

# Money and Capital in a Persistent Liquidity Trap<sup>\*</sup>

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## Abstract

In this paper we analyze the implications of a persistent liquidity trap in a monetary model with asset scarcity. We show that a liquidity trap may lead to an increase in real cash holdings and be associated with a decline in output in the medium term. This medium-term impact is a supply-side effect that may arise when agents are heterogeneous. It occurs in particular with a persistent deleveraging shock, leading investors to hold cash yielding a low return. Policy implications differ from shorter-run analyses implied by nominal rigidities. Quantitative easing leads to a deeper liquidity trap. Exiting the trap by increasing expected inflation or applying negative interest rates does not solve the asset scarcity problem.

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*Keywords:* Zero lower bound, liquidity trap, asset scarcity, deleveraging

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## 1. Introduction

Periods of persistent liquidity traps typically coincide with substantial increases in real cash holdings, as illustrated in Figure 6 for the U.S. and Japan. These periods are also characterized by disappointing levels of investment and of output growth. Can increased real money holdings be associated with lower physical investment and lower growth? Most macroeconomic models would give a negative answer to this question. In this paper, we argue that in a liquidity trap

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investment can be negatively correlated with investors' real money holdings. We consider a monetary model where prices are flexible in the medium run and where money is only held for transaction purposes in normal times. In a liquidity trap however, depending on the nature of the shocks hitting the economy, investors may allocate part of their saving to money holdings that have a low return. With agents heterogeneity, this lower return may then hamper aggregate investment capacity and have a long-lasting impact on output. Our paper therefore identifies a supply-side mechanism that may contribute to a slower recovery in a liquidity trap. The policy implications of these supply-side effects also differ from shorter-run analyses, where demand-side effects dominate due to nominal rigidities.

We focus on a liquidity trap generated by a deleveraging shock. Due to this shock, desired aggregate saving exceeds desired investment and the nominal interest rate cannot adjust downward because of the Zero Lower Bound (ZLB). In that case, excess desired saving translates into higher real cash holdings. We assume a persistent deleveraging shock, so that the liquidity trap can be persistent, even after prices have adjusted. This contrasts with the literature focusing on long-run demand effects and modeling long-lasting liquidity traps by assuming persistent nominal rigidities. Instead, we make the conventional assumption that prices are flexible in the medium run and analyze the longer-run implications of the liquidity trap.

More precisely, we introduce money in a model with scarce (liquid) assets due to the lack of income pledgeability, in the spirit of Woodford (1990) and Holmström and Tirole (1998).<sup>1</sup> Investors find investment opportunities every other period, so that they alternate between investing and saving phases. In their investing phase, they use past liquid saving and borrow to invest, but they face credit constraints. Agents can save in two liquid assets, bonds and money. As long as the nominal interest rate is positive, money is dominated as an asset and is held only for transaction purposes. At the ZLB, bonds and money become substitutes and money is held for saving purposes as well. In this framework, we consider a persistent deleveraging shock, modeled as in Eggertsson and Krugman (2012) by a tightening of the investors' borrowing constraints.<sup>2</sup> This shock generates a decrease in the nominal interest rate until it hits the ZLB.

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<sup>1</sup>See also Farhi and Tirole (2012) and Bacchetta and Benhima (2015) for more recent contributions.

<sup>2</sup>With nominal rigidities, the literature has already shown that such a deleveraging shock can lead to low levels of output and employment in the short run, due to lower demand. Eggertsson and Krugman (2012),

This creates a gap between the effective and the shadow real interest rate that would prevail without the ZLB. In our asset-scarce model, the fall in the shadow interest rate lasts as long as the deleveraging shock.

We show that the consequences of a deleveraging shock are very different outside or at the ZLB. Outside the ZLB, the shock has no effect on capital accumulation and output (in our benchmark specification) as the interest rate can adjust downward and offset the tighter borrowing constraint. However, deleveraging shocks that bring the economy to the ZLB have a negative effect on capital and output. Since the deleveraging shock reduces the investors' supply of assets, their excess saving is allocated to money in the absence of interest rate adjustment. Money holdings have then two effects on capital accumulation. First, saving in money rather than bonds means that fewer funds are channelled to investment—a negative crowding-out effect. Subsequently, however, money is a source of funds, as it can be liquidated to finance investment—a positive liquidity effect. A low return on money, however, implies a smaller amount of liquidity to finance investment. Therefore, the crowding-out effect dominates and investment decreases in the medium run. The channel through which deleveraging affects capital in the liquidity trap mainly comes from a Pigou-Patinkin real balance effect. Indeed, in our non-Ricardian model, real money holdings accumulated by investors in response to deleveraging are net wealth, which leads them to consume more and hence invest less. While real balance effects cannot arise in a Ricardian world, they are present in our framework due to credit constraints and to agent heterogeneity.

We start by focusing on the limiting case of a permanent liquidity trap. This case allows us to focus on steady states which is analytically tractable and gives important insights for transitory, but persistent, deleveraging shocks. We then analyze numerically transitory shocks by assuming that the deleveraging shock ends with a constant probability in each period. On impact, the shock has more negative effects than in the medium run. After a few years, the economy recovers, but only partially. The economy only recovers completely when the financial constraint parameter comes back to its initial state and the economy gets out of the liquidity trap. With nominal rigidities—in the form of downward wage rigidity, the impact effect is

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Werning (2012), Benigno et al. (2014), or Caballero and Farhi (2017) show this in New-Keynesian models.

stronger, but the medium-term effect is similar.

In our analysis, the medium-run investment slow-down is associated with an increase in *investors'* demand for cash, so that it is crucial that the deleveraging shock affects *investors*.<sup>3</sup> Indeed, tighter credit constraints among investors increase their net saving. This extra demand for saving is satisfied by money at the ZLB, and their capacity to finance investment is then directly affected by the low return on money. On the contrary, a deleveraging shock affecting only workers has no medium-run effects in the liquidity trap, because it does not alter the investors' demand for saving.<sup>4</sup> Besides, other types of shocks, such as an increase in the discount rate or a decrease in the growth rate of productivity, do not have a negative long-term effect on the investment rate. In these cases, the crowding-out of investment by cash is more than compensated by an increase in the aggregate saving rate. Our results therefore suggest that investors' deleveraging is an important factor of growth slowdowns in persistent liquidity traps.

The policy implications of our framework differ from traditional shorter-run analyses. In a liquidity trap, typical policies are quantitative easing (QE), negative interest rates, or an increase in expected inflation. These policies may have their merits in the short run, but they have drawbacks in the medium run. QE operations, by taking public bonds away from the market, decrease the shadow real interest rate and generate a deeper and potentially longer liquidity trap. Negative nominal interest rates or an increase in expected inflation help to exit the liquidity trap by lowering the effective real interest rate. However, these policies do not solve the asset scarcity problem but instead deteriorate the allocation of resources across time by further lowering the real interest rate. Instead, improving the supply of liquidity helps exiting the liquidity trap by increasing the shadow interest rate. This can be done through a higher supply of government debt.<sup>5</sup> However, while a higher supply of liquidity improves the

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<sup>3</sup>Section 1 of the Online Appendix shows that the rise in cash holdings in the US comes from the less constrained firms and households, which would correspond to investors in the model.

<sup>4</sup>The empirical literature shows that all sectors of the private economy suffer from deleveraging in the Great Recession.

<sup>5</sup>Such policies are also discussed in policy circles, e.g., Kocherlakota (2015). Acharya and Dogra (2018) examine the role of public debt and inflation policy to exit the ZLB in an overlapping-generation model, but the trade-offs are different from our framework with constrained investors.

allocation of resources across time, it can have undesirable redistributive effects by decreasing the capital stock and reducing wages.

Our asset-scarce environment is characterized by a low interest rate, so it is prone to rational bubbles. When we allow for bubbles that can be held by savers, we show that they play a role similar to money, generating crowding-out and liquidity effects. By sustaining a higher interest rate, the emergence of a bubble rules out money and brings the economy out of the ZLB.

*Related literature.* The paper is related to the recent literature on persistent ZLB equilibria. In this literature, liquidity traps usually arise when the natural rate of interest falls enough to make the nominal rate hit the ZLB (Krugman, 1998; Auerbach and Obstfeld, 2005; Eggertsson and Krugman, 2012; Werning, 2012). But even in a *persistent* liquidity trap, stagnation remains a demand-side phenomenon. Schmitt-Grohé and Uribe (2013) add permanent nominal rigidities (a non-vertical long-run Phillips curve) to Benhabib et al. (2001)'s multiple equilibrium model to get a lower output in the ZLB equilibrium. Similarly, Benigno and Fornaro (2018), in an endogenous growth model, assume permanent nominal rigidities to get a self-fulfilling ZLB steady state with low output and low growth. In the non-Ricardian models of Eggertsson and Mehrotra (2014), Caballero and Farhi (2017) and Michau (2018), long-run nominal rigidities also generate a persistently negative output gap at the ZLB.

Like us, Buera and Nicolini (2016), Guerrieri and Lorenzoni (2017) and Ragot (2016) examine the effects of a deleveraging shock at the ZLB in the absence of nominal rigidities.<sup>6</sup> Guerrieri and Lorenzoni (2017) focus on consumer spending in a model where households face borrowing limits, and Ragot (2016) studies optimal monetary policy in a model where money has redistributive effects due to limited participation. In both models, there is no capital accumulation. Closer to our approach, Buera and Nicolini (2016) consider a monetary model where producers need external funds to buy capital. While we focus on the negative relationship between cash holdings and capital, they study the reallocative effects of low real interest rates on total factor productivity and capital, and assume a moneyless economy in most of their paper. Like us,

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<sup>6</sup>Di Tella (2018) analyzes the role of money in a flexible price model with uncertainty shocks. Although he does not focus on the ZLB, he shows that the presence of money can reduce investment because of lower precautionary saving.

they discuss the trade-offs associated with the inflation policy but do not consider increases in public debt large enough to exit the ZLB.

The crowding-out and liquidity effects of money we emphasize are reminiscent of the effects of external liquidity in other models where investors' income is not fully pledgeable, such as Woodford (1990), Holmström and Tirole (1998), and more recently Covas (2006), Angeletos and Panousi (2009), Kiyotaki and Moore (2012), Kocherlakota (2009), and Farhi and Tirole (2012). The role of money as a saving instrument is also evocative of the literature on the value of fiat money (Samuelson (1958), Townsend (1980)). In our paper, transactions are not constrained by demography or spatial separation, but by the lack of income pledgeability.

