Fiscal Regimes and the (Non)stationarity of Debt *

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Abstract

This paper analyzes the sustainability of fiscal debt contingent on fiscal policy operating in two fiscal regimes. The first regime is characterized by active policy (not reacting to debt) and the other by passive fiscal policy (reacting to debt). The average duration for which either regime can be pursued in order to arrive at a long-run stable solution is dependent on the steady-state debt-to-GDP ratio and thus determines the cutoff point beyond which debt is non-stationary. We find that the longer an active policy regime is in force or, equivalently, the more likely fiscal policy is to remain in this regime, the lower the steady state debt-to-GDP ratio must be. This has repercussions for the overall business cycle, implying a higher volatility of inflation and output the longer fiscal policy is active for any given equilibrium debt-to-GDP level. Using the Markov-switching DSGE-model as the data generating process it is possible to apply the test by Bohn (1998) and find that it is prone to type 2 errors.

Keywords: DSGE, Markov-Switching, Fiscal Policy, Debt Sustainability
**JEL classification:** C62, E61, E62.

1 Introduction

Recent surges of public debt in many industrialized countries have brought back concerns about debt sustainability. Looking at fiscal data for the US from the Congressional Budget Office (CBO) with projections until 2050 (see Figure 1), it becomes obvious that, first, past periods of high and persistent deficits in percent of GDP were occasionally interrupted by surpluses and, second, the debt-to-GDP ratio would rise quickly if the gap between revenues and spending were to widen as projected. This evolution of debt seems not to comply with the notion of stationarity. In this paper, we ask for how long periods of inactive fiscal policy can generally persist before debt sustainability is jeopardized and what factors determine the average length of such periods. We allow fiscal policy to alternate between phases where it responds to debt and periods when fiscal policymakers are inattentive to rising debt levels. The probabilities of switching between these two regimes or, alternatively, remaining in any regime give rise to a cutoff point beyond which debt is no longer on a stationary path, enabling us to derive both quantitative and analytical solutions as to how long such phases can persist.

According to Leeper (1991), fiscal policy is passive if it responds sufficiently to debt

Figure 1: Deficit and Debt - Past and Projected in the US
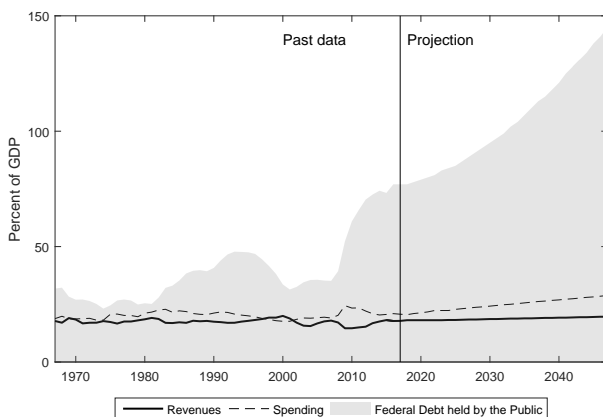
Source: Congressional Budget Office (Report Budget Outlook March 2017)

throughout. If fiscal policy deviates from this path, ie. it is active, debt is no longer on a stable path. Mauro, Romeu, Binder, and Zaman (2015) call the first behavior 'prudent' while the other is termed 'profligate'. They estimate the respective fiscal responses for a wide range of countries and over a long history of data. Their findings are that countries generally obey the notion of fiscal prudence, thereby satisfying the intertemporal budget constraint. Within a country, the response varies over time, however. This is consistent with the findings of Bohn (1998) and Mendoza and Ostry (2008) whose empirical results suggest that fiscal prudence need not be fulfilled at every point in time but only over an infinite horizon. Hence, fiscal policy may deviate from sustainable policy for some time as long as it is brought back to a sound path at a later date.

1

In our framework, we encompass and build on parts of the aforementioned papers by also allowing phases of active and passive fiscal policy to switch according to a Markov-switching process. Economic agents are rational and fully informed about the prevailing regime, as well as of the probability of each regime shifting or remaining the same. We start by building intuition and deriving the general conditions for debt to be stationary analytically using the linearized government budget constraint and the two possible regimes. These results are then quantified with a small New-Keynesian model which features a simple fiscal sector. Given the model setup and calibration, we identify debt stationarity contingent on all probabilities of either regime occurring and on different steady-state debt-to-GDP ratios.

We find that the lower the long-run level of the public debt-to-GDP ratio, the more likely it appears that an economy will be able to afford a longer period of active fiscal policy. Conversely, fiscal policy needs to remain passive for a longer period of time if the steady-state debt level is already elevated. Reaching the cutoff point beyond which debt is no longer stationary can be alleviated for any given length of active fiscal policy and any steady-state debt-to-GDP ratio, once fiscal policymakers generally pursue a passive fiscal policy for a longer time period once it enters this regime. These results hold irrespective whether expenditure-based or tax-based instruments react to debt in the passive regime. If passive fiscal policy is characterized by a stronger reaction to debt, the public sector can afford to react less to debt, while a slighter reaction requires shorter periods of active policy to keep fiscal policy sustainable. Simulating the model with different steady-state debt-to-GDP ratios and probabilities of staying in a passive policy regime, the volatility of major macroeconomic variables such as inflation and output increases, the higher is the probability of remaining fiscally active for any given long-run debt-to-GDP ratio. Hence, loose fiscal policy and an adverse fiscal position pose a drag on the overall economy and are welfare-detrimental. Lastly, another contribution of this paper is that we use the Markov-switching DSGE-model as a data-generating process and apply the generated time series to the test introduced by Bohn (1998). Therefore, we take the theoretical data stemming from the model economy and regress the surplus over GDP on the debt-to-GDP ratio and control for the business cycle. The comparison of the coefficient that should correctly identify whether government debt is sustainable or not according to the test is now contrasted with the true cutoff point. It turns out that the test yields the correct result when government debt is indeed sustainable. However, in a few instances it wrongly classifies debt as sustainable when it is not, thus identifying the cutoff point beyond the true one for all probabilities of remaining active. In other words, the test is prone to type 2 errors.

This paper is closely related to other papers in the literature. For example, Ghosh, Kim, Mendoza, Ostry, and Qureshi (2013) claim and show that a debt limit is reached when governments, because of fiscal fatigue, do not respond sufficiently to debt once debt levels attain high levels. Bi (2012) and Bi, Leeper, and Leith (2013) introduce the notion of a fiscal limit by looking at the maximum of the Laffer curve where the government is unable to raise any additional revenues. This limit is characterized by a probability distribution given underlying transfer and technology shocks. One example of regime switching in the fiscal sector is the paper by Davig (2004). He uses regime switching with respect to the debt-to-GDP ratio to answer the question of how a tax shock affects the overall economy. Davig and Foerster (2014) also apply regime switching, primarily of fiscal shocks, to look

at fiscal uncertainty and fiscal cliffs. A paper which is also close to ours is Aldama and Creel (2016) who let fiscal reaction functions be regime-dependent and estimate the respective fiscal regime using French data. Bonam and Hobijn (2016), finally, look at the stability conditions in a two-country monetary union DSGE model and also apply regime switching to both joint monetary policy and each fiscal policy.

The structure of the paper is as follows. Section 2 will provide some tentative empirical evidence. The fiscal sector, the assumptions regarding the Markov-switching as well as the analytical results are set out in Section 3. Section 4 presents the rest of the model and quantifies and applies the results. Section 5 concludes.

