Marriage and Divorce under Labor Market Uncertainty*

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January 15, 2018

Preliminary version

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Abstract

This paper studies match formation and dissolution in frictional marriage markets under labor market uncertainty. We propose a search model with transferable utility in which ex-ante heterogeneous men and women simultaneously search for partners in the marriage market and switch between employment and unemployment in the labor market. In the marriage market, individuals match assortatively on education and draw a match-specific shock component representing mutual affection. Divorces can happen due to both match-specific reasons and labor market transitions, e.g. job loss of one spouse. We structurally estimate our model using German micro data and decompose the observed flow of divorces into so-called "labor market divorces" and match-specific divorces. While more than 90% of divorces happen for match-specific reasons, the share of labor market divorces has increased significantly in Germany since the mid 2000s. Interestingly, most of this increase is driven by couples in which a previously unemployed woman starts working, more so for highly-educated women. At the same time, the share of labor market divorces triggered by unemployment has decreased significantly.

Keywords: Search, Matching, Sorting, Marriage, Divorce, Unemployment

JEL Classifications: J12, J31, J64, E24, E32

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*We thank Stéphane Bonhomme, Natalia Danzer, Matthias Doepke, Greg Kaplan, Helmut Rainer, Michèle Tertilt, as well as conference and seminar participants in Munich and Aarhus for inspirational discussions. This paper is part of the ifo Institute's research project "Economic Uncertainty and the Family" (EcUFam). We gratefully acknowledge financial support from the Leibniz Association. We also thank the ifo project "Calculation of returns on education in Germany" for helping with data access and Philipp Hochmuth and Patrick Schulze for excellent research assistance. The usual disclaimer applies.
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1 Introduction

Among the many choices individuals make during their lifetime marriage is one of the most, if not the most, important decision. The marriage vow to be true to each other in good times and in bad, in sickness and in health and to love and honor each other for as long as one shall live reminds both partners that this is truly a decision taken under uncertainty. Strokes of fate like unexpected unemployment and severe sickness can stress a partnership and cause partners to drift apart and divorce.

The relative importance of economic shocks compared to other shocks disrupting a marriage is still poorly understood. The economic literature has documented that unemployment, especially male unemployment, is associated with an increase in the divorce rate.[1] Also, we know that marriage and divorce rates are negatively correlated with the unemployment rate over the business cycle.[2] Additionally, we know that marriage rates declined since the 1970s and that assortative matching with respect to education has increased.[3] Researchers have proposed explanations based on improvements in household technology since World War II and increased female labor supply,[4] as well as increased incentives for females to invest in education.[5] Very little is known, however, about the nature of the channels that connect marriage market decisions to the underlying source of economic shocks.

To investigate the importance of economic shocks, we integrate labor market shocks into a two-sided marriage market model with transferable utility and ex-ante heterogeneous men and women (Shimer and Smith, 2000; Jacquemet and Robin, 2013; Goussé et al., 2017). Individuals search for partners in the marriage market and, at the same time, switch between employment and unemployment. The employment statuses of both partners influence utility flows and the sharing of resources within the household. A negative shock, i.e., job loss of one spouse, may decrease the marital surplus sufficiently to trigger a divorce. Additionally, an idiosyncratic component captures non-economic factors of marriage (e.g. mutual affection). It is subject to shocks and may lead to separations as well. A complementarity in the household production function induces the tendency to sort positively. Given the German context, we also consider benefits from joint taxation, which have the opposite effect and encourage negative sorting. In the model, the balance between all these forces determines marriage and divorce flows, differentially across heterogeneous men and women.

[1] See Jensen and Smith (1990), Hansen (2005), and Amato and Beattie (2011) among others.

[2] See Schaller (2013), González-Val and Marcén (2017a), and González-Val and Marcén (2017b) among others.

[3] Both Doepke and Tertilt (2016) and Greenwood et al. (2017) offer excellent literature overviews, the latter with some cross-country facts.

[4] See Greenwood et al. (2005a) and Greenwood et al. (2005b). More recently, Greenwood et al. (2016) use a search model to analyze these trends empirically for the U.S. with an emphasis on sorting.