The real balance effect that underlies the adjustment mechanism of our model has been originally studied by Pigou (1943) and Patinkin (1956). More recently, Weil (1991), Ireland (2005), Bénassy (2008) and Devereux (2011) have analyzed real balance effects in OLG models.

In our model, rational bubbles arise in asset-scarce environments with a low interest rate, as in Samuelson (1958), Tirole (1985), and more recently Martin and Ventura (2012) and Farhi and Tirole (2012). Closer to our approach, Asriyan et al. (2016) introduce bubbles in a monetary environment and analyse liquidity traps.

The rest of the paper is organized as follows. Section 2 presents the basic model with infinitely-lived entrepreneurs and workers. Section 3 studies the effect of a permanent deleveraging shock in a flexible price steady state before extending the analysis to persistent shocks with nominal rigidities. Section 4 examines policy options. Section 5 studies several extensions of the benchmark model: workers' deleveraging, bubbles, preference and growth shocks, financial intermediation, and inefficient saving technology. Section 6 concludes.

## **2. A Model with Scarce Assets and Money**

We consider a heterogenous-agents, non-Ricardian monetary model where the supply of bonds and the distribution of money holdings matters. We first assume flexible prices and will introduce nominal rigidities later in the simulated model. In normal times, bonds dominate money and the real interest rate adjusts to balance the supply and demand for bonds. In a liquidity trap, however, bonds and money become perfect substitutes. The supply and demand

of assets are then balanced by an adjustment in real money holdings (coming from either prices or money supply). These two adjustment mechanisms, through interest rates or money holdings, have different implications for investment and output, and therefore for policy. We show that in a liquidity trap real money holdings by investors tend to increase, which may be associated with a decline in capital and output in the medium run. This is in particular the case for a deleveraging shock, which we analyze in Section 3. In this section, we describe the model and the equilibrium. For expositional purposes, we focus here on perfect foresight. The model will be simulated later under uncertainty.

### 2.1. The Setup

We model a monetary economy with heterogeneous investors, workers, and firms. There are three types of assets: bonds, money, and capital. Bonds are nominal and promise to pay one unit of currency in the next period. Denote by  $i_{t+1}$  their gross real rate of return expressed in units of currency: a bond issued in period  $t$  is traded against  $1/i_{t+1}$  units of money. Under perfect foresight, the gross real return expressed in units of good is  $r_{t+1} = i_{t+1}P_t/P_{t+1}$ , where  $P_t$  is the price of the final good in units of currency in period  $t$ . While  $r_{t+1}$  represents the effective real interest rate, at the ZLB we will also consider the *shadow* real interest rate  $r_{t+1}^s$ , which is the real interest rate that would prevail if the ZLB were not binding.

Money bears no interest, but it provides transaction services by relaxing a cash-in-advance constraint faced by workers. Money holdings are non-negative. In normal times, when  $i > 1$ , money is strictly dominated by bonds as a saving instrument. Then, only workers hold money, for transaction purposes. However, when  $i = 1$ , money becomes as good a saving instrument as bonds and investors start holding money as well.

*Investors.* Following Woodford (1990), investors find investment opportunities every other period, so that they alternate between a saving period and an investment period. This simple approach is a convenient limit case allowing to capture idiosyncratic shocks in a very tractable way. Section 4.10 of the Online Appendix examines the more general case with idiosyncratic uncertainty on the occurrence of an investment opportunity and shows that the analysis is

similar.<sup>7</sup> Consequently, at each point in time there are two groups of investors, investing and saving every other period. We call investors in their saving phase S-investors, or simply savers, and denote them by  $S$ , while investors in their investment phase are called I-investors and are denoted by  $I$ . Each group is of measure 1. We assume logarithmic utility in order to get closed-form solutions. An individual investor  $i$  maximizes  $U_t^i = E_t \sum_{s=0}^{\infty} \beta^s \log(c_{t+s}^i)$ , where  $c_t^i$  refers to her consumption in period  $t$ , subject to a sequence of budget constraints and borrowing constraints.

In period  $t$ , I-investors start with wealth  $(A_t + M_t^S)/P_t$  where  $A_t$  and  $M_t^S$  are respectively nominal bond holdings and nominal money holdings inherited from their preceding saving phase. They get an investment opportunity, which consists in a match with a firm. They consume  $c_t^I$ , issue  $B_{t+1}$  nominal bonds, and invest  $k_{t+1}$  in the firm. We focus on real budget constraints, so we denote by  $b = B/P$  and  $a = A/P$  the real values of nominal bonds issued and held by investors. We abstract from the money demand by I-investors, as it is always zero in equilibrium. Their budget constraint is

$$\frac{b_{t+1}}{r_{t+1}} + a_t + \frac{M_t^S}{P_t} = c_t^I + k_{t+1}. \quad (1)$$

In period  $t$ , S-investors start with equity  $k_t$  and outstanding nominal debt  $B_t$  inherited from their preceding investment phase. They receive a dividend  $\rho_t k_t$ . Then, they consume  $c_t^S$ , buy  $A_{t+1}$  nominal bonds and save  $M_{t+1}^S$  in money. Their budget constraint is

$$\rho_t k_t = c_t^S + b_t + \frac{a_{t+1}}{r_{t+1}} + \frac{M_{t+1}^S}{P_t}. \quad (2)$$

In general, the return on capital is larger than the return on bonds. Thus, I-investors choose to leverage up when they receive an investment opportunity. But they face a borrowing

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<sup>7</sup>We consider a 2-state Markov process where an investor with no investment opportunity at time  $t - 1$  receives an investment opportunity at time  $t$  with probability  $\omega \in (0, 1]$ ; while an investor with an investment opportunity at time  $t - 1$  receives no investment opportunity at time  $t$ .



constraint as they can only pledge a fraction  $\phi_t$  of dividends:

$$b_{t+1} \leq \phi_t \rho_{t+1} k_{t+1}. \quad (3)$$

This constraint rules out default in equilibrium as it ensures that I-investors will not renegotiate their debt ex post, since creditors can always recover at least the value of the debt.

In this framework, where investment opportunities are lumpy and investors cannot fully pledge their future income, there is an asynchronicity between the investors' access to and their need for resources. This creates a demand for assets for liquidity purposes in the investors' saving phase.<sup>8</sup> Both bonds and money can satisfy this demand for liquidity, or demand for assets (we will use these two terms interchangeably). Capital, on the other hand, is illiquid, since it cannot be fully pledged.

*Firms.* There is a unit measure of 2-period-lived firms, each matched with an I-investor. Firms use their investor's funds to buy capital  $k_t$ . In the following period, they hire labor  $h_t$  at real wage  $w_t$ , produce output  $y_t$  with a Cobb-Douglas function  $F(k_t, h_t) = k_t^\alpha h_t^{1-\alpha}$  and distribute profits  $y_t + (1-\delta)k_t - w_t h_t$  to I-investors as dividends. As the labor market is competitive, profits are linear in  $k$  and equal to  $\rho_t k_t$ , with  $\rho$  the equilibrium return on capital.<sup>9</sup> For expositional clarity, we assume full depreciation,  $\delta = 1$ , which gives profits  $\rho_t k_t = \alpha y_t$  and a wage bill  $w_t h_t = (1 - \alpha)y_t$ . The model will be simulated later with partial depreciation. Analytical results are extended to the case  $\delta < 1$  in Section 4.7 of the Online Appendix.

*Workers.* There is a unit measure of workers who maximize  $U_t^w = E_t \sum_{s=0}^{\infty} \beta^s \log(c_{t+s}^w)$ , where  $c_t^w$  refers to workers' consumption, subject to a sequence of budget constraints, borrowing constraints, and cash-in-advance (CIA) constraints. They have a fixed unitary labor supply, so that  $h_t = 1$  and  $y_t = k_t^\alpha$  in equilibrium. Their budget constraint is:

$$c_t^w + \frac{M_{t+1}^w}{P_t} + l_t^w = w_t + \frac{T_t^w}{P_t} + \frac{M_t^w}{P_t} + \frac{l_{t+1}^w}{r_{t+1}}, \quad (4)$$

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<sup>8</sup>We use the term liquidity in the same spirit as Woodford (1990) and Holmström and Tirole (1998).

<sup>9</sup> $\rho$  is equal to  $F(1, 1/k(w)) + 1 - \delta - w/k(w)$  where  $k(w)$  is the equilibrium capital-labor ratio defined by  $w = F_h(k(w), 1)$ .

where  $P_t l_{t+1}^w$  is the amount of nominal bonds issued in  $t$ ,  $M^w$  money holdings, and  $T^w$  a monetary transfer from the government. Workers are subject to a CIA constraint: they cannot consume more than their real money holdings. Assuming the bond market opens before the market for goods, these holdings are the sum of money carried over from the previous period, monetary transfers from the government, and money borrowed on the bond market (net of debt repayment):

$$c_t^w \leq \frac{M_t^w + T_t^w}{P_t} + \frac{l_{t+1}^w}{r_{t+1}} - l_t^w. \quad (5)$$

Workers also face a borrowing constraint which limits the real value of their debt:<sup>10</sup>

$$l_{t+1}^w \leq \bar{l}_t^w y_{t+1}. \quad (6)$$

When  $\beta r < 1$ , which we will assume throughout the analysis, (6) is binding in the vicinity of a steady state. Workers would prefer to dissave and always hold the minimum amount of money, so that the CIA (5) is also binding. Together with their budget constraint (4), this implies that their money holdings are simply equal to the wage bill:  $M_{t+1}^w/P_t = w_t$ . Since the wage bill is equal to  $(1 - \alpha)y_t$  in equilibrium, money demand by workers is given by:

$$M_{t+1}^w = (1 - \alpha)P_t y_t. \quad (7)$$

*The government.* Denote by  $M_t$  the money supply at the beginning of period  $t$ . In period  $t$ , the government can finance transfers to agents by creating additional money  $M_{t+1} - M_t$  and by issuing nominal bonds  $P_t l_{t+1}^g$ . For simplicity, we assume that the government only makes transfers to workers. The budget constraint of the government is:

$$\frac{M_{t+1}}{P_t} + \frac{l_{t+1}^g}{r_{t+1}} = \frac{M_t + T_t^w}{P_t} + l_t^g. \quad (8)$$

Several fiscal and monetary policies can be considered. As a benchmark case, we assume that the fiscal authority provides a supply of nominal bonds that is proportional in real terms

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<sup>10</sup>We assume that the borrowing limit is linear in the wage bill and therefore proportional to output, since the equilibrium wage bill is a fraction  $1 - \alpha$  of output.

to output and that the monetary authority controls the growth of money

$$l_{t+1}^g = \bar{l}_t^g y_{t+1}, \quad M_{t+1}/M_t = \theta_{t+1}. \quad (9)$$

Transfers to households then adjust to satisfy (8). In a steady state, money growth is constant and equal to  $\theta$ , which pins down steady-state inflation to  $\theta$ . We make the following parametric assumption:

**Assumption 1.**  $\theta > \beta$ .

Assumption 1 implies that the economy can only hit the ZLB in a steady state where  $\beta r < 1$ , with binding borrowing constraints. Indeed, in the steady state, the nominal gross interest rate is  $i = r\theta$ . With Assumption 1,  $i = 1$  implies  $\beta r = \beta/\theta < 1$ . This assumption is naturally satisfied as long as  $\theta \geq 1$ , that is with a non-negative steady-state inflation.

