

Optimal Prudential Policy in Economies with Downward Wage Rigidity

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Abstract

This paper studies optimal policy in economies with downward nominal wage rigidity when only prudential instruments are available. The optimal prudential policy intervenes in the labor market while all other markets may clear competitively. In contrast to fiscal devaluation where labor is subsidized in recessions, the prudential policy taxes labor in expansions as this curtails unemployment in recessions. However, the economy produces below potential in expansions. We analyse this trade-off theoretically and quantitatively by applying the model to Greece. We find that prudential intervention before 2008 would have significantly reduced Greek unemployment after 2008. The welfare cost of downward wage rigidity is reduced by about half. Our results hold in a Walrasian labor market, and in a labor market with wage-setting firms.

Keywords: fiscal devaluation, constrained efficiency, monopsonistic competition, Ramsey problem, labor market, unemployment

JEL-Codes: E24, E32, F41

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1 Introduction

The sharp rise in wages in some euro countries before the Great Recession, followed by large-scale unemployment and slowly declining wages in the aftermath of the Great Recession, have reinvigorated concerns about the harmfulness of downward nominal wage rigidity for macroeconomic adjustment (Baldwin and Giavazzi, 2015). First-best policies to address downward wage rigidity are manifold. For example, monetary policy may attempt to raise price inflation to “grease the wheels” of the labor market (Tobin, 1972), or similarly, depreciate the nominal exchange rate (Na et al., 2018). Alternatively, fiscal policy may subsidize labor demand in recessions—so called “fiscal devaluation” (Farhi et al., 2014).

Instead, this paper is concerned with optimal *prudential* policies in economies with downward wage rigidity. The difference is that first-best policies operate in recessions when downward wage rigidity binds, whereas prudential policies operate in expansions when downward wage rigidity is slack. Prudential policies are useful once first-best policies are not available. First, monetary policy may be unable to combat downward (nominal) wage rigidity, for example if the country has no independent monetary policy. Second, payroll subsidies may be hard to implement in recessions, for example due to fiscal strain.¹

To characterize optimal prudential policies, we consider a small open economy model with a Walrasian labor market subject to downward nominal wage rigidity. Because all agents are wage takers, they are too small to internalize that their collective action generates too sharp wage increases in expansions (Schmitt-Grohé and Uribe, 2016). Formally, we show that the competitive equilibrium is *constrained inefficient*. That is, we show that a social planner can improve welfare even by respecting that downward wage rigidity binds in recessions. The fact that the equilibrium is constrained inefficient creates scope for prudential policy intervention (e.g., Bianchi, 2011; Bianchi and Mendoza, 2018; Dávila and Korinek, 2018).

We show that prudential intervention is required in the economy’s labor market, while all other markets may clear competitively (including capital markets). The optimal prudential policy reduces the firms’ labor demand in expansions, which can be decentralized by taxing the firms’ payroll in those periods. This is the *opposite* intervention as under first best, where labor demand is *subsidized* in *recessions*.

Intuitively, as the labor demand curve shifts in, this curtails wage increases as the economy moves down an upward-sloping labor supply curve. While lower wages reduce unemployment in recessions, the cost is that the economy produces below potential in expansions. This trade-

¹ Once a fiscal and an unemployment crisis occur jointly, it may be counterproductive to grant payroll subsidies as this adds to sovereign debt thereby intensifies the fiscal crisis (Bianchi et al., 2018). In a monetary union with decentralized fiscal policy, a fiscal crisis may result from short-sighted or free-riding government that over-accumulate sovereign debt (Beetsma and Uhlig, 1999; Chari and Kehoe, 2007).

off depends on the labor supply and demand wage elasticities, but also on the utility penalty suffered from unemployment, the degree of downward wage rigidity and the volatility of shocks in the economy. We derive a formula to compute the optimal prudential tax depending on model primitives (e.g., the labor share and Frisch elasticity of labor supply) and on the size of the unemployment spell expected for next period.

As a robustness and extension we depart from the Walrasian labor market and thus from the assumption of wage taking. As emphasized in [Elsby \(2009\)](#), once firms *actively set* wages they internalize that downward wage rigidity affects their workers as this has an effect on their profits. We find that, even though firms compress wage increases in expansions which mimics the constrained-efficient outcome, they do so to a socially insufficient extent. Intuitively, by facing competition with other firms for workers, firms have little leeway for prudential wage reductions: following (unilateral) wage reductions, firms lose workers as they substitute to better-paying competitors.² This provides an instructive caveat on [Elsby \(2009\)](#)'s result that once firms set wages, downward wage rigidity is consistent with weak macroeconomic effects. Formally, we show that the competitive equilibrium is still constrained inefficient. Therefore, the case for prudential policy intervention is alive and well.

In a quantitative application we consider the case of Greece, 1999-2016. The Greek cycle has been characterized by a strong increase in wages until 2008, followed by a slow decline in wages after 2008 and record-unemployment. We demonstrate that the competitive equilibrium can replicate the Greek experience rather well. Thereafter, we study the optimal prudential intervention during the initial phase of the cycle 1999-2008, where downward wage rigidity has been slack. We find that the optimal prudential payroll tax climbs to 14 percent during this period, significantly reducing wage inflation. As a result, the policy has significant positive effects on unemployment during the following contraction.

Finally, we show that the welfare gain from the prudential policy is large. In our application to Greece, it removes 44 percent of the total welfare cost of downward wage rigidity (it removes 1.1 of a total 2.5 percent loss of permanent consumption). This reflects a reduction in mean unemployment from 5.8 to 1.5 percent.

Related literature.—Modeling downward wage rigidity has recently become very popular. Building on the seminal contribution of [Akerlof et al. \(1996\)](#), recent influential contributions include [Benigno and Fornaro \(2018\)](#), [Bianchi et al. \(2018\)](#) and [Na et al. \(2018\)](#), but also the whole secular stagnation literature (e.g., [Fornaro and Romei \(2018\)](#), [Eggertsson et al. \(2018\)](#) and [Corsetti et al. \(2018\)](#)).

² Hence we consider a set-up of monopsonistic competition where firms retain workers even by paying a strictly lower wage than their competitors (i.e., substitution is imperfect, see [Manning \(2003\)](#)). Market power on the labor demand side is required for firms to set wages and thereby internalize downward wage rigidity.

One well-known result in the literature on downward wage rigidity is that, once firms set wages and internalize downward wage rigidity, downward wage rigidity is consistent with weak macroeconomic effects (Elsby, 2009). Here we reach a different conclusion: by studying wage-setting firms in general equilibrium and considering competition among firms, firms behave similar as in a Walrasian labor market. Competition among firms is monopsonistic, and is therefore modeled as in Manning (2003). We show that, even when firms actively compress wage increases which is privately efficient, the resulting allocation is not socially efficient, creating scope for policy intervention.

Specifically, we analyse optimal *prudential* intervention to address downward wage rigidity. This relates to Schmitt-Grohé and Uribe (2016) who show that a currency peg with downward wage rigidity benefits from capital controls. Relative to them, we show that the inefficiency from downward wage rigidity does not rely on inefficient movements in the country’s capital flows or real exchange rate that hinge on a non-tradable sector (indeed, the inefficiency would arise even in a closed-economy context). Moreover, we find that a different set of policies is required for decentralization: labor-market rather than capital-control policies.³ Finally, we additionally study the case of wage-setting firms.

On the theory side, we add to the literature on macro-prudential intervention. This literature is mostly concerned with externalities that arise from financial frictions (e.g., Dávila and Korinek, 2018; Farhi and Werning, 2016; Lorenzoni, 2008). Here we study macro-prudential intervention in the presence of a pecuniary externality that arises from a *nominal* friction: from downward nominal wage rigidity.

On the applied side, the differential wage developments across euro area countries have received considerable attention. Schmitt-Grohé and Uribe (2013) and Fahr and Smets (2010) make a case for temporary price inflation in the euro core in order to overcome downward wage rigidity in the euro periphery. Gilchrist et al. (2018) explain the high level of wages in the euro periphery by a combination of customer markets and financial frictions. Kuvshinov et al. (2016) show that debt deleveraging pressure can paralyze the adjustment of relative labor costs across currency union members. We make a case for prudential intervention in the labor market when currency union members suffer from downward wage rigidity.

Outline.—Section 2 introduces the model. Sections 3-4 discuss constrained efficiency and decentralization. Section 5 presents the model extension featuring wage-setting firms. Section 6 presents the quantitative application to Greece. Finally, Section 7 concludes.

³ Capital controls are ineffective to address downward wage rigidity in the present analysis. In Schmitt-Grohé and Uribe (2016) capital controls work because capital flows are linked to the dynamics of wages via their effect on the real exchange rate. In the present analysis, there is no such link and the need for prudential intervention arises instead in the labor market.

2 Model

We study a currency peg that is small enough to not affect developments outside the domestic economy. Households consume, work, and save in (incomplete) international financial markets. Nominal wages are downward rigid. Firms produce a single consumption good which is traded freely across borders. The economy is buffeted by demand-side shocks and by shocks to labor productivity.

2.1 Households

A representative household maximizes consumption utility net of disutility from work

$$E_0 \sum_{t \geq 0} \beta^t U(c_t - V(h_t^f - \delta u_t)), \quad \beta \in (0, 1), \quad \delta \in [0, 1], \quad (1)$$

where U is positive, increasing and concave and V is positive, increasing and convex, subject to the budget constraint

$$p_t c_t + \frac{\Lambda_{t+1}}{R} = w_t h_t + \Pi_t + \Lambda_t \quad (2)$$

and a no-Ponzi constraint. Here, p_t is the price of consumption, $w_t h_t$ is wage income, Π_t are firm profits and Λ_t are nominal bonds, traded across border at price $1/R$. Households take prices, wages, profits and the nominal interest rate R as given.

In (1) we assume that households have Greenwood-Hercowitz-Huffman (GHH) preferences. GHH preferences are commonly used in international business cycle models and also in the literature studying macro-prudential intervention (see for example [Bianchi, 2016](#); [Bianchi and Mendoza, 2018](#); [Mendoza and Yue, 2012](#)). As is well known, GHH preferences eliminate the wealth effect on labor supply, which prevents a counterfactual increase in labor supply during crises.

Because the consumption good c_t is traded internationally, its price p_t is exogenous from the vantage point of the small domestic economy (it is here that we use that the country has a fixed nominal exchange rate). In this context, p_t is often interpreted as the country's terms of trade ([Schmitt-Grohé and Uribe, 2016](#)).

Taking first order conditions with respect to consumption and bonds gives the consumption Euler equation

$$1 = \beta R E_t \frac{U'(t+1)}{U'(t)} \frac{p_t}{p_{t+1}}, \quad (3)$$

where we define $U'(t) \equiv U'(c_t - V(h_t^f - \delta u_t))$. The no-Ponzi condition must hold with equality in equilibrium, yielding a terminal (or transversality) condition.

2.2 Labor supply

In the objective (1), h_t^f denotes the amount of hours that the household would like to supply. Because of downward wage rigidity, the amount of hours h_t that are actually supplied may fall below this level, creating unemployment⁴

$$u_t \equiv h_t^f - h_t \geq 0. \quad (4)$$

In the objective (1), we allow for the possibility that voluntary leisure and unemployment are imperfect substitutes, $\delta < 1$. This implies that the disutility of work falls by only little in a recession even as working hours are reduced.⁵ This reflects the evidence that the unemployed are not any happier from the additional leisure, but instead suffer a direct utility loss from unemployment (Winkelmann and Winkelmann, 1998). It also reflects the evidence that, while not directly engaged in working-related activities, unemployed workers spend a lot of time searching for new employment (Krueger and Mueller, 2012).

