The demographics of structural change

Laurent Brembilla[*][†]

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Abstract

We study the impact of population aging on the process of structural change, defined as the composition shift of aggregate consumption. In this aim, we develop a multi-goods overlapping generations model with endogenous human capital accumulation. The model produces life cycle profile for the expenditure share on services consistent with the hump-shaped pattern that we document from the Consumer Expenditure Survey. We then conduct counterfactual experiments on the demographic variables. They reveal that aging exerts several counteracting forces on the sectorial allocation of resources, while its total effect on the aggregate consumption share of services is positive.

Keywords: population aging, structural change
JEL classification: O41, I15, E13

1 Introduction

In 1950, US households dedicated 39% of their consumption expenditure to services. In 2018, this proportion raised to 67% (see Figure 1).[1]This trend is a manifestation of the process of structural change, which defines as the sectorial reallocation of resources that occur as economies grow. It is a well-studied phenomenon by academics, which has has also gained interest in the public debate. The reason being that populations fear deindustrialization, one of the other facets of the rise of the expenditure share on services. This means that understanding the determinants of structural change is an important issue to tackle for economists. This paper studies whether the aging of the population is one of these determinants.
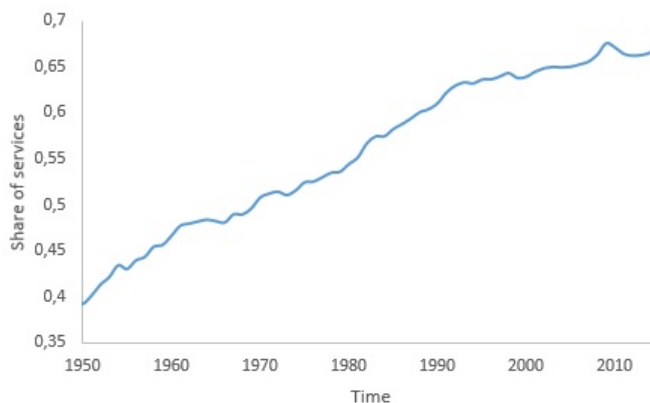

Figure 1: US expenditure share of services. Source: BEA

Aging is another notable trend of the postwar period in developed economies. It is the result of both fertility and mortality declines and is summarized by the increase of the ratio of people aged more than 65 to working age individuals from 8.75% in 1955 to 14.5% in 2015 in US (see Figure 2). This demographic shift is expected to significantly affect the growth path of developed countries. Thus it is natural to ask whether this phenomenon interacts with the process of structural change. To be more specific, there are three main reasons that suggest a role for aging in the process of structural change.

First, the consumption bundle of an individual varies with his age. Health consumption is an obvious example of this age-dependency of the consumption bundle. As aging shifts the age distribution of the economy, it modifies the aggregate demand for each good. We will refer to this effect as the *population effect*. Second, lifecycle theory implies that demographic variables affect the intertemporal allocation of individuals' resources. Individuals discount less future periods and so channel more resources, through higher savings or through higher

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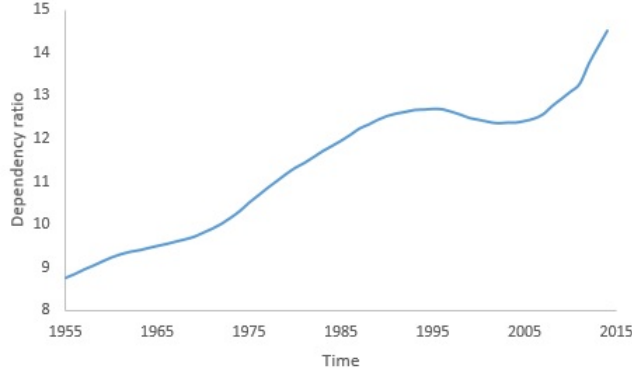[1]These numbers are obtained from the BEA.

Figure 2: US dependency ratio. Source: OECD database

contributions to a pension system, at older ages if their survival chances improve. Individuals also need to channel less resources during their parenthood ages if they have fewer children. These intertemporal reallocations are synonymous with intratemporal reallocations of resources once consumption goods feature different income-elasticity. Thus aging modifies individual consumption bundles, which also affects the aggregate demand for each good. This effect will be referred as the *allocation effect*. Third, aging influences the composition of aggregate consumption through its general equilibrium consequences. Particularly, aging has an impact on the income level, which is considered since Kuznets (1966) as an important determinant of structural change.

To answer our research question, we build a multi-goods overlapping generations model that take into account these three channels. To capture well the population effect, the life cycle model we use must produce consumption bundles by age consistent with individual data. To the best of our knowledge, no paper documents such life cycle profiles. Our first contribution is to fill this gap. Using data from the consumption expenditure survey (CEX), we determine the life cycle profile of the expenditure share of services (henceforth x_S). The latter follows an inverted U-shaped pattern, which is very similar to the well-documented life cycle profile of total consumption expenditures. This finding is in line with the structural change literature which emphasizes that services is a luxury good. Indeed, as the relative between goods and services is controlled for in our econometric analysis, the life cycle profile of x_S must mirror that of total consumption expenditures. This means that we can build the individual's problem of our general equilibrium model by incorporating preferences over multi-goods from the structural change literature. More precisely, we use the non-homothetic CES aggregator recently studied by Comin et al. (2017). Then, our model will replicate the empirical profile of x_S if it is successful in reproducing the consumption expenditure life cycle profile. To achieve this, we introduce two ingredients put forth by

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previous literature. First, individuals face idiosyncratic shocks to their human capital stock, which push them to save for precautionary motives (Gourinchas and Parker (2002)). Second, as in Bullard and Feigenbaum (2007), consumption and leisure are substitutes. As the price of leisure, hence the human capital stock, (endogenously) decreases from age 50, consumption expenditures diminish. This makes us confident that our model realistically captures the population and the allocation effect. We then incorporate this life cycle model into an OLG structure to take into account the impact of aging on the aggregate consumption share through general equilibrium effects. As our individual's problem includes saving and human capital decisions, our model integrates the main channels through which aging affects the economy. As highlighted by Bloom et al. (2003) and Chakraborty (2004), aging spurs individuals to save more to finance consumption during retirement period, which stimulates physical capital accumulation. And as demonstrated quantitatively by Ludwig et al. (2012), aging also stimulates human capital accumulation through the Ben-Porath effect. These two effects counteract the direct labor supply decline caused by the higher proportion of old individuals. The model is then used to make counterfactual experiments on demographic variables to gauge the quantitative effect of aging on the aggregate consumption share.

The first contribution of the paper is to complement the structural change literature by studying a new determinant of this process, aging, and its channels of interactions.[2] The growth process itself is considered as the main driver of structural change. Capital accumulation and technological progress change the relative price between goods and services and the expenditure level which both affect the relative demand. We demonstrate that aging also interacts with the structural change process through this channel. Here, given that we target individual expenditure data, we define our sectors in terms of final consumption expenditures. From the production function estimates of Valentinyi and Herrendorf (2008)), this implies that the relative price is likely not affected by aging. However, aging is likely to modify the expenditure level through the allocation effect and general equilibrium effects. Our third channel, the population effect, stems from the heterogenous dimension of our economy and so differs from the traditional price and expenditure channels. Aging changes the size of population groups with different consumption bundles, which shifts the relative aggregate demand. This relates to Alonso-Carrera and Felice (2018) who study the role of income inequality in a multi-sector economy. This also relates more generally to papers studying the implications of heterogeneity in economies featuring non-homotheticities such as Straub (2017).

Beyond the growth process, authors have put forth other determinants of structural change that include: trade (Swiecki (2017)) female employment (Ngai and Petrongolo (2017)),

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[2]This literature is thoroughly reviewed in Herrendorf et al. (2014).

technological progress in home production (Moro et al. (2017)) and population growth (Leukhina and Turnovsky (2016)). As Leukhina and Turnovsky (2016) also highlight a demographic factor, fertility, as a driver of structural change, it is the closest to this paper. Several differences distinguish our work from theirs. First, they focus on the fertility rate, while we examine the role of the fertility rate as well as that of mortality rates. Second, and more importantly, they study the process of structural change in England over 1750-1900, while we study this process in US during the post-war period. In Leukhina and Turnovsky (2016), the main channel of interaction relies on the use of land as a fixed factor of production of the farming good. A larger population makes land scarcer, rising the production costs of the farming sector and thus contributing to reallocating resources towards the manufacturing sector. Here we abstract from land to focus on the reallocation between manufacturing and services sectors. Then, the channels of interaction between demographic variables and sectorial sizes differ from those of Leukhina and Turnovsky (2016). As they use a representative agent model, the allocation and the population effect are both absent from their framework. Their general equilibrium effects also differ from ours as we include a human capital decision.