*Market clearing for bonds and money.* Equilibrium in the two markets is given by:

$$b_{t+1} + l_{t+1}^w + l_{t+1}^g = a_{t+1}. \quad (10)$$

$$M_{t+1}^S + M_{t+1}^w = M_{t+1}. \quad (11)$$

*Sequences of leverage.* The sequences of leverage  $\{\phi_t, \bar{l}_t^w, \bar{l}_t^g\}$  are exogenous and deterministic, consistent with our assumption of perfect foresight.

## 2.2. Equilibrium

*Asset scarcity and binding borrowing constraints.* We focus on equilibria where borrowing constraints for I-investors and workers are binding in every period. In such “asset-scarce” equilibria, borrowing constraints prevent borrowers from supplying the saving instruments needed by savers and steady states are characterized by  $\beta r < 1$ .

More precisely, consider an exogenous sequence of leverage  $\{\phi_t, \bar{l}_t^w\}_{t \geq 0}$ , an exogenous sequence of policy parameters  $\{\theta_{t+1}, \bar{l}_t^g\}_{t \geq 0}$ , and initial assets  $\{k_0, a_0, b_0, M_0, M_0^S, M_0^w, l_0^w, l_0^g\}$ . The associated asset-scarce equilibrium is an allocation  $\{y_t, c_t^I, c_t^S, c_t^w, k_{t+1}\}_{t \geq 0}$ , a vector of portfolio choices  $\{a_{t+1}, b_{t+1}, l_{t+1}^w, M_{t+1}^S, M_{t+1}^w\}_{t \geq 0}$ , a policy  $\{M_{t+1}, T_t^w, \bar{l}_t^g\}_{t \geq 0}$ , and a price vector

$\{r_{t+1}, \rho_{t+1}, P_t\}_{t \geq 0}$  solving the maximization problems of both groups of investors and workers with binding borrowing constraints (3) and (6), and satisfying the production function  $y_t = k_t^\alpha$ , the expression for profits  $\rho_t k_t = \alpha y_t$ , the government budget constraint (8) and policy rules (9), and the market-clearing conditions (10) and (11). We omit the gross nominal rate from that definition as it is simply given by  $i_{t+1} = r_{t+1} P_{t+1}/P_t$ . The full list of equilibrium conditions is given in Section 3.1 of the Online Appendix.

*A four-equation model.* An asset-scarce equilibrium can be reduced to a 4-dimensional system. Before doing so, it is useful to define  $m_t^S = M_t^S/P_t$ , real money holdings by S-investors, and  $\bar{l}_t = \bar{l}_t^g + \bar{l}_t^w$ , the exogenous total supply of bonds to investors as a share of output. In equilibrium,  $\bar{l}_t y_{t+1} = l_{t+1}^g + l_{t+1}^w$  is both equal to the real supply of bonds by workers and the government, and to the real net position of investors.

The dynamics of the model can be fully described by the set of variables  $\{r_{t+1}, m_{t+1}^S, k_{t+1}, P_t\}_{t \geq 0}$  which satisfies the following four equations:

$$m_{t+1}^S \left( r_{t+1} - \frac{P_t}{P_{t+1}} \right) = 0, \quad (12)$$

$$\beta \alpha (1 - \phi_{t-1}) y_t = \frac{1}{r_{t+1}} [(\phi_t \alpha + \bar{l}_t) y_{t+1} + m_{t+1}^S], \quad (13)$$

$$k_{t+1} + \frac{1}{r_{t+1}} \bar{l}_t y_{t+1} + \frac{P_{t+1}}{P_t} m_{t+1}^S = \beta [(\alpha + \bar{l}_{t-1}) y_t + m_t^S], \quad (14)$$

$$\frac{M_{t+1}}{P_t} = (1 - \alpha) y_t + \frac{P_{t+1}}{P_t} m_{t+1}^S, \quad (15)$$

where  $y_t = k_t^\alpha$ . The sequence  $\{\phi_t, \bar{l}_t, M_{t+1}\}$  is exogenous with  $M_{t+1} = \theta_{t+1} M_t$ , and there is an initial condition  $\{\bar{l}_{-1}, m_0^S, k_0, M_0\}$ .

Equation (12) is the complementary slackness condition (CSC) summarizing the optimal portfolio choice of S-investors. As long as  $i > 1$ , or equivalently  $r_{t+1} > P_t/P_{t+1}$ , money has a strictly lower expected return than bonds and investors do not hold it:  $m^S = 0$ . We refer to this case as “normal” periods. When  $i = 1$ , that is,  $r_{t+1} = P_t/P_{t+1}$ , bonds and money become perfect substitutes for savers, and investors also hold money for saving purposes, so  $m^S \geq 0$ . We refer to this case as “liquidity trap” periods.

Equation (13) directly derives from the Euler equation of S-investors. As they are uncon-

strained, their consumption satisfies the usual Euler condition:  $1/c_t^S = \beta r_{t+1}/c_{t+1}^I$ . With log-utility, consumption is a fraction  $1 - \beta$  of wealth for both types of investors:  $c_{t+1}^I = (1 - \beta)(a_{t+1} + m_{t+1}^S)$  and  $c_t^S = (1 - \beta)(\alpha y_t - b_t)$ .<sup>11</sup> Substituting these expressions into the Euler equation, and using the binding borrowing constraints (3) and (6), and the market clearing condition for bonds (10), we get (13). This equation can also be interpreted as an equilibrium condition for saving instruments. The left-hand side (LHS) is the demand for saving instruments by S-investors, which depends on current income. The right-hand-side (RHS) is the supply of saving instruments. The first term is the supply of bonds, which depends on future pledgeable income and on the leverage ratio  $\phi$  of I-investors. The second term depends on  $\bar{l}$ , and represents the supply of bonds by workers and the government. Finally, the last term on the RHS corresponds to money used by S-investors as a saving instrument.

Equation (14) is the aggregate budget constraint of I-investors and S-investors, which describes capital accumulation. It obtains by aggregating (1) and (2), substituting for consumption, and using the bond market clearing condition (10). In the aggregate, investors save a fraction  $\beta$  of profits, money holdings and maturing bonds (on the RHS), which they use to get capital, bonds and money (and the LHS).

Equation (14) provides some intuitions as to how money interacts with capital accumulation. First, on the LHS, an increase in  $m_{t+1}^S$  implies a lower capital stock, because other things equal the corresponding funds are not channeled to I-investors. This is the *crowding-out effect of money*. For a given level of  $m_{t+1}^S$ , the crowding-out effect is stronger if inflation, i.e. the price of (real) money, is larger. Second, from the RHS, a larger  $m_t^S$  enables to increase the capital stock, because it can be liquidated to finance investment. This is the *liquidity effect of money*. This effect is stronger if  $\beta$  is larger, because I-investors use a higher share of their wealth to invest. The bond's external position of investors has similar effects, except that the price of liquidity in the case of bonds is not inflation but  $1/r_{t+1}$ . Of course, inflation and the real interest rate are equilibrium objects which react to shocks affecting money holdings and capital, but we will show in the next section that these intuitions still apply when we solve for the equilibrium.

Finally, Equation (15) is the money market equilibrium (11), where  $M^w$  has been substituted

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<sup>11</sup>The proof of this property is available upon request.

for using (7). Money supply has to be equal to the demand for money for transaction purposes plus the demand for saving purposes. With flexible prices, this equation ensures that any real demand for money can be met through a price adjustment.

*Normal and liquidity-trap steady states.* In the next section, we will first focus on steady state equilibria. Suppose  $\phi, \bar{l}$  are constant and  $M_t$  grows at a constant gross rate  $\theta$ . A steady state can be characterized by constant  $r, m^S, k$ , and a constant inflation rate  $P_{t+1}/P_t = \theta$ , satisfying (12) to (14). The Euler equation (13) and the aggregate budget constraint (14) become

$$\beta r \alpha (1 - \phi) y = (\phi \alpha + \bar{l}) y + m^S, \quad (13')$$

$$k = \beta \alpha y - \left( \frac{1}{r} - \beta \right) \bar{l} y - (\theta - \beta) m^S, \quad (14')$$

with  $y = k^\alpha$ . The CSC (12) becomes  $m^S(r - \theta^{-1}) = 0$  and implies that there are two types of steady states: *normal steady states*, with  $r > \theta^{-1}$  (or  $i > 1$ ) and  $m^S = 0$ , and *liquidity-trap steady states*, with  $r = \theta^{-1}$  (or  $i = 1$ ) and  $m^S > 0$ . The path of prices  $P_t$  is determined by (15).

### 3. The Impact of Investors' Deleveraging

This section studies the effects of deleveraging, modeled by a drop in  $\phi$ . We first consider permanent shocks, which allows us to study analytically changes in steady states. This provides useful insights as to the asymptotic effects of very persistent deleveraging shocks. Then we simulate a persistent but non-permanent deleveraging shock in an extended version of the model with nominal rigidities.

