worth is exogenous to banks, hitting the bound forces banks to sell their assets. As long as $\eta > 0$ in (6), the fire sale depresses asset prices and further impairs the net worth, a consequence that banks do not internalise. This completes a vicious loop, which is known as the financial acceleration mechanism. In this case, $\lambda_t > 0$ implies $\nu_{\omega,t} > 0$ and $\nu_{n,t} > \nu_t$. Net worth is more valuable than deposits because the former helps relax the financial constraint. There is also a credit spread between the return on securities and the return on deposits. The spread, $\mathbb{E}_t \left(r_{k,t+1} - r_t \right)$, conveniently measures the efficiency of the financial sector.

Finally, consider the case in which the constraint is not binding but expected to bind in the future with some positive probability. Since the constraint binds in some states of the world, it must be true that $\mathbb{E}_t \lambda_{t+1} > 0$ and $\mathbb{E}_t \nu_{\omega,t+1} > 0$, suggesting

relatively lower investment. In fact, my numerical exercise shows that the OBC has a precautionary effect on banks such that banks try to stay away from being financially constrained by keeping their leverage relatively low.

D.   Government and unconventional policy

There are several unconventional measures that can be adopted to address the financial frictions. I focus on an asset purchase programme because it is the easiest to understand and present. In appendix B, I consider two alternative measures and show that, without further distortions introduced in the model[10], all three measures only differ from one another in a way that already noted in the literature.

The asset purchase programme involves central banks lending directly to domestic goods producers.[11] Central banks do not purchase foreign securities due to political reasons or a very high cost of evaluation and monitoring. Following the standard approach in the public finance literature, the specific agency that implements the policy is abstracted from the model. As argued in Del Negro and Sims (2015), to avoid central bank insolvency, it would be appropriate for central banks conducting unconventional policy receiving fiscal backing from fiscal authorities. I use the term "central bank" and the term "government" interchangeably. Let the value of the programme be V_t^p, which bears a rate of return $r_{k,t+1}$ and renders a cost Γ_t. The programme is financed by government bonds B_t (or reserves, the liabilities of the central bank) and a lump-sum tax T_t. There is also wasteful government spending G_t. The consolidated government budget is given by

$$G_t + \Gamma_t + r_t B_{t-1} + V_t^p = T_t + B_t + V_{t-1}^p r_{k,t}.$$

The easiest way to understand unconventional policy is to look at figure 1. The solid lines indicate the flow of funds via banks, subject to financial frictions. The dashed lines indicate the flow of funds via central banks. By using unconventional policy, central banks act as financial intermediaries. They are free from financial frictions but face possible policy cost (detailed shortly). Therefore, the optimal policy can be deemed as the optimal size of central banks' balance sheets relative to banks', which is shaped by the trade-offs between the policy cost and the benefit of making the economy financially less frictional. To reflect this point, the policy instrument is normalised as $\mathcal{P}_t = \frac{V_t^p}{q_t S_t} \in [0,1]$.

Specifically, the asset purchase programme benefits the economy in a financial crisis as follows. As discussed above, the financial constraint forces banks to sell their assets. However, the asset purchase programme supports asset prices, from which banks enjoy a capital gain and stronger net worth. The externality that banks do not internalise the effect of their asset selling on asset prices becomes irrelevant.

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[10]For instance, public asset purchases would still be very effective if banks' lending channel is frictional. But measures such as government lending to banks would be less effective in this case.

[11]Quint and Rabanal (2017) show how the model can be modified modestly to allow purchasing of long-term government bonds. These modifications should not change the main implications of this paper.

FIGURE 1. MODEL OVERVIEW
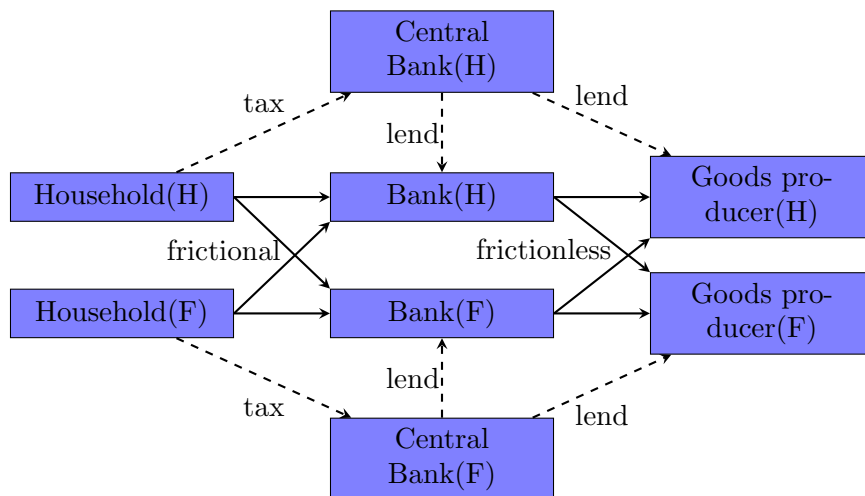
This is the capital channel of unconventional policy. Effectively, the policy crowds out inefficient financial intermediaries (banks) and replace them with efficient ones (governments). As ω_t doing down and n_t going up, the financial constraint is relaxed. It is easy to see the spillover effect of the policy from (12), which states that asset return must be synchronised across countries. Hence, foreign banks benefit from domestic interventions via the capital channel.

Unconventional policy also benefits the economy in normal times. If the government promises to address financial frictions when they appear, it raises banks' expectations of asset prices. Knowing that future financial crises will have smaller impacts on them, banks are willing to take higher leverage in normal times, resulting in more investment in the goods producers. This is referred to as an expectation channel. However, the higher leverage means a higher likelihood of hitting the financial bound. As a result, the government has to intervene and pay the policy cost Γ_t more often. Therefore, the government face a trade-off in affecting the expectations.

Turning to the policy cost. In the literature, policy cost is either abstracted from the analysis (Del Negro et al., 2017; Quint and Rabanal, 2017) or modeled in a reduced-form (Gertler and Karadi, 2011; Dedola, Karadi and Lombardo, 2013; Foerster, 2015). I follow the latter by assuming that the government must pay resource costs on its holding of securities[12]:

$$(14) \qquad\qquad\qquad \Gamma_{i,t} = \tau \left(V_t^p \right)^2 .$$

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[12]In an earlier version of this paper, I also consider policy at least partially being financed by a consumption or labour tax. In the current model, the distortion effect of the tax is too strong such that optimal unconventional policy becomes passive.