[5] See Nick and Walsh (2007); Chiappori et al. (2009)

The relative importance of each of these forces is an open empirical question. We thus take our model to the data. Using German micro data from various sources, we use our model as a tool to decompose marriage and divorce flows into the respective contributions of economic and non-economic forces. To this end, we develop a structural estimation procedure that allows us to back out key components of our model from the data. We estimate meeting rates, marriage probabilities, and separation rates, all differentiated according to individuals' education and labor market status.

The marriage rate depends on an individual's chance to meet somebody from a certain education group times the probability he/she is willing to marry. We show that the probabilities to marry (willingness to marry) upon meeting is highest for employed individuals with equally educated partners. A similar positive assortative matching pattern emerges for medium and highly educated individuals in all other labor market status combinations (male employed/female unemployed, male unemployed/female employed, and male unemployed/female unemployed). Low educated females still have reasonably high chances to marry with a medium or highly educated male if they stay out of the labor force (remain unemployed), most likely because of the high financial incentives provided by joint income taxation in Germany. Low educated, unemployed males have almost no chances to marry. Marriage rates are also driven by an individual's chance to meet somebody from a certain education group. Our estimates suggest that medium and highly educated individuals direct their search such that the number of meetings with individuals from the other sex with similar education level are higher than the number of meetings with lower educated individuals. Conversely, our estimates tend to suggest random meetings for low educated individuals.

We finally decompose the number of divorces into economic (labor market) factors and non-economic factors and show how their contributions evolve over time. The overall majority of divorces is driven by non-economic factors. Overall, less than 10% of divorces are due to labor market transitions of one spouse. However, the share of "labor market divorces" exhibits very interesting dynamics, it has increased by more than 20% since the mid 2000s. We take a granular view and investigate which types of heterogeneous couples have started to divorce more frequently in response to labor market transitions. Surprisingly, we find that positively sorted couples are the major contributor to this trend. In our sample, the largest and growing share of labor market divorces can be attributed to couples in which a previously unemployed highly educated female starts working. On the other hand, low education couples with a high likelihood of job loss contribute a shrinking number of labor market divorces. Both trends might be related to the booming German labor market. Low separation rates make marriages among low education individuals more stable. With high education, the option value of going on the marriage market with good employment perspectives can outweigh the value of staying married.

Our paper is organized as follows. Section 2 introduces our data sources. Section 3 discusses descriptive empirical evidence relevant to our hypothesis and modeling choices. In Section 4, we introduce our marriage market model and solve it. Section 5 presents a model-based, structural decomposition of marriage and divorce flows into their sources. Section 6 concludes and further results as well as technical details can be found in the Appendix.

2 Data Sources

The empirical content of this paper is based on three sources of German micro data:

1. The German Microcensus, a household survey.

2. The Sample of Integrated Labor Market Biographies, a matched employer-employee data set.

3. The German marriage and divorce registers.

We briefly introduce and describe each data source before presenting and discussing the respective empirical results relevant to our analysis.

2.1 The German Microcensus (MC)

The German Microcensus (henceforth MC) is an annual survey that yields representative statistics on the German population and labor force. Data access is provided by the Research data center (FDZ) of the statistical offices of the German federal states.

It samples 1% of the population, consisting of all persons legally residing in Germany. It is the largest household survey in Europe. Participation is mandatory[6] and only a subset of questions can be answered on a voluntary basis.

The MC survey design relies on single-stage stratified cluster sampling. The primary sampling units are artificially delimited districts with a number of neighboring buildings. All households residing in these buildings are interviewed (principal residence).[7] Typically, one household member responds to the survey for all individuals living in the household, including the spouse, children, and other cohabitants if applicable. The survey program of the MC consists of a set of core questions that remains the same in each wave, covering general socio-demographic characteristics like marital status, education, employment status, individual and household income, and many other things.

[6] According to the German Microcensus law, non-response may be fined.

[7] Since 1990 the average number of buildings has been 9, the targeted number of individuals is 15. Larger buildings are subdivided.

Data Preparation

We restrict our attention to adults of ages 18 to 68 living in private households, either as singles (alone or with cohabitants) or as heterosexual married partners in the same household. Our definition of singles includes never-married, divorced, and widowed individuals. To reliably identify couples, we have to condition on legal marriage, since we cannot distinguish cohabitation of non-married couples from shared apartments in the earlier MC waves. Married couples are legally required to have the same principal residence, if they want to file a joint tax statement in order to enjoy the benefits of joint income taxation.