2 Empirical Evidence

This section provides tentative evidence for the relationship between the fiscal policy stance and policy's perception of an implicit debt sustainability. Here, we concentrate on mere bivariate relationships to establish simple data-driven associations. Additionally, we do not rely on the primary surplus, as it does not account well for discretionary fiscal policy's behavior and use the fiscal stance instead. This variable captures the change in the primary surplus (in percent of GDP) adjusted for the cyclical component. As in the remainder of the paper,too, the government budget constraint is linearized and therefore symmetric around its long-run steady state, this specification is more appropriate for our purpose. Furthermore, it is also suitable for testing for fiscal sustainability under the assumption that the business cycle is symmetric.

Our first aim is to observe how the fiscal stance reacts to public debt (as a percentage

Figure 2: The relationship between fiscal stance and the debt-to-GDP ratio.
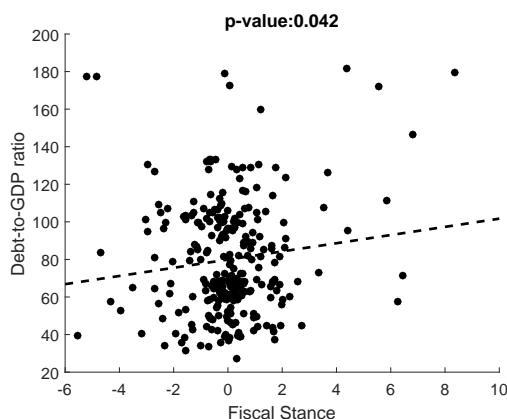
Note: the figure displays the relationship between the yearly fiscal stance (change in the structural primary surplus) and the respective debt-to-GDP ratio in nine countries. It can be observed that there is a significantly positive relationship. Whenever debt rises, the fiscal stance positively adjusts on average.

of GDP) in general. Countries that are included in this setup are Germany, France, Italy,

Spain, Greece, Portugal, the United Kingdom, Austria, Belgium and the Netherlands and the periods covered are 1995 up to (including the projections for) 2018. The frequency of these data is yearly and they are taken from the AMECO database of the European Commission.

Figure 2 shows the bivariate relationship between the fiscal stance and debt ratios of altogether 272 observations. Without being able to draw any conclusions about causality, there is a clearly positive relationship between the fiscal stance and the debt ratio. Whenever the debt ratio rises by 10 percentage points, the fiscal stance increases by roughly 4 percentage points on average. With a p-value of 0.042 this correlation is highly significant, too.

As we are interested in two different regimes of fiscal policy and how long they (can) last,

Figure 3: The duration of negative fiscal stance and the corresponding debt-to-GDP ratio.
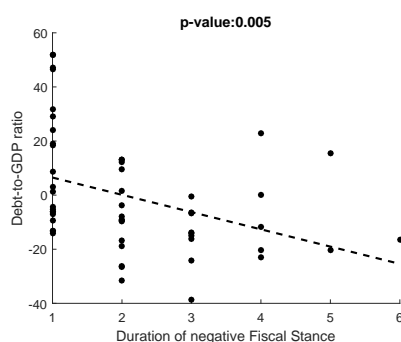
Note: the figure displays the duration of negative fiscal stance (change in the structural primary surplus) episodes in nine countries and the respective initial deviation of the debt-to-GDP ratio (from its country-specific long term mean). It becomes obvious that there is a clear negative and significant relationship between the duration of a negative fiscal stance and the deviation of the debt-to-GDP ratio from its long run mean.

we analyze, in an additional step, whether there is a relationship between the duration of the negative fiscal stance and the debt-ratio. Therefore, we take the 272 observations from above and identify negative fiscal stance instances and for how many years they last. To account for the difference in countries' overall debt ratios over time, we deduct the yearly debt ratio from its overall sample mean on a country-by-country basis. Then we relate beginning of the period (last period's debt deviation from its long-run mean) debt ratio to the number of consecutive negative fiscal stances. We are left with 53 instances of a consecutive negative fiscal stance that last from one period to six years (only one case) in total. Figure 3 shows that the higher is the deviation of the debt-to-GDP ratio from its long run trend, the shorter the negative fiscal stance instance lasts on average. The relationship that is described here is highly significant and hints at the presence of a clear negative connection between the duration of fiscal profligacy and size of the average debt ratio which may give rise to a perceived cutoff point beyond which debt is no longer stationary following the length of a negative fiscal stance.

4

3 Determining the Conditions for Debt Stationarity

This section describes the fiscal sector and provides analytical insight into which parameters are central to the determination of debt stationarity in our analysis.

3.1 Fiscal sector

The government's budget constraint takes the following form:

$$B_t = \left(\frac{B_{t-1}R_{t-1}}{\pi_t}\right) + G_t - \tau_t^l W_t N_t \tag{1}$$

The government can accumulate debt B_t. Every year, debt is increased by interest payments, ie the previous year's debt multiplied by the previous year's nominal interest rate which is given by R_{t-1}. The inflation rate is given by π_t and government expenditure in the current period by G_t [1]. Labor income taxes (with τ_t^l as the tax rate applied to wages W_t and hours worked N_t) reduce the deficit and hence government debt. In this model, both government expenditure and the tax rate are allowed to respond in a rule-based manner. For the purpose of exhibition in this section, we restrict our analysis solely to the government spending rule for now. It is persistent and has an anticyclical component which is linked to last period's debt level. If debt is higher than its long-run trend, government expenditure is cut back accordingly, in order to return to the long-run equilibrium path. If government debt (denoted in deviations from its own steady state and in absolute terms, not relative to GDP[2]) is below its long-term equilibrium value, government expenditure can be increased [3]. The rule can be expressed in linearized form (with small-case letters) as follows:

$$g_t = \rho_g g_{t-1} - \delta_b(s_t)(b_{t-1} - b^*) + \epsilon_t^g \tag{2}$$

The * as a superscript denotes the target of the variable, here the debt target, and ϵ_t^g is the fiscal policy shock. The parameters ρ_g, δ_b and denote the intensity of the response of government expenditure to, respectively, its own lag and the deviation of debt from its target. δ_b is dependent on the regime s_t in period t, which will be specified in the next subsection.

3.2 Analytical Approach

We assume throughout that fiscal policy can be subject to two possible regimes s_t in period t. In one of these regimes, government expenditure is sensitive to debt levels and is thus passive. In the other regime, it violates this principle and government expenditure is no longer permitted to respond to debt. In this regime, the government is thus pursuing

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[1]The focus of this analysis is exclusively on fiscal policy. Hence, this is not a model where inflation and debt are both jointly determined, as monetary policy is kept active at all times.

[2]The analysis does not change if the debt-to-GDP ratio is included as a target variable or if another variable such as GDP is included in the government spending rule.

[3]It is also assumed that the long-run structural growth of the economy is zero, and the interest growth differential is therefore assumed to be positive.

an active fiscal policy. Seen in isolation, the first regime leads to a stable and determined equilibrium, whereas the second regime is, as such, unstable. Hence, in this regime, debt is no longer stationary and can therefore potentially grow without limit. A temporarily active fiscal policy should also be stable if the probability of it occurring is sufficiently small and it is associated with an expected return at a later date to a passive fiscal policy. This outcome has been known for monetary policy since Davig and Leeper (2007) [4]. Fiscal policy, accordingly does not have to be passive at all times but can deviate slightly from a purely passive policy over a certain period of time. Both regimes seen together should hence still result in a stable equilibrium. The transition probabilities between the two regimes are given by the matrix below.

$$P = \begin{pmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{pmatrix} \tag{3}$$

Expressed in generally analytical terms, the two regimes follow a Markov chain which is described as

$$p_{ij} = P[s_t = j | s_{t-1} = i] \tag{4}$$

where i, j = 1,2 stand for both regimes. What this probability matrix intuitively means is that with the probability p_{11} the coefficient on debt was 0 in the last period and is still 0 in the contemporaneous period. On the other hand, the probability p_{22} measures the likelihood of fiscal policy being passive in the previous year and remaining so in period t. The other two entries in the matrix represent the probability of transitioning from one regime to the other. In this vein, $1 - p_{22}$ would be the probability of fiscal policy being active in t-1 and passive in period t. For $1 - p_{11}$, the reverse holds true. Over the long run, the probabilities p_{11} and p_{22} can also be interpreted as the average length of time for which each policy is pursued. This is expressed as the sample equation $1/(1 - p_{11})$ for the period for which fiscal policy remains active. If the probability p_{11} stood at 20%, fiscal policy would thus remain active for 1.25 years. A 90% probability would result in an interval of ten years.