This cost represents inefficient public activities in private financial markets or the costs of strengthened financial surveillance.[13]

E.   Aggregation and the market clearing conditions

The law of motion of aggregate net worth is

$$
\begin{aligned}
N_t &= (1 - r_{n,t,t}) \\
&\times \left(q_{t-1} S_{h,t-1} r_{k,t} + q^*_{t-1} S_{f,t-1} r^*_{k,t} - D_{h,t-1} r_{t-1} - D^*_{h,t-1} r^*_{t-1} \right) \\
&+ \varpi \left(\omega_{t-1} + V^p_{t-1} \right),
\end{aligned}
$$

where the last term is the start-up funds received by new banks. Finally, the model is closed by market clearing conditions on goods and security markets

$$
Y_t + Y^*_t = C_t + C^*_t + G_t + G^*_t + \Gamma_t + \Gamma^*_t + f\left(K_{t-1}, I_t\right) + f\left(K^*_{t-1}, I^*_t\right),
$$

$$
(15) \qquad\qquad\qquad q_t S_t = q_t \left(S_{h,t} + S^*_{h,t} \right) + V^p_t,
$$

$$
(16) \qquad\qquad\qquad q^*_t S^*_t = q^*_t \left(S_{f,t} + S^*_{f,t} \right) + V^{p*}_t.
$$

III.   Quantitative method

The model above and the expanded model characterising Ramsey equilibrium are calibrated, approximated around the deterministic steady state, and simulated numerically with occasionally binding constraints.

A.   Numerical solution

DEALING WITH OCCASIONALLY BINDING CONSTRAINTS (OBCs). — Stochastic models with OBCs are typically solved with global methods. However, our models contain too many state variables to be solved even by methods that are explicitly designed to deal with a large state space, such as that of Maliar and Maliar (2015). Guerrieri and Iacoviello (2015) provide a fast algorithm based on piecewise linearization which, however, gives certainty equivalent results.

I employ the approach of Holden (2016a,b). It supports second-order approximation to evaluate welfare. Furthermore, it captures precautionary effects due to OBCs, so we can study the expectation channel outlined above. DynareOBC[14] created by the same author is the toolkit to implement this approach, which roughly

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[13] Dedola, Karadi and Lombardo (2013) also add a linear term to the cost but they find only the coefficient on the quadratic term playing an important role. In this paper, the linear term implies a positive marginal cost regardless of V^p_t, so the Ramsey policy is nonzero in normal times.

[14] It is available at https://github.com/tholden/dynareOBC.

consists of 3 steps. First, the model is Taylor approximated up to a chosen order around the deterministic steady state, ignoring any OBCs. Then, all OBCs are added back to the approximated model. Second, the model is solved in a perfect-foresight manner using Holden (2016b)'s algorithm. We can stochastically simulate the model in the spirit of Fair and Taylor (1983)'s extended path (EP) algorithm. In doing so, we assume that the model's agents act today as if they knew the status of OBCs in every future period. For a model that is linear apart from OBCs (due to first-order approximation), the simulation is certainty equivalent. For a model that is non-linear apart from OBCs (due to higher-order approximation), the simulation captures the risk stemming from non-OBC nonlinearity. Third, to further capture the risk stemming from OBCs, Holden (2016a) applies a modified version of Adjemian and Juillard (2013)'s stochastic extended path (SEP) algorithm. This algorithm involves integrating the model over a certain number of periods of future uncertainty. I integrate over 50 periods and find no considerable change over longer periods. I refer to Holden's solutions based on the EP and the SEP algorithm as EP alike and SEP alike solutions, respectively. I compare these two solutions to show the precautionary effects of OBCs.

DEALING WITH PORTFOLIO INDETERMINACY. — An issue related to the perturbation-based method is the indeterminacy of equilibrium portfolio $\alpha_{p,t}$. According to Devereux and Sutherland (2011), a second (third, fourth, ...) order approximated model is generally enough to pin down up to zero (first, second, ...) order term(s) of the portfolio, while up to first (second, third, ...) order terms of the portfolio are needed to solve the second (third, fourth, ...) order approximated model. The zero-order term is the deterministic steady state. Devereux and Sutherland (2011) propose a general solution as follows. Conjecturing $\alpha_{p,t}$ as a (N-1)th order polynomial of the model's state variables, we can use this conjecture to replace (12). Then, we can solve the Nth order approximated model and search for parameters in the conjecture such that the (N+1)th order approximation of the portfolio's first order condition is satisfied.

Combining Devereux and Sutherland (2011)'s algorithm with Holden (2016a)'s algorithm is computationally very demanding for high order approximation. Therefore, I calculate a first-order approximation to the model in most exercises to focus on the nonlinearity induced by OBCs. A second-order approximation is calculated to evaluate welfare, in which case the portfolio is fixed to its steady state.

B. Parameterization

Table 1 shows the parameterization of the model. It is based on a second order approximation to the model with no government intervention. The behaviours of such a model are discussed in appendix A.

Parameters concerning the non-financial sector are standard in the literature and are borrowed from Dedola, Karadi and Lombardo (2013). There are three parameters in the financial sector, namely $\bar{r}_n$, $\bar{\theta}$, and ϖ. Following Gertler and Kiyotaki

<div align="center">Table 1—Parameterization</div>

Description	Parameter	Value
Steady-state discount factor	$\bar{\beta}$	0.99
Elasticity of discount factor	ψ_β	-0.001
Habit	h	0.815
Risk aversion	σ	1.5
Weight on labour disutility	χ	3.4
Inverse elasticity of labour supply	φ	0.276
Capital share	α	0.33
Inverse elasticity of investment	η	1.728
Depreciation rate	δ	0.025
Steady-state survival probability of banks	$1 - \bar{r}_n$	0.972
Transfer rate from households to new banks	ϖ	0.0045
Steady-state fraction of divertable assets	$\bar{\theta}$	0.2457
Persistence of financial shocks	ρ_θ	0.8
Standard deviation of financial shocks	σ_θ	0.1
Persistence of capital quality shocks	ρ_ξ	0.66
Standard deviation of capital quality shocks	σ_ξ	0.05
Persistence of productivity shocks	ρ_A	0.95
Standard deviation of productivity shocks	σ_A	0.0044

(2010), I choose the survival rate $1 - \bar{r}_n$ so that bankers survive for around 8 years on average. Next, I depart from the literature by choosing a steady state in which the financial constraints are slack.[15] I set the steady-state leverage ratio $\bar{\phi}$ to 4, which is considered by Gertler and Kiyotaki (2010) as an average across sectors with vastly different financial structures. The leverage pins down the start-up rate of new banks $\varpi = \left(1 - \frac{1 - \bar{r}_n}{\bar{\beta}}\right)/\bar{\phi}$. The steady-state proportion of divertable assets $\bar{\theta}$ is adjusted so that the financial constraints are close to their bounds in the steady state. This is to ensure reasonable accuracy of the approximation when the financial constraints are binding. Moreover, the financial constraints bind in most times, meaning that the model's quantitative results are comparable to those of Dedola, Karadi and Lombardo (2013) and other papers in the literature assuming always binding constraints. Despite the similar quantitative results, our OBC set-up matters for optimal policy.