#### 3.1. Steady-state Impact of Permanent Deleveraging

A deleveraging shock leads to an excess demand for saving instruments by investors. In normal equilibria, adjustment comes from a lower equilibrium interest rate which helps restore a higher supply of bonds. In the liquidity trap, as the interest rate cannot adjust, the higher net demand for saving instruments by investors takes the form of higher money holdings. This diverts resources away from investment and leads to lower capital in the medium run.

In the following, we focus on the case  $\bar{l} = 0$  where investors are in autarky: S-investors lend to I-investors. In addition to being simpler, this is also a realistic description of the US prior

to the crisis: we show in Section 2.1 of the Online Appendix that the net position in financial assets of non-financial corporate businesses was indeed close to 0 in the years 2000 prior to the crisis. Afterwards, we briefly describe how the analysis would change with  $\bar{l} < 0$ .

*Normal steady state.* Consider first a normal steady state with  $m^s = 0$ . When  $\bar{l} = 0$ , the aggregate budget constraint (14') determines the capital stock independently of leverage  $\phi$  and the real interest rate  $r$ :

$$k = \beta\alpha y = \beta\alpha k^\alpha. \quad (16)$$

While leverage matters for the distribution of wealth between investors, it has no effect on the capital stock. Indeed, for a given interest rate, the shock generates a decrease in the bond supply  $b$  by I-investors. Besides, as S-investors start the period with less debt, it increases their wealth and hence their demand for bonds  $a$ . Since the net supply of bonds by the rest of the economy remains unchanged at zero, adjustment to deleveraging takes place through a decrease in the interest rate, which equates the demand for bonds by S-investors with the supply by I-investors. Intuitively, savings need to be channeled to investment in equilibrium, whatever the level of  $\phi$ , and the decrease in interest rate achieves just that.<sup>12</sup> This is clear from equation (13'), which determines  $r$  in the normal steady state as

$$r = \frac{\phi}{\beta(1 - \phi)}. \quad (17)$$

Notice that a decrease in  $r$  implies a proportional decrease in  $i = r\theta$  for a given steady-state inflation rate  $\theta$ . Therefore, a strong contraction of credit may lead to the ZLB. This is the case when  $\phi/[\beta(1 - \phi)] \leq 1/\theta$ . Similarly, a high enough  $\phi$  brings the equilibrium interest rate at  $1/\beta$ . Beyond this, the credit constraint is not binding anymore.

*Liquidity trap.* If  $i$  hits the ZLB, the equilibrium becomes a liquidity trap. The effective real interest rate is simply  $1/\theta$ . We define the shadow real interest rate  $r^s$  as the rate that would

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<sup>12</sup>Note that the log-utility implies that the change in interest rate does not affect saving, as the intertemporal elasticity of substitution is equal to one. If this elasticity was larger (lower) than one, then saving would decrease (increase) and hence investment.

prevail if the ZLB were not binding. It is given by the RHS of (17), i.e.,  $r^s = \phi/[\beta(1 - \phi)]$ .<sup>13</sup> We then define the interest rate gap as the difference between the effective and the shadow interest rates:

$$\Delta \equiv r - r^s = \frac{1}{\theta} - \frac{\phi}{\beta(1 - \phi)}$$

We think of the magnitude of this gap as the depth of the liquidity trap.

In a liquidity trap steady state, the Euler equation (13') determines real money holdings:

$$m^S = \alpha \left[ (1 - \phi) \frac{\beta}{\theta} - \phi \right] y. \quad (18)$$

The ratio  $m^S/y$  is decreasing in  $\phi$ : an increase in investors' net demand for saving instruments triggered by a deleveraging shock is now accommodated by an increase in their real money holdings  $m^S$ . It is also interesting to notice that  $m^S$  is proportional to the interest rate gap  $\Delta$ :  $m^S = \kappa \Delta y$ , where  $\kappa = \alpha\beta(1 - \phi)$ . The magnitude of investors' real money demand is therefore also a measure of the depth of the liquidity trap.

This switch to money takes out resources from investment, as suggested by (14'), which becomes in a liquidity trap

$$k = \beta\alpha y - (\theta - \beta)m^S. \quad (19)$$

From Assumption 1, we have  $\theta > \beta$  and holding additional money entails a net resource cost that decreases the steady-state capital stock. Indeed, in the steady state, the cost of saving in money for S-investors,  $P_{t+1}/P_t = \theta$ , is larger than the I-investors' propensity to use money holdings for investment  $\beta$ . This implies that the crowding-out effect of money overcomes its liquidity effect.

Notice that asset scarcity is crucial here. First, it generates a persistent drop in interest rate, making the liquidity trap persistent. Second, asset scarcity means that the return on bonds, and hence the return on money in the liquidity trap, is below  $1/\beta$ , so bond or money accumulation in the liquidity trap is costly.

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<sup>13</sup>The shadow rate goes to 0 when  $\phi$  goes to 0. This is an extreme situation where savers, absent money, would have no instruments to trade intertemporally. Section 5 introduces an alternative inefficient saving technology, which puts a strictly positive lower bound on the shadow rate.



The net resource cost for investors arises because of a real balance effect together with an inflation tax, as can be seen by rewriting Equation (19):

$$k = \beta\alpha y - \underbrace{(\theta - 1)m^S}_{\text{inflation tax}} - \underbrace{(1 - \beta)m^S}_{\text{extra consumption}} .$$

Because cash is considered as net wealth by investors (a consequence of the non-Ricardian structure of the model), they consume a fraction  $1 - \beta$  of it. Consequently, as more financial wealth is accumulated by investors through real money balances, they consume more, and hence invest less, out of their revenues. In addition, a fraction  $\theta - 1$  of cash is lost as an inflation tax, which decreases investors' revenues and further decreases investment.<sup>14</sup>

The upward adjustment in investors' real money holdings  $m^S$  takes place through disinflation. From (15) taken in the steady state, we have  $M/P = (1 - \alpha)y/\theta + m^S$ . Since workers' money holdings always equal their wage bill, total real money supply  $M/P$  has to increase. For a given path of money supply, given by (9), this obtains through a downward shift in the path of prices  $P_t$ . Using this analysis, we establish the following Proposition:

**Proposition 1 (Steady state with autarkic investors).** *Define  $\phi_T = \beta/(\theta + \beta)$  and  $\phi_{max} = 1/2$ . If  $0 < \phi < \phi_{max}$ , then there exists a locally constrained steady state with  $r < 1/\beta$ .*

- (i) *If, additionally,  $\phi \geq \phi_T$ , then the steady state is normal.*
- (ii) *If  $\phi < \phi_T$ , then the steady state is a liquidity trap.*
- (iii) *In the normal steady state, the real interest rate  $r$  and the nominal interest rate  $i$  are increasing in  $\phi$ ,  $m^S = 0$  and  $k$  is invariant in  $\phi$ .*
- (iv) *In the liquidity-trap steady state, the real interest rate  $r$  is invariant in  $\phi$ ,  $m^S/y$  is decreasing in  $\phi$  and  $k$  is increasing in  $\phi$ .*

**Proof.** See proof in the Online Appendix. ■

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<sup>14</sup>This tax is redistributed to workers through transfers. This second effect would be lower if investors also received transfers from the government.

This Proposition establishes under which condition on  $\phi$  the steady state is normal or a liquidity trap. It is illustrated in Figure 2. The solid lines show the levels of  $k$ ,  $r$ , and  $m^S$  as a function of  $\phi$ , while the broken lines show the levels of the shadow rate  $r^s$  and of  $k$  and  $m^S$  if the ZLB were not binding. For intermediate values of  $\phi$  (between  $\phi_T$  and  $\phi_{max}$ ), the normal real interest rate  $r$  is higher than  $1/\theta$ , and the steady state is normal as the nominal interest rate  $i$  is above the ZLB, as is illustrated by equilibrium  $C$ . When  $\phi$  falls below  $\phi_T$ , the steady state becomes a liquidity trap where the effective interest rate is  $r = 1/\theta$ , larger than the shadow rate  $r^s$ . It is characterized by positive real money holdings among investors, for saving purposes, as illustrated by point  $T$ .

As long as the economy is in the normal steady state (when  $\phi > \phi_T$ ), a permanent deleveraging shock on investors (a decrease in  $\phi$ ) has no effect on capital, but it has a negative effect on the real interest rate  $r$ . But a deleveraging shock large enough to make the economy fall into a liquidity trap ( $\phi < \phi_T$ ) has negative steady-state effects on capital and output. A permanent shock bringing the economy from  $C$  to  $T$  is then consistent with a lower output. The effects come from the disinvestment due to the resource cost of money, thus from the supply side of the economy. This contrasts with the recent literature, where long-run stagnation is driven by a fall in consumption demand in the presence of persistent nominal rigidities.

The fact that higher money holdings come with lower capital and output in the steady state does not imply that investors would be better off if money did not exist. By putting a lower bound on the real rate of interest, money helps investors better smooth consumption across time. Under a mild assumption on the degree of decreasing returns to scale to capital,  $\alpha$ , this can be shown to make both groups of investors better off in a liquidity trap steady state than they would be in the corresponding normal steady state, despite the lower capital stock (see Section 5.1 of the Online Appendix). Workers may however be hurt by lower wages.

When investors are net debtors, we have  $\bar{l} < 0$ . The real interest rate is then increasing in  $\bar{l}$ :

$$r = \frac{\phi + \bar{l}/\alpha}{\beta(1 - \phi)}. \quad (20)$$

Moreover,  $r$  has a redistributive effect between investors and workers, which affects capital accumulation: when  $\bar{l} < 0$ , a lower interest rate reduces the cost of debt and allows investors

to accumulate more capital. This implies that a deleveraging shock actually *increases* the steady-state capital stock in the normal economy.<sup>15</sup>

In a liquidity trap however, a deleveraging shock still has a negative effect on capital. In that case, as money and bonds are perfect substitutes, capital accumulation is affected by the total amount of net liquidity  $s = m^S + \bar{l}y$ , which plays the same role as cash holdings in the autarky case. Notice that we still have  $m^S = \kappa\Delta y$ . We therefore refer to  $\bar{l}y$  as shadow liquidity, since  $s = \bar{l}y$  when  $\Delta = 0$ . Further details of this case are found in the Online Appendix.