3.3   Analytical result

With the ingredients from the fiscal sector given before, we derive the parameter constellations that are responsible for the existence of debt stationarity in general terms. The result is the following:

Proposition: Given the government budget constraint, the government spending rule (with the regime-dependent coefficient on debt) and the Markov-chain, it holds that the probability that the debt becomes non-stationary
i) increases with p_{11}: the probability of remaining in a fiscal active regime
ii) increases with $\frac{B_{ss}}{Y_{ss}}$: the debt-to-GDP ratio in steady state
iii) decreases with p_{22}: the probability of remaining in a fiscal passive regime
iv) increases with $\frac{\pi_{ss}}{\beta}$: the steady-state interest rate
v) decreases with $\delta_b(s_2)$: the reaction to public debt in the passive regime

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[4]With regard to monetary policy, this would have to be active most of the time, or the possibilities of this happening would be sufficiently high. Bianchi and Melosi (2013) and Leeper and Davig (2011) also use Markov-switching to account for the regime changes in both branches of government.

Proof: $\frac{\pi_{ss}}{\beta}$ is the real interest rate with β as the parameter governing the time preference of households. For simplicity, we assume that inflation is kept constant at its zero steady state, thereby ensuring that inflationary pressures cannot help to stabilize the real value of government debt. The linearized government budget constraint with government spending (the linearized government spending rule is already inserted) and with tax income being held constant (at its steady state, which is denoted by ss) is given by

$$b_t = \frac{\pi_{ss}}{\beta}\left(b_{t-1} + r_{t-1} - \pi_t\right) + \frac{G_{ss}}{B_{ss}}\left(\rho_g g_{t-1} - \delta_b(s_t)b_{t-1}\right) - \frac{T_{ss}}{B_{ss}}t_t \tag{5}$$

Including the two states and the respective probabilities, the difference equation can take two different forms:

$$\begin{bmatrix} b_{1,t} \\ b_{2,t} \end{bmatrix} = \begin{bmatrix} \frac{\pi_{ss}}{\beta} \\ \frac{\pi_{ss}}{\beta} \end{bmatrix} - \begin{bmatrix} \frac{G_{ss}}{B_{ss}}\delta_b(s_1) & \frac{G_{ss}}{B_{ss}}\delta_b(s_1) \\ \frac{G_{ss}}{B_{ss}}\delta_b(s_2) & \frac{G_{ss}}{B_{ss}}\delta_b(s_2) \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} b_{1,t-1} \\ b_{2,t-1} \end{bmatrix} + ... \tag{6}$$

where the last terms are neglected as they either do not influence the dynamics of the system or, in the case of the autoregressive component, are left out to restrict the system to being an AR(1) process. Rearranging then yields:

$$\begin{bmatrix} b_{1,t} \\ b_{2,t} \end{bmatrix} = \begin{bmatrix} \frac{\pi_{ss}}{\beta} - \frac{G_{ss}}{B_{ss}}\delta_b(s_1)p_{11} & \frac{\pi_{ss}}{\beta} - \frac{G_{ss}}{B_{ss}}\delta_b(s_1)(1-p_{22}) \\ \frac{\pi_{ss}}{\beta} - \frac{G_{ss}}{B_{ss}}\delta_b(s_2)(1-p_{11}) & \frac{\pi_{ss}}{\beta} - \frac{G_{ss}}{B_{ss}}\delta_b(s_2)p_{22} \end{bmatrix} \begin{bmatrix} b_{1,t-1} \\ b_{2,t-1} \end{bmatrix} \tag{7}$$

where stability of debt in both states is given by the eigenvalues of A. If one of the two eigenvalues exceeds one, then debt is no longer on a sustainable path and explodes. Debt becomes non-stationary when both eigenvalues $\lambda_{1,2}$ are on or below the unit circle.

$$\begin{bmatrix} b_{1,t} \\ b_{2,t} \end{bmatrix} = \begin{bmatrix} \frac{\pi_{ss}}{\beta} - \lambda & \frac{\pi_{ss}}{\beta} \\ \frac{\pi_{ss}}{\beta} - \frac{G_{ss}}{B_{ss}}\delta_b(s_2)(1-p_{11}) & \frac{\pi_{ss}}{\beta} - \frac{G_{ss}}{B_{ss}}\delta_b(s_2)p_{22} - \lambda \end{bmatrix} \begin{bmatrix} b_{1,t-1} \\ b_{2,t-1} \end{bmatrix} \tag{8}$$

after some algebra and setting $\delta_b(s_1) = 0$, ie. active fiscal policy, the two respective λs are now given by the following equations:

$$\lambda_1 = 0.5\left[2\frac{\pi_{ss}}{\beta} - \frac{G_{ss}}{B_{ss}}\delta_b(s_2)p_{22}\right] + 0.5\sqrt{\left(2\frac{\pi_{ss}}{\beta} - \frac{G_{ss}}{B_{ss}}\delta_b(s_2)p_{22}\right)^2 p_{22}^2 + 4\left[\left(\frac{\pi_{ss}}{\beta}\right)^2 - \frac{G_{ss}}{B_{ss}}\delta_b(s_2)(1-p_{11}) - \left(\frac{\pi_{ss}}{\beta} - \frac{G_{ss}}{B_{ss}}\delta_b(s_2)p_{22}\right)\right]}$$

$$\lambda_2 = 0.5\left[2\frac{\pi_{ss}}{\beta} - \frac{G_{ss}}{B_{ss}}\delta_b(s_2)p_{22}\right] - 0.5\sqrt{\left(2\frac{\pi_{ss}}{\beta} - \frac{G_{ss}}{B_{ss}}\delta_b(s_2)p_{22}\right)^2 p_{22}^2 + 4\left[\left(\frac{\pi_{ss}}{\beta}\right)^2 - \frac{G_{ss}}{B_{ss}}\delta_b(s_2)(1-p_{11}) - \left(\frac{\pi_{ss}}{\beta} - \frac{G_{ss}}{B_{ss}}\delta_b(s_2)p_{22}\right)\right]}$$

Note that if $\delta_b(s_2)$ were also zero, the resulting eigenvalues would both be $\frac{\pi_{ss}}{\beta}$ and hence bigger than one. We restrict our analysis here to λ_1 which is more likely to yield a value bigger than one. Here, we identify that an increase in the steady state interest rate $\frac{\pi_{ss}}{\beta}$, either by a higher steady state inflation or smaller time preference parameter value leads ceteris paribus to a rise in the eigenvalue. The case is also unambiguous in the case of both p_{22}, i.e. a higher probability of staying in a passive fiscal policy regime and the coefficient $\delta_b(s_2)$. Both parameters enter negatively which makes it easy to conclude that increasing both parameters lowers the eigenvalue λ_1. That is to say, if fiscal policy either responds more strongly to debt in regime 2 or remains in this regime with a higher probability, the eigenvalue is ceteris paribus lower, thereby lowering the probability of unsustainable

debt. On the other hand, increasing the probability p_{11}, i.e. staying fiscally active for longer, leads to a clear increase in the eigenvalue. Another steady state ratio that enters the analysis is government spending over GDP $\frac{G_{ss}}{B_s}$ which is negatively connected with the debt-to-GDP ratio (by dividing it by one and multiplying it by the government spending over GDP ratio $\frac{G_{ss}}{Y_{ss}}$ and, hence, a higher debt ratio implies also a higher eigenvalue. This proves the individual parameters and their connection to the overall eigenvalue of the government budget constraint.