There are three shocks in each country, affecting productivity A_t, capital quality ξ_t, and the fraction of divertable assets θ_t, respectively. The last one is referred

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[15]With integrated security markets and slack financial constraints in the steady state, there is indeterminacy between banks' total assets in Home and Foreign, ω_t and ω_t^*. To pin them down, it is sufficient to introduce an asset adjustment cost to banks:

$$\frac{\psi_\omega}{2} \left(\omega_t + V_t^p - SteadyState\right)^2,$$

where $\psi_\omega = 10^{-5}$ in practice.

to as a financial shock. Each shock follows an uncorrelated AR(1) process. Parameters for the productivity shock are taken from the estimate of Heathcote and Perri (2002). Parameters for the capital quality shock follow Gertler, Kiyotaki and Queralto (2012), the working paper version of which provides a microfoundation. Parameters for the financial shock are calibrated to make the mean of annualized credit spreads about 2.35%.[16] However, without features such as liquidity premia and true default risk,[17] I inevitably overestimate the standard deviation of the spreads. Or I would underestimate the mean if I calibrated the model to match the standard deviation. With our calibration, the model generates financial crises by an unconditional probability of 5.28%. A financial crisis is defined as an occasion in which the spreads are two standard deviations above its mean. This definition corresponds to the dot-com bubble and the 2007 - 2008 financial crisis in the US since 1983.

The calibration above is meant as a benchmark. To check the robustness, I consider alternative parameterizations. For example, $\bar{r}_n$ is set to match a dividend rate of 5.15% made by the largest 20 U.S. banks during 1965–2013.[18] The steady-state leverage can be set to 16, the estimate of Quint and Rabanal (2017) in which the authors use GMM to estimate a similar model with nominal frictions, a Taylor rule, and an always binding financial constraint. Other parameters are adjusted accordingly. I find that my main conclusions remain though these alternative parameterizations change quantitative results.

IV. Cooperative Ramsey policy

In this section, I present the results when Home and Foreign governments cooperate. The government in each country jointly maximizes a single objective function - the lifetime utilities averaged across countries - by committing to a state-contingent plan of unconventional policy. Policymakers solve the following problem:

$$\min WEL_g \equiv \min -\frac{1}{2}\mathbb{E}_t\sum_{j=0}^{\infty}$$
$$\left\{\beta_{t,t+j}\left[\frac{(c_{t+j}-hc_{t+j-1})^{1-\sigma}}{1-\sigma} - \chi\frac{l_{t+j}^{1+\varphi}}{1+\varphi}\right] + \beta_{t,t+j}^*\left[\frac{(c_{t+j}^*-hc_{t+j-1}^*)^{1-\sigma}}{1-\sigma} - \chi\frac{l_{t+j}^{*1+\varphi}}{1+\varphi}\right]\right\}$$

subject to all the private sector equilibrium conditions. These conditions include two inequalities, $\lambda_t \geq 0$, $OBC_t \geq 0$, a slackness condition $\lambda_t OBC_t = 0$, and their Foreign counterparts. It can be verified that $\lambda_t \geq 0$ is a redundant constraint. Intuitively, $\lambda_t \geq 0$ roughly means $r_{k,t+1} - r_t \geq 0$ according to (11), which a benevolent

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[16]This figure is calculated from quarterly data of Moody's seasoned Bbb corporate bond yield relative to the yield on 10-year treasury constant maturity, 1983q1-2017q1, retrieved from FRED, Federal Reserve Bank of St. Louis. By contrast, Gertler and Kiyotaki (2010) and Dedola, Karadi and Lombardo (2013) target at 1%, which is roughly the mean of Moody's seasoned Aaa corporate bond yield relative to the yield on 10-year treasury constant maturity.

[17]Recently Gertler, Kiyotaki and Prestipino (2017) are working on a similar model where banks can default on their debts possibly due to a bank run.

[18]This number is calculated by Swarbrick, Holden and Levine (2017) using Baron (2017) dataset.

policymaker would never violate. In solving for the optimal policy, I follow the "timeless" perspective advocated by Woodford (2003).[19]

I focus on a benchmark case in which the policy cost parameter τ is small. As $\tau \to 0^+$, it is not difficult to see that the governments want to fully exploit the policy benefit by making $\nu_{\omega,t} = \mathbb{E}_t\left(r_{k,t+1} - r_t\right) = 0$ for all t.[20] This strategy may also approximate the circumstances when policymakers promise to do whatever it takes to preserve the economy. In this case, the real economy achieves its first best allocation, the behaviours of which are documented in appendix A. What remains to be seen is the trajectories of policy instruments and variables the financial sector in response to adverse shocks. These trajectories do not change considerably when the policy cost is larger. However, the policy cost is essential to noncooperative policy, which will be examined in section V.