In principle we could use all MC waves from 1976 to 2013 for our analysis.[8] We carefully clean and properly weight the cross-sectional data sets in order to represent the German population. This enables us to study the composition of the German married and non-married population conditional on gender, education, and employment over time.

For our analysis we use the data starting from 1993. With this short sample we avoid complications related to the German reunification and, in turn, can analyze the German population as a whole. Another reason is that the SIAB data (see below) does not fully cover the East German labor market before 1993.

Unfortunately, the MC is not a panel. In contrast to Goussé et al. (2017), who use the British Household Panel Survey (BHPS), we cannot follow individuals over time and directly observe them switching between states of singlehood and marriage as well as employment and unemployment. This complicates connecting the model to German data. To tackle this issue, we categorize the individuals in each cross-section into 84 classes based on gender (male or female), education (low, medium, high), employment status (employed or unemployed), marital status and, if married, the education and employment status of the partner. We use this aggregated data to study the German marriage market.

Our theoretical sorting model is based on the presumption that the value of the spouse's labor in home and market production (labor productivity) is an important determinant of matching and separation decisions in the marriage market (in addition to non-economic forces like love and companionship). The empirical part of this paper uses education and wages as a proxy for labor productivity. The MC includes detailed information on individuals' school education and vocational degrees. The way this information is collected in the survey varies across waves. To construct a time-consistent measure, we rely on the ISCED-1997 scale[9] and, accordingly, define three education categories:

[8]The MC has been conducted in West Germany since 1957 and in East Germany since 1991. The waves before 1976 contain no information on individual education. Before 1995 we have one wave every two or three years (1976, 1978, 1980, 1982, 1985, 1987, 1989, 1991, 1993). This is due to the fact that the MC was not always a yearly survey and, once it was, not all waves asked for education information. From 1995 onwards we have all waves at an annual frequency.

[9]The International Standard Classification of Education (ISCED) of the UNESCO intends to make

1. Low education: individuals, who at most graduated from lower secondary schools with or without a vocational degree (ISCED categories 1 & 2).

2. Medium education: individuals, who graduated from upper secondary schools with or without a vocational degree (ISCED categories 3 & 4).

3. High education: individuals with a tertiary degree (ISCED categories 5 & 6).

The second important dimension of individual heterogeneity in our analysis is the labor market status. Some details about job search behavior in the labor market are available in our data, but only for a subsample in the later years. In order to ensure that employment and unemployment are defined consistently over the whole time horizon, we pool unemployment and non-participation and do not subdivide the non-employed into job-seekers and inactive persons.

Our final MC data set (1993 - 2013) contains information on 8,426,756 individuals[10] of whom 47% are men and 53% women. 72% of men and 64% of women are married. Across all ages in our sample, from 18 to 68 years, the labor force participation rate is 62% for men and 46% for women, respectively.[11] In the period after German reunification, the individuals in our sample are representative of a roughly constant population of about 53 million adults.[12]

2.2 Sample of Integrated Labor Market Biographies (SIAB)

To construct labor market transition rates and wage measures, we rely on German matched employer-employee data. We use the Sample of Integrated Labor Market Biographies (henceforth SIAB) provided by the Institute for Employment Research (IAB) in Nuremberg, Germany.[13] These data cover the years 1975 to 2014. The SIAB is a 2 percent random sample drawn from the universe of employment and unemployment spells registered at the Federal Employment Agency (Bundesagentur für Arbeit) within the German social security system.[14] Individuals who are not subject to social insur-

education systems internationally comparable.

[10] The average number of observations per wave is 443,513.

[11] The participation-age profiles are hump-shaped. In the 2006 MC wave, participation for men is highest in the age bracket 35-39 (88%) and the maximum for women (77%) is reached for ages 40-44.

[12] We use the MC sample weights to scale our sample. The population increases somewhat after reunification and reaches a maximum of almost 55 million people in 2007, afterward it starts declining. The mean population between 1993 and 2013 is about 53 million people with a standard deviation of 1 million.

[13] We use the factually anonymous Sample of Integrated Labor Market Biographies (File: `SIAB_7514`). Data access is provided via a Scientific Use File supplied by the Research Data Center (FDZ) of the German Federal Employment Agency (BA) at the IAB, project no. 101693. See also Ganzer et al. (2016) for more details on the data set.