End of Proof:

The cutoff point of one is, of course, determined by the combination of all parameters simultaneously. In addition, concentrating on the budget constraint alone is a stark simplification, as, in reality, households form expectations about both possible regimes and optimize accordingly. The underlying conditions, however, also holds in a fully-fledged DSGE-model, which we will use in the next section for a quantitative assessment of the stationarity of debt.

4 Quantification of Debt Stationarity

This section sets out the model and then quantifies the regions where debt is sustainable under diverse assumptions with a simple New-Keynesian model (the fiscal part was already introduced in section 3.1) and a standard calibration.

4.1 Rest of Model

Households:

Household i maximizes its expected life-time utility where the period utility function is given by

$$U_t^i = \left(ln(C_t) - \frac{N_t(i)^{1+\phi}}{1+\phi}) \right) \tag{11}$$

the household derives utility from consumption C_t and disutility from labor N_t. Households maximize their utility subject to the budget constraint:

$$C_t + B_t = W_t(i)N_t(i,j)(1-\tau_t^l) + \frac{R_{t-1}B_{t-1}}{\pi_t} + T_t \tag{12}$$

where all remaining variables have been initially explained in section 3.1.

Firms:

Firm j produces its output with the production function

$$Y_t(j) = exp(A_t)N_t(j) \tag{13}$$

where Y_t denotes the output produced with a given level of technology A_t and hours $N_t(j)$ as input factors. An AR(1) process describes the exogenous process for technology:

$$log(A_t) = \rho_a log(A_{t-1}) + \epsilon_t^A \tag{14}$$

We assume that price setting is carried out by retailers according to the Calvo (1983) mechanism, i.e. each period the fraction $(1 - \theta)$ of all firms is allowed to reset their prices $(P_t(j))$ optimally. There is no indexation of those firms that cannot reoptimize their prices. Profits of firm j are then given by (in nominal terms):

$$\Pi_t(j) = (P_t(j) - MC_t(j)) \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon^d} Y_t(j) \tag{15}$$

with real marginal costs given by the following expression:

$$MC_t(j) = \frac{W_t}{A_t} \tag{16}$$

and the demand for good j is expressed by

$$Y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\frac{1+\epsilon^d}{\epsilon^d}} \tag{17}$$

where ϵ^d is the demand elasticity for good j.

Market Clearing and Monetary Policy:
Demand on the part of the government and households in the form of consumption must fully absorb the output of the firms:

$$Y_t = C_t + G_t \tag{18}$$

where G_t denotes government spending with the respective rule given in section 3.2. Market clearing in the bond market implies that all bonds issued by the government are bought by the households in the economy. Monetary policy is conducted by the central bank which follows a Taylor-type rule and reacts to its own lag as well as to deviations of inflation and output from its target[5]:

$$\frac{R_t}{R_{ss}} = \left(\frac{R_{t-1}}{R_{ss}} \right)^{\rho_r} \left(\frac{\pi_t}{\pi_{ss}} \right)^{\phi_\pi} \left(\frac{y_t}{y_{ss}} \right)^{\phi_y} \epsilon_t^R \tag{19}$$

with ϵ_t^R as the normally distributed monetary policy shock. The distribution is given by N(0,σ_r)

4.2 Calibration and Solution

In order to quantify the point beyond which debt is no longer stationary, we have to calibrate all model parameters. These will be explained in this section. $\delta_b(s_t)$ is the key coefficient in the government spending rule which determines the elasticity of government expenditure to debt. Put differently: Across regimes that coefficient may vary and what

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[5]As monetary policy is not allowed to vary in this paper, it is assumed to always follow an active policy, i.e. satisfies the Taylor principle. In terms of stability conditions, it would be equivalent to our exercise in a purely real business cycle model.

weight is given to deviations of debt from its target is of importance when setting government expenditure in period t. In the baseline calibration, if fiscal policy is active, this parameter is set to a value that is lower than the real interest rate (here, $\delta_{b,s=1} = 0$) and, in a passive regime, to the standard value of 0.1 (hence $\delta_{b,s=2} = 0.1$). The economy as a

Table 1: Calibrated Parameters of the model

Description	Parameter	Value
impatience	β	0.98
Disutility of labor	ϕ	1
Calvo Prices	θ	0.70
Steady State Tax Rate	τ_{ss}	0.3
coeff. on inflation in TR	ϕ_π	1.5
coeff. on output in TR	ϕ_y	0.5
coeff. on debt in gov.spending	$\delta_b(s)$	$s_1 = 0$
		$s_2 = 0.1$
AR parameter tax	ρ_t	0.6
AR parameter gov. Spending	ρ_g	0.6
AR parameter technology	ρ_a	0.6
AR parameter interest rate	ρ_r	0.6
Steady state ratios:	C_{ss}/Y_{ss}	0.6
	G_{ss}/Y_{ss}	0.4
	G_{ss}/B_{ss}	$[0.2 : 1]$
Std.deviation technology	σ_a	0.01
Std.deviation gov. spending	σ_g	0.01
Std.deviation interest rate	σ_r	0.01
Std.deviation labor tax rate	σ_{tau^l}	0.01

Note: this table gives an overview of all calibrated parameters. Most of them are taken from the literature. δ_b can take two values depending on what regime fiscal policy is following. The steady state ratio G_{ss}/B_{ss} is varied to arrive at different steady-state debt-to-GDP ratios.

whole is simulated with the calibrated parameters from Table 1. The model is calibrated to a yearly frequency where most parameters are taken from the literature. The coefficients of the interest rate rule are set to standard 1.5 for inflation and 0.25 for output. The Calvo-parameter is chosen to be 0.7, which means that 30% of all firms can choose to reset their prices each year (or, correspondingly, close to 10% each quarter). This gives rise to a flatter Phillips curve than in many models, but is consistent with recent estimates of the US economy (see Kulish, Morley, and Robinson (2017)). The autoregressive parameters are all set to uniform values of 0.6 and the standard deviations to 0.01.[6] Our simulation now rests on calculating, for all potential probabilities p_{11} and for various long-run debt-to-GDP ratios B_{ss}/Y_{ss}, whether a stable long-run equilibrium can be identified even though the two regimes would separately indicate an unstable regime (for s_1) and a stable regime (for s_2).[7] In the current two-regime system, economic agents do not assess these regimes in isolation but weight them together with the respective probability of occurrence. Households believe at any time t with a certain probability that fiscal policy will respond to debt and, with a certain probability, that it will not.

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[6]As the focus of this paper is not necessarily on the matching of certain moments and replicating past data, the uniformity of parameters in the AR processes is not of greater interest for the mostly qualitative analysis.

[7]The solution of the Markov-switching problem is achieved using the code developed by Farmer, Waggoner, and Zha (2009). The initial scale is set to 0 and the convergence criterion to $1 * 10^-9$.