A.   Impulse response analysis
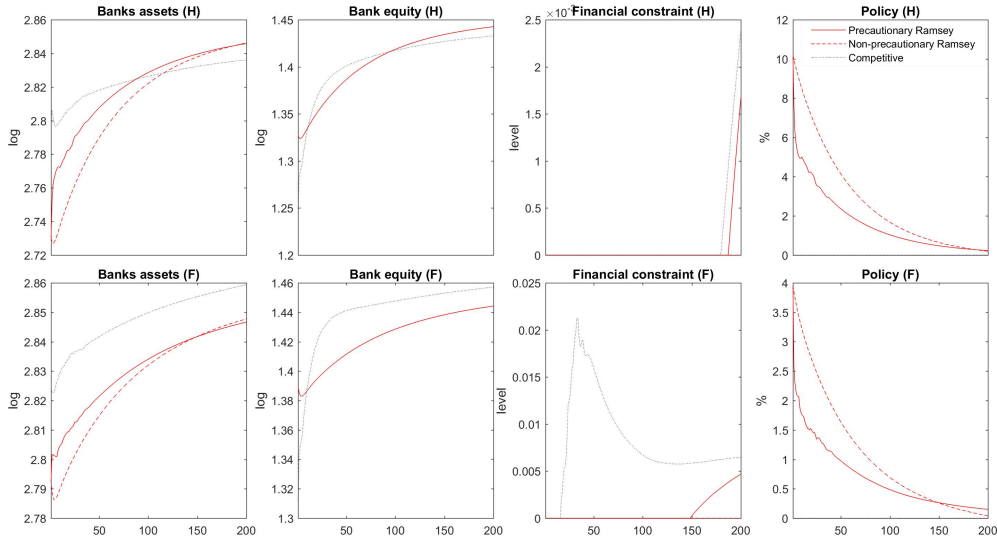

FIGURE 2. COOPERATIVE POLICY RESPONSES TO A HOME CAPITAL QUALITY SHOCK

Figure 2 and 3 plots impulse responses of banks and policy to, respectively, a negative Home capital quality shock and a positive Home financial shock of one standard deviation. The impact of the capital quality shock can be decomposed into two stages. In the first stage, There is a real impact similar to that of a productivity shock in a standard RBC model. Because the return on bank assets is

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[19]The Ramsey policy is time inconsistent in the same way discussed in Bianchi (2016)

[20]Using this argument, we can imposing $\nu_{\omega,t} = 0$ to reduces the number of OBCs and to find the solution quicker.
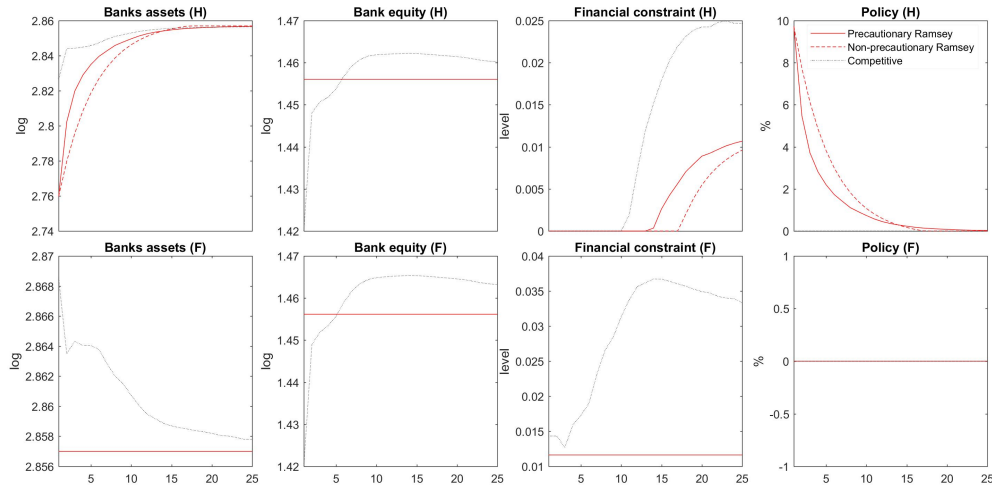
Figure 3. Cooperative policy responses to a Home financial shock

unexpectedly low, banks' net worth drops by a multiplier of their leverage. Consequently, the financial constraints may be binding and the second stage impact of financial accelerator takes place. The financial shock has only the second stage impact.

Let us focus on the main results based on the SEP alike algorithm. The Ramsey equilibrium is represented by red solid lines, and the competitive equilibrium is represented by black dotted lines. Both figures demonstrate the capital gain channel outlined in section II.D. In the wake of the shock, the losses on net worth are smaller (zero in the case of a financial shock) with interventions than it would be without interventions. However, public asset purchases crowd out banks from the security markets. The crowding-out effect results in bank equity growing at a rate slower than it would be without policy, at least in the near term. To stabilize the financial sector, the governments must exit slowly from the policy until banks accumulate enough equity to carry fund intermediation on their own. The exit path needs to be consistent with the path of banks deleveraging. Due to the real impact, the capital quality shock generates particularly slow deleveraging and persistent interventions.

Furthermore, the exit from unconventional policy must be slower than deleveraging because of the precautionary motivation arising from the OBCs. This can be seen by comparing the SEP alike Ramsey solution to the EP alike Ramsey solution (red dashed lines). If the model's agents do not take into account the risk of hitting the financial bounds in the future, unconventional policy ends in the same period when the economy escapes from the constraint. That is about 200 quarters following a capital quality shock and 17 quarters following a financial shock. By contrast, with the precautionary motivation, the policy is more persistent in its tail, in which time the economy has escaped from the constraints sooner than it does in the former case. Intuitively, the governments should give some precautionary protection

to the economy for a few periods after a crisis, during which banks, though having enough net worth to escape from financial constraints, are vulnerable to future adverse shocks. In models where there are true default risks and bank runs, such as that in Coimbra and Rey (2017) and Gertler, Kiyotaki and Prestipino (2017), the precautionary effects are arguably stronger and hence the exit from policy is slower.

On the international dimension, the policy responses are asymmetric because the two countries are ex-post asymmetric. Following the Home capital quality shock, Foreign interventions are roughly half as strong as Home interventions. This is because banks hold a portfolio that consists of more domestic assets ($\alpha^p = 0.4$). Since Home banks are more affected by the shock, they benefit more from the government's purchases of Home assets. Following a Home financial shock, however, Foreign government needs not intervene at all. Foreign banks would only be affected by depressed asset prices through the financial accelerator. Home interventions fully stabilize asset prices in both countries so Foreign banks enjoy a free ride. Note that, however, the timings by which each country escapes from their financial constraints are synchronised.

B. Simulations

I simulate the economy twice, with and without unconventional policy. Both simulations share the same realisation of 5000 shocks. These shocks are small enough so that financial constraints never bind and policy is never actually used. However, the standard deviations of shocks remain as calibrated. So unconventional policy affects the economy only via the expectation channel.

Given our calibration, the existence of unconventional policy raises average consumption, labour, and investment by 0.65%, 0.04%, and 3.05%, respectively. Banks raise their leverage by 12.8%. Measured by the variable OBC_t, banks are about 4 times closer to the financial bounds. Therefore the probability $P(OBC_t = 0)$ must be higher in the presence of unconventional policy. This is a risk that the government and banks are willing to take in exchange for better economic performance in normal times.