[14] The data consist of individuals which are characterized as follows: employed subject to social security, marginal part-time employed, unemployed benefit recipient, officially registered job-seeker, (planned) participant in programs of active labor market policy.

ance contributions i.e. self-employed workers, civil servants, students, and non-employed persons are not included in the sample.

Every observation in the SIAB corresponds to an employment or unemployment spell lasting between one day and one year in accordance with the reporting rules of the German social security system. This allows us to measure the employment status of an individual exact to the day. We observe individuals switching between different employers, employment and unemployment, as well as (with severe limitations) non-employment. For every employment spell we observe the nominal gross daily wage. In case of unemployment, the wage variable contains the amount of benefits paid to the worker. Since the SIAB is a sample of the labor force, we do not have the number of individuals not participating in the labor force. The SIAB is simply not representative for this part of the population. We are therefore unable to calculate transition rates out of inactivity and only use the transition rates from employment into unemployment and vice versa.[15]

The SIAB data contain a wide array of individual characteristics including gender, age, educational attainment, details on they type of employment (part/full-time, marginal/subject to social security) as well as occupation and some information on the employer. Unfortunately, only unemployment spells contain the information whether an individual is married or not. Since this is a non-representative fraction of the data, we cannot condition on marital status when estimating wage measures and labor market flows. The German social security data are collected at the individual level so they contain no information on the spouse that we could condition our estimations on.[16]

Data Preparation

In order to create a sample comparable to the MC data, we use data for both men and women in East and West Germany. The available age range in our SIAB sample, 17 to 62 years, is slightly narrower than in the MC. We conduct our estimations with the data from 1993 onwards, the same start year as in our primary MC sample. As mentioned before, the East German labor market was not completely covered by the institutional data sources before 1993. We drop all spells of marginal employment because they are not included in the data before 1999. In case an individual has multiple jobs at a given point in time we always define the highest paying job as the primary one and discard all other employment spells.

[15]Since we are interested in transition rates between labor market states, we have to divide the number of individuals changing the labor market status by the stock of individuals in the state from which the respective individuals exited.

[16]This limitation will be mitigated in a future version of the IAB data. Goldschmidt et al. (2017) develop a method for identifying married couples in the German matched employer-employee data by using confidential address and name data. The resulting couple identifier will be made available to other researchers and we will incorporate it in a future version of this paper. For now, we are forced to estimate wage distributions and labor market transitions without controlling for marital status and the partner's labor market attachment.

We use the SIAB education information to construct a variable that resembles the three ISCED categories in the MC data. Education information in the SIAB suffers from inconsistencies and missing values.[17] To solve this problem, we follow Fitzenberger et al. (2006) and impute missing and inconsistent values in the education variable. Spells with missing education after imputation are dropped.

Regarding wages, we start by deflating nominal gross daily wages using the German consumer price index with base year 2010. The German social security system tracks earnings only up to a certain limit. Beyond this threshold, further earnings are not taken into account for the calculation of social security contributions. We follow Dustmann et al. (2009) and impute the upper tail of the wage distribution by running a series of Tobit regressions, fitted separately for years, education levels, and age groups.

After data preparation, our SIAB sample consists of 18,623,471 employment spells from 968,215 individuals, 58% of which are male. The male share of all employment spells is similar (57%).

2.3 Marriage and Divorce Registers (MDR)

The marriage and divorce register data (henceforth MDR) originates from the German civil registry offices and the divorce courts. It is compiled by the Research Data Center of the statistical offices of the German federal states. The data are organized at the level of the married couple and contain information on the exact birth dates of both spouses, the exact date of marriage, and, if applicable, the date of divorce. Additionally, the data contain various covariates including religion, citizenship, place of residence, number of children (before marriage and at the time of divorce), as well as who filed for divorce and the court's ruling. Unfortunately, there is no information about education, so we have to rely on age differences as a proxy for marital sorting.

Data Preparation

The marriage and divorce data are separate yearly files and we have access for the waves from 1991 - 2013 (marriage registers) and 1995 - 2013 (divorce registers). We clean the yearly files from missings and inconsistent observations. The waves of marriage and divorce data are then merged to get two big data sets. For one, we are interested in the aggregate yearly flows of marriages and divorces. We need these numbers for the structural decomposition of marriage and divorce flows in Section 5. We then proceed to link the two register data sets in order to estimate a series of hazard models.[18] 21.3% of

[17]Employers are not forced to report an employee's highest educational degree and might in some cases not even know about it, for instance when a worker switches occupations.