Regimes are always observable, as perfect information and rational expectations exist. This invites the question as to how long the respective regime can prevail, depending on other factors, such as the size of the debt-to-GDP ratio in the long-run equilibrium.

The probabilities are naturally endogenous in reality and are influenced by the respective policy regime and, above all, by the underlying fiscal position that may trigger higher risk premia at capital markets. The less stabilizing a policy was in the past, the more likely it is that the policy will be reversed at some time. However, this is beyond the scope of this paper; our goal is, rather, to identify the limit for debt sustainability for every conceivable probability. Probabilities are thus exogenously given ex ante. Equally decisive elements for the determination of debt stationarity is how monetary policy is conducted, i.e. whether inflation is allowed to stabilize the real value of debt (see Leeper (1991) for the concept of fiscal dominance) and the possibility of outright default (see, for example, Uribe (2006)). These issues are also beyond the scope of this paper. In particular, the model economy is solved for 100 different probabilities of staying in an active fiscal policy regime ranging from $p_{11} = 0$ to $p_{11} = 0.99$ and 80 different debt-to-GDP ratios (with $\frac{G_{ss}}{Y_{ss}}$ given to be 0.4 and $\frac{G_{ss}}{B_{ss}}$ in a range from 0.2 to 1). Each solution is checked in terms of whether it is stationary or not. The exact stationarity condition we use is the mean-square stability criterion discussed by Bianchi (2013) and Farmer et al. (2009).[8]

4.3 Results

4.3.1 Debt ratio and active fiscal policy

The shaded areas in Figure 4 depict the non-stationary solutions to the model. Starting out with the probability of $p_{22} = 0.8$ it can be observed that, given the selected set of parameters for the coefficients in the government spending rule ($\delta_{b,s=1} = 0$ and $\delta_{b,s=2} = 0.1$), unstable equilibria generally materialize only if the probabilities of an active fiscal policy are very high. Thus, at a debt-to-GDP ratio of 40%, the explosive equilibria would be reached only at a probability of roughly 0.92 of an active fiscal policy, representing an average duration of around 12 periods [9]. If we now raise the steady state debt-to-GDP ratio in increments, this (maximum) probability of just reaching a stable equilibrium diminishes more and more. At the maximum assumed debt-to-GDP ratio of 200%, the probability p_{11} which would be just about permissible for ensuring a stable debt-to-GDP ratio would shrink to around 0.7. This is equivalent to an average lifetime of an active fiscal policy of only just over three years. Thus, the level of the long run debt-to-GDP ratio is negatively related to the possibility of a long period of deficits. The higher the steady state level of debt already is, the shorter the episodes of bad fiscal policy must be. Conversely, if the debt-to-GDP ratio is low enough, fiscal policy can afford to be active even for fairly long periods of time. This holds for the probability of remaining in a passive regime of 80%. If the parameter p_{22} is reduced by steps of 20pp to 20% we acknowledge that the threshold of debt stationarity is reached earlier for all probabilities p_{11} and debt-to-GDP ratios. Keeping everything else constant, a fiscal policy that remains shorter in a

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[8]For a more detailed discussion about different concepts of stationarity in Markov-switching DSGE models, we refer the reader to those two papers.

[9]The quantitative derivation of the exact region of debt sustainability is, of course, contingent on the calibration and on the model chosen. Therefore, the cutoff points may differ quantitatively in an otherwise specified model.

fiscally responsible regime lowers the probability of having non-stationary debt overall. As before, at low debt-to-GDP ratios of, for instance, 40%, higher probabilities of fiscal policy remaining active before unstable equilibria set in are permissible. The threshold here is around 80% or, on average, five periods. Under this set of circumstances, the threshold of maximum permissible probabilities rises nearly out of proportion to the debt-to-GDP ratio. At a debt-to-GDP ratio of 100%, the probability p_{11} already diminishes to 0.5%. This equals an average active fiscal policy regime lifetime of two years. If long-run debt to GDP stands at 200% of GDP, the threshold already drops to 0% and indicates that fiscal policy cannot be allowed to be active at all if government debt is to remain sustainable. What becomes visible from both figures is that fiscal policy can remain active longer, the lower the debt-to-GDP ratio is and the more frequent or longer the episodes of fiscal responsibility are.

Figure 4: Debt sustainability regions for different probabilities of remaining active (and also passive) depending on debt ratios.
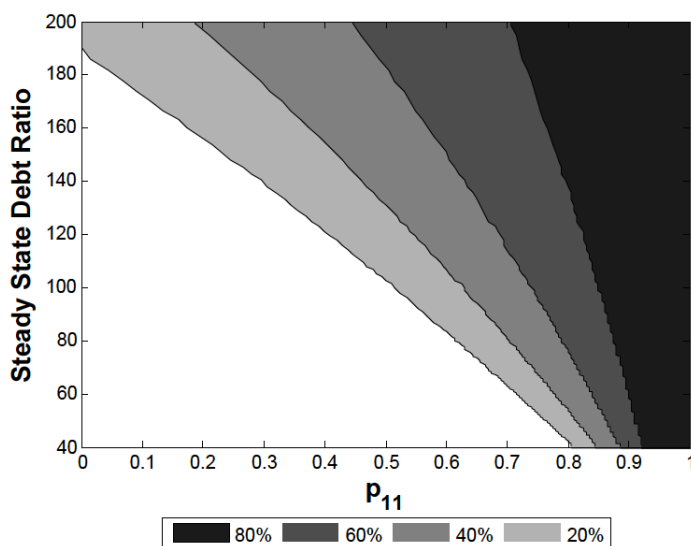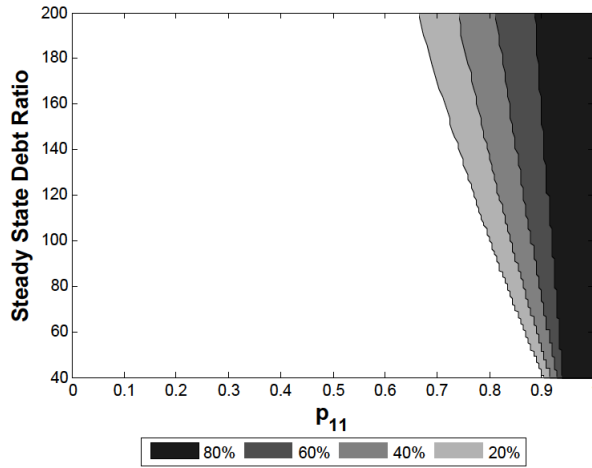

Note: the figure depicts unstable regions depending on the size of the steady-state debt ratio and the probability of remaining in a fiscally active regime. Different shaded areas show the cases when p_{22} (the probability of staying in an passive regime) is varied. The higher the probability of remaining in an active regime is, the lower is the probability of staying in a passive regime, and the higher the debt-to-GDP ratio is, the earlier debt becomes non-stationary.

This result holds for the case where the coefficient on debt in the passive fiscal policy regime is held constant at 0.1. To gain more insight into the way the cutoff points of debt stationarity evolves once this response is changed, Figures 5 and 6 provide robustness checks where all other parameters are held constant and the exercise is conducted in the same way as above. Figure 5 looks at a response coefficient in the case of passive policy which is set to 0.2. Thus, for any deviation of debt from its long-run steady state,

12

government spending aims at stabilizing debt twice as strongly as before. This results in a threshold that is pushed outwards for any debt-to-GDP ratio as well as for any probability of staying in an active regime. In other words, if it is known that fiscal policy, once it enters a passive regime, is responding strongly to debt, debt sustainability does not pose as big a threat as with a smaller response to debt, and the average duration even of active fiscal policy is allowed to last much longer. The opposite outcome occurs once the fiscal policy reaction to debt is halved compared to the baseline scenario. Figure 6 presents the regions of debt sustainability under the same assumptions as above with $\delta_{b,s=1} = 0.05$. The cutoff point is now heavily pushed inwards and fiscal policy becomes unsustainable both at relatively low probabilities p_{11} and also at lower debt-to-GDP ratios than before.