V. Noncooperative Ramsey policy

In the absence of cooperation, each government maximizes domestic welfare using domestic instruments, taking the entire path of foreign instruments as given. In solving this problem, each government is subject to all the private sector equilibrium conditions in both countries and choose all the endogenous variables in both countries' private sectors. The outcome is an open-loop dynamic Nash equilibrium. Following Coenen et al. (2007), the chosen strategy space (past and future instruments) is unrealistic but meant as a necessary simplification to the problem. I compare the open-loop result to the closed-loop result of Dedola, Karadi and Lombardo (2013) in which the strategy space of the game is the parameters in feedback policy rules. Note that the strategy space in Dedola, Karadi and Lombardo (2013)

is narrower than that here. As in Dedola, Karadi and Lombardo (2013), I highlight how the noncooperative policy changes subject to policy cost τ. The noncooperative equilibrium is very difficult to calculate, so I only calculate an EP alike solution and ignore the precautionary effect arising from OBCs.
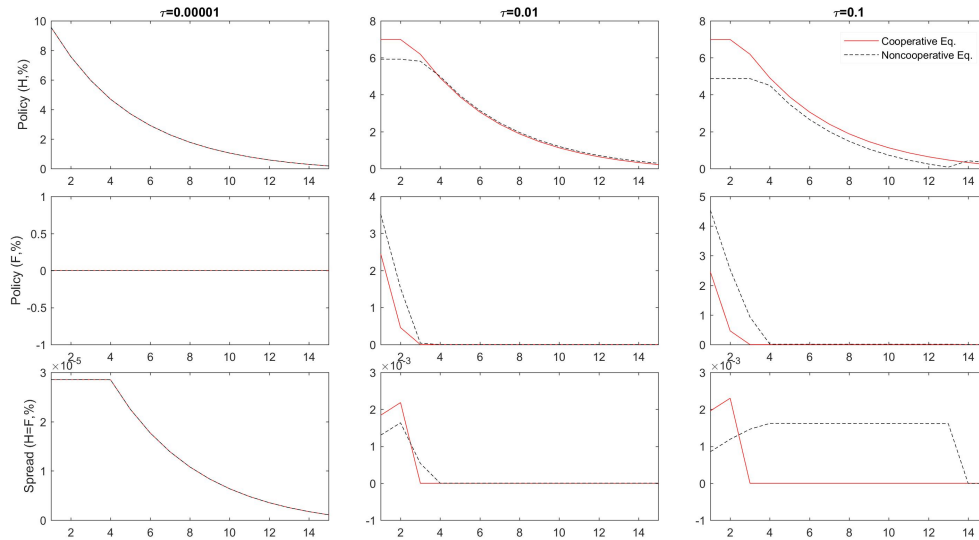

FIGURE 4. NONCOOPERATIVE POLICY RESPONSES TO A HOME FINANCIAL SHOCK

In figure 4, I report impulse responses of policy and the measure of financial market efficiency (credit spread). Only the responses to a financial shock are reported as the difference between cooperation and noncooperative is similar in response to other shocks. The cooperative equilibrium is shown in red solid lines, and the noncooperative equilibrium is shown in black dashed lines.

The two equilibria are identical when unconventional policy is very cheap to use ($\tau = 0.00001$). Increasing τ makes it favourable to divide the necessary interventions more equally across countries. Given a Home shock, this means fewer interventions by the Home government and more by the Foreign government. This is because the marginal intervention cost is increasing in domestic policy but independent to foreign policy (Recall 14). Meanwhile, the domestic economy enjoys a large positive spillover effect of foreign policy. However, the spillover effect is not perfect, so credit spreads are increasing in τ. Furthermore, increasing τ affects the noncooperative policy more than the cooperative policy, implying a positive cooperation gain. The noncooperative equilibrium features excessive interventions in Foreign and insufficient interventions in Home.[21] By contrast, in Dedola, Karadi and Lombardo (2013), increasing τ always reduces the policy responses in both countries.

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[21]In case of a capital quality shock, we would need a much larger τ to find a noticeable difference between the cooperative and noncooperative equilibria.

After a certain point, policy responses in the noncooperative equilibrium quickly approaching zero in both countries. Further increasing the cost makes cooperative policy unresponsive.

VI. Simple Rules

It is well known that Ramsey policy is silent regarding implementation. In extreme cases, for instance, the capital control policy analysed in Schmitt-Grohé and Uribe (2016), there could be no policy intervention in Ramsey equilibrium. Whether optimal outcomes can be supported by such policy, however, depends on policy implementation. In this section, I examine the extent to which various simple feedback rules can approximate the Ramsey policy. I focus on the cooperative case.

The benchmark rule proposed in the literature is

$$(17) \qquad \mathcal{P}_t = \kappa \mathbb{E}_t \left(r_{k,t+1} - r_t \right)$$

in Home and similarly in Foreign where κ determines the policy responsiveness. Given asymmetric countries, κ in each country must be the same. I refer to (17) as the spread rule. Foerster (2015) proposes an improvement by adding an autoregressive term (AR spread rule):

$$(18) \qquad \mathcal{P}_t = \kappa \left(1 - \rho_{\mathcal{P}} \right) \mathbb{E}_t \left(r_{k,t+1} - r_t \right) + \rho_{\mathcal{P}} \mathcal{P}_{t-1}.$$

Our discussion on the capital gain channel suggests a price rule:

$$(19) \qquad \mathcal{P}_t = -\kappa \left(\ln q_t - \ln q_{t,potential} \right),$$

where $q_{t,potential}$ is the asset price that would occur in a financially frictionless world, and $\ln q_t - \ln q_{t,potential}$ is the asset price gap. The negative sign before κ reflects the fact that asset prices are lower in periods of financial distress. The unconditional welfare losses and the optimised parameters are reported in table 2. Adding an autoregressive term improves the spread rule, but both spread-based rules generate much larger welfare losses than the price rule. The price rule needs to be very aggressive to maximise welfare.

Table 2—Unconditional welfare losses under optimised simple rules

	Spread rule	AR spread rule	Price rule
Unconditional welfare losses (0.01%)	6.84	5.80	3.83
Optimised parameter(s)	150	150, 0.9	450

To better understand how these rules differ, it is useful to study their impulse responses, which are plotted in figures 5 and 6. The policy cost is $\tau = 0.01$.