In the absence of downward wage rigidity, unemployment is zero at all times. In this case, we obtain the amount of hours that households would voluntarily supply

$$V'(h_t^f) = \frac{w_t}{p_t}, \quad (5)$$

where consumption c_t does not appear because of GHH preferences. Instead, for downward wage rigidity we follow Akerlof et al. (1996) and assume

$$w_t \geq \psi(u_t)w_{t-1}. \quad (6)$$

Nominal wages may fall by an amount $\psi(u_t)$ per period, which may depend on the extent of unemployment. Specifically, we assume that $\psi(0) = \bar{\psi} \leq 1$, and that $\psi' \leq 0$ but $\psi \geq 0$. Unemployment can arise only if the wage rigidity binds:

$$u_t(w_t - \psi(u_t)w_{t-1}) = 0. \quad (7)$$

We have $V'(h_t^f - \delta u_t) \leq w_t/p_t$ from (5) and from $u_t \geq 0$; i.e., the marginal rate of substitution may fall strictly below the real wage. Therefore, if downward wage rigidity binds, the labor market is rationed on the labor supply side.

⁴ Therefore, as in Galí (2011) and others, we treat an involuntary reduction in the amount of hours supplied as representing unemployment. Throughout the paper, we shall sometimes use the alternative definition for unemployment $\hat{u}_t \equiv (h_t^f - h_t)/h_t^f$, see for example Section 4.

⁵ To see this, insert (4) into the disutility of work: $V(h_t^f - \delta u_t) = V(h_t + (1 - \delta)u_t)$. In a recession, h_t falls. However, to the extent that this represents unemployment ($u_t > 0$), and if unemployment and leisure are imperfect substitutes ($\delta < 1$), the households' disutility of work falls not to the same extent.

2.3 Firms

A representative firm maximizes profits, taking the nominal wage and the sales price of the consumption good as given. It uses only labor for production

$$y_t = a_t F(h_t), \quad (8)$$

where F is positive, increasing and concave, and where a_t is the productivity of labor, which is exogenous. Profits are given by $\Pi_t = \max\{p_t y_t - w_t h_t\}$. The labor demand curve is

$$p_t a_t F'(h_t) = w_t. \quad (9)$$

2.4 Competitive equilibrium

A competitive equilibrium can be defined as follows. Given initial conditions w_{-1} and an exogenous process for $\{a_t, p_t\}$, a competitive equilibrium is a process for $\{w_t, h_t, u_t\}$ such that the following conditions are satisfied

$$\begin{aligned} i) & & w_t/p_t &= V'(h_t + u_t) & \text{(labor supply)} \\ ii) & & w_t/p_t &= a_t F'(h_t) & \text{(labor demand)} \\ iii) & & w_t &\geq \psi(u_t) w_{t-1}, \quad u_t \geq 0, \quad \cdot \times \cdot = 0 & \text{(inequality and slackness).} \end{aligned}$$

Equilibrium output follows residually: $y_t = a_t F(h_t)$.

Given initial conditions Λ_0 , equilibrium consumption and assets $\{c_t, \Lambda_{t+1}\}$ are determined residually from the following equations (as well as from a terminal condition)

$$\begin{aligned} iv) & & 1 &= \beta RE_t(U'(t+1)/U'(t))(p_t/p_{t+1}) & \text{(consumption Euler)} \\ v) & & p_t c_t + \Lambda_{t+1}/R &= p_t y_t + \Lambda_t & \text{(resource).} \end{aligned}$$

Equilibrium welfare follows residually: $\mathcal{U} \equiv E_0 \sum_{t \geq 0} \beta^t U(c_t - V(h_t + (1 - \delta)u_t))$.

Notice the two-block structure of the competitive equilibrium. Equations i)-iii) pin down equilibrium in the labor market and therefore the economy's equilibrium production, without reference to consumption and therefore the capital market. In turn, equations iv)-v) determine consumption and assets as residuals.

3 Constrained efficiency

Is the competitive equilibrium maximizing welfare? It is clearly not first best, due to downward wage rigidity. However, it may still be second best, or *constrained efficient*, referring to efficiency conditional on downward wage rigidity.

We now show that the competitive equilibrium is *constrained inefficient*. That is, even by respecting that downward wage rigidity binds in recessions, a social planner can increase welfare.

3.1 The planning problem

A social planner maximizes welfare (1) subject to technology (8) and by respecting downward wage rigidity (6). The planner also takes as given (2), as this constitutes the country's resource constraint. To see this, note that from the firm side, profits and the wage bill must always add to total output, $p_t y_t = w_t h_t + \Pi_t$. Inserting this in (2) yields

$$p_t c_t + \frac{\Lambda_{t+1}}{R} = p_t y_t + \Lambda_t, \quad (10)$$

where Λ_t constitutes the country's net foreign assets.

This yields the following planning problem

Definition 1. [CONSTRAINED-EFFICIENT EQUILIBRIUM] *The welfare optimal equilibrium in the presence of downward wage rigidity solves*

$$\mathcal{U}(w_{t-1}, \Lambda_t, a_t, p_t) = \max_{\{c_t, \Lambda_{t+1}, h_t, u_t, w_t\}} \{U(c_t - V(h_t + (1 - \delta)u_t)) + \beta E_t \mathcal{U}(w_t, \Lambda_{t+1}, a_{t+1}, p_{t+1})\}$$

subject to the set of constraints

$$\begin{aligned} i) & & w_t/p_t &\leq a_t F'(h_t) \\ ii) & & w_t/p_t &= V'(h_t + u_t) \\ iii) & & u_t &\geq 0 \\ iv) & & w_t &\geq \psi(u_t)w_{t-1} \\ v) & & p_t c_t + \Lambda_{t+1}/R &= p_t a_t F(h_t) + \Lambda_t, \end{aligned}$$

for given exogenous $\{a_t, p_t\}$.

Constraints iii)-v) are easily understood: unemployment cannot be negative; by definition of constrained efficiency, the planner respects downward wage rigidity; the planner respects the economy's resource constraint. Instead, constraints i)-ii) deserve further comment.

The planner respects the households' voluntary labor supply (5), which, once combined with (4), is constraint ii). That is, the planner lets labor supply be determined competitively.⁶ This is because constraint ii) determines the nominal wage as a function of the equilibrium

⁶ The literature studying constrained efficiency commonly assumes that the social planner lets some markets clear competitively, while intervening in others. See for example Bianchi (2011), Dávila and Korinek (2018) or Bianchi and Mendoza (2018).

allocation. The planner needs to be able to infer the nominal wage, because he is required to respect downward wage rigidity, constraint iv).

In turn, constraint i) imposes that the planner cannot raise labor demand relative to the competitive equilibrium. If i) were not a constraint, he could always implement the first-best amount of employment: in a period when lagged wages are high, hence w_t is determined by iv), choose h_t according to $a_t F'(h_t) = V'(h_t)$, then let $w_t/p_t > a_t F'(h_t)$. That is, firms hire the first-best amount of workers, even as the marginal product lies below the marginal cost. Implicitly, this is therefore assuming that the planner can subsidize the firms' hiring, which from the literature on fiscal devaluation is well understood to deliver the first best (Farhi et al., 2014). By imposing i), we therefore shift attention to optimal *prudential* intervention, as the planner *cannot* intervene in recessions by subsidizing labor demand.

We now characterize the optimal prudential intervention by proceeding in three steps. We first show that the problem in Definition (1) can be reduced to a problem purely in the labor market while the capital market clears competitively. Second, we illustrate the optimal prudential intervention in the labor market by using a labor market diagram. Third, we explore formally the optimal prudential intervention (Section 3.2).

The following proposition is verified in the Appendix A

Proposition 1. [LABOR MARKET REPRESENTATION] *The welfare optimal allocation can be obtained as the solution to the following problem in the labor market*

$$\tilde{U}(w_{t-1}, a_t, p_t) = \max_{\{h_t, w_t, u_t\}} \{U'(t)(a_t F(h_t) - V(h_t + (1 - \delta)u_t)) + \beta E_t \tilde{U}(w_t, a_{t+1}, p_{t+1})\}$$

subject to the set of constraints

$$\begin{array}{ll} i) & w_t/p_t \leq a_t F'(h_t) \\ ii) & w_t/p_t = V'(h_t + u_t) \\ iii) & u_t \geq 0 \\ iv) & w_t \geq \psi(u_t)w_{t-1} \end{array}$$

for given exogenous $\{a_t, p_t\}$, where $U'(t)$ is determined in (3) and (10).

Proposition 1 is a first key result, as it shows that the planner does not intervene in capital markets. Capital flows (and therefore consumption) are determined by the consumption Euler equation (3) and the resource constraint (10)—as in the competitive equilibrium. This reflects a dichotomy: the planner intervenes in the labor market and hence on the *production side* of the economy, but not on its consumption side. Given the optimal intervention, consumption

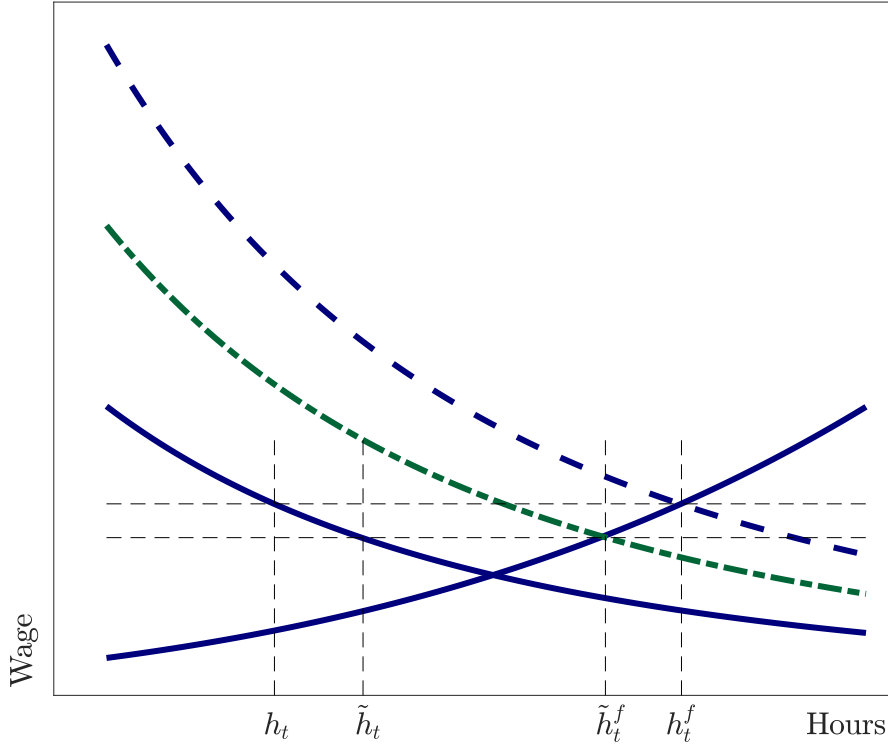


Figure 1: Labor market diagram: labor supply and demand (blue solid). Labor demand shifts rightwards following a rise in technology (blue dashed). Intervention: shift labor demand back to the left (green dashed-dotted). Here it is assumed that wages are fully downward rigid.

and capital flows are determined residually. Note that as a result, capital controls are neither required nor useful to decentralize the optimal prudential intervention.⁷

One reading of this result is as follows. Policy intervenes in the labor market to maximize the present value of production (net of the disutility of hours worked)—this is Proposition 1. Consumption smoothing spreads the present value of production over time. However, at this stage no further intervention is required: once the optimal intervention in the labor market is in place, the capital market allocates consumption over time efficiently.

Details on the intervention in the labor market (Proposition 1) are worked out formally in Section 3.2. We foreshadow the formal analysis by using a labor market diagram, see Figure 1. Shown is a temporary rise in technology. Unemployment is zero. Labor demand shifts to the

⁷ As it turns out, this property hinges on our assumption of GHH preferences. Otherwise, the planner would intervene both on the consumption and on the production side. However, in this case the optimal policy problem would become much less tractable, because the labor market problem in Proposition 1 would feature the consumption Euler equation (3) as implementability constraint.

right, inducing a wage rise along the labor supply curve (dashed). Once the shock dissipates in the next period, labor demand moves back, but wages remain high. Employment falls to h_t and there is unemployment, reflecting that h_t is low, but also that h_t^f is high (recall that $u_t = h_t^f - h_t$). The intervention is dashed-dotted in green: *shift labor demand slightly back to the left* in the first period (set $w_t/p_t < a_t F'(h_t)$). Wages increase by less, which reduces the cut in employment \tilde{h}_t and unemployment $\tilde{h}_t^f - \tilde{h}_t$ in the next period.