Finally, we complement a large literature on the economic consequences of aging. Economists consider aging as a source of various economic changes. Prettner and Trimborn (in press) and Heer and Irmen (2014) and Acemoglu and Restrepo (2017) show that aging spurs technological progress. Ludwig et al. (2012) study the welfare consequences of aging, while Eggertson et al. (in press) and Gagnon et al. (2016) show that aging causes the real interest rate to decline. Cooley and Henriksen (2018) and Aksoy et al. (in press) examine the growth consequences of aging. Our contribution is to highlight a novel economic effect of aging that we investigate in a rich economic model that could be used in future works. Indeed, thanks to the computational technique developed by Druedahl and Jorgensen (2017), our environment includes most economic decisions linked to aging such as the labor supply, the saving and the human capital decisions.

Section 2 emphasizes the channels of interaction between aging and the composition of aggregate consumption. In section 3, we determine the life cycle profile of the expenditure share of services using CEX data. In section 4, we present the quantitative model and the results of the counterfactual experiment. Section 5 concludes.

2   Theoretical considerations

The goal of this section is to highlight the channels through which aging affects the aggregate consumption share. For this, we discuss a standard multi-goods model embedded in a realistic demographic structure. We also discuss other possible channels of interaction.

We start by examining the role of aging in the individual decision problem. An individual lives up to age $T > 0$. His survival probability between ages a and $a + 1$ is q_a. Parenthood ages, $\mathcal{P}$, are a subinterval of $[\![0, T]\!]$, during which the individual lives with n children. The individual derives utility from the consumption of $N \in \mathbf{N}$ different goods. He also possibly directly derives utility from the number of his children, however we assume that preferences over the number of children are separable from the preferences over the consumption goods. During parenthood ages, the individual derives utility for each good i only from a fraction $s(n)$ of his consumption level of good i, with $s(n) < 1$ and $s'(n) > 0$. He takes the prices as given. His return on savings at age a is $\frac{1+r_a}{\lambda + (1-\lambda)q_{a,a+1}}$. Where $\lambda \in [0,1]$ captures the degree of imperfection of the annuity market, r_a is the interest rate at time a.[3]

To maximize his lifetime expected utility under the budget constraints, the individual first chooses at each age his consumption bundle given his total level of consumption expenditures. Then, at each age a, the expenditure share on good i writes: $x_{a,i} = f_a(E_a, \mathcal{P}_a)$, where E_a is the level of consumption expenditure at age a, $\mathcal{P}_a$ is the price vector of the N goods at time a. The function $f_a(., .)$ possibly depends on a if we allow utility per period to differ with age. It would directly depend on n if the scale factor $s(n)$ was different across goods. It would also directly depend on n if preferences over goods and children were not separable because the marginal rate of substitution between consumption goods would depend on n. Thus, here the only possible impact of aging on the consumption expenditure share $(x_{a,i})_{a \leq T, i \leq N}$ is through its impact on the total level of consumption expenditures:

$$\frac{\partial \ln(x_{a,i})}{\partial q_a} = \frac{\partial E_a}{\partial q_a}(\zeta_{a,i} - 1) \tag{1}$$

and

$$\frac{\partial \ln(x_{a,i})}{\partial n} = \frac{\partial E_a}{\partial n}(\zeta_{a,i} - 1) \tag{2}$$

With $\zeta_{a,i}$ the expenditure-elasticity at age a of good i. Hence the impact of aging on consumption shares is *opposite* for luxury and necessity goods. The direction of the impact is given by the impact of the demographic variables on the level of consumption expenditures at age a, which in turn is determined by solving the intertemporal problem of the individual. Suppose first that there is no annuity market, $\lambda = 1$. The Euler equation states that the growth rate of E_a between two periods positively depends on the survival probability between the two periods, q_a. In our framework and in the absence of an annuity market, the lifetime resources of the individual do not depend on survival probabilities. If $q_{a,a+1}$ increases, then the growth rate of E_a between a and $a+1$ increases, while other growth rates are unchanged.

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[3]For notational convenience, we assume that the individual is born at time 0, so much that age and time are equal.

This necessitates a decrease of the intital expenditure level, E_0, for the lifetime budget constraint to be fullfilled. Consequently there exists a pivot age $\tilde{a}$ such that $E_{a'}$ decreases (resp. increases) if a' is smaller (resp. greater) than $\tilde{a}$ following an increases of q_a. Hence a survival probability shift at a particular age modifies consumption bundles at any age, and the direction of the change is age-dependent. It spurs young individuals to consume more necessary goods and old individuals to consume more luxury goods. In presence of a perfect annuity market, $\lambda = 0$, the direction of the change is no more age-dependent. The Euler equation is now independent on the survival probabilities, hence the growth rate of consumption expenditures between two periods is unaffected by changes of survival proabilities. The amount of lifetime resources diminish with higher survival probabilities, because these ones reduce the return on assets. Thus, an increase of the survival probability at a particular age reduces consumption expenditures at any age in the same proportion. In the imperfect annuity case, $\lambda \in (0,1)$, the two scenari can occur. The growth rate of consumption expenditures between two consecutive ages increases in the survival probability and the lifetime resources diminish with survival probabilities. Thus consumption expenditures at young ages diminish. If they diminish sufficiently and if future consumption growth rates increase sufficiently, consumption expenditures at old age can increase.

We now examine the impact of the number of children, n, on the consumption expenditures profile $(E_a)_{a \in [\![0,T]\!]}$. This impact can only occur through the scaling factor $s(n)$. Suppose first the individual is no more parent, $a \in [\![\max(\mathcal{P}) + 1, T - 1]\!]$. Then, the Euler equation at this age is no more dependent on n because the scaling factor is equal to 1. Thus, for two consecutive periods not belonging to $\mathcal{P}$, the growth rate of consumption expenditures does not depend on n. This is also the case for two consecutive periods during the parenthood period. Indeed, the scaling factor is similar to a discount factor of consumption in parenthood periods. As this additional discount factor is the same for all periods in parenthood, the Euler equation is independent on the scaling factor. However, the Euler equation does depend on the scaling factor for the transition between non-parenthood to parenthood and inversely. Suppose $\min(\mathcal{P}) > 0$, hence the individual is not initially a parent. At age $\min(\mathcal{P}) - 1$, an increase of n spurs the individual to increase his next period consumption expenditures to offset the increase of the scaling factor. Thus the growth rate of consumption expenditures increases in n at age $\min(\mathcal{P}) - 1$. The reverse occurs at age $\max(\mathcal{P})$, if it is assumed smaller than T. These growth rate shifts imply that n modifies the profile $(E_a)_{a \in [\![0,T]\!]}$. Using the fact that lifetime resources do not depend on n, we deduce that consumption expenditures increase during parenthood ages, while either consumption expenditures before parenthood or consumption expenditures after parenthood must decrease. Therefore, a fertility change modifies consumption bundles, and the direction of this change is age-dependent.

The main insight of this analysis lies in equations (1) and (2): the intertemporal reallocation of resources due to aging create an intratemporal reallocation of resources once intratemporal preferences are non-homothetic. Non-homothethicity is a common and documented feature of preferences over different goods (Herrendorf et al. (2014)). Thus, aging modifies the consumption bundle of individuals. The direction of these consumption changes can be broadly caracterized in our stylized multi-goods lifecycle framework. However, additional effects can be at play. They are linked to the fact that lifetime resources of an individual can change with demographic variables, even in the absence of an annuity market. First, an individual can modify his labor supply at the intensive margin. In section 4, we include such a decision in our quantitative analysis. Second, an individual can increase his labor supply at the extensive margin by postponing his retirement age if his life expectancy increases. This tends to increase consumption expenditures at any age. In our quantitative section, we do not include a retirement decision, because over our period of interest, 1950-2015, the retirement age has not changed by much. Consequently it is unlikely that it plays a large role in the change of consumption shares. Third, the Ben-Porath effect stipulates that individuals invest more in their human capital if they live longer. This implies that consumption expenditures increase if survival probabilities increase. Ludwig et al. (2012) show it is important to take into account this effect to examine the economic consequences of population aging. Thus in our quantitative model we include a human capital decision as in Huggett et al. (2011) in order not to neglect this channel of interaction.