Consider the spread rule shown in red with circles. In response to both shocks, the spread rule is not aggressive enough to stabilize the financial sector, leaving a significant gap in asset prices and spreads. The inefficient financial intermediaries, i.e. banks, are holding too many assets. As a result, there are also substantial fluctuations in consumption gaps. However, when the governments are existing from their policy, the speed is roughly the same following the spread rule or following the Ramsey path.

The AR spread rule (shown in blue with asterisks) implies stronger and more persistent interventions, particularly in response to the financial shock. Following this rule, central banks can pin down expectations that spreads and asset price gaps will be low in the long term. However, this comes at a cost that spreads and asset price gaps surge in the first few periods after the shock. The tough short-term financial environment forces foreign banks to deleverage heavily in response to the financial shock. If central banks careful balance the long- and short-run effects, they can reduce fluctuations in consumption and improve welfare.

As expected, the price rule (shown in green with triangles) roughly mimics the Ramsey outcomes. There is barely any fluctuation in consumption gaps, asset price gaps, and spreads. However, the price rule is not practicable in response to capital quality shocks or any other real shock because the potential asset prices are not observable. This rule still generates welfare losses for three reasons. First, interventions seem little too strong following the price rule. This occurs probably because the search for κ is taken on a rather crude grid. Second, the Ramsey policy features asymmetric interventions across countries, which is not allowed by simple rules. Third, I consider unconditional welfare so the parameters are not customised to different shocks.

It may be natural to consider a policy shock. For example, there have been concerns that excessive unconventional interventions may create asset bubbles. I left this exercise to further research because policy shocks bring no interesting dynamics in this model. A positive shock on policy perfectly crowds out banks and hence can not push asset prices beyond their optimal level.

VII. Conclusions

I study the Ramsey optimal unconventional monetary policy in a two-country version of Gertler and Kiyotaki (2010), with and without cross-country policy cooperation. In this model, banks face occasionally binding constraints on their leverage. The optimal policy is the optimal size of central banks' balance sheets relative to banks'. There are two policy trade-offs. Ex-ante, the policy needs to balance excessive risk-taking and economic efficiency. Ex-post, the trade-off is between a real resource cost and the benefit of relaxing financial constraints. One important quantitative finding is that the exit from unconventional policy must be slower than banks deleveraging due of the precautionary effect arising from the OBCs. This leads to positive balance sheets of central banks after a crisis. I find that the Ramsey policy can be characterised by the simple rule that responds to asset price gaps.
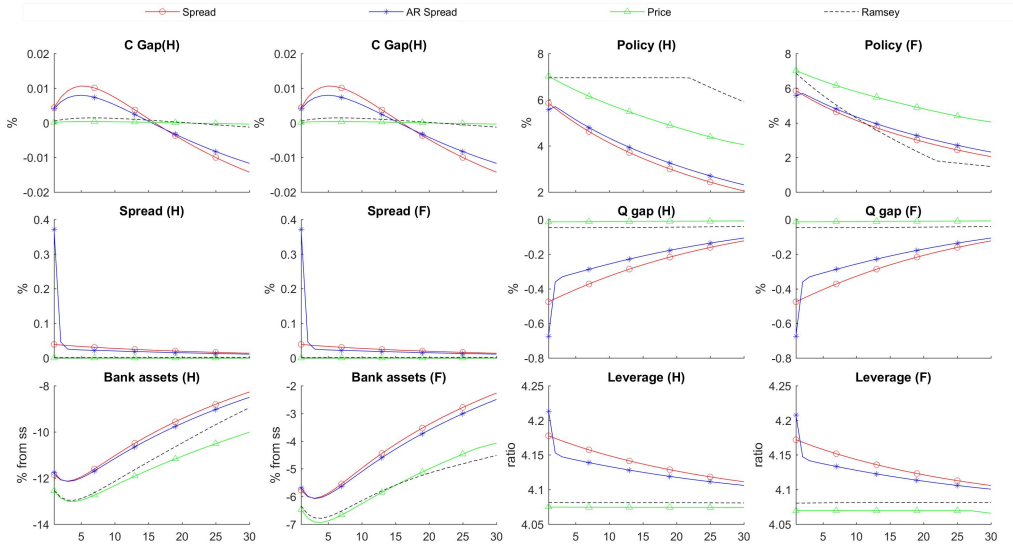
FIGURE 5. IMPULSE RESPONSES TO A HOME CAPITAL QUALITY SHOCK, UNDER DIFFERENT POLICY RULES

Note: Gap variables are the difference between the variables themselves and their counterparts that would be realized in a world without the financial constraints. C denotes consumption and Q denotes the asset price.

Traditional rules based on credit spreads struggle to pine down the expectations of long-term interest rates. Hence, they imply substantial welfare losses.

On the international dimension, I find the lack of cooperation reduces domestic interventions and increases foreign interventions in response to a domestic shock, or vice versa in response to a foreign shock. This is in contrast to the literature suggesting that noncooperative simple rules feature insufficient interventions in both countries. In consistency with the literature, I find the key determinant of cooperation gain to be the real resource cost.

Naturally, this paper is subject to several limitations. First, banks in this model are stylised financial intermediaries, lacking many important features of modern banks. Consequently, reducing credit spreads automatically boosts lending and investment. This is not necessarily true in data according to Chakraborty, Goldstein and MacKinlay (2017) and Acharya et al. (2017). Second, both the policy trade-offs are ultimately pinned down by the real resource cost. This is a standard simplification in the literature. However, the cost of unconventional policy is still little understood. A recent paper by Kandrac (2014) summarises several potential costs that have been discussed in the Federal Reserve. These costs may introduce new policy trade-offs and shade further light on optimal policy. At last, public asset purchases and alternative measures are similar in our model. An interesting extension would be to differentiate the measures employed by the Fed, the ECB, and other central banks and to study their relative effectiveness.
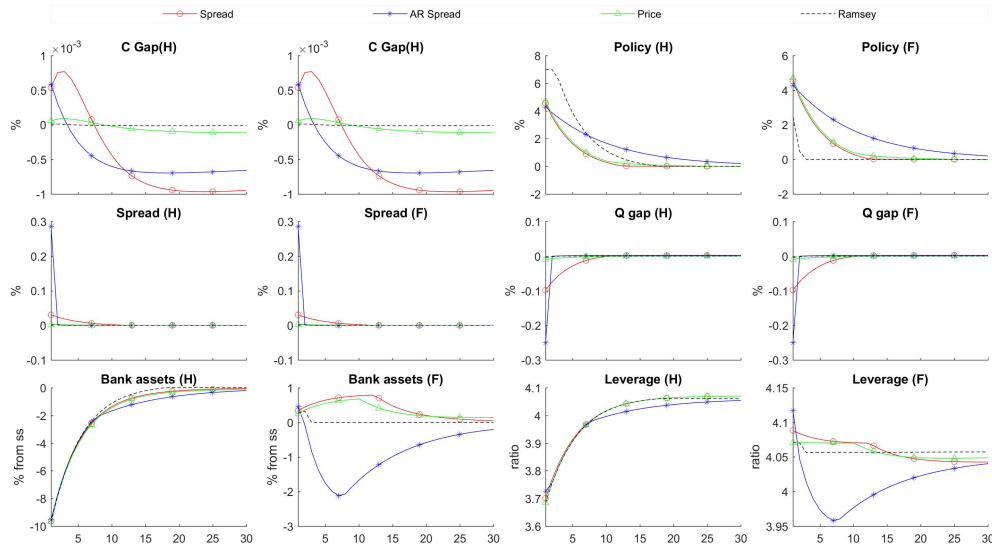
FIGURE 6. IMPULSE RESPONSES TO A HOME FINANCIAL SHOCK, UNDER DIFFERENT POLICY RULES