### 3.2. Simulated Impact of Transitory Deleveraging

Steady state comparisons are helpful to derive closed-form solutions and facilitate the analysis, but they imply a permanent liquidity trap and abstract from transition dynamics. We now consider a transitory deleveraging shock, using an extended version of the model described in Section 3.2 of the Online Appendix. There are three main differences with the benchmark model. First, capital only partially depreciates. Second, the deleveraging shock is persistent, but not permanent. Leverage  $\phi_t$  is now a stochastic variable that can take two values:  $\phi^H$  in normal times and  $\phi^L$  for deleveraging. After a deleveraging shock hits, there is a probability  $\lambda$  in each period to switch back to  $\phi^H$  and stay there. This introduces aggregate uncertainty in the model. Third, to discuss transition dynamics in a meaningful way, we introduce a downward nominal wage rigidity, in the spirit of Schmitt-Grohé and Uribe (2016). The nominal wage, defined by  $W_t = P_t w_t$ , must satisfy

$$W_t = \max \{ \gamma W_{t-1}, W_t^* \} \tag{21}$$

where  $\gamma \in (0, \theta)$  is the degree of nominal rigidity and  $W^*$  is the nominal wage that would satisfy full employment:  $W_t^* = p_t(1 - \alpha)k_t^\alpha$ . If  $W_t^* \geq \gamma W_{t-1}$ , wages can adjust and there is full employment. Otherwise, there is unemployment:  $h_t < 1$ , where  $h_t$  is determined by  $\gamma W_{t-1} = p_t(1 - \alpha) \left( \frac{k_t}{h_t} \right)^\alpha$ . These rigidities are not active in the steady state where prices grow

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<sup>15</sup>The positive effects on capital accumulation of financial frictions is not an uncommon result: uninsurable risk and credit constraints in Bewley-Aiyagari models notoriously leads to an over-accumulation of capital. See Aiyagari (1994), Krusell and Smith (1997), Covas (2006), and Dávila et al. (2012).

at rate  $\theta$ , which is by assumption larger than  $\gamma$ , so our steady state analysis is still valid. But nominal rigidities can affect the short-term adjustment to a deleveraging shock.

With nominal rigidities, a deleveraging shock large enough to move the economy to the ZLB creates a negative output gap in the short run, as in the New Keynesian literature. The intuition is best described by Equation (15), the market-clearing condition for money:  $M_{t+1} = (1 - \alpha)P_t y_t + M_{t+1}^S$ . When the economy hits the ZLB, money demand by investors  $M^S$  increases. If the monetary authority does not react, adjustment has to come from a lower nominal output  $P_t y_t$ . If prices adjust slowly, adjustment in the short run requires a drop in output. This takes place through the labor margin, which will not be at full employment.

*Calibration.* The model is calibrated to fit the recent experience of the US at the ZLB. The time period is a year. We calibrate the balance sheet parameters  $\bar{l}^g$  and  $\bar{l}^w$  to match their empirical counterparts in the US in 2006. We show in Section 2.2 of the Online Appendix that the net position of the general government and the monetary authority in interest-bearing assets was about 40% of GDP. However, the net position of the rest of the world in these instruments was about -40% of US GDP. The net supply available to the domestic economy is thus approximately 0. With the assumption of autarkic entrepreneurs, this implies  $\bar{l}^g = \bar{l}^w = 0$ .

The discount factor  $\beta$  is set to 0.96 and  $\phi^H$  to 0.495 in order to match a real interest rate of 2%, consistent with the 10-year TIPS before the crisis, and a real rate of return on capital of 4% which implies a realistic 200 bp corporate spread. We make conventional choices for the capital share  $\alpha = 0.33$ , the depreciation rate  $\delta = 0.10$ , and we set  $\theta = 1.02$  to get a steady state inflation of 2%.

To discipline the choice of  $\phi^L$ , which gives the extent of deleveraging, and the degree of nominal rigidity  $\gamma$ , which drives the increase in unemployment, we match the response of investment and unemployment during the crisis in the US. We set  $\phi^L = (1 - 0.039)\phi^H$  and  $\gamma = 1.01$  to reproduce the 20% peak-to-trough variation of non-residential investment and the 5.5 pp increase in civilian unemployment of the data.<sup>16</sup> Finally, we set  $\lambda$  at 10% per year, which implies a 10 year average duration of liquidity traps.

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<sup>16</sup>Our calibration of  $\gamma$  implies a 1% lower bound on wage inflation. With 2% steady-state inflation, this implies that real wages downwardly adjust by at most 1% per year.

*Results.* The impact of a transitory deleveraging shock is shown in Figure 3. The shock hits in period 1 and is reversed after 10 years in period 11. The dashed blue line represents the baseline case without nominal rigidities. The thin black solid line represents the outcome in the absence of ZLB. The drop in  $\phi$  generates both a drop in the supply of and a rise in the demand for assets by investors. In the absence of ZLB, the real interest rate accommodates this excess demand for assets by dropping substantially (panels a and b). The large decrease in real rate compensates the tightened financial constraint, allowing borrowing to increase (panel c), and accommodates higher saving. With autarkic investors, capital and output are unaffected. In contrast, with the ZLB,  $i_t$  is stuck at its lower bound, which prevents the real rate from adjusting. The excess demand for saving by investors is then channeled to money:  $M^S$  increases (panel d). The increase in the demand for money is accommodated by the fall in prices (panel e). Because the real rate decrease does not compensate the tightening of the financial constraint, borrowing decreases (panel c) and the capital stock drops on impact (panel f).

After the initial drop, the capital stock recovers somewhat, but does not go back to its initial value. As long as the economy stays in the liquidity trap, it remains persistently low. This medium-run effect corresponds to the effect of deleveraging due to the cost of liquidity highlighted in the steady state analysis. Output shows the same pattern as capital: an initial drop followed by a capped recovery (panel h).

Consider now the thick red line, which is drawn under the assumption of downward wage rigidity. This rigidity prevents the decrease in nominal wage needed to clear the labor market. As a result, there is unemployment, which amplifies the output drop. It also worsens the financing capacities of investors further and hence lowers the capital stock even more. On impact, labor, output and capital are more strongly hit than with flexible wages (panels f, g, and h). These demand effects are strong, but they only take place in the short run. As time goes by, real wages adjust and all variables converge toward their level under flexible wages.

The short-run impact of deleveraging is thus stronger than in the medium run, and even more so in the presence of nominal rigidities. But in the medium run, the effects caused by the scarcity of assets prevail. Contrary to the New Keynesian literature, the economy lingers at the ZLB with a lower capital stock and output level, even after wages have adjusted and the

output gap has closed.

#### 4. Policy

There is a range of macroeconomic policies that have been implemented or considered in the context of a liquidity trap. In this section, we consider the most relevant policies and study their implications in the current model. We start by showing that monetary policy can be very effective in the short run. We then take a medium-run perspective, abstracting from nominal rigidities, and consider various policies aimed at exiting the liquidity trap: negative interest rates, higher inflation target, QE, or an increase in government bonds. We show that these policies have distributional effects and generate trade-offs inside and outside the liquidity trap.

Exiting from a liquidity trap implies driving the interest rate gap to zero. We have:

$$\Delta = \frac{i}{\theta} - \frac{\phi + \bar{l}/\alpha}{\beta(1 - \phi)}$$

While a strict ZLB implies  $i = 1$ , we can allow  $i < 1$  to analyze the impact of negative rates. The authorities can eliminate the interest rate gap either by decreasing the effective rate or by increasing the shadow rate. These two approaches have different implications as they imply a different real interest rate level. Besides the potential distributional effects on lenders and borrowers, the interest rate level affects intertemporal allocations and investment efficiency. For example, a low effective rate promotes investment, but distorts intertemporal consumption choices. We discuss the welfare implications and the first-best policy. We assume a constant  $\phi$  throughout the section.

##### 4.1. Helicopter Money

Previous studies focus on short-term effects in the presence of nominal rigidities so that demand-side policies are paramount. These effects are similar in our context when we introduce nominal rigidities. However, there is a policy that is particularly efficient. We can show that a monetary expansion taking the form of transfers to workers (“helicopter money”) can almost replicate the flexible wage equilibrium (see Section 4.4 of the Online Appendix). While this policy has no effect at the ZLB in standard models, it is efficient in our non-Ricardian

framework.<sup>17</sup> By increasing money supply, the monetary authority accommodates the demand for money by investors, making it unnecessary to decrease output or the price level. Monetary policy is then potent to mitigate the short-run impact of deleveraging, but is unable to address its medium-run impact (unless it changes the inflation target itself, see below). Since these monetary transfers can correct the impact of nominal rigidities, we can examine the medium-term impact of policies by assuming wage flexibility.

#### 4.2. Lowering the Effective Real Interest Rate

Decreasing the effective rate could be done by increasing expected inflation through an increase in  $\theta$ . This is a natural solution mentioned in the literature (e.g. Krugman, 1998). Alternatively, there could be a negative nominal interest rate.<sup>18</sup> A lower effective real rate can make the economy exit the ZLB, restoring a higher level of capital and output. Workers would then be better off thanks to higher wages, but the lower interest rate would impair consumption smoothing. Exiting the ZLB by reducing the effective real interest rate drives out monetary liquidity without providing alternative liquidity and solving the underlying asset scarcity problem.

If the effective rate is not lowered completely to the shadow rate, this has an ambiguous effect on capital and output. Consider a negative interest rate.<sup>19</sup> Besides a decline in the effective real interest rate, this also increases the cost of holding money. This negative effect dominates when money holdings are large. In fact, we show in Section 4.3 of the Online Appendix that, in a liquidity trap, a lower effective rate has a negative effect on capital at the margin if both  $\phi$  and  $\theta$  are low, as they induce high money holdings. However, a decrease in the effective rate that is large enough to drive the economy out of the liquidity trap has a positive effect on the capital stock, as it suppresses the need to hold money.

Figure 4 illustrates this by showing the effect of a negative interest rate policy implemented when the shock hits and for as long as it lasts. In the benchmark case of panel A, only a very

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<sup>17</sup>In Krugman (1998), for instance, money creation taking the form of transfers has no effect at the ZLB with pre-set prices (see footnote 11 of this work).