As shown in Section 4, reducing labor demand in expansions can be decentralized by taxing the firms' payroll or by taxing the firms' sales revenue. The prudential policy is therefore the opposite of fiscal devaluation where labor demand is subsidized in recessions. The benefit of the intervention is that wage inflation is reduced in expansions, which reduces unemployment in subsequent recessions. The cost is that in expansions, the economy produces below potential (see again Figure 1).

3.2 Properties of the optimal prudential intervention

As shown in the Appendix A, in a period when downward wage rigidity is slack, labor demand and supply in the constrained-efficient equilibrium are

$$a_t F'(h_t) = \frac{w_t}{p_t} + \frac{1}{U'(t)} \varepsilon_t^V \frac{w_t}{h_t} \beta E_t \psi(u_{t+1}) \lambda_{t+1} \quad (\text{demand}) \quad (11)$$

$$V'(h_t) = \frac{w_t}{p_t} \quad (\text{supply}) \quad (12)$$

and unemployment is zero ($u_t = 0$). In the labor demand curve, $\varepsilon_t^V > 0$ denotes the wage elasticity of labor supply, and $\lambda_t \geq 0$ denotes the non-negative Lagrange multiplier associated with downward wage rigidity (constraint iv) in Proposition 1).⁸

This is intuitive. Combining (11)-(12) yields

$$U'(t)(a_t F'(h_t) - V'(h_t)) = \varepsilon_t^V \frac{w_t}{h_t} \beta E_t \psi(u_{t+1}) \lambda_{t+1}. \quad (13)$$

The left hand side is the utility loss from reducing employment below potential in the current period (recall that in the current period, by assumption, downward wage rigidity is slack). Instead, the right hand side is the gain from doing so: the expected reduction in the utility loss suffered from downward wage rigidity in the next period. It is composed of $\varepsilon_t^V (w_t/h_t) > 0$ which measures by how much wages can be reduced by moving labor demand back to the left—by sliding down the labor supply curve (recall again Figure 1). In turn, this is multiplied with the expected loss from downward wage rigidity in the next period, as measured by the compounded term $\beta E_t \psi(u_{t+1}) \lambda_{t+1} \geq 0$.

⁸ By denoting a generic variable \bar{h}_t and letting $V'(\bar{h}_t)p_t = w_t$, this is therefore $\varepsilon_t^V \equiv (\partial w_t / \partial \bar{h}_t)(\bar{h}_t / w_t)$.

This implies a stronger intervention when the labor supply curve is steep (w_t is moving strongly with h_t), as in this case the intervention is cheap: a small reduction in employment restrains wage inflation to a strong extent. At the same time, a larger intervention is required when wages are more downward rigid (when $\psi(u_{t+1})$ is expected to be larger), and when the utility loss associated with downward wage rigidity, as measured by the multiplier $\lambda_{t+1} \geq 0$, is expected to be larger. Notice that, because the function $\beta\psi(u_{t+1})\lambda_{t+1} \geq 0$ is kinked (and therefore convex), more uncertainty *per se* also justifies a stronger intervention.⁹

The utility loss $\lambda_t \geq 0$ is an equilibrium object, determined in periods when downward wage rigidity binds. Again as shown in the Appendix A, in this case labor demand and supply in the constrained-efficient equilibrium are

$$a_t F'(h_t) = \frac{w_t}{p_t} \quad (\text{demand}) \quad (14)$$

$$U'(t)V'(h_t + (1 - \delta)u_t) \left(\delta + (1 - \delta) \frac{\varepsilon_t^F}{\varepsilon_t^V} \frac{h_t + u_t}{h_t} \right) = \varepsilon_t^F \frac{w_t}{h_t} (\lambda_t - \beta E_t \psi(u_{t+1}) \lambda_{t+1}) \\ + U'(t) \frac{w_t}{p_t} + \lambda_t \psi'(u_t) w_{t-1} \left(1 - \frac{\varepsilon_t^F}{\varepsilon_t^V} \frac{h_t + u_t}{h_t} \right) \quad (\text{supply}) \quad (15)$$

and there is unemployment ($u_t > 0$). This can be read as follows: w_t is determined by downward wage rigidity (constraint iv) in Proposition 1). Given w_t , employment h_t is determined by labor demand (14), and unemployment $u_t > 0$ is determined by the condition for voluntary labor supply, constraint ii) in Proposition 1. Finally, the utility loss associated with downward wage rigidity $\lambda_t \geq 0$ is determined residually in (15).

To understand how the utility loss is determined, it helps to focus on the special case of the model where unemployment does not carry a special penalty ($\delta = 1$), and where downward wage rigidity is flat in unemployment ($\psi(u_t) = \bar{\psi}$ such that $\psi'(u_t) = 0$). In this case, (15) simplifies considerably and by combining it with (14) we obtain

$$\lambda_t = -(\varepsilon_t^F)^{-1} \frac{h_t}{w_t} U'(t) (a_t F'(h_t) - V'(h_t)) + \beta \bar{\psi} E_t \lambda_{t+1}. \quad (16)$$

When downward wage rigidity binds, employment h_t is rationed and hence inefficiently low. The term $U'(t)(a_t F'(h_t) - V'(h_t)) > 0$ measures by how much utility would rise by increasing employment by a marginal unit. By how much must wages decline for firms to be willing to hire the marginal unit? This is given by $-(\varepsilon_t^F)^{-1}(h_t/w_t) > 0$, which is the (negative of the inverse) slope of labor demand F' (here, $\varepsilon_t^F < 0$ denotes the wage elasticity of labor demand, which is negative because labor demand slopes downwards).¹⁰ Therefore, the product of these

⁹ In this respect, our analysis is linked to recent contributions on the interaction of risk and nominal rigidities. See for example [Basu and Bundick \(2017\)](#).

¹⁰ Along the lines of the definition of the elasticity of labor supply, for a generic variable \bar{h}_t and $a_t p_t F'(\bar{h}_t) = w_t$, this is given by $\varepsilon_t^F \equiv (\partial w_t / \partial \bar{h}_t)(\bar{h}_t / w_t)$.

two terms is the utility value of decreasing wages by a marginal unit if downward wage rigidity binds—which is exactly λ_t . Finally, λ_t depends on its own expected value in the next period. This is because when downward wage rigidity binds in the current period, because wages are slow to be reduced, it is likely to also bind in the next period.

One can verify that the multiplier $\lambda_t \geq$ determined in (15) rises once unemployment carries an additional utility penalty (once $\delta < 1$), and that it falls once wages become more downward flexible with unemployment (once $\psi'(u_t) < 0$). Both of these results are intuitive. Then, from (11), a value of $\delta < 1$ strengthens the optimal intervention in expansions, whereas a rigidity function $\psi'(u_t) < 0$ weakens the intervention in those periods.

The Appendix A contains further details and a step-by-step derivation of the constrained-efficient equilibrium.

4 Decentralization

As discussed above, in the constrained-efficient allocation, the firms' hiring is reduced relative to the competitive equilibrium in periods when downward wage rigidity is slack. This points to taxing the firms' payroll as one option to decentralize the constrained-efficient allocation, which we verify next. Formally, define a regulated competitive equilibrium as

Definition 2. [REGULATED COMPETITIVE EQUILIBRIUM] *The government sets non-negative taxes $\{\tau_t^w \geq 0\}$ and rebates them lump-sum to households. Given initial conditions $(w_{-1} > 0, \Lambda_0)$ and an exogenous process for $\{a_t, p_t\}$, a regulated competitive equilibrium is a path for $\{c_t, \Lambda_{t+1}, w_t, h_t, u_t\}$ such that the following conditions are satisfied*

- i) $w_t/p_t = V'(h_t + u_t)$ (labor supply)
- ii) $w_t(1 + \tau_t^w)/p_t = a_t F'(h_t)$ (labor demand)
- iii) $w_t \geq \psi(u_t)w_{t-1}$, $u_t \geq 0$, $\cdot \times \cdot = 0$ (inequality and slackness)
- iv) $1 = \beta RE_t(U'(t+1)/U'(t))(p_t/p_{t+1})$ (consumption Euler)
- v) $p_t c_t + \Lambda_{t+1}/R = p_t y_t + \Lambda_t$ (resource).

We obtain the following proposition (see the Appendix A for a proof)

Proposition 2. [RAMSEY PROBLEM: DECENTRALIZATION] *The regulated competitive equilibrium that maximizes welfare $\mathcal{U} \equiv E_0 \sum_{t \geq 0} \beta^t U(c_t - V(h_t + (1 - \delta)u_t))$ coincides with the constrained-efficient equilibrium.*

Recall that, if the government were able to set *negative* taxes, it would choose to subsidize firms in recessions, thereby effectively *undo* downward wage rigidity (as discussed above).

However, to the extent that subsidies are not available, the government may use payroll taxes in expansions in order to implement the constrained-efficient equilibrium.¹¹

Clearly, an alternative is to tax the firms' sales revenue, as this implies the labor demand curve $w_t/(p_t(1 - \tau_t^p)) = a_t F'(h_t)$. Therefore, in this case the constrained-efficient equilibrium can be decentralized by setting $\tau_t^p = \tau_t^w/(1 + \tau_t^w) \geq 0$.

The implied tax τ_t^w can be obtained from (11)

$$\tau_t^w = \left(\frac{w_t}{p_t}\right)^{-1} \frac{1}{U'(t)} \varepsilon_t^V \frac{w_t}{h_t} \beta E_t \psi(u_{t+1}) \lambda_{t+1}, \quad (17)$$

whenever the labor market is not rationed, and $\tau_t^w = 0$ else. Here, λ_{t+1} is defined in (15) (or, in the special case $\psi(u_t) = \bar{\psi}$ and $\delta = 1$, in (16)).

To gauge the empirical importance of the optimal tax, consider the following illustration. Assume that downward wage rigidity is slack in the current period, but that it may bind in the next period (and again slack thereafter). Specialize to $\psi(u_t) = \bar{\psi}$ and $\delta = 1$. Denote $\hat{u}_t \equiv (h_t^f - h_t)/h_t^f$ unemployment expressed relative to potential, and specialize to the functional forms for production technology and disutility of work introduced in Section 6. The optimal prudential tax can be expressed as¹²

$$\tau_t^w = \frac{\varphi}{1 - \alpha} \bar{\psi} E_t \xi_{t,t+1} \left(\frac{w_{t+1}}{w_t}\right)^{\frac{1}{\varphi}} \left(\frac{p_t}{p_{t+1}}\right)^{\frac{1}{\varphi}} (1 - \hat{u}_{t+1})(1 - (1 - \hat{u}_{t+1})^\varphi), \quad (18)$$

where $\xi_{t,t+1} \equiv \beta(U'(t+1)/U'(t))(p_t/p_{t+1})$ is the one-period-ahead nominal stochastic discount factor. The labor share α and the (inverse) Frisch elasticity φ appear in (18) as these determine the labor demand and supply wage elasticities (recall the discussion in Section (3.2), and see Section 6 below). The formula in (18) conveniently determines the optimal tax as a function only of the stochastic discount factor, of price inflation and of unemployment expected for the next period.¹³

Assume, for example, that $\varphi = 4$, $\alpha = 2/3$, $\bar{\psi} = 0.96$, that there is no inflation to next period ($p_{t+1} = p_t$), that the discount factor $\xi_{t,t+1} = 0.96$, and that with a probability of 10 percent, a crisis is expected for next period whereby unemployment climbs to 10 percent. In this case, the implied optimal tax is $\tau_t^w = 0.1095$, or about 11 percent.

This example shows that the optimal tax can be quite large. Nonetheless, recall that (18) is only valid for a special case of the model. Moreover, it ignores general equilibrium effects:

¹¹ Note that since the problem of the constrained-efficient planner is time consistent, the Ramsey policy problem is also time consistent (Bianchi, 2016).

¹² Here we have combined (16) and (17), and used (14), (4) and (5). Details are in the Appendix A.