Figure 5: Debt sustainability regions for different probabilities of remaining active (and also passive) depending on debt ratios - Robustness
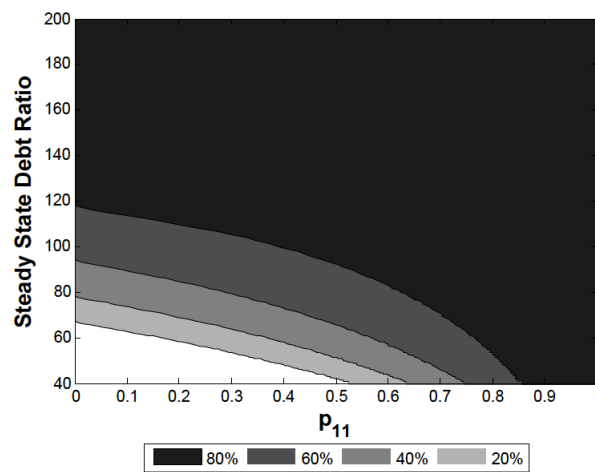

Note: the figure depicts unstable regions depending on the size of the debt ratio and the probability of remaining in a fiscally active regime. Different shaded areas show the cases when p_{22} (the probability of staying in an passive regime) is varied. The difference to figure 4 is that the fiscal reaction in the passive regime on debt is doubled.

So far, the only instrument that was allowed to react to debt was government spending. If the fiscal sector were to use labor income taxes to respond to debt instead, the linearized tax rate would look as follows:

$$\tau_t^l = \rho_t^t \tau_{t-1}^l + \delta_b^\tau(s_t)(b_{t-1} - b^*) + \epsilon_t^t \tag{20}$$

Once again, the reaction to the lagged value of debt is regime-dependent with $\delta_{b,s_t=1}^\tau = 0$ and $\delta_{b,s=2}^\tau = 0.05$. In regime $s_t = 1$ the tax rate follows an autoregressive process, whereas as before regime $s_t = 2$ denotes the passive fiscal policy regime and taxes sufficiently

Figure 6: Debt sustainability regions for different probabilities of remaining active (and also passive) depending on debt ratios - Robustness

Note: the figure depicts unstable regions depending on the size of the debt ratio and the probability of remaining in a fiscally active regime. Different shaded areas show the cases when p_{22} (the probability of staying in an passive regime) is varied. The difference from figure 4 is that the fiscal reaction in the passive regime on debt is halved.

respond to debt[10]. Government spending is assumed to follow only an autoregressive process without any reaction to debt. As can be seen in Figure 7, the result in terms of debt limits looks alike. Although quantitatively slightly different, qualitatively the exact same pattern emerges as in the case when government spending was used to respond to debt. In conclusion, the question of whether debt remains stationary depends non-linearly

Figure 7: Debt sustainability for different probabilities of remaining active (and also passive) depending on debt ratios - Taxes respond to debt
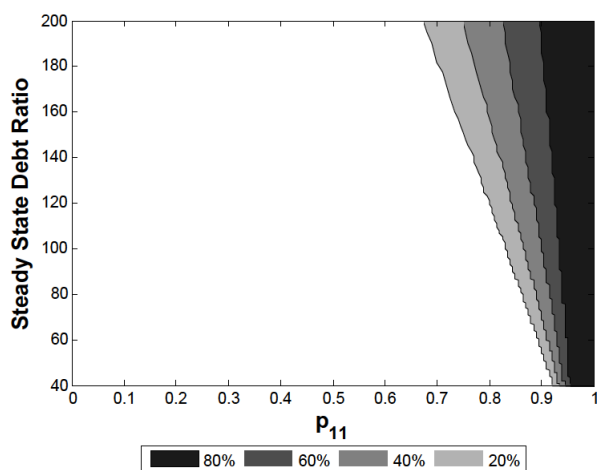
Note: the figure depicts unstable regions depending on the size of the debt ratio and the probability of remaining in a fiscally active regime. Different shaded areas show the cases when p_{22} (the probability of staying in an passive regime) is varied. The difference from figure 4 is that the tax rate reacts to government debt.

on the probability of remaining fiscally active, on the probability of remaining fiscally passive and the long-run structural debt-to-GDP ratio. They are similar irrespective of what fiscal instrument is used to respond to debt in the passive fiscal regime.

4.3.2 Business cycles under different fiscal policies and positions

We use the insights of the previous subsections and look at what these imply for the business cycle overall. Therefore, the model economy is simulated for 1,000 periods. It is assumed that it starts in period one in the active policy regime. From then on, the regime switches (or remains the same) depending on the underlying probability. Each simulation is, in turn, carried out for seven different debt-to-ratios (50% to 200%) and for ten different probabilities of remaining passive (ranging from 0.50 to 0.95). After obtaining time series for inflation and output, the respective volatilities are computed and expressed relatively to the 'anchor' fiscal position with the lowest debt-to-GDP ratio of 50% and the lowest

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[10]The calibration of different steady-state debt-to-GDP levels runs in that case through the ratio of taxes to debt.

probability of remaining in an active regime, $p_{11} = 0.5$. The probability of remaining in a passive regime is set throughout to be 0.8.

As can be seen in Table 2, the inflation volatilities compared to the base volatility

Table 2: Volatilities of Inflation

p_{11} \ $\frac{B_{ss}}{Y_{ss}}$	50%	75%	100%	125%	150%	175%	200%
0.5	1	0.9821	0.9702	0.9626	0.9589	0.9585	0.9605
0.55	1.0013	0.9849	0.9740	0.9675	0.9652	0.9662	0.9694
0.6	1.0113	0.9913	0.9797	0.9751	0.9765	0.9822	0.9900
0.65	1.0097	0.9941	0.9884	0.9921	1.0048	1.0236	1.0439
0.7	1.0169	1.0048	1.0043	1.0168	1.0392	1.0617	1.0675
0.75	1.0208	1.0082	1.0108	1.0296	1.0551	1.0649	1.0297
0.8	1.0282	1.0158	1.0232	1.0490	1.0611	1.0136	1.0074
0.85	1.0462	1.0296	1.0714	1.1343	1.0794	5.2199	>10
0.9	1.0929	1.1405	1.4564	1.8361	>10	>10	>10
0.95	1.6791	>10	>10	>10	>10	>10	>10

Note: this table displays the relative volatilities of inflation after 1,000 periods relative to the scenario of remaining active with a probability of 50% and a debt-to-GDP ratio of 50%. In general, it holds that the higher the probability of staying in the fiscal active regime given any debt ratio, the larger is the volatility of inflation.

becomes bigger once the probability of remaining in an active regime is raised given any debt ratio. For example, inflation is more than 9% more volatile over the simulation horizon if the probability is increased from 0.5 to 0.9 and the long run debt-to-GDP ratio is 50%. At low percentages, higher steady-state debt-to-GDP ratios can deliver slightly lower volatilities compared to the baseline scenario. It still holds, however, that once the probability is sufficiently large, the volatility of inflation rises. For a given probability of 70%, for instance, the volatility is 2% higher compared to the baseline if debt stands at 50% and close to 7% if the debt ratio is at 200%. If both probabilities and debt ratios are increased above the threshold where debt is unsustainable, volatilities increase without bounds. Table 3 shows the volatilities for output relative to the same base as before. The pattern is generally the same. Increasing only the probabilities for a given debt-ratio almost always renders output more volatile. At the lowest long run debt-to-GDP ratio of 50% output volatility increases by 14% if the probability of staying passive is 0.85 compared to only 0.5. As with inflation, if a certain threshold of probabilities is surpassed, volatilities also increase with a higher steady-state debt-to-GDP ratio.