<sup>18</sup>Suppose that cash is replaced by Central Bank digital money, on which a negative interest rate can be charged. There would then be no ZLB on the nominal interest rate and we could have  $i < 1$ . A liquidity trap with a negative interest rate is a situation currently observed in several countries.

<sup>19</sup>A similar analysis holds for small increases in the inflation rate  $\theta$ .

negative interest rate of  $-4\%$  (thick dashed red line) allows the economy to avoid the liquidity trap. More timid policies of  $-0.5\%$  and  $-1\%$  (solid red lines) keep the economy in the trap but support capital and output. For a stronger and more persistent deleveraging shock (panel B), lower interest rates only supports capital and output in the short run. Eventually, a negative rate of  $-4\%$  even lead to slightly lower capital and output.

### 4.3. Enhancing Shadow Liquidity

The alternative to eliminate the interest rate gap is to raise the public supply of liquidity. Even though this increase has no impact at the ZLB, it increases the shadow interest rate. We consider both an increase in the supply of government debt and QE policies.

*Public Debt and the ZLB.* An increase in the supply of government bonds, by increasing  $\bar{l}$ , can bring the economy out of the liquidity trap by increasing the shadow interest rate and shadow liquidity. However, marginal changes in  $\bar{l}$  only affect shadow values as long as the economy remains in the liquidity trap, consistently with the “irrelevance result” highlighted in the literature and earlier in this paper.

Interestingly, an adequate supply of liquidity enables the economy to reach a Pareto-*efficient* equilibrium, as shown in Section 5.2 of the Online Appendix. Indeed, by raising the real interest rate, this enables optimal consumption smoothing by all agents as well as the optimal level of capital (see Proposition 9 in the Online Appendix).<sup>20</sup> However, as we will see, this does not imply that transition dynamics are Pareto-efficient, nor that the new steady state Pareto-*improves* on the initial one.

*Quantitative Easing.* The above analysis implies that QE has no effect per se in the liquidity trap steady state. QE consists in creating money through open market operations, i.e., increasing  $M$  by decreasing  $Pl^g$ . Since money and government bonds are perfect substitutes, this has no effect in our setting.<sup>21</sup> However, QE entails a decrease in the available amount of government

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<sup>20</sup>The proposition shows that the efficient level of capital is given by  $k = \beta\alpha y$ . This level obtains when  $r = 1/\beta$ , which also corresponds to perfect consumption smoothing, and requires a high enough public debt  $\bar{l} = \bar{l}_{max}(\phi)$ . In the case where investors are net debtors out of the ZLB, capital is too high compared to a Pareto-efficient allocation and a higher public debt crowds out this inefficiently high capital stock.

<sup>21</sup>Note that we abstract from some potential channels of QE. In particular, the perfect substitutability of money and bonds means that there is no broad portfolio balance channel that could lower term or risk premia.



bonds  $l^g$ , which decreases shadow liquidity and the shadow interest rate. QE therefore leads to a deeper liquidity trap.

Figure 5 illustrates the effect of QE with a late or early exit. We suppose that the central bank implements QE by buying bonds worth 10% of GDP when the shock hits in period 1.<sup>22</sup> The dashed blue line displays the case of early exit where the central bank sells the bonds in period 11 when deleveraging stops. The thick red line represents the case of late exit where the central bank announces in period 11 that it will hold the bonds for four additional years, and does so.<sup>23</sup> As a benchmark, the thin black line reproduces the case without QE.

As QE reduces the level of public debt available to investors (panel a), it increases the interest rate gap  $\Delta$  (panel d). Inside the liquidity trap, this has no effect on real variables and only changes the composition of assets held by investors. When exit is early, QE has therefore no real effects. A late exit, by contrast, has a substantial impact on the economy. Absent QE or with an early exit, the interest rate would increase to slow down investment as the economy relevels, bringing the capital stock close to its steady state level. Since QE deprives the economy of bonds, the interest rate stays stuck at the ZLB (panel b) and the investment boom goes unhampered (panel g). Instead, adjustment comes again from the price level, leading to stronger inflation than with an early exit (panel f). This is only temporary. As agents expect QE to be eventually reversed, the price level slowly decreases back to its steady state level, increasing the real rate and hurting capital and output. The interest rate only leaves the ZLB when QE is finally undone. Therefore, with a late exit from QE, the economy overheats, before plummeting again, instead of quickly going back to normal.

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It could also be that QE goes hand-in-hand with credit easing aimed at improving credit conditions for the private sector, which can alleviate the effect of deleveraging. This would consist in the government issuing new debt to lend to credit constrained investors, effectively relaxing their constraint. This does not affect government net debt but can help in getting out of the liquidity trap by increasing the shadow interest rate through a re-leveraging by investors.

<sup>22</sup>According to H.4.1 Federal Reserve statistical releases, the large-scale asset purchase programs of 2010–2014, usually referred to as QE2 and QE3, increased the amount of securities held at the Federal Reserve by 9 percent of GDP. The total increase since 2006 amounts to 17 percent of GDP.

<sup>23</sup>In the simulation presented in Figure 5, this announcement comes as a surprise. Section 4.11 of the Online Appendix presents the case where late exit is expected from the start.

*Public Debt and Capital.* Getting out of the liquidity trap through a higher public supply of liquidity, while leading to better consumption smoothing thanks to a higher interest rate, has an ambiguous impact on capital accumulation and output. This impact depends on the level of liquidity  $\bar{l}$  that prevails at the exit of the liquidity trap and of the size of the deleveraging shock. We show in Section 4.2 of the Online Appendix that in a normal equilibrium capital is a U-shaped function of liquidity  $\bar{l}$ . In our benchmark case where investors are initially in autarky, a moderate increase of public debt can lead to lower capital as the economy exits the liquidity trap when the deleveraging shock is not too strong. For a large enough increase of debt or for a strong enough deleveraging shock, capital always increases as the economy exits the trap.

This is illustrated in Figure 6. Panel A illustrates a policy that increases debt by 5% of GDP in 2 years.<sup>24</sup> This is enough to lift the economy out of the ZLB (panel A.2) but it leads to lower capital and output as long as deleveraging lasts (panels A.3 and A.4). Panel B shows the example of a stronger and more persistent shock, with a larger debt increase (by 18% of GDP). In that case, the economy exits the ZLB without a negative effect on capital and output.

#### 4.4. *Welfare and Pareto Efficiency*

While an increase in liquidity makes the economy *converge* to a Pareto-efficient steady state, the whole equilibrium including transition dynamics is not in general a Pareto equilibrium. The higher interest rate indeed initially hurts borrowers and temporarily decreases investment even lower than its liquidity trap level. In addition, the new steady state does not always Pareto-*improve* on the initial one, as in the case mentioned above where capital decreases in the medium run, which lowers wages and hurts workers.

Addressing these two problems requires many additional policy instruments. Section 5.3 of the Online Appendix shows how three additional taxes/subsidies make it possible for the policy maker to implement a Pareto-efficient equilibrium path (including the transitory dynamics) that Pareto-improves on the initial liquidity trap.

## 5. Extensions

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<sup>24</sup>The budget is balanced by making a corresponding transfer to workers.

*Workers' Deleveraging.* We have considered so far a deleveraging shock on investors, modeled as a decline in  $\phi$ . Likewise, a deleveraging shock on workers can be modeled by a drop in  $\bar{l}$ , coming from a drop in  $\bar{l}^w$ .<sup>25</sup> Such a shock limits the economy's supply of assets and has a similar effect on the interest rate  $r$  as a deleveraging shock on investors, as can be seen from Equation (20). Workers' deleveraging can therefore also lead to the zero lower bound. However, once the economy is in a liquidity trap, changes in  $\bar{l}$  have no effect as they are offset by changes in money demand. In contrast to the deleveraging shock on investors, this does not affect the investors net *demand* for assets, which is the source of disinvestment.<sup>26</sup>

*Bubbles.* In our framework with scarce assets, rational bubbles can provide additional saving instruments to accommodate the demand for assets by S-investors. A bubble, when it emerges, provides enough liquidity to exit the ZLB. But it also constrains the real interest rate and prevents the natural equilibrium adjustment. Section 4.5 of the Online Appendix shows that if a bubbly steady state exists, it has a zero *real* interest rate:  $r = 1$ . With positive steady state inflation ( $\theta > 1$ ), this is higher than  $1/\theta$ , the real of return of money, so the bubble strictly dominates money as a saving instrument. The bubble then raises the nominal interest rate from  $i = 1$  to  $i = \theta$ , and S-investors substitute the bubble for money in their portfolio. For a given money supply, this also reflate the economy as the price level increases to accommodate the lower money demand. However, the bubbly steady state is qualitatively similar to a liquidity trap: the bubble plays the same role as investor-held money in the liquidity trap, but with a higher real return. As with money, holding the bubble takes out resources from investment and output is lower in the bubbly equilibrium than in the normal steady state.

*Preference and Growth Shocks.* In the existing literature, the shock that brings the economy to the ZLB is often assumed to be an increase in the factor of time preference. This shock, by increasing the agents' propensity to save, has a negative effect on the interest rate. A reduction in the average growth rate of productivity has also been put forward as an explanation for

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<sup>25</sup>Note that this shock might imply a positive net position of workers ( $\bar{l}^w < 0$ ). This is consistent with a high proportion of wealthy hand-to-mouth households, that is, households who own sizeable amounts of illiquid assets (like retirement accounts) but hold little liquid assets, as documented by Kaplan et al. (2014).

<sup>26</sup>Outside the liquidity trap, workers' deleveraging has a positive effect on capital as investors become net debtors. See Section 4.2 of the Online Appendix.

the secular decrease in the interest rate and for hitting the ZLB. In fact, in an infinite-horizon model, the effect of a growth slowdown is isomorphic to an increase in the factor of time preference. We therefore restrict our analysis to the latter. We find that a permanent increase in  $\beta$  (alternatively, a permanent fall in steady-state growth), cannot generate a fall in the investment rate when the economy falls into a liquidity trap.