¹³ Instead, knowledge of wage inflation is not required. In a period when unemployment opens up, the wage rigidity must bind. Hence, we can replace $w_{t+1}/w_t = \bar{\psi}$. Instead, in states of the world where unemployment is zero, the right hand side of (18) is equal to zero. Therefore, in those states of the world we do not require knowledge of w_{t+1}/w_t .

per effect of charging the prudential tax, the probability (and severity) of the unemployment spell in the next period is reduced. These general equilibrium effects are taken care of in our quantitative application in Section 6.

5 Model extension: The case of wage-setting firms

As a robustness and extension, we now depart from the assumption of Walrasian labor market but instead consider the case of wage-setting firms. Intuitively, once firms are not wage takers they internalize that downward wage rigidity affects their workers as this has an effect on their profits. As famously shown by [Elsby \(2009\)](#), firms react by compressing wage rises (perform prudential wage reductions) accordingly. The case for prudential policy intervention may then be strongly reduced.

We find the case for prudential intervention to be alive and well. In a nutshell, if firms face competition with other firms for workers, unilateral prudential wage reductions entail substitution of workers to competitors. As a result, firms will not perform (large) unilateral wage reductions. In the limit as competition becomes perfect, firms will set wages always as their competitors, and we are back in a Walrasian labor market.

For ease of exposition, in this section we specialize to the version of the model where unemployment does not carry a special penalty ($\delta = 1$) and where downward wage rigidity does not depend on unemployment ($\psi(u_t) \equiv \bar{\psi}$). To introduce active wage setting, we break the assumption of firms being wage takers, but instead grant them market power. Specifically, as in [Manning \(2003\)](#), we assume that the labor market is characterized by *monopsonistic competition*.^{14,15}

Let aggregate employment be a CES-composite of firm-specific employment. By denoting $i \in [0, 1]$ the firm index on the unit interval and $\eta > 0$ the elasticity of substitution of firm-specific employment, this is

$$h_t = \left(\int_0^1 h_t(i)^{1+\frac{1}{\eta}} di \right)^{1/(1+\frac{1}{\eta})}. \quad (19)$$

In the households' budget (2), the wage income $w_t h_t$ is replaced by the integral expression $\int_0^1 w_t(i) h_t(i) di$. Households attempt to divide employment between firms so as to maximize

¹⁴ Whereas we assume market power on the labor demand side, models with wage rigidity often assume market power on the labor supply side, yielding *monopolistic competition* (e.g., [Galí, 2011](#); [Galí and Monacelli, 2016](#)). Monopsonistic competition is required for firms to actively set wages thereby internalize that downward wage rigidity affects their workers. If instead the market power were with the workers, the firms' problem would be static and indeed the same as in the Walrasian labor market. Both cases need again to be distinguished from monopolistic competition in the *goods* market (rather than in the labor market), which is the standard assumption made in New Keynesian price stickiness analyses.

¹⁵ Paul Krugman has argued that the combination of monopsony power and downward nominal wage rigidity may be useful also to explain the recent wage experience in the US ([Krugman, 2018](#)).

their wage income, subject to aggregate employment being h_t . This yields a set of firm-specific labor supply curves

$$h_t(i) \leq \left(\frac{w_t(i)}{w_t} \right)^\eta h_t, \quad (20)$$

for all $i \in [0, 1]$. A firm which pays a higher wage than its competitors, $w_t(i) > w_t$, receives a larger share of the aggregate labor supply. Conversely, a firm may pay less than its competitors and still not lose all of its workers. This is the source of market power which makes firms wage setters rather than wage takers. Notice that equation (20) holds only with a weak inequality. Because of downward wage rigidity, an individual firm may face a very large labor supply, yet decide to employ only a fraction of the workers (see below).

The households' consumption/leisure choice is unchanged from the baseline model. As a result, we again obtain the (aggregate) labor supply curve

$$V'(h_t) \leq \frac{w_t}{p_t}, \quad (21)$$

inequality which may be strict when downward wage rigidity binds.¹⁶

The heart of the extended model is the problem of the firms. As in the baseline model, we assume that the firms' technology is $a_t F(h_t(i))$, where firm index $i \in [0, 1]$ is now made explicit. We have the following dynamic program

Definition 3. [WAGE-SETTING FIRMS] *In the extended model, the problem of individual firm $i \in [0, 1]$ is to solve the following dynamic program*

$$\mathcal{P}_t(w_{t-1}(i)) = \max_{\{h_t(i), w_t(i)\}} \left\{ a_t F(h_t(i)) - \frac{w_t(i)}{p_t} h_t(i) + \beta E_t \frac{U'(t+1)}{U'(t)} \mathcal{P}_{t+1}(w_t(i)) \right\}$$

subject to the set of constraints

$$\begin{aligned} i) & \quad h_t(i) \leq (w_t(i)/w_t)^\eta h_t, \\ ii) & \quad w_t(i) \geq \bar{\psi} w_{t-1}(i), \end{aligned}$$

for given exogenous variables $\{a_t, p_t, h_t, w_t, U'(t)\}$.

In Definition 3, the value function \mathcal{P}_t denotes the present value of (real) period-profits, which has time index t for it depends on aggregate states. We focus on symmetric equilibria and therefore, after solving the problem in Definition 3, impose the symmetry condition $w_t(i) = w_t$ and $h_t(i) = h_t$ for all firms $i \in [0, 1]$.

¹⁶ In equilibrium, all firms are identical and index i disappears. Therefore, in equilibrium, equation (20) always holds with equality, whereas the rationing of employment arises from equation (21) (as in the baseline model). However, specifying (20) as a weak inequality is important for correctly specifying the problem of the firms, see Definition 3.

Before turning to the characterization of equilibrium, it is important to recognize that the extended model has the *same* constrained-efficient equilibrium as the baseline model. This is because the planner will impose symmetry $w_t(i) = w_t$ and $h_t(i) = h_t$ for all firms $i \in [0, 1]$ from the start, such that the “new” equations (19)-(20) disappear. Then, regarding aggregate variables the planner’s problem is unchanged from the baseline model.

Turn now to equilibrium in the extended model. As shown in the Appendix A, in a period when downward wage rigidity is slack, labor demand and supply are

$$a_t F'(h_t) = \frac{\eta + 1}{\eta} \frac{w_t}{p_t} + \frac{1}{U'(t)} \frac{1}{\eta} \frac{w_t}{h_t} \beta \bar{\psi} E_t \lambda_{t+1} \quad (\text{demand}) \quad (22)$$

$$V'(h_t) = \frac{w_t}{p_t} \quad (\text{supply}). \quad (23)$$

In equation (22), the variable $\lambda_t \geq 0$ denotes the multiplier associated with downward wage rigidity, constraint ii) in Definition 3. By combining equations (22)-(23) we obtain

$$U'(t) \left(a_t F'(h_t) - \frac{\eta + 1}{\eta} V'(h_t) \right) = \frac{1}{\eta} \frac{w_t}{h_t} \beta \bar{\psi} E_t \lambda_{t+1} \quad (24)$$

which has to be compared with (13) in the constrained-efficient equilibrium.

The firms in the extended model charge a mark-up, reflecting monopsonistic competition. Besides this conventional effect, two results are noteworthy. First, once firms are wage setters they reduce their labor demand in expansions, which mimics the constrained-efficient outcome and therefore the optimal prudential intervention (the right hand side of (24)). This echoes the main result in [Elsby \(2009\)](#).

However, second, the extent to which labor demand and wages are reduced is not maximizing social welfare. The reason is that elasticity ε_t^V enters equation (13), whereas elasticity $1/\eta$ enters equation (24). The role of elasticity ε_t^V for the optimal intervention was discussed in Section 3.2, and is briefly repeated here for convenience. A steep (aggregate) labor supply curve, reflected in a large ε_t^V , warrants a large prudential intervention because by reducing wages, the resulting drop in employment below potential is only small.

Yet, the labor supply curve which is relevant for the individual firm is (20), having elasticity $1/\eta$, rather than (21), having elasticity ε_t^V . To the extent that $\varepsilon_t^V > 1/\eta$, the individual firm faces a labor supply curve that is *flat* compared with the aggregate labor supply curve. Then, the individual firm will find it optimal to compress wage rises only *weakly* compared to the optimal intervention. It is easy to see that this is the empirically relevant case. In the application below, we will specialize to the disutility of labor supply $V(h) = h^{1+\varphi}/(1+\varphi)$, implying an elasticity of aggregate labor supply $\varepsilon_t^V = \varphi$. A plausible value for the (inverse) Frisch elasticity φ is 4. Thus, $\varepsilon_t^V > 1/\eta$ imposes the restriction $\eta > 1/4$. Clearly, $\eta = 1/4$

reflects very strong market power for the firms.¹⁷ Conversely, for more reasonable values of η such as 5, elasticity ε_t^V is about 20 times as large as elasticity $1/\eta$.

Intuitively, the reason why wage-setting firms behave almost as under perfect competition is that, by lowering wages unilaterally, they face a substitution of their workers to competing firms. Indeed, as $\eta \rightarrow \infty$ (perfect competition), we recover the baseline model as (22) reduces to (9). This mechanism arises because, in choosing $w_t(i)$, individual firms take the aggregate wage w_t as given. Instead, were individual firms to collectively agree on a wage policy, they would compress wage rises strongly because the crowding out in aggregate employment would be understood to be comparatively small.

To finish, we state the equation determining the multiplier λ_t in the extended model in periods when downward wage rigidity binds.¹⁸ As shown in the Appendix A, this is

$$\lambda_t = \frac{U'(t)}{p_t} h_t + \beta \bar{\psi} E_t \lambda_{t+1}, \quad (25)$$

which has to be compared with (16) in the constrained-efficient equilibrium. The multiplier λ_t is different because here it reflects the (shadow) increase in the firms' real profits by relaxing downward wage rigidity by a marginal unit, whereas for the planner it reflects the (shadow) rise in households' utility. An unambiguous ranking of the size of the two multipliers is not possible. However, in simulations one can verify that the effect of elasticity $1/\eta$ versus ε_t^V by far dominates any differences in multiplier λ_t .

The Appendix A contains further details and a step-by-step derivation of the extended-model equilibrium.

6 Quantitative analysis

Here we assess the quantitative relevance of the optimal prudential tax by applying the model to Greece, 1999-2016. We ask the following questions: How large is the optimal prudential tax during 1999-2008? How much does the tax reduce wage inflation during this period? How much is unemployment reduced after 2008 due to the policy intervention? And what are the welfare gains of the optimal prudential policy?

In this section, for simplicity, we use the baseline model from Section 2. This reflects that, as explained above, for reasonable values of firm competition $\eta > 0$ the baseline and extended model produce very similar equilibrium dynamics.¹⁹

¹⁷ From (22), the implied monopsonistic mark-up $(\eta + 1)/\eta$ is 500 percent.

¹⁸ Instead, labor demand in periods when downward wage rigidity binds is given by (14), as in the baseline model and the constrained-efficient equilibrium. See the Appendix A for details.

¹⁹ Using the baseline model also makes the welfare analysis transparent. In the extended model, the social planner charges the prudential tax as well as removes the monopsonistic mark-up. As a result, the welfare

6.1 Functional forms, parameters, and data

We solve the non-linear model numerically by using a global solution method.²⁰ To do so, we choose functional forms as well as specialize to a set of parameters. While some parameters are standard and therefore will be taken from other studies, other parameters are tailored to our application at hand.

We specialize to the following conventional functional form for technology

$$F(h_t) = h_t^\alpha$$

and set the standard value of $\alpha = 0.66$ to capture a labor share of two thirds. For the disutility of labor supply we assume that

$$V(h_t^f - \delta u_t) = \frac{(h_t^f - \delta u_t)^{1+\varphi}}{1+\varphi}$$

and set the (inverse) Frisch elasticity to the conventional value of $\varphi = 4$. Notice that both functional forms imply constant elasticities of labor demand and supply, given by $\varepsilon^F = \alpha - 1 = -0.33 < 0$ and $\varepsilon^V = \varphi = 4 > 0$, which had been used in Sections 4-5 above.