We now turn briefly to the impact of aging on the relative aggregate demand between goods in partial equilibrium. The previous analysis underlines that demographic factors, $\tilde{q} = (q_a)_{a<T}$ and n, affect consumption levels. So we explicitely mention this dependance by writing the consumption level of good i by an aged a individual belonging to cohort c as $c^i_{c,a}(n, \tilde{q})$. then the aggregate relative demand between good i and j at time t writes:

$$D_t^{i,j} = \frac{\sum_{c+a=t} L_{c,a} c^i_{c,a}(n, \tilde{q})}{\sum_{c+a=t} L_{c,a} c^j_{c,a}(n, \tilde{q})} \tag{3}$$

Where $L_{c,a}$ is the number of cohort-c individuals aged a. (3) allows to visualize the two partial equilibrium effects mentioned in the introduction. Aging affects $D_t^{i,j}$ because it affects individual consumption levels $c^i_{c,a}(n, \tilde{q})$ (allocation effect). Aging also affects $D_t^{i,j}$, because demographic variables determine $L_{c,a}$ the size of the different groups of the population (population effects). Caracterizing the dependence of $D_t^{i,j}$ with respect to demographic variables is out of reach without further assumptions. However, there is an extreme case that it is worth mentioning. It is well-known that $D_t^{i,j}$ simplifies in case intratemporal preferences are homothetic *and* identical among all individuals. The first assumption implies that for any

individual (c, a), the ratio of consumption levels of goods i and j only depends on the relative price between good i and j, $\frac{c^i_{c,a}(n,\tilde{q})}{c^j_{c,a}(n,\tilde{q})} = f_{c,a}(\frac{P^i_{c+a}}{P^j_{c+a}})$. The second assumption implies that the function $f_{c,a}(.,.)$ is the same across all individuals (c, a). Then, $D^{i,j}_t = f(\frac{P^i_t}{P^j_t})$ and the relative aggregate demand between goods is independent on demographic variables. This result is the aggregate counterpart of equations (1) and (2), which state that the individual lifecycle problem is independent on demographic variables if and only if individual preferences do not depend on age and are homothetic. Hence $D^{i,j}_t$ does not depend on demographic variables under identical and homothetic preferences because the two partial equilibrium effects of aging are neutralized under this assumption. The composition change of the population due to aging does not affect $D^{i,j}_t$ because all individuals choose the same consumption bundle. On the contrary, if we abstract from homothetic preferences, then the ratio of consumption levels of goods i and j for an individual (c, a) is also a function of the total consumption expenditure level, $E_{c,a}$: $\frac{c^i_{c,a}(n,\tilde{q})}{c^j_{c,a}(n,\tilde{q})} = g(E_{c,a}, \frac{P^i_{c+a}}{P^j_{c+a}})$. Hence this ratio is different across individuals for two reasons. $E_{c,a}$ is age-dependent, individuals at a different point of their life cycle have different expenditure levels. $E_{c,a}$ is cohort-dependent, because individuals belonging to different cohorts face different prices. Hence the compositon change of the population effect operates under non-homothetic preferences. Relaxing the assumption of identical preferences across individuals is obviously the alternative way to make $D^{i,j}_t$ dependent on demographic variables. The following proposition summarizes these results:

Proposition 1 *The relative aggregate demand between goods is invariant with respect to aging for any price vector if and only if intratemporal preferences are homothetic and identical across individuals.*

Given that the non-linearity of Engel curves is well-documented (see among others Herrendorf et al. (2013)), Proposition 1 justifies our research question that is to quantitatively assess the contribution of aging to the process of structural change.

3   The lifecycle profile of the expenditure share of services

We use data from the Consumer Expenditure Survey (CEX), more precisely the waves from 1986 to 2011. The dataset reports households'consumption expenditures on 46 categories. We followBoppart (2014) to classify consumption expenditures into two categories: goods and services.[4] As mentioned in the introduction, health expenditures are a component of

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[4]According to the BEA: 'A good is a tangible commodity that can be stored or inventoried. A service is a commodity thant cannot be stored and inventoried and that is usually consumed at the place where it is

services that need to be carefully examined. The CEX only reports out-of-pocket medical expenditures but not total health expenditures. So we choose to remove these out-of-pocket medical expenditures to estimate an expenditure share of services without any health expenditures. As explained by Aguiar and Hurst (2013), individuals'decisions with respect to their health expenditures are different from their consumption decisions, which justifies to treat health expenditures separately. In our quantitative model of section 4, we introduce exogenous health expenditures, with a age-profile directly drawn from the literature.

A well-known multicollinearity problem prevents from estimating our age-profile while simultanously controlling for time and cohort effects. Yet there are reasons to concern that both time and cohort effects affect the age-profile of x_S. Suppose we only control for cohort effects and consider two individuals from the same cohort c with ages a and a', with $a < a'$. Then the individual of age a' faces a higher relative price of goods to services than does the individual of age a because he is observed at later date. Hence his age-estimate of x_S is downward biased relative to that of the individual of age a. Suppose now we only control for time effects and consider two individuals belonging to cohorts c and c', with $c < c'$, who are observed at the same date. Then, individual c is likely to have a small expenditure level at any age than individual c' because his lifetime resources are smaller. Hence his age-estimate of x_S is downward biased relative to that of the individual c'.

Fortunately, Schulhofer-Wohl (2018) proposes a strategy to remove both cohort and time effects to estimate life cycle profiles. This is the strategy we apply to estimate the life cycle profile of x_S, that we report in Figure 3. The expenditure share of services is an inverted-U shaped function of age. This is reminiscent of the findings of Fernndez-Villaverde and Krueger (2007), Aguiar and Hurst (2013) who estimate the life cycle profile of total consumption expenditures. To compute this profile, we deflate the total consumption expenditure by the price index and we divide it by the OECD scale equivalence to control for households's size changes along the lifecycle. In Figure 4, we report this profile, which is also hump-shaped.

The results of Figures 3 and 4 are in line with the structural change literature, which points out that services is a luxury good. Hence x_S positively depends on the expenditure level which implies that x_S and the expenditure level must follow a similar pattern over the life cycle.

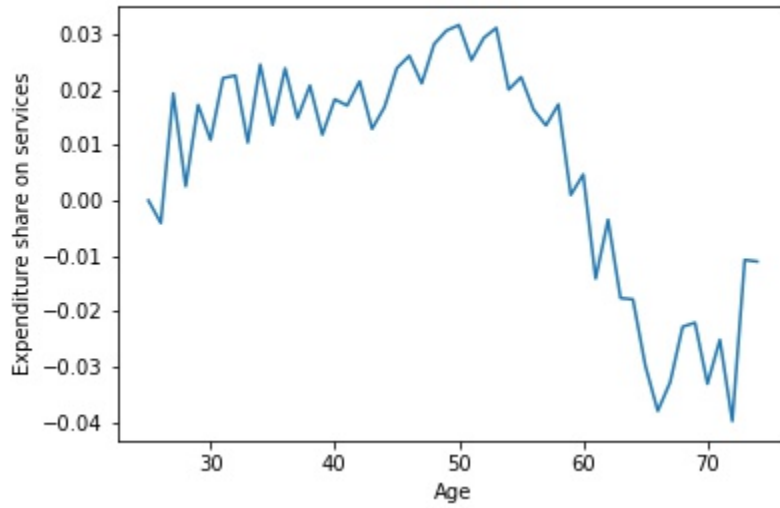purchased.' The exact classification is given in Appendix A.

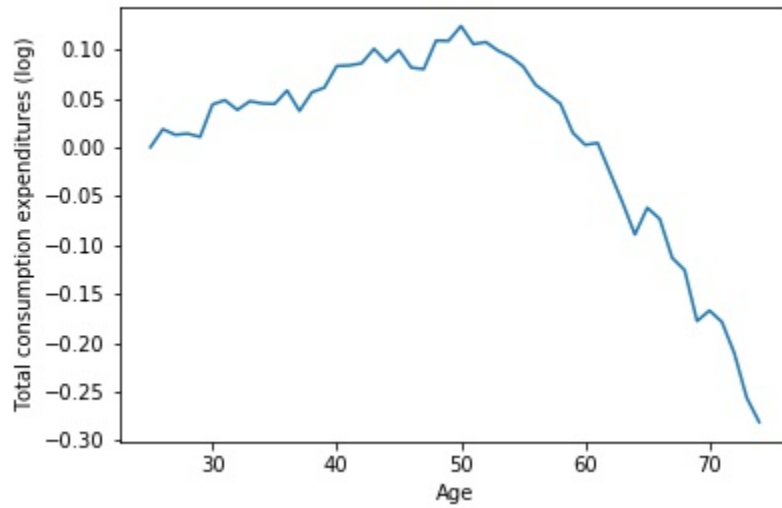Figure 3: Life cycle profile of the expenditure share of services

Figure 4: Life cycle profile of total consumption expenditures

4   The quantitative model

In this section, we develop a multi-goods OLG model in which demographic variables are exogenous. We then quantify their impact on the composition of aggregate consumption.