Note: Gap variables are the difference between the variables themselves and their counterparts that would be realized in a world without the financial constraints. C denotes consumption and Q denotes the asset price.

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A. Competitive Equilibrium

In this appendix, I document the quantitative behaviours of the model in the absence of unconventional policy, highlighting the role of occasionally binding financial constraints.

Figure A1 and A2 plots impulse responses to, respectively, a negative Home capital quality shock and a positive Home financial shock of one standard deviation. The main results are plotted by red solid lines and the frictionless economy is represented by black dashed lines.

The capital quality shock creates a deep and persistent global recession similar to that of Dedola, Karadi and Lombardo (2013) in which financial constraints always bind. The Home shock propagates internationally via the cross-country synchronisation of asset return according to (12) and banks' diversified portfolio. Since banks invest more in domestic assets($\alpha_p = 0.4$), Foreign banks are less affected by the Home shock than Home banks. The financial shock forces domestic banks to fire sell their assets. Foreign banks would like to pick up those assets when the prices are low. However, Foreign banks have a limited ability to do so due to their own financial constraint. Overall, global investment drops. Consumption and output movements are perfectly synchronised across countries.

When financial constraints bind, the financial acceleration mechanism clearly has a big impact on the financial sector but little on consumption and output. This is because our model has a simple structure. Del Negro et al. (2017) suggest the role of nominal rigidity and a zero lower bound on monetary policy, without which financial frictions account for a drop in investment but not in output thanks to a rise in consumption. In a small open economy RBC model with an occasionally binding collateral constraint, Mendoza (2010) also finds small precautionary effects on long-run business cycle.

A2. Precautionary effects

Table A1—Unconditional Mean (StD) of Home variables

	$p(Crisis)$	Bank assets	Annualized spread	Consumption
EP alike	5.48%	4.72	2.89%	0.316
		(0.90)	(5.34)	(0.22367)
SEP alike	5.28%	4.47	2.33%	0.313
		(0.76)	(4.62)	(0.22375)

If the financial constraints are always binding, banks always hold the maximum level of assets permitted by their net worth. However, if financial constraints bind occasionally, the amount of bank assets depends on the probability of hitting the bound in the future. I simulate the model with and without precautionary effects, using the SEP alike solution and the EP alike solution respectively. The first and second moments from the simulation is presented in table A1. First, the precautionary effects reduce the probability of a financial crisis (defined in section III.B) by
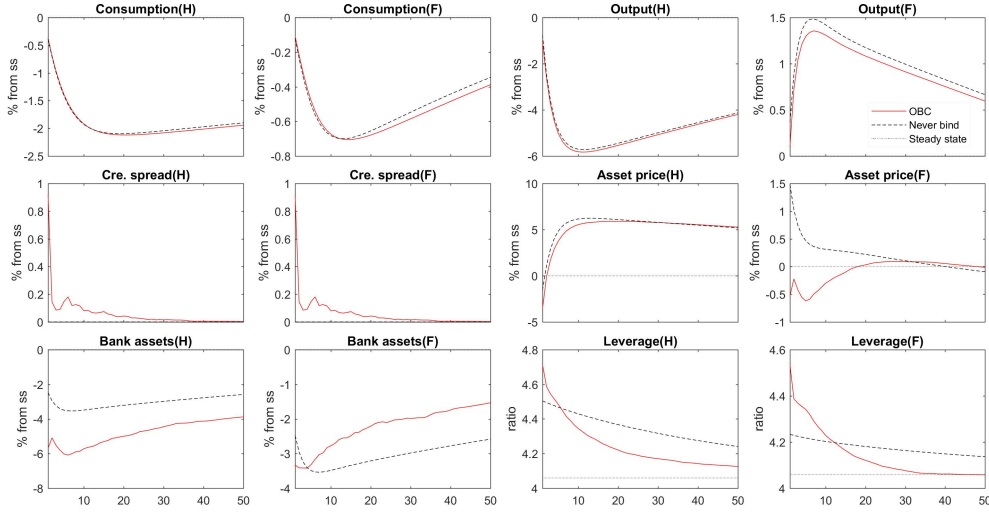
Figure A1. Impulse responses to a Home capital quality shock, competitive equilibrium

Note: Black dashed lines represent variables that would be realized in a financially frictionless world (potential variables), red solid lines represent actual variables, and the dotted line represents the steady state.

0.2%. To avoid being financially constrained, banks hold fewer assets on average, which also benefits them with a smaller volatility. The precautionary effects also reduce both the mean and the standard deviation of the credit spread. Similar to the impulse response analysis, the precautionary effects are small on non-financial variables, e.g. consumption.