For consumption utility we choose the functional form

$$U(c_t - V(h_t^f - \delta u_t)) = \frac{(c_t - V(h_t^f - \delta u_t))^{1-\sigma}}{1-\sigma}$$

and set for risk aversion $\sigma = 2$ —once again a conventional value in the international business cycle literature (e.g., [Bianchi, 2011](#)).

For the no-Ponzi constraint we use the natural borrowing limit. We then calibrate for the time discount factor $\beta = 0.957$ to obtain a mean ratio of external debt to GDP of 80 percent, in line with the average external debt-to-GDP in Greece since its joining the euro area.²¹

Downward wage rigidity is governed by the function $\psi(u_t)$, which we specialize to

$$\psi(u_t) = \bar{\psi} - \kappa u_t.$$

Here, $\kappa \geq 0$ determines how downward wage rigidity is affected by unemployment, and $\bar{\psi} \leq 1$ determines downward wage rigidity when unemployment is zero.

For the parameter governing downward wage rigidity in the absence of unemployment, we set $\bar{\psi} = 0.96$ such that wages can fall by at most one percent per quarter. This corresponds to (conservative) estimates in [Schmitt-Grohé and Uribe \(2016\)](#) for Greece.

gain from the prudential intervention becomes entangled with the welfare gain from removing distortions due to the firms exercising their market power.

²⁰ The equilibrium in all three models (baseline, extended and constrained-efficient) can be obtained by using fixed-point iteration. As an alternative, the constrained-efficient equilibrium can be obtained by using value function iteration, see Definition 1.

²¹ The borrowing limit is generous enough to be practically never binding in the asset ergodic distribution.

Below we will employ annual data and therefore set for the interest rate $R = 1.04$.

Another parameter to be determined is δ , measuring the utility penalty from unemployment. This parameter is hard to pin down as it lacks a clear empirical counterpart. We take a pragmatic approach and set $\delta = 0.5$, implying that leisure derived from unemployment carries half as much utility value as voluntary leisure. Notice that, under an (equally plausible) value of $\delta = 0$, whereby households do not receive any pleasure from unemployment, the optimal prudential intervention would be correspondingly larger.

A remaining set of parameters are κ and the parameters governing the stochastic properties of the two stochastic processes $\{a_t, p_t\}$. These parameters will be determined as part of the empirical strategy, to be described below.

We rely on a set of four time series from 1999-2016, used to proxy for the hourly wage w_t , labor productivity a_t , unemployment u_t and real GDP y_t .

To proxy for w_t we use Eurostat’s *labor cost index*. The labor cost index measures hourly wage costs of firms, inclusive of taxes minus subsidies. We also use *hourly productivity*, from OECD, to proxy for labor productivity a_t . From Eurostat we extract *total unemployment* as a fraction of the participating population to proxy for \hat{u}_t , and from OECD we extract *real GDP* to capture y_t . All series (except for the unemployment series) are detrended by using a euro-zone average, to take account of technology growth as well as trend inflation which are not modeled. Further details on the data are available on request.

6.2 Empirical strategy and model performance

In this class of models, p_t is commonly interpreted as the country’s terms of trade. In turn, a_t may be interpreted as labor productivity. Therefore, wage dynamics in the model are either driven by the “demand side” following terms of trade shocks, or by the “supply side” following technology shocks. In what follows, we will interpret p_t more broadly than just the terms of trade. Rather, p_t will be used to capture demand shocks more generally, or any variation in wages that is not productivity-related.²²

Specifically, we use the following empirical strategy. From solving the competitive equilibrium we obtain a policy function for wages $w(w_{t-1}, a_t, p_t)$ (the policy function is independent of assets, due to the two-block structure of the competitive equilibrium, see Section 2). Because the sequences for $\{w_t\}$ and $\{a_t\}$ are observed, at each time t , this policy function can be inverted to solve for the implied value of p_t .²³ That is, we back out a series of demand

²² A similar strategy is pursued in [Berka et al. \(2017\)](#), who decompose real exchange rate variation into productivity-related factors and other factors which capture real exchange rate variation that is independent of productivity.

²³ This inversion step is only possible if $\kappa > 0$. Otherwise, the policy function for w_t would be flat in the region where downward wage rigidity binds, and therefore could not be inverted.

shocks in order to explain the observed wage series perfectly. Importantly, in the Greek crisis after 2008 when w_t falls and downward wage rigidity binds, parameter κ governs the extent to which wage deflation translates into unemployment. We therefore adjust parameter κ so as to generate a rise in unemployment of 18 percentage points, which (as shown below) corresponds to the rise in unemployment in Greece during the crisis period.

Before turning to the results, two remarks are in order. First, to carry out the previous exercise, no information about the process governing $\{a_t, p_t\}$ is needed. This is because the competitive equilibrium’s labor market is *static*. Therefore, the policy functions are identical for any stochastic process governing $\{a_t, p_t\}$. Second, the previous exercise requires that the series for $\{w_t, a_t\}$ is observable in levels, not as an index. This is not the case. Therefore, to be able to continue we make the following assumption²⁴

Assumption 1. *In 1999 before Greece joined the euro, $w_{-1} = w_{ss}$.*

This assumption is not unreasonable. As is well known, when the euro was incepted real exchange rates were set to minimize the distance of each country’s real exchange rate from purchasing power parity (see for example the discussion in [Berka et al., 2017](#)). It is therefore not implausible that, in 1999, Greece did not experience any severe up- or downward pressure on its nominal wages.

Turn now to the result (Figure 2). The upper left panel shows that productivity follows hump-shaped dynamics during the sample period, and that a declining path for demand shocks is required to capture the hump-shaped dynamics for hourly wages (upper right panel). The lower left panel shows the evolution of unemployment in the model and data. By construction, model-implied unemployment rises by 18 percentage points in the crisis (the implied $\kappa = 0.23$). Finally, as an external validation, the lower right panel shows that the model captures the evolution of real output quite well.

It should be noted that the model fails to capture the unemployment dynamics before the crisis. Clearly, this is because we abstract from any frictional unemployment, but only consider “rationing” unemployment. In this regard, the model successfully predicts an unemployment spike after 2008.²⁵ A decomposition of unemployment into frictional versus rationing for the US during the Great Recession is done in [Michaillat \(2012\)](#).

²⁴ Without this assumption, the initial level for wages in the estimation is not determined. One could attempt to estimate it by using a non-linear filter. However, the computational burden is likely to become overwhelming given the numerical complexities involved in solving the (non-linear) model.

²⁵ The 2005 unemployment spike that the model predicts results from the fact that wages decline strongly in this year—and hence that downward wage rigidity binds (see the upper right panel). At the same time, because the rate of wage deflation subsides after 2013, the model fails to predict that unemployment remains high during this period. Recall that all data is de-trended by using a euro-zone average.

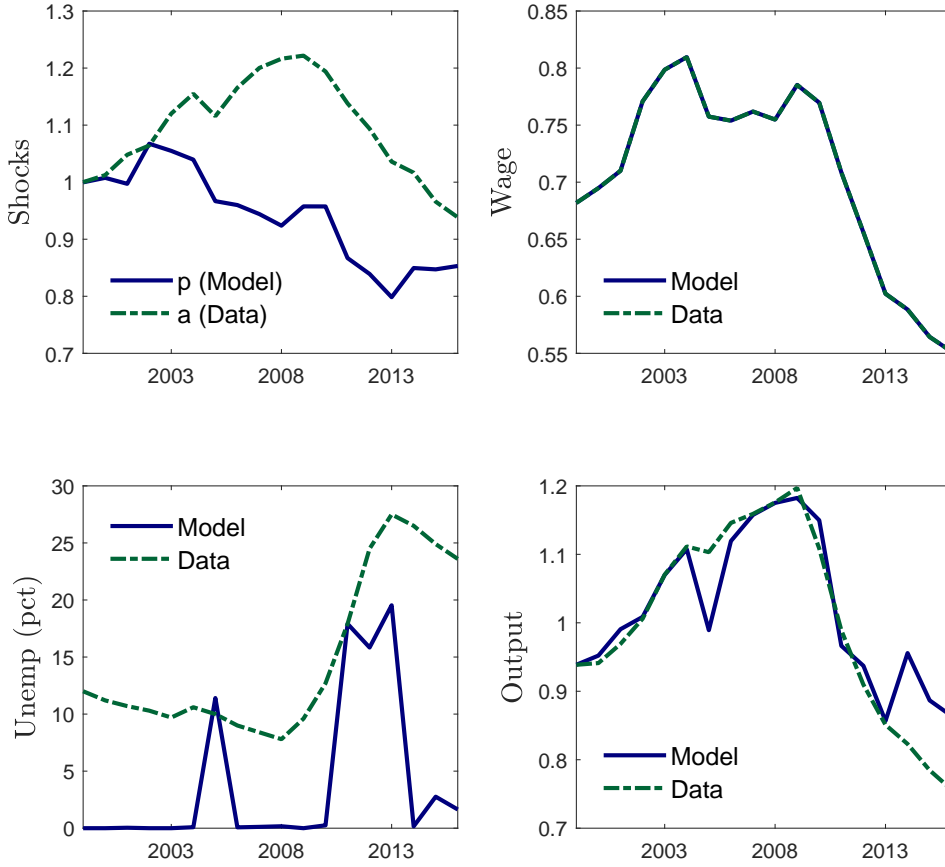


Figure 2: Model performance. Data green dashed, model predictions solid blue. Data sources, methodology and parameters are described in the text. Unemployment shown in the figure is $\hat{u}_t \equiv (h_t^f - h_t)/h_t^f$, expressed in percent.

6.3 Optimal prudential intervention

We now study counterfactual dynamics of Greek variables induced by the optimal prudential policy, as well as the implied prudential (payroll) tax. To do so, we specify a stochastic process for the two exogenous variables $\{a_t, p_t\}$. This is because the constrained-efficient equilibrium is dynamic and non-linear, and therefore depends not only on the shocks' realization, but also on their stochastic properties.

As is common in the international business cycle literature, we assume that the shocks obey a first-order bivariate autoregressive process in logs (e.g., [Bianchi, 2011](#))

$$\log([a_t, p_t]') = \boldsymbol{\rho} \times \log([a_{t-1}, p_{t-1}]') + v_t,$$

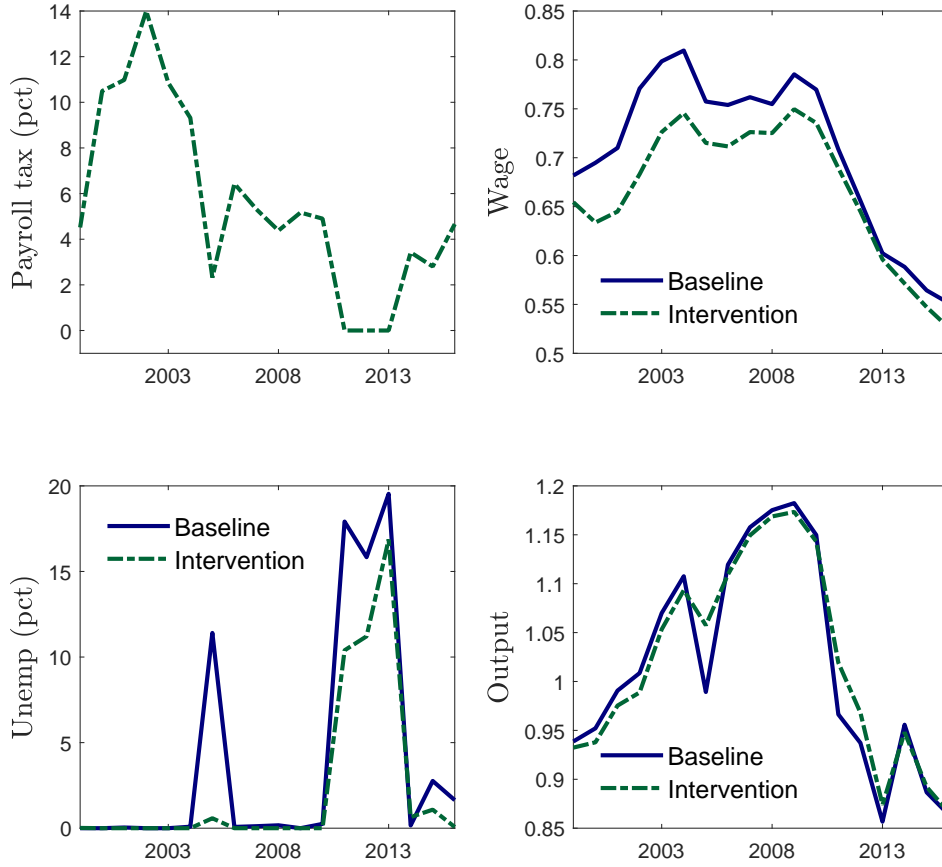


Figure 3: Model counterfactual. The policy intervention is green dashed, the baseline dynamics are solid blue. The methodology and parameters are described in the text. Unemployment shown in the figure is $\hat{u}_t \equiv (h_t^f - h_t)/h_t^f$, expressed in percent.

where $v_t \sim \mathcal{N}([0, 0]', \Sigma)$, with ρ and Σ conformable matrices.