Given that both inflation and output volatilities increase in general with longer average fiscal profligacy, one can safely conclude that in this type of model, under the assumptions imposed above, welfare is negatively affected by looser fiscal policy. No matter what weighting scheme is applied between both variables, it is obvious that welfare decreases the longer fiscal policy does not respond to the movement of debt from its target given a certain debt-to-GDP ratio. Unlike Reinhart and Rogoff (2010) who argue that GDP is negatively affected once the debt ratio surpasses a threshold of 90%, our analysis points

Table 3: Volatilities of Output

$\frac{B_{ss}}{Y_{ss}}$ p_{11}	50%	75%	100%	125%	150%	175%	200%
0.5	1	0.9020	0.8509	0.8264	0.8208	0.8289	0.8453
0.55	1.0056	0.9104	0.8614	0.8400	0.8389	0.8522	0.8734
0.6	1.0400	0.9397	0.8959	0.8894	0.9104	0.9496	0.9954
0.65	1.0240	0.9449	0.9283	0.9594	1.0313	1.1316	1.2372
0.7	1.0494	0.9853	0.9990	1.0764	1.1972	1.3135	1.3430
0.75	1.0525	0.9851	1.0172	1.1278	1.2662	1.3204	1.1485
0.8	1.0793	1.0371	1.1124	1.2660	1.3319	1.0894	1.1224
0.85	1.1388	1.1386	1.4228	1.7599	1.6197	>10	>10
0.9	1.4245	1.9508	3.8617	5.7485	>10	>10	>10
0.95	5.2741	>10	>10	>10	>10	>10	>10

Note: this table displays the relative volatilities of output after 1,000 periods relative to the scenario of remaining active with a probability of 50% and a debt-to-GDP ratio of 50%. In general, it holds that the higher the probability of staying in the fiscal active regime given any debt ratio, the larger is the volatility of output.

more to a joint threshold of fiscal profligacy and long-run debt. Primarily, the threshold of increased business cycle volatilities seems to be given by the probability of 0.7 or a period of slightly more than three years. If fiscal policy remains active for longer, both output and inflation are more volatile for any debt level. The overall policy conclusion is straightforward. Volatility and negative effects on the business cycle seem to advocate shorter periods of fiscal profligacy for any debt level and avoiding very high debt levels altogether.

4.4 Empirical application

The seminal paper by Bohn (1998) showed that US government debt was sustainable, once the primary surplus regressed on government debt is controlled for by the business cycle and military spending. This along with subsequent papers such as Mauro et al. (2015) conclude that this specific econometric regression therefore serves as an appropriate test for government sustainability. The fact that the test performs well for one or even more countries over a given period of time is not necessarily a yardstick for measuring whether the test can be successfully applied under any circumstances, however. The advantage of this paper is that we know exactly when the cutoff point is reached and government debt is no longer sustainable contingent on the underlying steady-state debt-to-GDP ratio and conduct of fiscal policy. Therefore, we use our model with Markov switching as our data-generating process and generate respective time series for all variables in the model. We simulate the different model economies for 1,000 periods each. The differences in the DGPs lie in both varying long run debt-to-GDP ratios and the probabilities of staying active p_{11}. The probability of staying passive p_{22} is set to 0.8 throughout. As the model is not detailed enough to include different forms of government spending and does

not account for military spending, the only control variable we use is the business cycle condition. The regression we estimate (which is very close to Bohn (1998)) is therefore given by

$$PS_t = \rho d_t + \alpha_0 + \alpha_Y YVAR_t + \epsilon_t \tag{21}$$

where PS_t is the primary surplus to GDP ratio, d_t denotes the ratio of debt to GDP and α_0 is the intercept. In the construction of YVAR we closely follow Bohn (1998), who relied on the measure of Barro (1986). The variable is transformed such that

$$YVAR_t \equiv (1 - y_t/y_t^*) * (g_t^*/y_t) \tag{22}$$

with $1 - y_t/y_t^*$ as the temporary shortfall of output, which is given in our setup by the level of output divided by its steady-state value weighted by the long-term government spending ratio over total output. Below, we obtain a coefficient ρ for every possible probability p_{11}

Figure 8: Stationarity results from the empirical test and the "true" threshold.
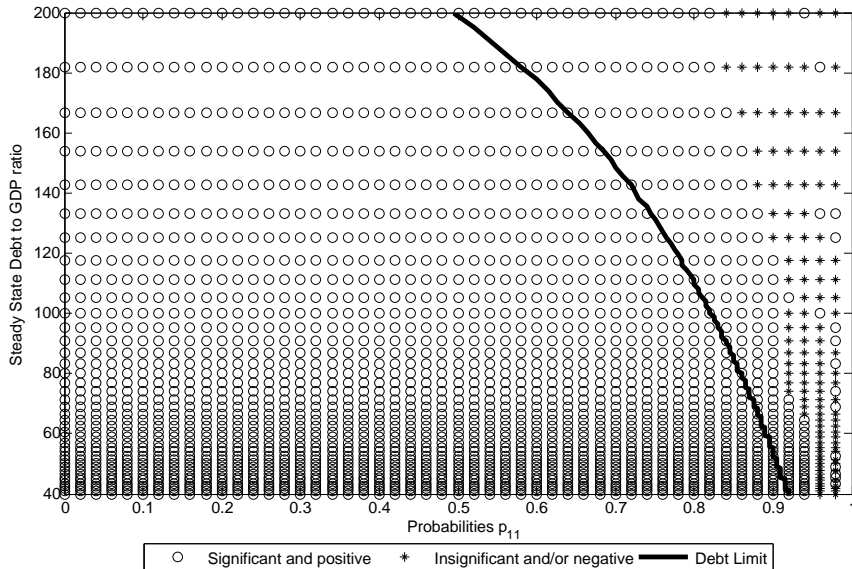
Note: *the figure depicts whether the regression coefficient that decides stationarity in the empirical test is positive and significant or otherwise. In the true sustainability region, the test performs very well, while, in the unsustainability region the test wrongly declares government debt as sustainable in a number of instances.*

which ranges from 0 to 0.99 and different debt-to-GDP ratios ranging from 40% to 200%. If ρ is positive and significant (at the 95% significance level), the empirical test concludes that government debt is sustainable. In Figure 8 these coefficients are depicted as circles, while negative or insignificant coefficients are shown as stars. Furthermore, the cutoff point is added in order to compare where the true region of unsustainable government debt commences. As can be seen in the graph on the left of the threshold, the empirical test correctly identifies government debt as stationary. Beyond the cutoff points to the