4.1 The model economy

4.1.1 Firms

There are three sectors in the economy: the goods producing sector (G), the investment sector (I) and the sector of services (S). The investment good is taken as the numéraire. As our interest is to study the composition of aggregate consumption, we define our sectors in terms of final consumption expenditures. Valentinyi and Herrendorf (2008) estimate Cobb-Douglas production functions for this approach that we borrow here:

$$Y_G = A_G K_G^\alpha L_G^{1-\alpha} \tag{4}$$

$$Y_S = A_S K_S^\alpha L_S^{1-\alpha} \tag{5}$$

$$Y_I = A_I K_I^{\alpha_I} L_I^{1-\alpha_I} \tag{6}$$

Where Y_i is the final output of good $i \in \{G, S, I\}$, K_i the capital stock, L_i effective labor, and A_i the total factor productivity (TFP). As documented by Valentinyi and Herrendorf (2008), the capital intensity of good and services sectors is equal, and is noted $\alpha \in (0,1)$. α_I is the capital intensity of the investment sector. We assume that the TFP levels are exogenous. Factors of production are perfectly mobile across sectors and are paid at their marginal product. This implies the following equalities:

$$w = P_G A_G (1-\alpha) k_G^\alpha = P_S A_S (1-\alpha) k_S^\alpha = A_I (1-\alpha_I) k_I^{\alpha_I} \tag{7}$$

$$r + \delta = P_G A_G \alpha k_G^{\alpha-1} = P_S A_S \alpha k_S^{\alpha-1} = A_I \alpha_I k_I^{\alpha_I-1} \tag{8}$$

Where k_i is capital to labor ratio of sector i and δ is the depreciation rate of capital. (7) and (8) imply that the capital to labor ratios are equal across the two consumption sectors (hence $k_1 = k_2$). Moreover, the relative price of consumption goods is equal to the ratio of TFP levels: $\frac{P_G}{P_S} = \frac{A_S}{A_G}$.

4.1.2 Timing and demographics

A period of the model corresponds to five years. There is a continuum of individuals entering the economy at the age of 20 and living up to 17 periods, or age 105. Given that the mean retirement age has not changed by much during our period of interest (1950-2015), we fix the retirement age to 65, which corresponds to period 9.[5] Each individual lives with

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[5]Prettner and Canning (2014) show that in a perpetual youth model, the retirement age increases with life expectancy contrary to what happenned in US and other OECD countries. They conclude that political constraints impede these adjustments. Then we take here these political constraints as given.

his n children from age 30 to age 50 or equivalently from period 2 to period 5. During these parenthood ages, he derives utility from a fraction $s(n) = \frac{1}{(1+\phi n)^\omega}$ of the household consumption level, with $0 < \phi, \omega < 1$. This expression of the scale factor, borrowed from Greenwood et al. (2003), introduces a scale effect in household consumption. The probability to survive between period a and period $a+1$ is noted q_a, while the unconditionnal probability to reach period a is noted Q_a. The mass of individuals who reach period a is noted L_a.

4.1.3 Individuals

Individuals derive utility from leisure, consumption of services and goods and bequests. We consider the following felicity function:

$$u(C, z) = \frac{(C^{\alpha_L} z^{1-\alpha_L})^{1-\sigma}}{1 - \sigma} \tag{9}$$

with z the time dedicated to leisure. C is the non-homothetic CES aggregator recently studied by Comin et al. (2017). It is implicitly defined by the following equation:

$$\Omega_G^{\frac{1}{\gamma}} C^{\frac{\epsilon_G - \gamma}{\gamma}} C_G^{\frac{\gamma-1}{\gamma}} + \Omega_S^{\frac{1}{\gamma}} C^{\frac{\epsilon_S - \gamma}{\gamma}} C_S^{\frac{\gamma-1}{\gamma}} = 1 \tag{10}$$

Where Ω_S, Ω_G are positive weights. γ is the elasticity of substitution. ϵ_G and ϵ_G are parameters that control the expenditure-elasticity of each consumption category. If $\epsilon_G < \epsilon_S$, then services is luxury good. We provide some comments on our functional form choice. First, our felicity function features non-separability between leisure and consumption. Bullard and Feigenbaum (2007) demonstrate that this ingredient is helpful for life cycle models to generate a realistic life cycle profile of total consumption expenditures. Second, to define our preferences over goods and services, we choose the CES aggregator recently studied by Comin et al. (2017). Two other options would have been possible. Herrendorf et al. (2013) show that Stone-Geary preferences are consistent with the joint evolution of aggregate consumption expenditure, consumption shares and relative prices for US over the postwar period. However, as argued by Comin et al. (2017), they imply that the expenditure-elasticity vanishes as the expenditure level grows. An alternative choice is to use the PIGL preferences recently studied by Boppart (2014). However, these preferences have a drawback for our purpose. When these preferences are modified to introduce non-separable leisure, the expenditure share of services positively depends on the price of leisure.This prevents the model from producing a life cycle profile of the expenditure share of services in line with the data.

Then, the objective of the individual is to maximise his expected lifetime utility which

writes:

$$E_0[\sum_{a=0}^{16} \beta^a Q_{a-1}(q_a u(c_a, z_a) + (1 - q_a)v(x_a))] \tag{11}$$

Where $v(.)$ is the utility from bequests, x_a the wealth at the beginning of period a. The individual faces four constraints:

1) A time constraint that requires the amount of time available to split between leisure, labor and schooling: $1 = S_a + z_a + l_a$. Where S_a is the time devoted to schooling l_a is the labor supply.

2) A budget constraint, which by introducing the expenditure function $E(.)$, writes:[6]

$$x_{a+1} = (1 + r_a)x_a + I_a + (1 - \tau)wh_a l_a - E(c_a) \tag{12}$$

Where x_a is the asset level at age a, h_a is the human capital level, τ is the tax rate on labor income and I_a collects the bequests received and other sources of income and of expenditures that will be detailed below.

3) A human capital law of accumulation similar to that of Huggett et al. (2011):

$$h_{a+1} = e^{\zeta_a}(h_a + A(h_a S_a)^\theta) \tag{13}$$

Where $A > 0$ and $\theta \in [0, 1]$. $(\zeta_a)_{0 \leq a}$ is the history of human capital shocks. These are independent normal random variables.

4) A borrowing constraint which imposes individuals to hold a positive amount of assets.

4.1.4 Retirement system

The pension system is a simple pay-as-you-go system. Independently on his age, each retiree receives a pension income, B, which is proportional to the current mean labor income in the economy. Given a replacement rate, ψ, the tax rate is determined so as to balance the budget. More precisely, let Λ be the state of an individual and $\mu(d\Lambda)$ its distribution. The total contributions to the pension system is $\tau \int wl(\Lambda)h(\Lambda)\mu(d\Lambda)$, the pension level for each retiree is $B = \psi \frac{\int wl(\Lambda)h(\Lambda)\mu(d\Lambda)}{\sum_{a=0}^{9} L_a}$. This implies that the tax rate is given by $\tau = \psi \frac{\sum_{a=0}^{9} L_a}{\sum_{a=10}^{16} L_a}$.

4.1.5 Health expenditures

At each period, individuals have to spend some resources for their health. These health expenditures take the form $mg(a)$ where $g(a)$ is a non-decreasing function that captures their dependency with respect to the age of individuals. m is a technological term common

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[6]In Appendix B, we derive the expression of the expenditure function

to all individuals that will be computed for the aggregate ratio of health expenditures to total expenditures to match its empirical counterpart. Part of these health expenditures are publicly financed for retired individuals. They are financed through a lump-sum tax on all individuals, which is determined to balance the budget.

4.1.6 Equilibrium

Our definition of a stationary equilibrium is standard. Each individual maximizes the objective (11) subject to the time constraint, the budget constraint, the borrowing constraint and the law of human capital accumulation. Firms of the three sectors maximize their profits. Pension and health budgets balance. Bequests are distributed equally among survivors. All markets clear.

4.2 Calibration

We calibrate the model for the stationary equilibrium to replicate key moments of the US economy in 2015.

Demographic data: There are three types of demographic data we feed in the model: the survival probabilities $(q_a)_{a=0,..16}$, the number of children individuals live with at each age $(n_a)_{a=0,..,16}$ and the age population shares $(L_{a=0,..,16})$. Using the population share from the data instead of computing them from survival probability and the fertility rate allows to take into account migrants and thus not to overestimate the aging of the population. We use period-survival probabilities for males in 2010 from Bell et al. (1992). We obtain the share of births by age of the mother from the Census database. Then we assume a total fertility rate equal to its 2015 level to compute the number of children individuals live with at each age. The population shares for 2015 are obtained in the Census database. Note that our model is invariant to the population level.

Firms: The values of α and α_I are obtained or computed from Valentinyi and Herrendorf (2008). These authors estimate production functions with capital and labor as inputs for five sectors: agriculture, manufactured consumption, services, equipment investment and construction investment. Consistenly with our final consumption approach, we use their capital share in purchaser price with their aggregated input-output table to compute the capital share of the sector producing goods (hence agriculture and manufactured consumption). We find a capital share of the goods sector equal to 0.35, which is also that of services, as specified by production functions (4) and (5). Hence $\alpha = 0.35$. We obtain $\alpha_I = 0.28$ directly in Valentinyi and Herrendorf (2008). We normalize the TFP level of the good sector

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to 1. Then we compute the TFP level for the relative price between goods and services to equal its 2015 value. We choose the TFP level in the investment sector to match the relative price between the capital good and the manufacturing sector. The depreciation rate δ is set to target a capital to output ratio equal to 3.