From the earlier analysis, a time series for $\{a_t, p_t\}$ is directly available. The matrices ρ and Σ can thus be estimated by using standard econometric methods. The result is

$$\rho = \begin{pmatrix} 1.0312 & -0.1667 \\ 0.2097 & 0.9161 \end{pmatrix}, \quad \Sigma = 0.0001 \times \begin{pmatrix} 8.8148 & 7.0084 \\ 7.0084 & 16.0398 \end{pmatrix}.$$

As can be cross-checked against Figure 2, the shocks are strongly autocorrelated as well as positively correlated. To implement this process numerically, we use the quadrature-based procedure of [Tauchen and Hussey \(1991\)](#).

To construct a counterfactual, we assume that the constrained-efficient planner takes over the economy in the initial period. By facing the same sequence of shocks as the competitive

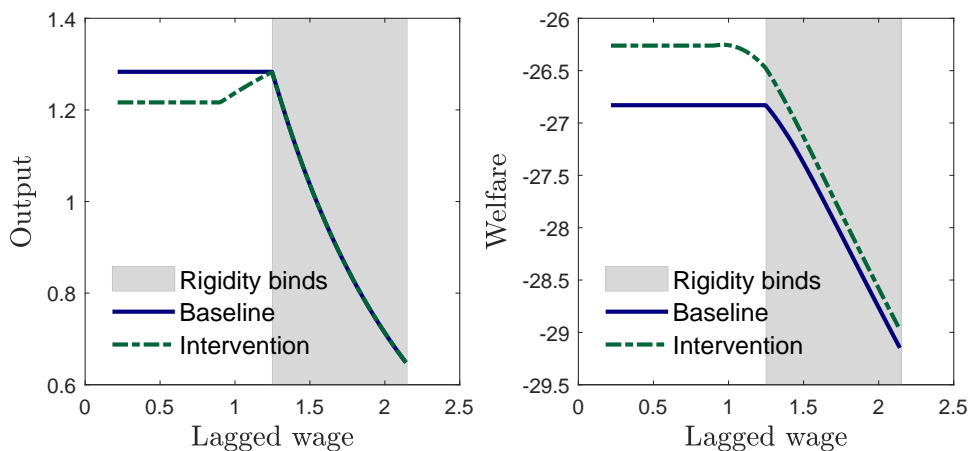


Figure 4: Output y_t and welfare \mathcal{U}_t in the competitive versus constrained-efficient equilibrium. Shown are policy functions against the lagged wage w_{t-1} , by keeping assets Λ_t at their (stochastic) steady state and the exogenous disturbances a_t and p_t constant at +1 standard deviation. “Rigidity binds” refers to the states w_{t-1} where downward wage rigidity binds in the competitive equilibrium (“Baseline”).

equilibrium, we study the resulting path for wages, unemployment, real GDP and the payroll tax that is needed to decentralize the constrained-efficient allocation.

The result is shown in Figure 3. The peak response of the payroll tax is 14 percent, which distinctly reduces wage inflation during the early years of the Greek cycle. As wages are lower at the onset of the Greek crisis, the unemployment spike post 2008 is markedly reduced—in the years 2011-2012 by almost half. However, in the year 2013 Greek unemployment is high even under the optimal prudential intervention. This reflects that the negative shocks which hit Greece in the crisis were quite severe—and persistently so. As a result, even under the optimal policy unemployment could not be avoided entirely.

6.4 Welfare effects and summary statistics

Under the optimal policy, the country faces fewer and shallower recessions. However, it produces below potential in expansions. This balancing act between “static losses” and “dynamic gains” is visualized in Figure 4, which contrasts policy functions for output y_t and welfare \mathcal{U}_t among the competitive and constrained-efficient allocation.

The first observation is that output among the two allocations coincides when downward wage rigidity binds. This reflects that the optimal policy is inherently *prudential*: conditional on having entered a recession, the policy is powerless (in recessions, the prudential tax equals zero). Second, we observe that output is *lower* under the optimal intervention when downward wage rigidity is slack, representing a “static loss”: the economy produces below potential in

$mean(\hat{u}_t)\%$	$frac(\hat{u}_t > 0)$	$mean(\hat{u}_t > 0)\%$	$mean(\hat{u}_t^*)\%$	$frac(\hat{u}_t^* > 0)$
5.8	0.47	12.3	1.5	0.14
	$mean(\hat{u}_t^* > 0)\%$	$mean(\tau_t^w)\%$	$mean(\iota_t)\%$	$mean(\iota_t^*)\%$
	10.3	15.3	1.1	2.5

Table 1: Summary statistics. Reported are: mean unemployment, frequency of unemployment being strictly positive and mean unemployment conditional on unemployment being strictly positive, competitive equilibrium (columns 1-3) and constrained-efficient equilibrium (columns 4-6). Column 7 contains the mean optimal prudential payroll tax, in percent. Mean consumption equivalent: columns 8-9, column 9 relative to the constrained-efficient equilibrium where it is additionally imposed that $\psi(u_t) \equiv 0$.

expansions (in expansions, the prudential tax is strictly positive).

Still, welfare under the optimal policy is higher, reflecting “dynamic gains”: while conditional on state variables, the economy under the optimal policy performs worse, the stationary distribution of nominal wages is shifted towards more favorable states, such that overall welfare increases.²⁶ This gain is more pronounced when downward wage rigidity is slack, for this is the region of the state space where policy actively intervenes.²⁷

Table 1 contains summary statistics. The first three columns are unemployment statistics in the competitive equilibrium: mean unemployment, the fraction of periods in which unemployment is strictly positive, and unemployment conditional on unemployment being strictly positive. As in Figures 2-3, here we define unemployment as $\hat{u}_t \equiv (h_t^f - h_t)/h_t^f$, and express it in percent. The mean unemployment rates are 5.8 and 12.3 percent respectively, and the fraction of periods in which the labor market is rationed is 0.47.

Columns 4-6 show the same unemployment statistics in the constrained-efficient equilibrium. In this case, the means drop to 1.5 and 10.3 percent, respectively, and the fraction of periods in which the labor market is rationed is only 0.14. The optimal prudential intervention hence reduces unemployment fluctuations significantly, and this operates mainly through a reduction in the *frequency of periods* in which the labor market is rationed.

This also implies that in most periods, the economy bears the “static loss” of the intervention (whereas the periods of crisis in which the economy actually benefits are relatively rare). Column 7 contains the source of this static loss: the mean payroll tax, expressed in percent, that is required in order to decentralize the constrained-efficient equilibrium. It amounts to

²⁶ The stationary distribution of assets is also shifted by implication of the optimal prudential intervention. As crises are less frequent and shallower, the economy under the optimal intervention takes on more debt, such that the stationary distribution of assets is wider and shifted to the left (in line with the results in [Schmitt-Grohé and Uribe \(2016\)](#)).

²⁷ Instead, in the region where downward wage rigidity binds, the welfare gain arises from *expectations* that in the future, once downward wage rigidity turns slack again, the policy will optimally intervene.

15.3 percent, and therefore is in fact higher than in the particular episode preceding the Greek crisis that was considered earlier above (recall Figure 3).

Despite the “static loss”, the welfare gain from the intervention is large. The last two columns contain permanent consumption statistics. Here we compute the percentage increase in period-consumption that is necessary in order to make the household indifferent between staying in the competitive equilibrium and moving to the constrained-efficient equilibrium.²⁸ We report the mean of the stationary distribution of the consumption equivalent. It is 1.1 percent—a large number compared to traditional estimates of the cost of business cycles (see also [Schmitt-Grohé and Uribe, 2016](#)).

In turn, the last column shows the consumption equivalent in case downward wage rigidity is absent in the constrained-efficient equilibrium (that is, the policy maker has the ability to subsidize labor demand in recessions which, as explained above, restores the first best). This number therefore isolates the cost of downward wage rigidity *per se*. The loss is 2.5 percent of permanent consumption. Therefore, the optimal prudential intervention removes $1.1/2.5 \approx 44$ percent of the total welfare cost of downward wage rigidity.

7 Conclusion

This paper studies optimal policy in economies with downward nominal wage rigidity when only prudential instruments are available. The prudential intervention strikes a balance between reducing labor demand and therefore output and wages in expansions, and reducing unemployment in subsequent recessions. All other markets may clear competitively, including capital markets. Our results apply to the case where the labor market is Walrasian, but also to a labor market with wage-setting firms.

We have explored this trade-off theoretically and quantitatively. The theory analysis yields a formula for the optimal prudential tax. The numerical analysis suggests that the welfare gains from the prudential intervention are large. In an application to Greece, the intervention removes close to half of the total welfare cost of downward wage rigidity, and it reduces mean unemployment significantly.

We hence conclude that prudential labor market policies are an important tool in a policy maker’s toolkit in order to reduce the economic cost of downward wage rigidity.

²⁸ The consumption equivalent in percent $\iota_t\%$ is defined as

$$\mathcal{U}(w_{t-1}, \Lambda_t, a_t, p_t) = U(c_t(1 + \iota_t\%/100\%) - V(h_t + (1 - \delta)u_t)) + \beta E_t \mathcal{U}(w_t, \Lambda_{t+1}, a_{t+1}, p_{t+1}),$$

where \mathcal{U} is welfare in the constrained-efficient equilibrium, and where $[c_t, h_t, u_t, w_t, \Lambda_{t+1}]'$ are policy functions in the competitive equilibrium.

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A Appendix: Technical Appendix

This Appendix contains proofs and derivations. We derive the constrained-efficient equilibrium and the equilibrium in the extended model, and we provide the proofs of Propositions 1-2. We also derive the special case of the optimal tax (18).

A.1 Proof of Proposition 1

From Definition 1), the constrained-efficient equilibrium solves

$$\mathcal{U}(w_{t-1}, \Lambda_t, a_t, p_t) = \max_{\{c_t, \Lambda_{t+1}, h_t, w_t, u_t\}} \{U(c_t - V(h_t + (1 - \delta)u_t)) + \beta E_t \mathcal{U}(w_t, \Lambda_{t+1}, a_{t+1}, p_{t+1})\}$$

subject to the set of constraints

$$\begin{aligned} i) \quad & w_t/p_t \leq a_t F'(h_t) && \text{(multiplier: } \gamma_t) \\ ii) \quad & w_t/p_t = V'(h_t + u_t) && \text{(multiplier: } \zeta_t) \\ iii) \quad & u_t \geq 0 && \text{(multiplier: } \kappa_t) \\ iv) \quad & w_t \geq \psi(u_t)w_{t-1} && \text{(multiplier: } \lambda_t) \\ v) \quad & p_t c_t + \Lambda_{t+1}/R = p_t a_t F(h_t) + \Lambda_t && \text{(multiplier: } \iota_t) \end{aligned}$$

for given exogenous $\{a_t, p_t\}$.