B. Alternative measures

In this appendix, I show how alternative measures, i.e. lending to and equity injections in banks, differ from public asset purchases. Each of these measures targets one of the three components in a balance sheet, namely assets, liabilities, and equity. Lending to banks can be interpreted as the ECB's longer-term refinancing operations. I consider cooperative Ramsey policy as in section IV. Let a bank's balance sheet be

$$\omega_t = d_{h,t} + d_{h,t}^* + d_{g,t} + e_{h,t} + e_{g,t}$$

where $d_{g,t}$ is the banking borrowing from the government, $e_{h,t}$ and $e_{g,t}$ are equity held respectively by households and the government. Net worth is evenly distributed to all equities[22], so $e_{h,t} + e_{g,t} = n_t$. Net worth is given as:

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[22]Note that in Gertler and Karadi (2011), government equity has the same payout stream as banks' assets. The government acquire equity by a price higher than the market price. They also assume that government equity is non-divertable. Due to these assumptions, equity injections are effectively public asset purchases
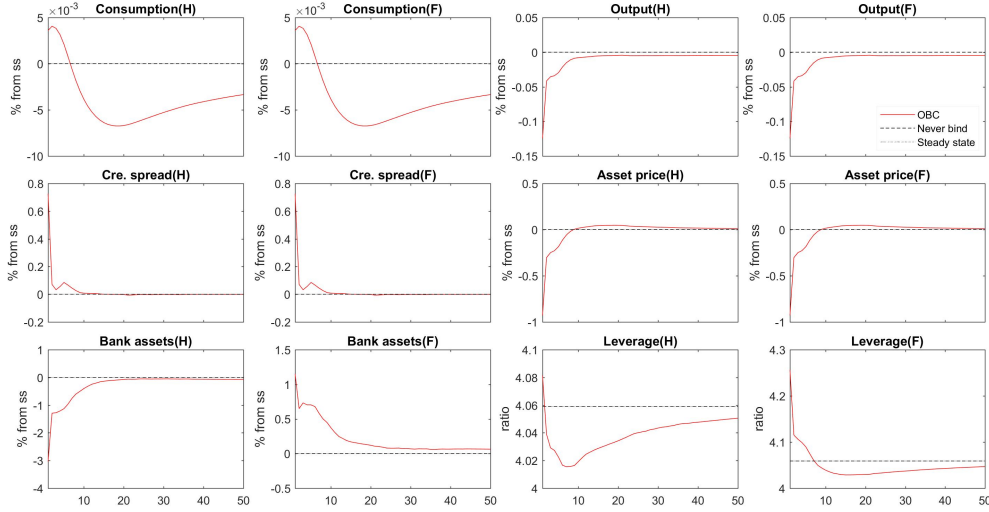
FIGURE A2. IMPULSE RESPONSES TO A HOME FINANCIAL SHOCK, COMPETITIVE EQUILIBRIUM

Note: Black dashed lines represent variables that would be realized in a financially frictionless world (potential variables), red solid lines represent actual variables, and the dotted line represents the steady state.

$$n_t = q_{t-1}s_{h,t-1}r_{k,t} + q^*_{t-1}s_{f,t-1}r^*_{k,t} - d_{h,t-1}r_{t-1} - d^*_{h,t-1}r^*_{t-1} - d_{g,t-1}r_{g,t-1},$$

where $r_{g,t}$ is the interest rate on $d_{g,t}$. I modify leverage as $\phi_t = \frac{\omega_t - d_{g,t} - e_{g,t}}{e_{h,t}}$ and the financial constraint as

$$\nu_{n,t}e_{h,t} - [\theta_t(\omega_t - \theta_g d_{g,t}) - \nu_{n,t}e_{g,t}] \geq 0,$$

where a proportion $\theta_g \in [0,1]$ of borrowing from the government is non-divertable, and government equity is non-divertable. Thanks to the government's superior ability to enforce repayment, replacing one unit of deposits by one unit of $d_{g,t}$ or $e_{g,t}$ relaxes the financial constraint and hence allows banks to expand investment.

I discuss how these three measures differ through the lens of the impulse responses to a capital quality shock, which are plotted in figure B1. Again, policy spending is normalised by $q_t s_{h,t}$ and the policy cost (14) applies. All measures share two common roles. First, policy provides public funds to the economy when banks are constrained to borrow from households and hence are constrained to lend. all measures work in the same way through the capital gain channel. In particular, private equity grows

plus a lump-sum transfer to banks. By contrast, our assumption that government equity is identical to private equity makes equity injections similar to that studied in He and Krishnamurthy (2013).

at the same rate regardless of measures, as shown by the middle panel. This is because government equity, lending, and assets (purchased by the government as an opportunity cost for banks) are equally valued in the Ramsey equilibrium.

On the other hand, however, these measures differ in terms of a crowding-out effect as shown by the left panel. A small crowding-out effect must be explained by the fact that banks can raise more deposits thanks to more relaxed financial constraints. The second role of unconventional policy is to relax financial constraints. Evidently, equity injections have the smallest crowding-out effect, which in turn means that equity injections constitute the least costly policy. Lending to banks is at most as efficient as public asset purchases in the extreme case of $\theta_g = 1$.
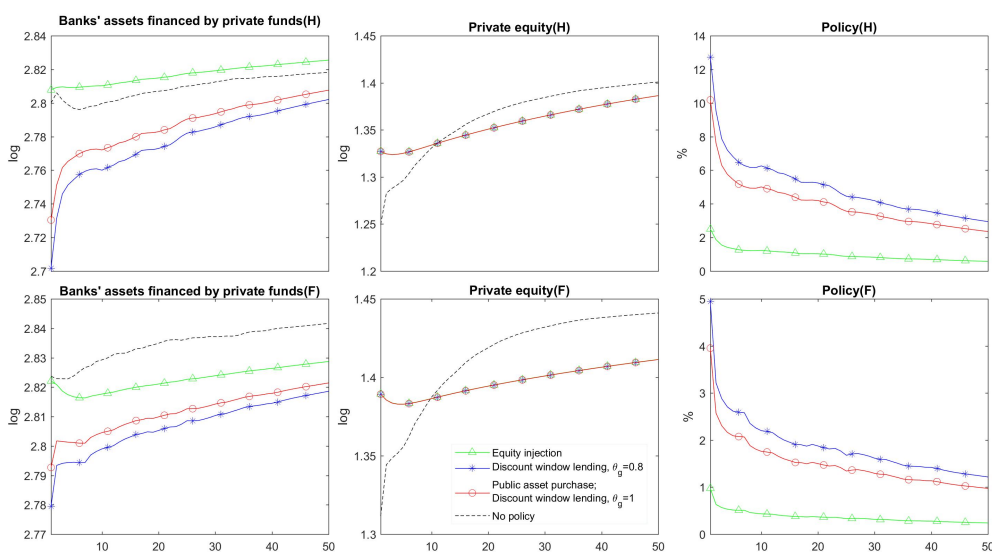

FIGURE B1. COOPERATIVE POLICY RESPONSES TO A HOME CAPITAL QUALITY SHOCK, DIFFERENT MEASURES