right, however, the test does not perform particularly well. It wrongly identifies the threshold as being further to the right for all different long-run debt-to-GDP ratios. In other words, the regression delivers positive and significant coefficients while government debt is actually not stationary. At very high probabilities the test is then once again, able to differentiate quite well between sustainable and unsustainable debt. The reason for this may be that the true threshold is the cutoff point for the eigenvalues at precisely one. Looking at the generated data, however, this kind of unit root behavior cannot be easily and uniquely determined. Thus, even beyond the true debt limit, the process for debt still behaves as a unit root with an eigenvalue slightly above one. Therefore, the test only improves when the eigenvalue is significantly bigger than one and, hence, incorrectly identifies a debt limit which is beyond the true one. This result is also robust to the variation in the probability of remaining fiscally passive and the strength of the reaction to government debt. Figure 9 in the appendix shows qualitatively the same result with $p_{22} = 0.4$, i.e. a debt limit which is shifted inwards for all debt levels compared to the baseline result. Also, in this case the test wrongly selects many cases of de facto unsustainable fiscal behavior as being stable. In another robustness check, we set $\delta_b = 0.2$ and show the results in Figure 10. These confirm the overall conclusion that this test is not able to differentiate clearly between stable and unsustainable policies and find the correct threshold between them. In this case, there are additionally a few cases where at low steady state debt-to-GDP values the test wrongly also declares sustainable fiscal policy to be unsustainable (type one error), i.e. clearly mis-specifying the debt threshold. In summary, it can be concluded that the empirical test is a nice approximation, but that it would not always reject the hypothesis of unsustainable government finances if they actually are unsustainable. The type 2 error is therefore quite high.

5 Conclusion

This paper analyzed the relationship between the debt-to-GDP ratio and two types of fiscal policy regimes: active and passive fiscal policy. We find that the length of time for which fiscal policy is allowed to be active correlates negatively with the size of the long-run debt-to-GDP ratio if unsustainability of public finances is to be avoided. Keeping this debt-to-GDP ratio low is, accordingly, a fundamental precondition should fiscal policy temporarily depart from its goal - which is to stabilize debt. This can be shown to have advantageous effects on the volatility of the business cycle as well. The lower the steady state debt-to-GDP ratio is and the shorter the times of fiscal profligacy are, the lower in general are the volatilities both for inflation and output. Applying the model with Markov switching as the data-generating process, it is possible to run a popular empirical test and determine whether this test truly differentiates between sustainable and unsustainable debt. It turns out that the test is prone to type 2 errors and still wrongly classifies fiscal policies beyond this debt threshold as stationary. Routes for future research could be to include non-zero inflation targets and deduce how those would affect long-run debt limits. Furthermore, the introduction of endogenous regime switching could give rise to reputation effects for the government and therefore yield endogenous time-varying risk premia.

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Appendix

A Additional Results - Robustness

Figure 9: Robustness Check: Lower probability of remaining passive ($p_{22} = 0.4$).
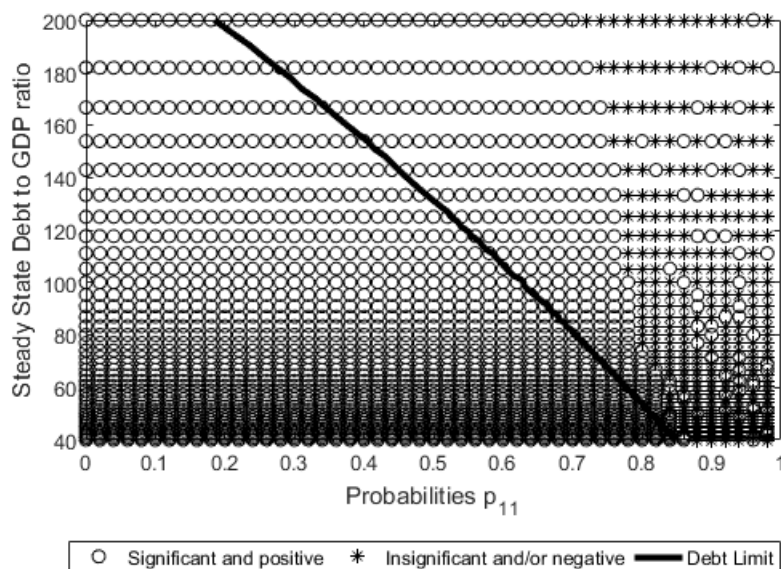
Note: the figure depicts whether the regression coefficient that decides stationarity in the empirical test is positive and significant or otherwise. The baseline results holds that, in the unsustainability region, the test wrongly declares government debt as sustainable in a number of instances and misses the debt limit by pushing it beyond the true one.

Figure 10: Robustness Check: Stronger reaction of fiscal policy ($\delta_b = 0 - 2$).
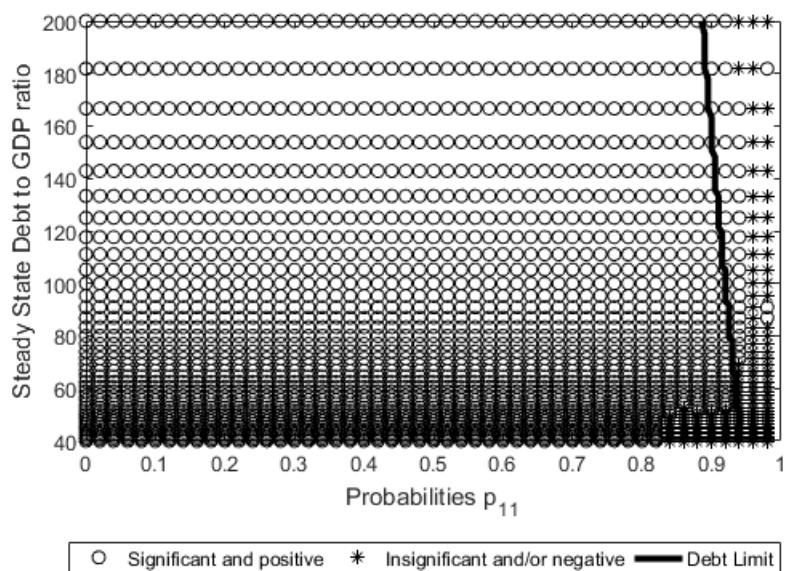
Note: the figure depicts whether the regression coefficient that decides stationarity in the empirical test is positive and significant or otherwise. The baseline results holds that, in the unsustainability region, the test wrongly declares government debt as sustainable in a number of instances. At low values the test wrongly also declares sustainable fiscal policy to be unsustainable (type one error).

B Log-Linearized Model

$$c_t = c_{t+1} - (r_t - E_t\pi_{t+1})$$

$$n_t = w_t - c_t$$

$$w_t = y_t + mc_t - n_t$$

$$n_t = y_t - a_t + \frac{\tau_{ss}^l}{1 - \tau_{ss}^l}$$

$$y_t = a_t + n_t$$

$$a_t = \rho_a a_{t-1} + \epsilon_t^a$$

$$\pi_t = \beta\pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}mc_t$$

$$y_t = \frac{C_{ss}}{Y_{ss}}c_t + \frac{G_{ss}}{Y_{ss}}g_t$$

$$b_t = \frac{1}{\beta}(b_{t-1} - \pi_t + r_{t-1}) + \frac{G_{ss}}{B_{ss}}g_t - \frac{\tau_{ss}^l W_{ss} N_{ss}}{B_{ss}}\tau_t^l w_t n_t$$

$$\tau_t^l = \rho_t^t \tau_{t-1}^l \left(+\delta_b^\tau(s_t)(b_{t-1})\right) + \epsilon_t^t$$

$$g_t = \rho_g g_{t-1} - \delta_y y_t - \delta_b b_{t-1} + \epsilon_t^g$$

$$r_t = \rho_r r_{t-1} + \phi_\pi \pi_t + \phi_y y_t + \epsilon_t^r$$