The first order conditions are

$$\begin{aligned} U'(t) - \iota_t p_t &= 0 \\ \beta E_t \frac{\partial}{\partial \Lambda_{t+1}} \mathcal{U}(w_t, \Lambda_{t+1}, a_{t+1}, p_{t+1}) - \frac{\iota_t}{R} &= 0 \\ -U'(t)V'(h_t + (1 - \delta)u_t) + \gamma_t \varepsilon_t^F \frac{w_t}{h_t} - \zeta_t \varepsilon_t^V \frac{w_t}{h_t + u_t} + \iota_t p_t a_t F'(h_t) &= 0 \\ \beta E_t \frac{\partial}{\partial w_t} \mathcal{U}(w_t, \Lambda_{t+1}, a_{t+1}, p_{t+1}) - \gamma_t + \zeta_t + \lambda_t &= 0 \\ -U'(t)V'(h_t + (1 - \delta)u_t)(1 - \delta) - \zeta_t \varepsilon_t^V \frac{w_t}{h_t + u_t} + \kappa_t - \lambda_t \psi'(u_t)w_{t-1} &= 0 \end{aligned}$$

and the Envelope conditions are

$$\begin{aligned} \frac{\partial}{\partial \Lambda_t} \mathcal{U}(w_{t-1}, \Lambda_t, a_t, p_t) &= \iota_t \\ \frac{\partial}{\partial w_{t-1}} \mathcal{U}(w_{t-1}, \Lambda_t, a_t, p_t) &= -\lambda_t \psi(u_t). \end{aligned}$$

Combining the first two first order conditions and the first Envelope condition gives the consumption Euler equation (3). By replacing $\iota_t = U'(t)/p_t$ and using the second Envelope

condition, the remaining first order conditions can be written as

$$U'(t)(a_t F'(h_t) - V'(h_t + (1 - \delta)u_t)) + \gamma_t \varepsilon_t^F \frac{w_t}{h_t} - \zeta_t \varepsilon_t^V \frac{w_t}{h_t + u_t} = 0 \quad (+)$$

$$-\beta E_t \psi(u_{t+1}) \lambda_{t+1} - \gamma_t + \zeta_t + \lambda_t = 0 \quad (++)$$

as well as

$$-U'(t)V'(h_t + (1 - \delta)u_t)(1 - \delta) - \zeta_t \varepsilon_t^V \frac{w_t}{h_t + u_t} + \kappa_t - \lambda_t \psi'(u_t) w_{t-1} = 0. \quad (+++)$$

From Proposition 1, the constrained-efficient equilibrium equivalently solves

$$\tilde{\mathcal{U}}(w_{t-1}, a_t, p_t) = \max_{\{h_t, w_t, u_t\}} \{U'(t)(a_t F(h_t) - V(h_t + (1 - \delta)u_t)) + \beta E_t \tilde{\mathcal{U}}(w_t, a_{t+1}, p_{t+1})\}$$

subject to the set of constraints

$$\begin{aligned} i) & & w_t &\leq p_t a_t F'(h_t) & (\text{multiplier: } \gamma_t) \\ ii) & & w_t &= p_t V'(h_t + u_t) & (\text{multiplier: } \zeta_t) \\ iii) & & u_t &\geq 0 & (\text{multiplier: } \kappa_t) \\ iv) & & w_t &\geq \psi(u_t) w_{t-1} & (\text{multiplier: } \lambda_t) \end{aligned}$$

for a given exogenous path $\{a_t, p_t\}$, where $U'(t)$ is exogenous to this maximization problem and determined in (3) and (10).

We have already established that $U'(t)$ is determined in (3) and (10). That is, we have already established that the planner lets the asset market clear competitively (equation (10) appeared as constraint v) in the maximization from Definition 1).

Therefore, to prove Proposition 1, it remains to be shown that the previous problem has the first order conditions (+)-(+++). The first order conditions of the equivalent representation are

$$\begin{aligned} U'(t)(a_t F'(h_t) - V'(h_t + (1 - \delta)u_t)) + \gamma_t \varepsilon_t^F \frac{w_t}{h_t} - \zeta_t \varepsilon_t^V \frac{w_t}{h_t + u_t} &= 0 \\ \beta E_t \frac{\partial}{\partial w_t} \tilde{\mathcal{U}}(w_t, a_{t+1}, p_{t+1}) - \gamma_t + \zeta_t + \lambda_t &= 0 \\ -U'(t)V'(h_t + (1 - \delta)u_t)(1 - \delta) - \zeta_t \varepsilon_t^V \frac{w_t}{h_t + u_t} + \kappa_t - \lambda_t \psi'(u_t) w_{t-1} &= 0 \end{aligned}$$

and the Envelope condition is

$$\frac{\partial}{\partial w_{t-1}} \tilde{\mathcal{U}}(w_{t-1}, a_t, p_t) = -\lambda_t \psi(u_t).$$

By inserting the Envelope condition into the first order conditions, one can see that the first order conditions coincide with (+)-(+++).

This completes the proof of Proposition 1.

A.2 Properties of the constrained-efficient equilibrium

From the previous section, the first order conditions describing the labor market problem in the constrained-efficient equilibrium are

$$\begin{aligned} U'(t)(a_t F'(h_t) - V'(h_t + (1 - \delta)u_t)) + \gamma_t \varepsilon_t^F \frac{w_t}{h_t} - \zeta_t \varepsilon_t^V \frac{w_t}{h_t + u_t} &= 0 \\ -\beta E_t \psi(u_{t+1}) \lambda_{t+1} - \gamma_t + \zeta_t + \lambda_t &= 0 \end{aligned}$$

as well as

$$-U'(t)V'(h_t + (1 - \delta)u_t)(1 - \delta) - \zeta_t \varepsilon_t^V \frac{w_t}{h_t + u_t} + \kappa_t - \lambda_t \psi'(u_t)w_{t-1} = 0.$$

Using the fact that multiplier ζ_t is always non-zero, we may combine these to

$$\begin{aligned} U'(t)(a_t F'(h_t) - V'(h_t + (1 - \delta)u_t)) + \gamma_t \varepsilon_t^F \frac{w_t}{h_t} &= \varepsilon_t^V \frac{w_t}{h_t + u_t} (\gamma_t - \lambda_t + \beta E_t \psi(u_{t+1}) \lambda_{t+1}) \quad (*) \\ &= -U'(t)V'(h_t + (1 - \delta)u_t)(1 - \delta) + \kappa_t - \lambda_t \psi'(u_t)w_{t-1}. \end{aligned}$$

We now derive the labor demand and supply curves in the constrained-efficient equilibrium when downward wage rigidity is slack. Thereafter, we derive both curves when downward wage rigidity is binding.

A.2.1 Downward wage rigidity is slack

We first show that, when downward wage rigidity is slack, there can be no unemployment. That is, we show that the complementary slackness condition (7) from the competitive equilibrium is also an equilibrium condition in the constrained-efficient allocation.

Assume that condition iv) is slack, such that $\lambda_t = 0$.²⁹ Assume also that $u_t > 0$ such that $\kappa_t = 0$. From (*), this implies that

$$\varepsilon_t^V \frac{w_t}{h_t + u_t} (\gamma_t + \beta E_t \psi(u_{t+1}) \lambda_{t+1}) = -U'(t)V'(h_t + (1 - \delta)u_t)(1 - \delta).$$

The right hand side is strictly negative. However, the left hand side is weakly positive ($\gamma_t \geq 0$ and $E_t \psi(u_{t+1}) \lambda_{t+1} \geq 0$; recall that $\varepsilon_t^V > 0$ because labor supply is upward sloping)—this is a contradiction. Therefore, $w_t > \psi(u_t)w_{t-1}$ such that $\lambda_t = 0$ implies that $u_t = 0$, such that

$$u_t(w_t - \psi(u_t)w_{t-1}) \tag{7}$$

is an equilibrium condition also in the constrained-efficient allocation. Inserting this in condition ii) yields equation (12) in the main text. Furthermore, using $u_t = 0$ and condition ii),

²⁹ Here and in the following, we refer to the constraints in the maximization problems in Definition/Proposition 1—written out in Section A.1.

we can express (*) as

$$U'(t)(a_t F'(h_t) - \frac{w_t}{p_t}) = \gamma_t \left(\varepsilon_t^V \frac{w_t}{h_t} - \varepsilon_t^F \frac{w_t}{h_t} \right) + \varepsilon_t^V \frac{w_t}{h_t} \beta E_t \psi(u_{t+1}) \lambda_{t+1}.$$

We proceed by showing that the multiplier γ_t is equal to zero. Assume it is strictly positive. In this case, from condition i), it must be that $w_t/p_t = a_t F'(h_t)$. Inserting this in the previous equation we obtain

$$-\gamma_t \left(\varepsilon_t^V \frac{w_t}{h_t} - \varepsilon_t^F \frac{w_t}{h_t} \right) = \varepsilon_t^V \frac{w_t}{h_t} \beta E_t \psi(u_{t+1}) \lambda_{t+1}.$$

Because $\gamma_t > 0$ by assumption and $\varepsilon_t^F < 0$ and $\varepsilon_t^V > 0$ (recall that $\varepsilon_t^F < 0$ because labor demand is downward sloping), the left hand side is strictly negative. However, the right hand side is weakly positive—this is a contradiction. Therefore, it must be that $\gamma_t = 0$.

Using this we obtain the labor demand curve when downward wage rigidity is slack

$$a_t F'(h_t) = \frac{w_t}{p_t} + \frac{1}{U'(t)} \varepsilon_t^V \frac{w_t}{h_t} \beta E_t \psi(u_{t+1}) \lambda_{t+1},$$

which is equation (11) in the main text.

A.2.2 Downward wage rigidity is binding

We now turn to the case where downward wage rigidity binds. That is, we turn to the region where $\lambda_t > 0$. Here we need to make a case distinction that was not discussed in the main text. Namely, we distinguish the two cases $u_t = 0$ versus $u_t > 0$.

In the main text we have only shown the case $u_t > 0$, because this holds in most parts of the state space where $\lambda_t > 0$. Intuitively, the case of $\lambda_t > 0$ but $u_t = 0$ can arise in an intermediate region: if the wage rigidity binds only slightly, by reducing the size of the intervention but still remaining in the region $w_t/p_t < a_t F'(h_t)$, the planner can maintain full employment.

Formally, $\lambda_t > 0$ implies that condition iv) holds with equality. Therefore, w_t is determined from this condition. As additionally $u_t = 0$, h_t is determined from condition ii). Since both w_t and h_t are determined, condition i) can only hold with an inequality.³⁰ Therefore, $\gamma_t = 0$. Using this, the fact that $u_t = 0$ and condition ii) we obtain from (*)

$$a_t F'(h_t) = \frac{w_t}{p_t} + \frac{1}{U'(t)} \varepsilon_t^V \frac{w_t}{h_t} (\beta E_t \psi(u_{t+1}) \lambda_{t+1} - \lambda_t).$$

When downward wage rigidity starts to bind, λ_t turns positive. However, as the previous equation shows, even a $\lambda_t > 0$ may be consistent with $w_t/p_t < a_t F'(h_t)$. That is, the planner

³⁰ The case where it holds with equality is the knife-edge case where $w_t = \psi(u_t) w_{t-1}$ corresponds exactly to the wage under full employment. Therefore, $\lambda_t = 0$, and this case was dealt with earlier above.

still actively intervenes. However, the size of the intervention is reduced relative to the case where $\lambda_t = 0$.

Instead, once the recession turns severe enough

$$\lambda_t > \beta E_t \psi(u_{t+1}) \lambda_{t+1},$$

from the last equation, $a_t F'(h_t)$ would fall below w_t/p_t , which is impossible. Therefore, in this case, the planner sets $a_t F'(h_t) = w_t/p_t$ (which is equation (14) from the main text), and $\gamma_t > 0$ turns positive. In turn, once condition i) holds with equality, condition ii) implies that unemployment turns positive, too, $u_t > 0$, implying $\kappa_t = 0$ from condition iii). Using this in (*), we obtain

$$\begin{aligned} U'(t) V'(h_t + (1 - \delta)u_t) \left(\delta + (1 - \delta) \frac{\varepsilon_t^F}{\varepsilon_t^V} \frac{h_t + u_t}{h_t} \right) &= \varepsilon_t^F \frac{w_t}{h_t} (\lambda_t - \beta E_t \psi(u_{t+1}) \lambda_{t+1}) \\ &+ U'(t) \frac{w_t}{p_t} + \lambda_t \psi'(u_t) w_{t-1} \left(1 - \frac{\varepsilon_t^F}{\varepsilon_t^V} \frac{h_t + u_t}{h_t} \right), \end{aligned}$$

which is equation (15) in the main text.