Indeed, we show in Section 4.6 of the Online Appendix that an increase in  $\beta$  makes the interest rate fall, and eventually hit the ZLB. In both the normal and liquidity-trap steady states, an increase in  $\beta$  increases the investors' propensity to save, which increases the capital stock in the medium run. As a result, whereas an increase in  $\beta$  can explain the emergence of a liquidity trap, it cannot explain the persistent slowdown of investment. In the presence of trend growth, the same conclusions would hold in case of a growth slowdown. In particular, with lower trend growth, less investment is required to keep the capital stock on its trend. Therefore a given amount of saving leads to an upward shift in the capital intensity of production, and hence in the investment rate.

*Financial Intermediation.* In the benchmark model, money is modeled as *outside money* directly supplied by the government. However, in practice, cash holdings usually take the form of deposits, which are a liability of banks, and could in principle be intermediated to capital investment. We show in Section 4.8 of the Online Appendix that this is not the case. At the ZLB, banks are unable to channel deposits to credit-constrained I-investors for the same reason that savers are unable to do so in the benchmark model. Instead, banks increase their excess reserves at the central bank.

*Inefficient saving technology.* The benchmark model assumes that bonds and money are the only available saving instruments. In Section 4.9 of the Online Appendix, we extend the model by allowing for an inefficient storage technology, with a rate of return  $\sigma \in (\theta^{-1}, \beta^{-1})$  and concave installation costs. This technology starts being used by savers when the interest rate falls down to  $\sigma$ . Then, a moderate deleveraging shock reallocates saving to the storage technology, which crowds out “good” capital even in the normal equilibrium. This reallocative effect is similar to the one studied by Buera and Nicolini (2016). With a large enough deleveraging shock, the economy falls into the liquidity trap, the use of inefficient storage is pinned down by the

real rate of interest  $1/\theta$ , and higher money holdings crowd out capital as in the benchmark model. One difference with the benchmark model is that the shadow rate now has a strictly positive lower bound as  $\phi$  goes to 0, since the storage technology prevents a complete collapse of intertemporal trade, arguably a more realistic feature.

## 6. Conclusions

We explore the medium-term implications of a liquidity trap and find that a deleveraging shock may lead to a negative comovement between capital and investors' cash holdings. We analyze policies in a liquidity trap by examining their impact on the wedge between the effective real interest rate and the shadow rate. While most of our analysis is conducted in a stylized benchmark model, the main mechanism is robust to many extensions. Our theoretical results are derived with a permanent deleveraging shock for investors, but we show in simulations that they also obtain in the medium run for persistent shocks with nominal rigidities.

Medium-term output declines in a liquidity trap only with investors' deleveraging. Other positive shocks to saving, like workers' deleveraging or an increase in the discount rate, may also lead to a liquidity trap, but they do not depress output in the medium run. Therefore it is crucial to determine the factors that have led to a liquidity trap. Interestingly, Galí et al. (2012) suggest that financial shocks have played a key role in the slow recovery.

Overall, our approach is complementary to Keynesian analyses that stress the role of insufficient demand in a liquidity trap. While they describe a situation of negative output gap when the adjustment of prices is hampered by nominal rigidities, we show that low investment demand leads to lower potential output even after prices have fully adjusted. Our framework also enables to examine policies that are complementary to more standard demand management. In this context, we find that quantitative easing can deepen and possibly lengthen the liquidity trap. We also discuss the respective trade-offs of policies aiming at decreasing the effective real interest rate and policies aiming at increasing the shadow rate.

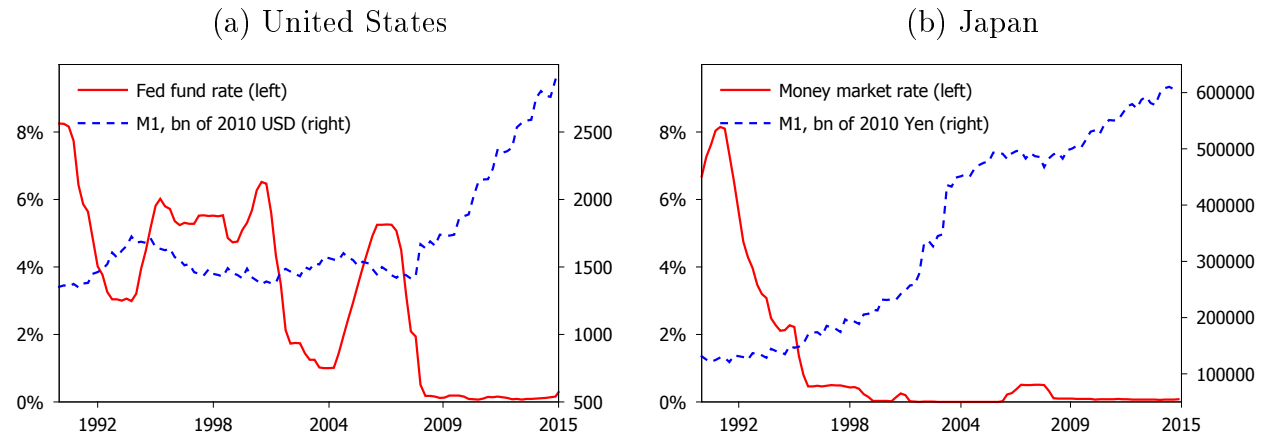
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Source: International Financial Statistics. M1 is deflated by the CPI.

Figure 1: Policy rates and M1 in the US and Japan.

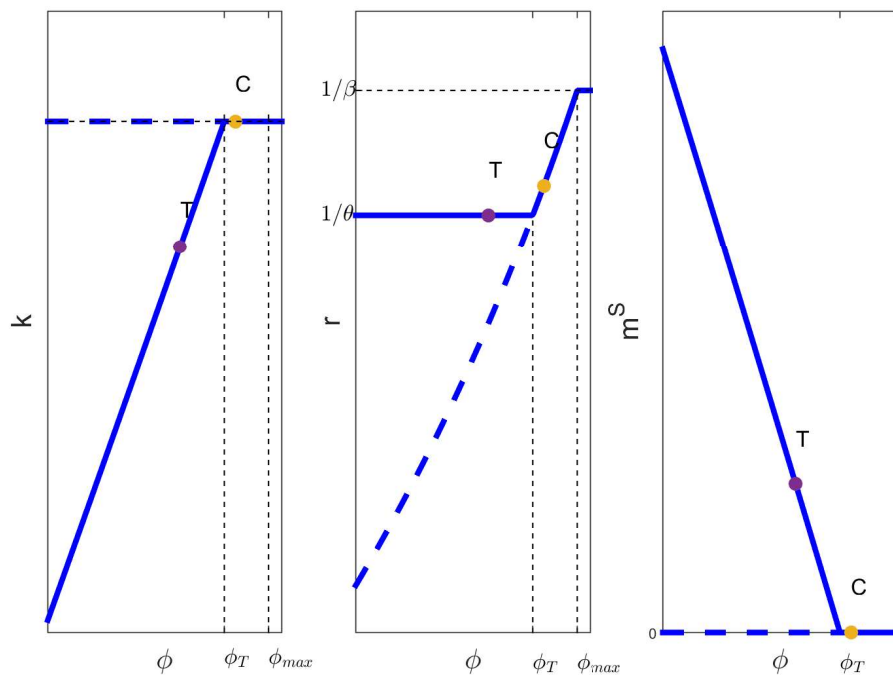


Figure 2: Steady states - Comparative statics w.r.t.  $\phi$ , with  $\bar{l} = 0$

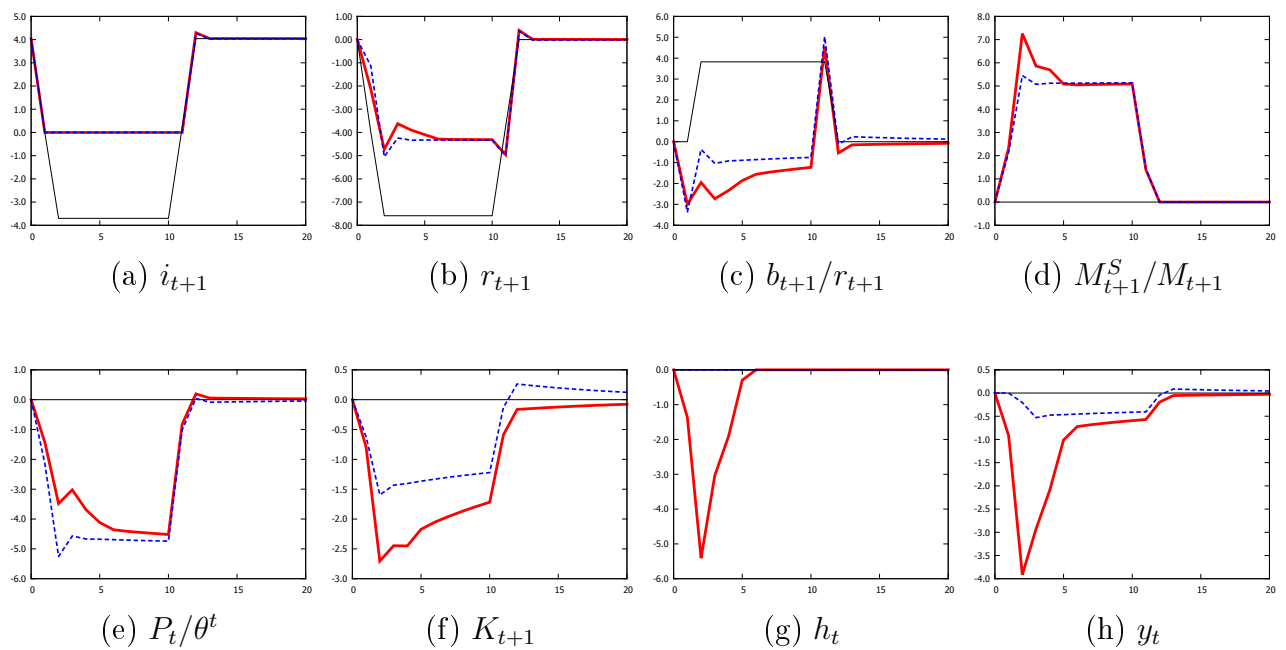
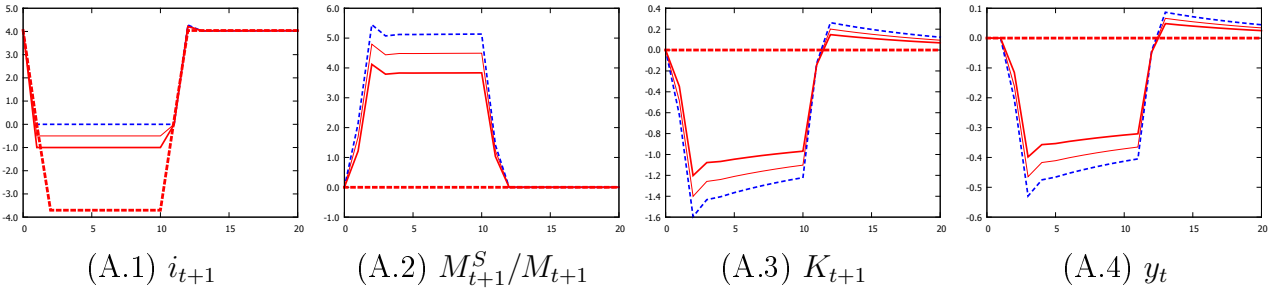


Figure 3: Transitory dynamics after a deleveraging shock. The shock hits in period 1 and lasts for 10 years. Thick red line: downwardly-rigid wages. Dashed blue line: flexible wages. Thin black line: flexible wages and no ZLB. All variables are in relative deviation from initial steady state, in percent, except interest rates and  $M^s/M$  which are in absolute deviation from initial steady state, in percent.