Preferences: We rely on Comin et al. (2017) to parametrize the non-homothetic CES aggregator. These authors estimate $\gamma, \epsilon_G, \epsilon_S$ from the CEX, so we directly use their values, which are reported in Table 4.2. Relative to bequests, we consider the following functional form for $v(.)$: $v(b) = B_{eq} \frac{b^{1-sigma}}{1-\sigma}$. We choose the parameter B_{eq} to target a bequest to GDP ratio equal to 2%

Health expenditures: Dalgaard and Strulik (2014) compute the growth rate of health expenditures over the lifecycle in several countries and obtain a value equal to 2%. Thus we assume that the age-dependent component of health expenditures, $g(a)$, is: $g(a) = 1,02^{5a}$. The medical technology term m is computed for the model to reproduce the ratio of total health expenditures to total consumption expenditures in 2015. The value is obtained from the BEA, $16, 5\%$. For the elderly, 65% of health expenditures are publicly financed as suggested by Nardi et al. (2016). The replacement rate of the pension system, ψ, is set to 0.4 as in Aguiar and Hurst (2013).

Parameters	Symbol	Value	Source-Target
Expenditure-elasticity of services	$\epsilon_S - \epsilon_G$	0.65	Comin et al. (2017)
Parameter CES aggregator	ϵ_G	1	Normalization
Elasticity of substitution	γ	0.28	Comin et al. (2017)
Consumption weights	Ω_G, Ω_S	1,1	Aggregate expenditure share on services (BEA)
Utility exponent	σ	2.5	see text
Consumption share	α_L	0.45	see text
Discount factor	β	0.9725^5	see text
Human capital production function	A	0.65	Life cycle profile of hours worked
Exponent Human production function	θ	0.6	Huggett et al. (2011)
Mean and variance of ζ	$\mu_\zeta, \sigma_\zeta^2$	$(-0.029 * 5, 0.111 * \sqrt{5})$	Huggett et al. (2011)
capital shares	α, α_I	$0.35, 0.28$	Valentinyi and Herrendorf (2008)
TFP goods	A_G	1	Normalization
TFP services	A_S	1.	$\frac{P_S}{P_G}$ (BEA)
TFP services	A_I	1.	$\frac{P_I}{P_G}$ (BEA)
Depreciation rate	δ	0.3	$\frac{K}{Y} = 3$
Bequest weight	B_{eq}	0.15	Bequests to GDP ratio = 2%
Replacement rate	ψ	0.4	Aguiar and Hurst (2013)

Other calibrated parameters: There remains to assign a value to $(\sigma, \beta, \alpha_L)$. Note that σ and α_L do not have the usual interpretation. Because our consumption aggregator is non-homothetic, the intertemporal elasticity of substitution is not constant. Also the ratio of consumption to leisure time is non-constant. This means that we cannot use the standard values for σ and α_L reported by the literature. Therefore, we choose $(\sigma, \beta, \alpha_L)$ over a large grid to minimize the distance between the life cycle profiles estimated in section 3 and their model counterparts. Let $(\hat{e}_a, \hat{s}_a)_{a=0..11}$ be the expenditure and the share profile estimated. Let $(e_a, s_a)_{a=0..11}$ be their model counterpart. Then $(\sigma, \beta, \alpha_L)$ are chosen to minimize the following objective:

$$\sum_{a=1}^{11} ((\hat{e}_a - e_a)^2 + (\hat{s}_a - s_a)^2) \tag{14}$$

Resolution: The algorithm to solve the model is standard. We make a guess for the values of the capital stock per worker in the investment sector k_I, the pension income B, the amount of unexpected bequests and the total level of consumption expenditures. With the level of consumption expenditures and the share of health expenditures in consumption expenditure, we can compute the aggregate level of health expenditures and then we can compute the lifecycle profile of health expenditures each individual faces. We then solve the individual's problem. Given that our problem features two continous state variables, we apply an endogenous grid method (Carroll (2006)) and more precisely the multidimensional interpolation set up by Druedahl and Jorgensen (2017) to speed up the computation. We then update our guesses until convergence. We provide details of our solution method in Appendix B.

4.3   Results

Backfitting: We examine the performance of the model with respect to data. Figure 6 and Figure 5 compare the life cycle profile of x_S and that of total consumption expenditures to those obtained in section 3. The model produces life cycle profiles for x_S and for total consumption expenditures in line with those estimated. As indicated, it is not surprising that our model generates a life cycle profile for total consumption expenditures in line with the data. One of the mechanisms operates through the decline of the price of leisure that occurs from age 50, which causes consumption to decline as it is a substitute to leisure. Here the price of leisure is an endogenous variable, so it is natural to check whether the life cycle profile for human capital is consistent with previous studies. Figure 7 reports this profile, which is hump-shaped as found by previous literature (see Huggett et al. (2011)). Therefore our model appears as a reliable laboratory to examine the impact of demographic variables on the aggregate expenditure share of services.

Experiment: Our main experiment is as follows. We keep all the parameters to their baseline values and we set the demographic variables to their values in 1950. These values are obtained from the same sources as those of 2015. Figure 8 plots the survival curves of 1950 and 2015, Figure 9 plots the population shares. Then we solve the model and compute the new aggregate expenditure share on services. The aggregate expenditure on services is now 2.3% smaller than that of the baseline economy. This corresponds to 3.3% of the total variation between 1950 and 2015.

To understand the result, we need to assess how each of the three effects previously outlined affects the expenditure share of services. The first effect we analyze is the allocation
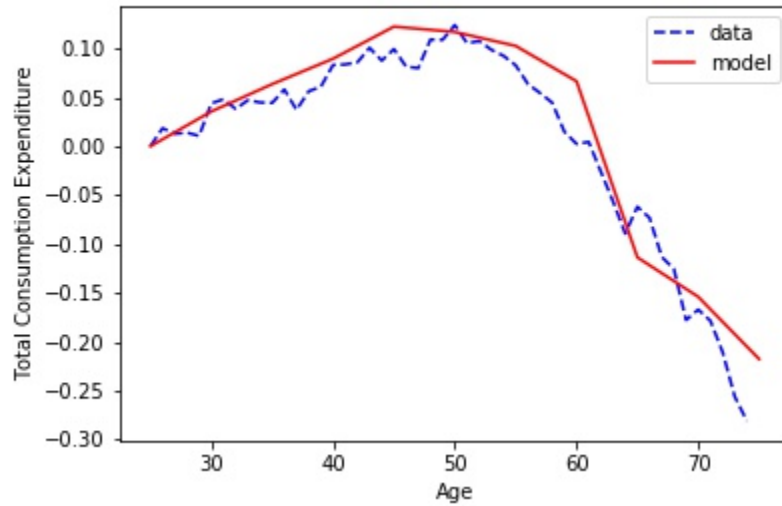
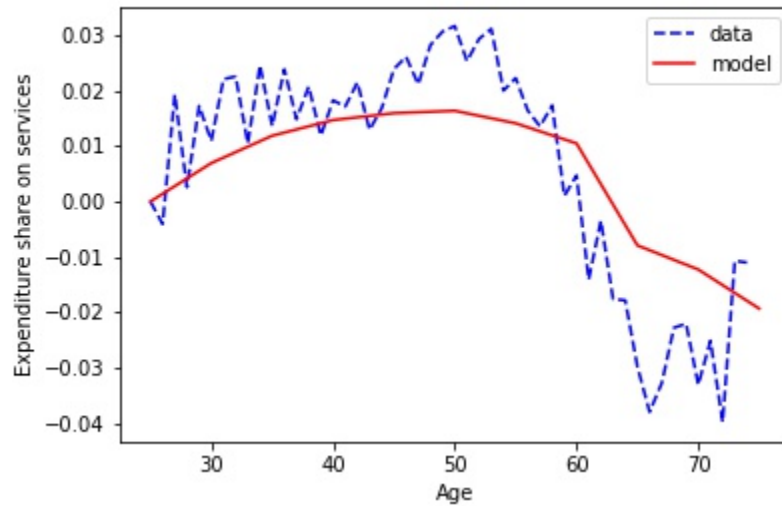Figure 5: Expenditure profile: data versus model