A.3 Proof of Proposition 2

The maximization in Proposition 2 can be written as

$$\mathcal{U}(w_{t-1}, \Lambda_t, a_t, p_t) = \max_{\{c_t, \Lambda_{t+1}, h_t, w_t, u_t, \tau_t^w \geq 0\}} \{U(c_t - V(h_t + (1 - \delta)u_t)) + \beta E_t \mathcal{U}(w_t, \Lambda_{t+1}, a_{t+1}, p_{t+1})\}$$

subject to the set of constraints

$$\begin{aligned} i) & w_t(1 + \tau_t) = p_t a_t F'(h_t) \\ ii) & w_t = p_t V'(h_t + u_t) \\ iii) & u_t \geq 0 \\ iv) & w_t \geq \psi(u_t) w_{t-1} \\ v) & u_t(w_t - \psi(u_t) w_{t-1}) = 0 \\ vi) & 1 = \beta R E_t (U'(t+1)/U'(t)) (p_t/p_{t+1}) \\ vii) & p_t c_t + \Lambda_{t+1}/R = p_t y_t + \Lambda_t, \end{aligned}$$

for given exogenous $\{a_t, p_t\}$.

By recognizing that $\tau_t^w \geq 0$, we note that constraint i) can be replaced by $w_t \geq p_t a_t F'(h_t)$. Therefore, the maximization problem is identical as the maximization problem of the constrained planner (see Definition 1 and Section A.1), except for conditions v)-vi) which constitute two additional constraints.

We proceed as in [Bianchi \(2016\)](#), by showing that the constraints v)-vi) are always slack. That is, while v)-vi) formally constitute two additional constraints in the maximization, at the optimum, the two constraints are never binding. To do so, we consider the maximization without constraint v)-vi) and show that the constraints v)-vi) are implied as an optimality condition.

The fact that the consumption Euler equation, constraint vi), is an optimality condition in the constrained-efficient equilibrium was a central part in the proof of Proposition 1. In Section A.2 we had further shown that equation (7), which is constraint v), is an optimality condition in the constrained-efficient equilibrium.

This completes the proof of Proposition 2.

A.4 Optimal prudential tax

Here we show how to obtain the special case for the optimal prudential tax (18).

Start with (17) which we write as

$$\tau_t^w = \varphi \frac{1}{U'(t)} \frac{p_t}{h_t^f} \beta \bar{\psi} E_t \lambda_{t+1},$$

where we have canceled w_t , used our assumption $\psi(u_t) = \bar{\psi}$, used that $h_t = h_t^f$ when downward wage rigidity is slack ((4) when $u_t = 0$), and replaced $\varepsilon_t^V = \varphi$.³¹

To replace λ_{t+1} , we use (16) and set $\lambda_{t+2} = 0$ (by assumption, downward wage rigidity is slack from period $t + 2$ onwards). Furthermore, we replace $a_t F'(h_t) = w_t/p_t$ from (14) and use that $V'(\bar{h}) = \bar{h}^\varphi$, the latter from our assumption on the functional form for the dis-utility of labor supply. We also replace $-(\varepsilon_t^F)^{-1} = 1/(1 - \alpha)$

$$\lambda_{t+1} = \frac{1}{1 - \alpha} \frac{h_{t+1}}{w_{t+1}} U'(t + 1) \left(\frac{w_{t+1}}{p_{t+1}} - h_{t+1}^\varphi \right).$$

Now use (5) and (4) to re-write the last equation as

$$\lambda_{t+1} = \frac{1}{1 - \alpha} \frac{h_{t+1}^f - u_{t+1}}{w_{t+1}} U'(t + 1) \left((h_{t+1}^f)^\varphi - (h_{t+1}^f - u_{t+1})^\varphi \right).$$

Now use the definition of $\hat{u}_t \equiv (h_t^f - h_t)/h_t = u_t/h_t^f \Leftrightarrow u_t = \hat{u}_t h_t^f$ to obtain

$$\lambda_{t+1} = \frac{1}{1 - \alpha} \frac{h_{t+1}^f}{w_{t+1}} U'(t + 1) (1 - \hat{u}_{t+1}) (h_{t+1}^f)^\varphi (1 - (1 - \hat{u}_{t+1})^\varphi).$$

Use again (5) to replace $(h_{t+1}^f)^\varphi$ and cancel the w_{t+1} to arrive at

$$\lambda_{t+1} = \frac{1}{1 - \alpha} \frac{h_{t+1}^f}{p_{t+1}} U'(t + 1) (1 - \hat{u}_{t+1}) (1 - (1 - \hat{u}_{t+1})^\varphi).$$

³¹ As shown in Section 6, we specialize to $V(h_t) = h_t^{1+\varphi}/(1+\varphi)$ and $F(h_t) = h_t^\alpha$. As a result, we obtain labor supply and demand $w_t = p_t h_t^\varphi$ and $w_t = p_t a_t \alpha h_t^{\alpha-1}$. Hence, the elasticities of labor supply and demand are $\varepsilon_t^V = \varphi$ and $\varepsilon_t^F = \alpha - 1$.

Insert this equation into the optimal prudential tax

$$\tau_t^w = \varphi \bar{\psi} \beta E_t \frac{U'(t+1)}{U'(t)} \frac{p_t}{p_{t+1}} \frac{1}{1-\alpha} \frac{h_{t+1}^f}{h_t^f} (1 - \hat{u}_{t+1}) (1 - (1 - \hat{u}_{t+1})^\varphi).$$

Use (5) one last time to replace h_{t+1}^f/h_t^f to end up with (18) from the main text.

A.5 Model extension: The case of active wage setters

We repeat the dynamic program of the firms from Definition 3

$$\mathcal{P}_t(w_{t-1}(i)) = \max_{\{h_t(i), w_t(i)\}} \left\{ a_t F(h_t(i)) - \frac{w_t(i)}{p_t} h_t(i) + \beta E_t \frac{U'(t+1)}{U'(t)} \mathcal{P}_{t+1}(w_t(i)) \right\}$$

subject to the set of constraints

$$\begin{aligned} i) \quad & h_t(i) \leq (w_t(i)/w_t)^\eta h_t, & (\text{multiplier: } \kappa_t/U'(t)) \\ ii) \quad & w_t(i) \geq \bar{\psi} w_{t-1}(i), & (\text{multiplier: } \lambda_t/U'(t)) \end{aligned}$$

for given aggregate state variables $\{a_t, p_t, h_t, w_t\}$. We scale the multipliers by $U'(t)$ such that they are expressed in utils, as in the constrained-efficient allocation.

The first order conditions are

$$a_t F'(h_t(i)) - \frac{w_t(i)}{p_t} - \frac{\kappa_t}{U'(t)} = 0 \quad (**)$$

for hours $h_t(i)$ as well as

$$-\frac{1}{p_t} h_t(i) - \frac{1}{U'(t)} \beta \bar{\psi} E_t \lambda_{t+1} + \frac{\lambda_t}{U'(t)} + \frac{\kappa_t}{U'(t)} \eta (w_t(i)^{\eta-1}/w_t^\eta) h_t = 0 \quad (***)$$

for wages $w_t(i)$, where we have already used the Envelope condition $(\partial/\partial w_{t-1}(i)) \mathcal{P}_t(w_{t-1}(i)) = -\lambda_t \bar{\psi}$. We proceed by distinguishing the cases where downward wage rigidity is slack and binding, respectively, then by studying the symmetric equilibrium.

A.5.1 Downward wage rigidity is slack

Assume that constraint i) in the maximization is slack. In this case, it must be that downward wage rigidity, constraint ii), is binding. Namely, if not, it were always possible to choose the same $h_t(i)$ but a lower $w_t(i)$, which is feasible because constraint i) is slack, and which raises \mathcal{P}_t because production is the same, but the wage bill is reduced.³²

Therefore, once downward wage rigidity ii) is slack, it must be that constraint i) is binding. By using that $\psi_t = 0$ in (**) and that $\kappa_t > 0$, we can combine (**) and (***) to obtain

$$a_t F'(h_t(i)) = \frac{\eta + 1}{\eta} \frac{w_t(i)}{p_t} + \frac{1}{U'(t)} \frac{1}{\eta} \frac{w_t(i)}{h_t(i)} \beta \bar{\psi} E_t \lambda_{t+1},$$

where we have used that constraint i) is binding to rewrite (***)

³² And there would be a non-negative effect on the continuation value, because \mathcal{P}_t is weakly decreasing in the individual state $w_{t-1}(i)$.

A.5.2 Downward wage rigidity is binding

Turn now to the case where downward wage rigidity ii) binds, such that $\psi_t > 0$.

Here we have to make a case distinction which is identical as the one in the constrained-efficient equilibrium (see Section A.2 above). Namely, we have to distinguish the case where constraint i) is binding (as in the case where downward wage rigidity is slack), versus the case where constraint i) is slack. In the main text we have only discussed the second case, as this is the relevant case for the most part of the state space.

Intuitively, when the wage rigidity just about binds, by eating into their monopoly rents and reducing their precaution against downward wage rigidity, firms can still face $a_t F'(h_t(i)) > w_t(i)/p_t$ (that is, the marginal worker still adds to current profits):

$$\lambda_t < \frac{U'(t)}{p_t} h_t(i) + \beta \bar{\psi} E_t \lambda_{t+1},$$

where we have imposed $\kappa_t > 0$ in equation (***)³³.

Instead, if downward wage rigidity binds too strongly, $\lambda_t > 0$ becomes large enough such that the implied κ_t would turn negative. In this case, we set $\kappa_t = 0$ which yields for λ_t

$$\lambda_t = \frac{U'(t)}{p_t} h_t(i) + \beta \bar{\psi} E_t \lambda_{t+1}.$$

In turn, imposing $\kappa_t = 0$ in (**) we obtain the labor demand curve

$$a_t F'(h_t(i)) = \frac{w_t(i)}{p_t}.$$

To sum up, $w_t(i)$ is determined from downward wage rigidity ii), $h_t(i)$ is determined from labor demand (the previous equation), $\kappa_t = 0$ and the shadow value λ_t is determined from the second to last equation.

A.5.3 Symmetric equilibrium

We now impose that $w_t(i) = w_t$ and that $h_t(i) = h_t$ for all $i \in [0, 1]$.

In the region where downward wage rigidity is slack, this directly yields labor demand (22) from the main text. Moreover, if downward wage rigidity is slack, households will choose to sell hours according to (23).

Turn now to the case where downward wage rigidity binds. As shown above, in an intermediate region where downward wage rigidity binds only lightly, an equilibrium obtains with h_t determined by labor supply (23), and λ_t determined in

$$a_t F'(h_t) = \frac{\eta + 1}{\eta} \frac{w_t}{p_t} + \frac{1}{\eta} \frac{w_t}{h_t} [\beta \bar{\psi} E_t \lambda_{t+1} - \lambda_t],$$

³³ Thus, in this region, $w_t(i)$ is determined by downward wage rigidity ii), $h_t(i)$ is determined by constraint i), and κ_t and λ_t are determined residually from (**) and (***), and are both strictly positive.

which holds as long as $a_t F'(h_t) > w_t/p_t$. Intuitively, in this region firms eat into their monopoly rents and reduce their precaution against downward wage rigidity by shifting rightwards labor demand. They are willing to do so as long as $a_t F'(h_t) > w_t/p_t$, as in this case the marginal worker still adds to current profits.

Instead, if the implied $a_t F'(h_t) < w_t/p_t$, firms would stop hiring the additional workers. In this case, hours h_t are determined by labor demand

$$a_t F'(h_t) = w_t/p_t,$$

and labor supply is strictly rationed: $V'(h_t) < w_t/p_t$. In turn, the implied λ_t is determined as shown above:

$$\lambda_t = \frac{1}{p_t} h_t + \beta \bar{\psi} E_t \lambda_{t+1},$$

which is equation (25) in the main text.