A. Baseline deleveraging shock ( $\phi$  drops by 3.9% with  $\lambda = 10\%$ )



B. Strong and persistent deleveraging shock ( $\phi$  drops by 8% with  $\lambda = 5\%$ )

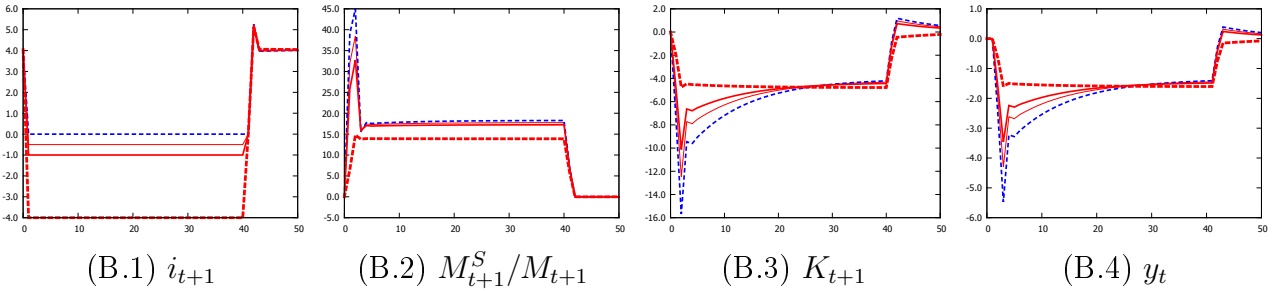


Figure 4: Transitory dynamics after an unexpected deleveraging shock. The shock hits in period 1 and lasts for 10 years. Red lines: negative interest rates of  $-0.5$ ,  $-0.1$ , and  $-4\%$ . Dashed blue line: no policy. Panel A: baseline deleveraging shock of  $3.9\%$  lasting 10 years with  $\lambda = 10\%$ . Panel B: stronger and more persistent deleveraging shock of  $8\%$  lasting 20 years with  $\lambda = 5\%$ . All variables are in relative deviation from initial steady state, in percent, except interest rates and  $M^s/M$  which are in absolute deviation from initial steady state, in percent.

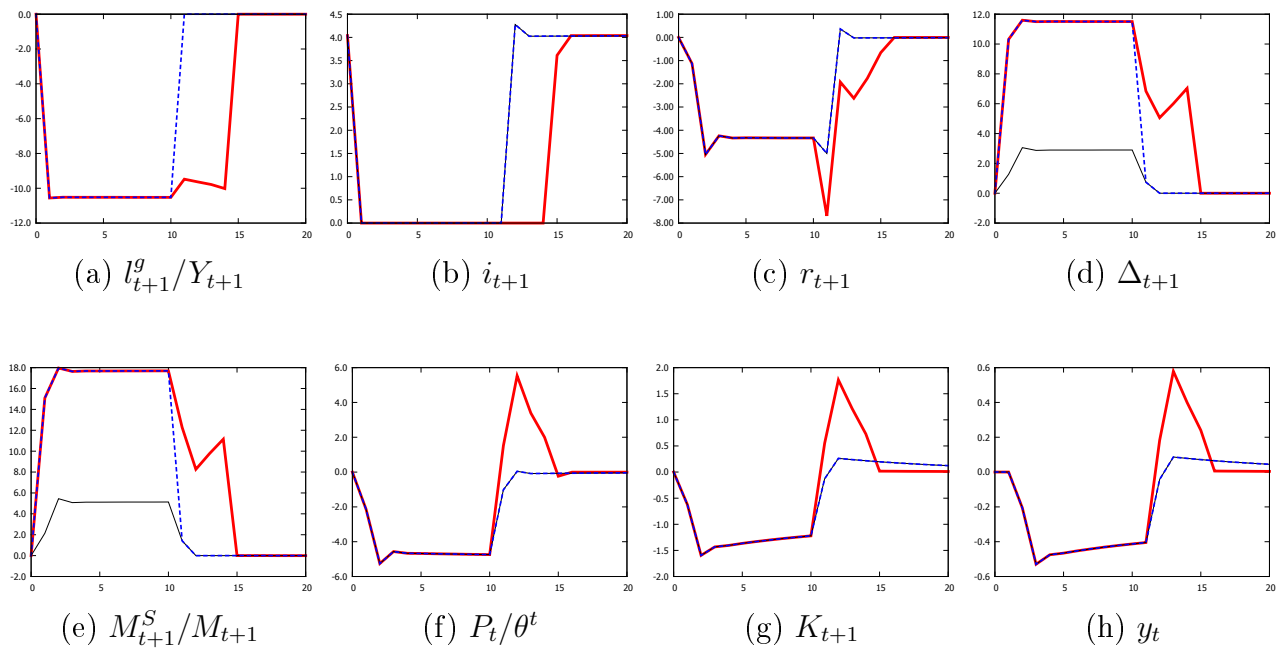
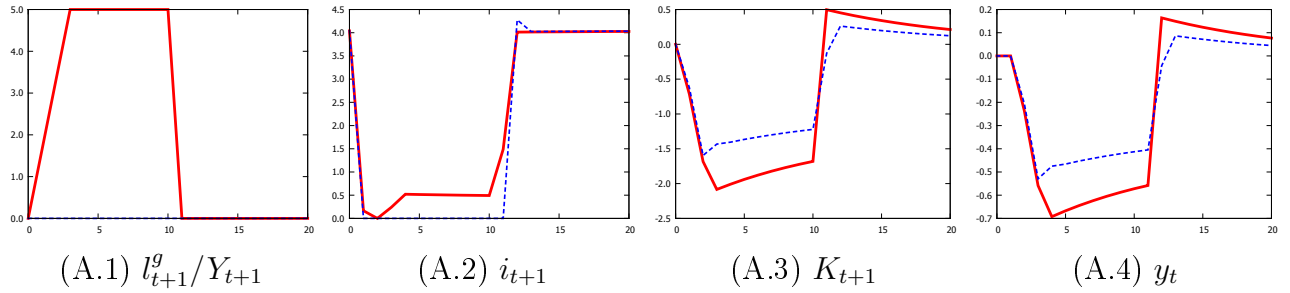


Figure 5: Transitory dynamics after an unexpected deleveraging shock. The shock hits in period 1 and lasts for 10 years. Thick red line: quantitative easing with late exit. Dashed blue line: quantitative easing with early exit. Thin black line: no quantitative easing. All variables are in relative deviation from initial steady state, in percent, except interest rates,  $l^g/Y$  and  $M^s/M$  which are in absolute deviation from initial steady state, in percent.

A. Baseline deleveraging shock ( $\phi$  drops by 3.9% with  $\lambda = 10\%$ )



B. Strong and persistent deleveraging shock ( $\phi$  drops by 8% with  $\lambda = 5\%$ )

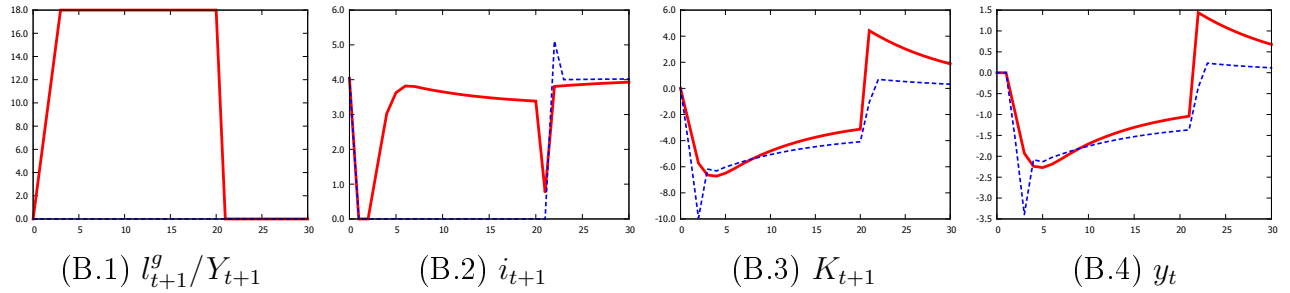


Figure 6: Transitory dynamics after an unexpected deleveraging shock. The shock hits in period 1 and lasts for 10 years. Thick red line: increase in public debt. Dashed blue line: no policy. Panel A: baseline deleveraging shock of 3.9% lasting 10 years with  $\lambda = 10\%$ , public debt increases by 5% of GDP in 2 years. Panel B: stronger and more persistent deleveraging shock of 8% lasting 20 years with  $\lambda = 5\%$ , public debt increases by 18% of GDP in 2 years. All variables are in relative deviation from initial steady state, in percent, except interest rates,  $l/Y$  and  $M^s/M$  which are in absolute deviation from initial steady state, in percent.