Figure 6: Expenditure share profile: data versus model

effect. Aging spurs young individuals to reallocate resources from young ages to old ages. As services are a luxury good, the expenditure share on services of old individuals increases, while that of young individuals declines. Hence the allocation effect exerts two opposite effects on the aggregate expenditure share on services. We compute the contribution of the allocation effect as follows. We keep the prices of the baseline economy and we use the population shares of 2015, while we use the survival probailities and the number of children
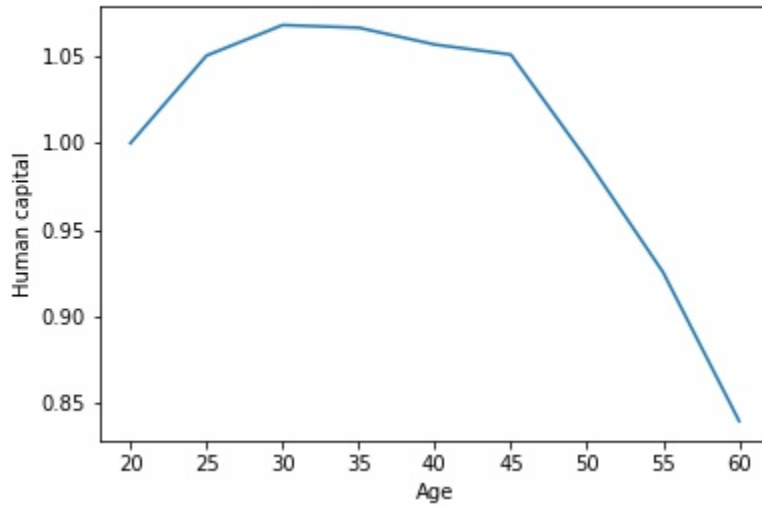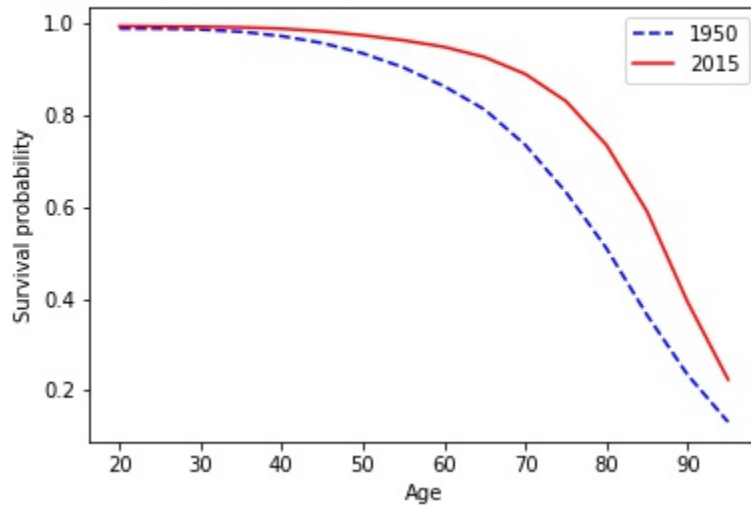
Figure 7: Human capital life cycle profile

Figure 8: US survival probabilities in 1950 and 2015

of 1950. Then we solve the individual's problem and compute the aggregate expenditure share on services. We find that this variable decreases by less than 1%, confirming that the allocation effect is close to be neutral because it implies two counteracting forces on the aggregate expenditure on services. The second effect we analyze are general equilibrium effects. Aging has an unambiguous positive effect on the capital stock as it spurs individual to save more and it reduces the dilution effect through a smaller population growth rate. The
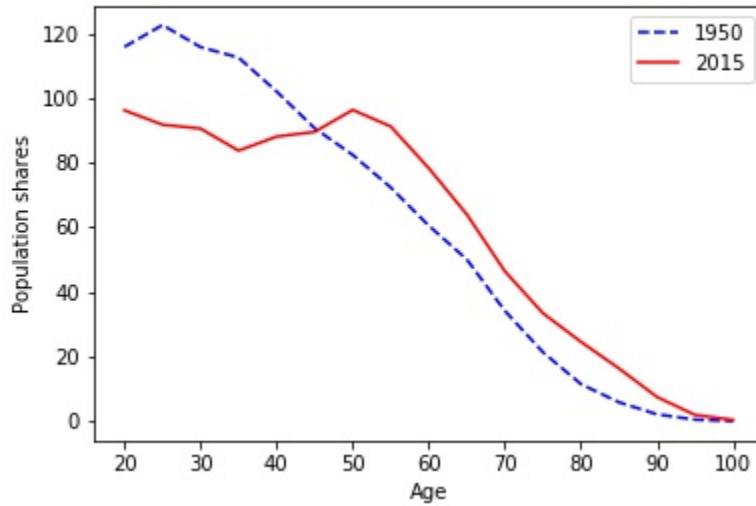
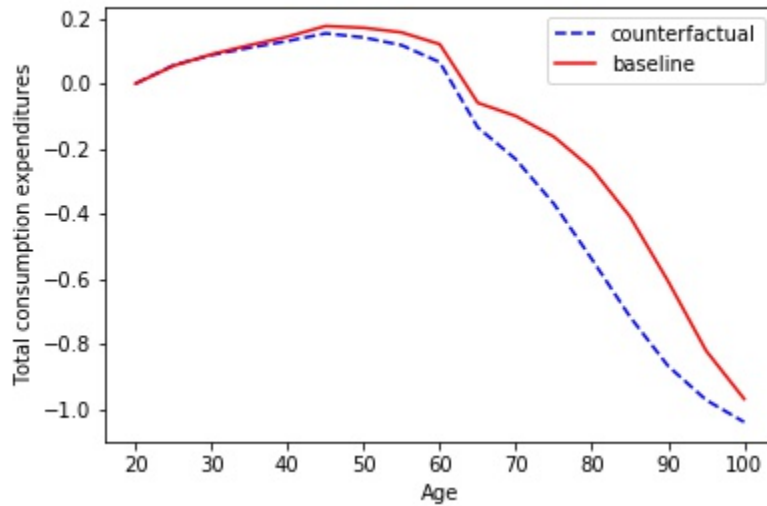Figure 9: US population shares in1950 and 2015

Figure 10: Expenditure profile: baseline versus counterfactual

baseline economy's capital stock is 4.83% greater than that of the counterfactual economy. The impact of aging on the laborforce is less clear as it is the result of two opposite forces. On the one hand, aging spurs individuals to invest more in their human capital as their survival chances improve (see Figure 12). On the other hand, aging reduces the working age population. We find that the laborforce of the baseline economy is less than 1% greater that of the counterfactual economy. Then, the general equilibrium effects are summarized by a
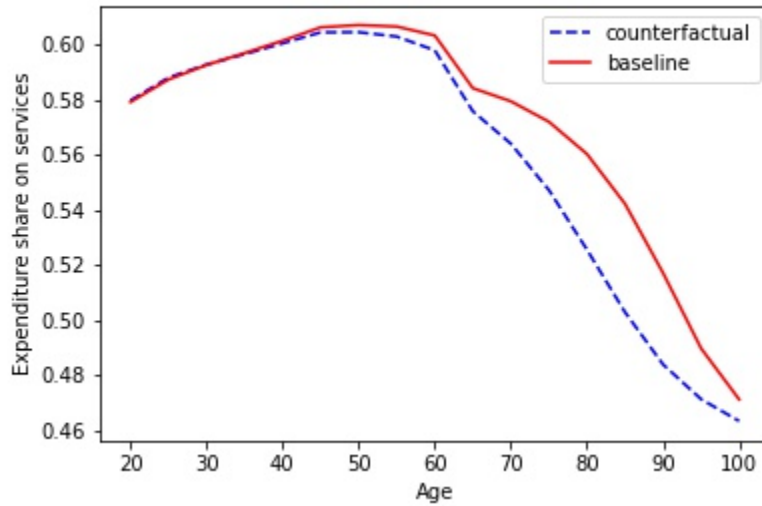
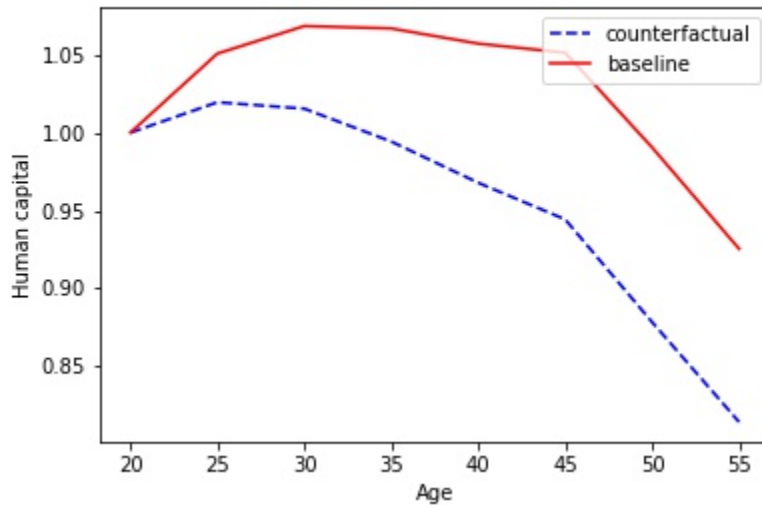Figure 11: Expenditure sharel profile: baseline versus counterfactual

Figure 12: Human capital profile: baseline versus counterfactual

GDP per capita 1.2% greater in the baseline economy than in the counterfactual economy. It implies that individual have more resources, hence can spend more on consumption, which increases their expenditure share on services. Figures 10 and 11 respectively plot the expenditure and the share life cycle profile in the baseline and in the counterfactual economy. Note that the graphs combine the result of both the allocation effect and the general equilibrium effects. General equilibrium effects imply that individuals spend more resources at

any any age, while the allocation effect implies that young individuals spend less resources on consumption. Figure 10 shows that at young ages, because of these two opposite effects, the net effect on the expenditure level, and equivalently on the expenditure share, is close to be null. At older ages, the total effect on the expenditure level and on the expenditure share is unambiguously positive as the two effects both have a positive effect.

We now analyze the role of the population effect. In the baseline economy, there are more old individuals than in the counterfactual economy (see Figure 9). From Figure 6, we see that old individuals have a smaller expenditure share on services than young and middle-aged individuals. This means that aging has a negative effect on the aggregate expenditure share on services through the population effect. However, this neglects the role of health expenditures, whose life cycle profile is upward sloping. Hence aging increases the aggregate expenditure share on services through an increase of health expenditures. Therefore the population effect creates two opposite effects. To compute the contribution of the population effect, we keep the consumption levels of the baseline economy and we use the population shares of 1950 to compute the aggregate expenditure share on services. We find that this variable is 1% smaller than that of the baseline economy. This means that the population effect is the main channel through which aging changes the aggregate expenditure share on services. The overall conclusion of this experiment is that aging creates several counteracting forces on the aggregate expenditure share on services, which implies that its total effect is small.

5   Conclusion

This paper assesses the impact of demographic variables, fertility and mortality rates, on the composition of aggregate consumption. We identify three channels through which demographic variables affect the composition of aggregate consumption. First, consumption bundles varies with the age of an individual. We document from the CEX that the life cycle profile of the expenditure share of services is hump-shaped. As aging modifies the age distribution, it creates a change of the relative demand between goods and services. Second, aging spurs individuals to reallocate resources from young ages to older ages. This modifies consumption bundles as it implies that young individuals consume more ordinary goods, while old individuals consume more services. Third, aging is the source of general equilibrium effects. On the one hand, aging reduces labor supply. On the other hand, individuals save more and invest more in their human capital. These effects change the resources of individuals and then their consumption bundles.

We build a rich multi-sector OLG model that includes these effects. The model generates

life cycle profiles for the expenditure share of services in line with the one we document. We then conduct counterfactual experiments on demographic variables. They reveal that aging has a positive impact on the aggregate consumption share of services. The magnitude of the total effect is small because the outlined effects act in opposite directions. First, old indivduals consume more health services than young individuals, yet their expenditure share on other services is smaller than that of younger individuals. Second, aging implies that young individuals spend less resources on consumption, so they consume less services, while old individuals increase their consumption expenditures, so they consume more services. Third, aging spurs individuals to invest more in their human capital through the Ben-Porath effect, yet aging also reduces the number of working aged individuals. Therefore aging has important consequences on the sectorial distribution of resources, which are masked by only looking at its total impact on the aggregate consumption share on services.

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6 Appendix A

Consumption categories that are considered as goods in the CEX data are: food-off premise, tobacco products, alcohol off-premise, clothing and shoes, jewelry and watches, toilet articles and preparations, furniture and durable household equipment, nondurable household supplies and equipment, fuel oil and coal, ophtalmic products and orthopedic appliances, new and used motor vehicles, tires, tubes accessories and other parts, gasoline and oil, books and maps, magazines, newspapers, other nondurable toys, recreation and sports equipment. Other categories are considered as services.

7 Appendix B

In this section, we explain how to solve the intratemporal problem of the individual, that is to derive the consumption of services C_S and goods C_G from the aggregate consumption level C. C_S and C_G minimize the consumption expenditure level subject to the utility level being equal to C. Noting λ the Lagrange multiplier on the constraint (10), the FOCs write:

$$P_S = \lambda \Omega_S^{\frac{1}{\gamma}} \frac{\gamma - 1}{\gamma} C^{\frac{\epsilon_S - \gamma}{\gamma}} C_S^{\frac{-1}{\gamma}} \tag{15}$$

$$P_G = \lambda \Omega_G^{\frac{1}{\gamma}} \frac{\gamma - 1}{\gamma} C^{\frac{\epsilon_G - \gamma}{\gamma}} C_G^{\frac{-1}{\gamma}} \tag{16}$$

This implies that:

$$C_S = \frac{\Omega_S}{\Omega_G} C_G (\frac{P_G}{P_S})^{\gamma} C^{\epsilon_S - \epsilon_G} \tag{17}$$

With C_G given by:

$$C_G = \Omega_G^{\frac{1}{1-\gamma}} C^{\frac{\epsilon_G - \gamma}{1-\gamma}} (1 + \frac{\Omega_S}{\Omega_G} (\frac{P_S}{P_G}^{1-\gamma} C^{\epsilon_S - \epsilon_G})^{\frac{\gamma}{1-\gamma}} \tag{18}$$

And we can compute the expenditure function $E(.)$:

$$E(C) = P_S C_S + P_G C_G = (\Omega_G P_G^{1-\gamma} C^{\epsilon_G - \gamma} + \Omega_S P_S^{1-\gamma} C^{\epsilon_S - \gamma})^{\frac{1}{1-\gamma}} \tag{19}$$

8   Appendix C

In this section, we explain how we solve the individual's problem. During the retirement period, the Bellman equation writes:

$$v_a(x) = \max_{0 \le C, x'} (u(C, 1) + \beta q_a v_{a+1}((1+r)x + I_a - E(C)) + \beta(1 - q_a)v((1+r)x + I_a - E(C))) \quad (20)$$

The Inada condition on $v(.)$ implies that the constraint $0 \le x'$ never binds. Then, the first-order condition on C writes:

$$\alpha_L C^{(1-\sigma)\alpha_L - 1} = E'(C)\beta(q_a \frac{\partial v_a}{\partial x'}(x') + (1 - q_a)v'(x')) \quad (21)$$

This writes:

$$\alpha_L C^{(\sigma-1)\alpha_L + 1} E'(C) = \frac{\alpha_L}{\beta(q_a \frac{\partial v_a}{\partial x'}(x') + (1 - q_a)v'(x'))} \quad (22)$$

We then apply an endogenous grid method (Carroll (2006)). We work with a grid over the post-decision asset level x': $(x'(n))_{n \in N}$. For each gridpoint $x'(n)$, the RHS of (22) is known and we can solve for the consumption level $C_a(n)$. Then, the pre-decision asset level is given by: $x_a(n) = \frac{x'(n) + E(C_a(n)) - I_a}{1+r}$. Then, we interpolate on the pre-decision grid $(x_a(n))_{n \in N}$ to determine the optimal consumption level for any asset level.

During the working period, the Bellman equation writes:

$$v_a(x, h) = \max_{0 \le C, x', S, l} (u(C, 1 - l - S) + \beta q_a E[v_{a+1}((1+r)x + I_a + whl - E(C), e^{\zeta_a}(h + A(hS)^\theta))]$$
$$+ \beta(1 - q_a)v((1+r)x + I_a + whl - E(C))) \quad (23)$$

Note μ the Lagrange multiplier of the constraint $0 \le l$ and $f = h + (hS)^\theta$. The first-order conditions write:

$$\alpha_L C^{(1-\sigma)\alpha_L - 1}(1 - S - l)^{(1-\alpha_L)(1-\sigma)} = E'(C)\beta(q_a E[e^{\zeta_a}\frac{\partial v_a}{\partial x'}(x', e^{\zeta_a}f)] + (1 - q_a)v'(x')) \quad (24)$$

$$(1 - \alpha_L)C^{(1-\sigma)\alpha_L}(1 - S - l)^{(1-\alpha_L)(1-\sigma)-1} = wh\beta(q_a E[e^{\zeta_a}\frac{\partial v_a}{\partial h'}(x', e^{\zeta_a}f)] + (1 - q_a)v'(x')) + \mu \quad (25)$$

$$(1 - \alpha_L)C^{(1-\sigma)\alpha_L}(1 - S - l)^{(1-\alpha_L)(1-\sigma)-1} = \beta q_a A\theta h^\theta S^{\theta-1} E[e^{\zeta_a}\frac{\partial v_a}{\partial h'}(x', e^{\zeta_a}f)] \quad (26)$$

We now work with two grids: one over the two post-decision states x' and f and one over the pre-decision-state x and h. Given $x'(n)$ and $f(m)$ we solve the last system of equations. It is convenient to define $Z(n, m) = \beta(q_a E[e^{\zeta_a}\frac{\partial v_a}{\partial h'}(x'(n), e^{\zeta_a}f(m))] + (1 - q_a)v'(x'))$. We first consider an interior solution for l. Then, $\mu = 0$ and equations (25) and (26) imply:

$$whZ(n, m) = \beta q_a A\theta h^\theta S^{\theta-1} E[e^{\zeta_a}\frac{\partial v_a}{\partial h'}(x'(n), e^{\zeta_a}f(m))] \quad (27)$$

Hence:

$$hS = \left(\frac{\beta q_a A\theta E[e^{\zeta_a}\frac{\partial v_a}{\partial h'}(x'(n), e^{\zeta_a}f(m))]}{wZ(n,m)}\right)^{\frac{1}{1-\theta}} \tag{28}$$

Using the definition of f, we obtain:

$$f(m) = h + A\left(\frac{\beta q_a A\theta E[e^{\zeta_a}\frac{\partial v_a}{\partial h'}(x'(n), e^{\zeta_a}f(m))]}{wZ(n,m)}\right)^{\frac{\theta}{1-\theta}} \tag{29}$$

This gives the pre-decision level of human capital $h(n,m)$:

$$h(n,m) = f(m) - A\left(\frac{\beta q_a A\theta E[e^{\zeta_a}\frac{\partial v_a}{\partial h'}(x'(n), e^{\zeta_a}f(m))]}{wZ(n,m)}\right)^{\frac{\theta}{1-\theta}} \tag{30}$$

And the time dedicated to schooling $S(n,m)$:

$$S(n,m) = \frac{\left(\frac{\beta q_a A\theta E[e^{\zeta_a}\frac{\partial v_a}{\partial h'}(x'(n), e^{\zeta_a}f(m))]}{wZ(n,m)}\right)^{\frac{1}{1-\theta}}}{h(n,m)} \tag{31}$$

We now use (24) and (25) to obtain:

$$1 - l - S(n,m) = \frac{1-\alpha_L}{\alpha_L}\frac{CE'(C)}{wh(n,m)} \tag{32}$$

We introduce this relationship in (24), this gives the following equation for C:

$$CE'(C)^{\frac{1+(1-\alpha_L)(\sigma-1)}{\sigma}} = \left(\frac{wh(n,m)}{1-\alpha_L}\right)^{\frac{(1-\alpha_L)(1-\sigma)}{\sigma}}\frac{\alpha_L^{\frac{1+(1-\alpha_L)(\sigma-1)}{\sigma}}}{Z(n,m)^{\frac{1}{\sigma}}} \tag{33}$$

$C(n,m)$ is the unique solution to the previous equation. The time dedicated to labor is given by:

$$l(n,m) = 1 - S(n,m) - \frac{1-\alpha_L}{\alpha_L}\frac{C(n,m)E'(C(n,m))}{wh(n,m)} \tag{34}$$

To accept our solution, we must verify that $l(n,m) > 0$. From (33), we obtain that this condition is satisfied if and only if the following inequality holds:

$$E'(\tilde{c}) < \alpha_L(1 - S(n,m))^{\frac{\sigma}{\alpha_L(1-\sigma)}}Z(n,m)^{\frac{1}{\alpha_L(\sigma-1)}}\left(\frac{wh(n,m)}{1-\alpha_L}\right)^{1+\frac{1}{\alpha_L(\sigma-1)}} \tag{35}$$

Where $\tilde{c} = \left(\frac{1-\alpha_L}{wh(n,m)Z(n,m)}\right)^{\frac{1}{\alpha_L(\sigma-1)}}\frac{1}{(1-S(n,m))^{\frac{1+(1-\alpha_L)(\sigma-1)}{\alpha_L(\sigma-1)}}}$. If this condition holds and if $h(n,m) > 0$ and $S(n,m) \leq 1$, then $C(n,m), l(n,m), S(n,m)$ satisfy the FOCs associated to the post-decision states $x'(n)$ and $f(m)$ and the pre-decision states $x(n,m)$ and $h(n,m)$. We now

examine a possible corner solution for the time dedicated to labour. From (24):

$$1 - S = \left(\frac{\alpha_L}{Z(n,m)E'(C)}\right)^{\frac{1}{(1-\alpha_L)(1-\sigma)}} \frac{1}{C^{\frac{1+\alpha_L(1-\sigma)}{(1-\alpha_L)(1-\sigma)}}} \tag{36}$$

We substitute this expression for S in (26) to obtain an expression of h as a function of C:

$$h = C_1^{\frac{1}{\theta}} E'(C)^{\frac{1+(1-\alpha_L)(\sigma-1)}{\theta(1-\alpha_L)(\sigma-1)}} C^{\frac{\sigma}{\theta(1-\alpha_L)(\sigma-1)}} \left(1 - \left(\frac{\alpha_L}{Z(n,m)C^{1+\alpha_L(1-\sigma)}E'(C)}\right)^{\frac{1}{(1-\alpha_L)(1-\sigma)}}\right)^{\frac{1-\theta}{\theta}} \tag{37}$$

Where $C_1 = \frac{1-\alpha_L}{\beta q_a A\theta E[e^{\zeta_a}\frac{\partial v_a}{\partial h'}(x'(n),e^{\zeta_a}f(m))]}\left(\frac{Z(n,m)}{\alpha_L}\right)^{\frac{1+(1-\alpha_L)(\sigma-1)}{(1-\alpha_L)(\sigma-1)}}$. We insert this expression in the definition of f:

$$f(m) = C_1^{\frac{1}{\theta}} E'(C)^{\frac{1+(1-\alpha_L)(\sigma-1)}{(1-\alpha_L)(\sigma-1)}} C^{\frac{\sigma}{(1-\alpha_L)(\sigma-1)}} \left(1 - \left(\frac{\alpha_L}{Z(n,m)C^{1+\alpha_L(1-\sigma)}E'(C)}\right)^{\frac{1}{(1-\alpha_L)(1-\sigma)}}\right)^{\frac{1-\theta}{\theta}}$$
$$+ AC_1 E'(C)^{\frac{1+(1-\alpha_L)(\sigma-1)}{(1-\alpha_L)(\sigma-1)}} C^{\frac{\sigma}{(1-\alpha_L)(\sigma-1)}} \left(1 - \left(\frac{\alpha_L}{Z(n,m)C^{1+\alpha_L(1-\sigma)}E'(C)}\right)^{\frac{1}{(1-\alpha_L)(1-\sigma)}}\right) \tag{38}$$

The RHS of (38) is an increasing function of C ranging from 0 to $+\infty$. Hence (38) has a unique solution $C(n,m)$. We must now check that μ is non-negative. From (25), this equivalent to:

$$C_1 E'(C(n,m))^{\frac{1+(1-\alpha_L)(\sigma-1)}{(1-\alpha_L)(\sigma-1)}} C(n,m)^{\frac{\sigma}{(1-\alpha_L)(\sigma-1)}} \left(1 - \left(\frac{\alpha_L}{Z(n,m)C^{1+\alpha_L(1-\sigma)}E'(C(n,m))}\right)^{\frac{1}{(1-\alpha_L)(1-\sigma)}}\right)$$
$$\leq \left(\frac{\beta q_a A\theta E[e^{\zeta_a}\frac{\partial v_a}{\partial h'}(x'(n),e^{\zeta_a}f(m))]}{wZ(n,m)}\right)^{\frac{\theta}{1-\theta}} \tag{39}$$

To sum up, for each couple $(x'(n), f(m))$, we first check if the necessary conditions for an interior solution to exist are satisfied. If this is the case, we compute the decision rules as explained above. Then, we rootfind (38) and check whether the solution satisfies the condition (39). If this is the case, we also store this decision rule. Note that for the moment it is possible that two decision rules are associated to one post-decision state. This possibly happens because our objective function is not necessarily concave, meaning that the FOCs are only necessary. The rest of the procedure consists of computing the decision rule for each state in the pre-decision grid. For this, we follow Druedahl and Jorgensen (2017). We consider triangles $((x'(n), f(m)), (x'(n+1), f(m)), (x'(n+1), f(m+1)))$ in the post-decision state space, for which we compute the associated triangles $((x(n,m), h(n,m)), (x(n+1,m), h(n+1,m)), (x(n+1, m+1), h(n+1, m+1)))$ in the pre-decision state. We look for points of our regular grid that lie in these triangles. For such points, we interpolate the decisions rules. If a point is associated to two decision rules, then we keep the one for which the value function is greater. Pratically, at the end of this procedure, the decision rules are not computed for 2% of the points of our regular grid. For these points, we use a gloal maximizer to directly solve the Bellman equation.