[18]Due to the strict German data protection regulations it is not allowed to match the marriage and divorce observations at the level of the individual couple. We aggregate the marriage and divorce data to yearly cells containing the number of individuals with equal observable characteristics, particularly age.

the 17,166,070 marriages we observe ended in divorce. The rest survived until the end of our observation period. This data set enables us to study marriage duration conditional on the observable characteristics available to us, in particular age.

3 Empirical Results

3.1 Sorting in the Marriage Market

The aim of our analysis is to connect an equilibrium search model of the marriage market with heterogeneous men and women to German micro data. Our theoretical model allows for positive as well as negative assortative matching. A complementarity in the household production function induces homophily—the love of the same—and encourages positive assortative matching. Benefits from joint income taxation, which increase with the wage gap between spouses, encourages negative assortative matching. We use our MC and MDR data to document the extent of assortative matching on the German marriage market. Our model will have to match the empirical patterns we find.

Results from MC data

In this section we study how the homogeneity of married couples in terms of education, employment, and income has evolved over time. Overall, it has increased significantly. The correlations are depicted in Figure 1. In 1993, the correlation between spouses' education levels was just above 0.4. For comparison, this is somewhat lower in magnitude than what Greenwood et al. (2016) report for the U.S.[19] However, they have only two education categories, college and less than college, so the correlation should be somewhat higher than what we find with three education categories. The education correlation increases and reaches a maximum of 0.5 in 1999. Afterwards, it levels off and starts decreasing slightly in the second half of the 2000s. It remains well above its initial level, however. The leveling-off could be driven by supply factors, e.g. a limited number of highly educated individuals looking for a partner.[20] Also, the changing macroeconomic environment in Germany, especially the booming labor market, might have led to a change in matching patterns. We will pick up this thought in our structural empirical analysis in Section 5.

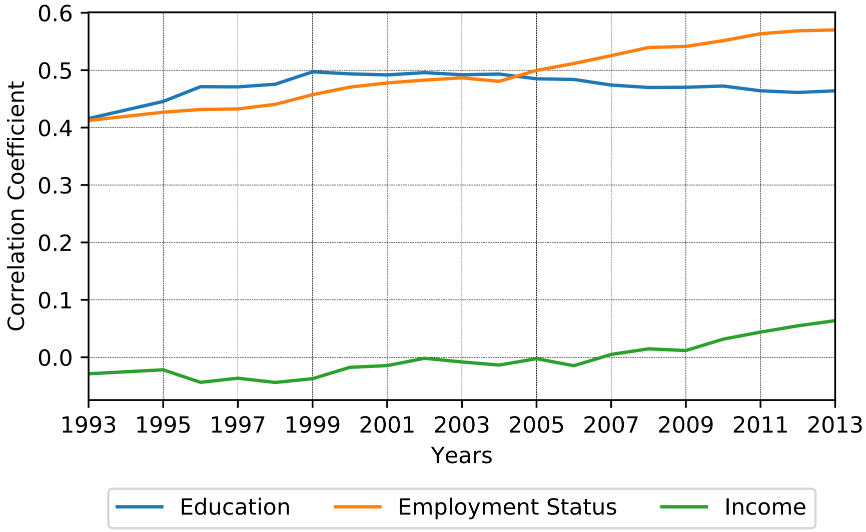
We do not observe a similar hump-shape for employment or income. The correlation between spouses labor market attachment (employed or unemployed) has increased

We then merge the cells based on the marriage year and "unpack" the linked data-set into individual marriage spells.

[19]The long-run analysis of Greenwood et al. (2016) reveals a correlation of 0.41 in 1960 which rises to 0.52 in 2006.

[20]Figure A.1 in the appendix shows the shares of education groups for men and women and their evolution over time.

Figure 1: Correlations of education, employment, and income within marriages



Note: Yearly Pearson correlation coefficients of the within-couple levels of education (three categories: low, medium, high), employment (two categories: employed, unemployed) and income (between 15 and 24 categories, depending on the wave). Source: Research Data Center of the Statistical Offices of the Länder and the Federal State, Microcensus, 1993-2013, own calculations.

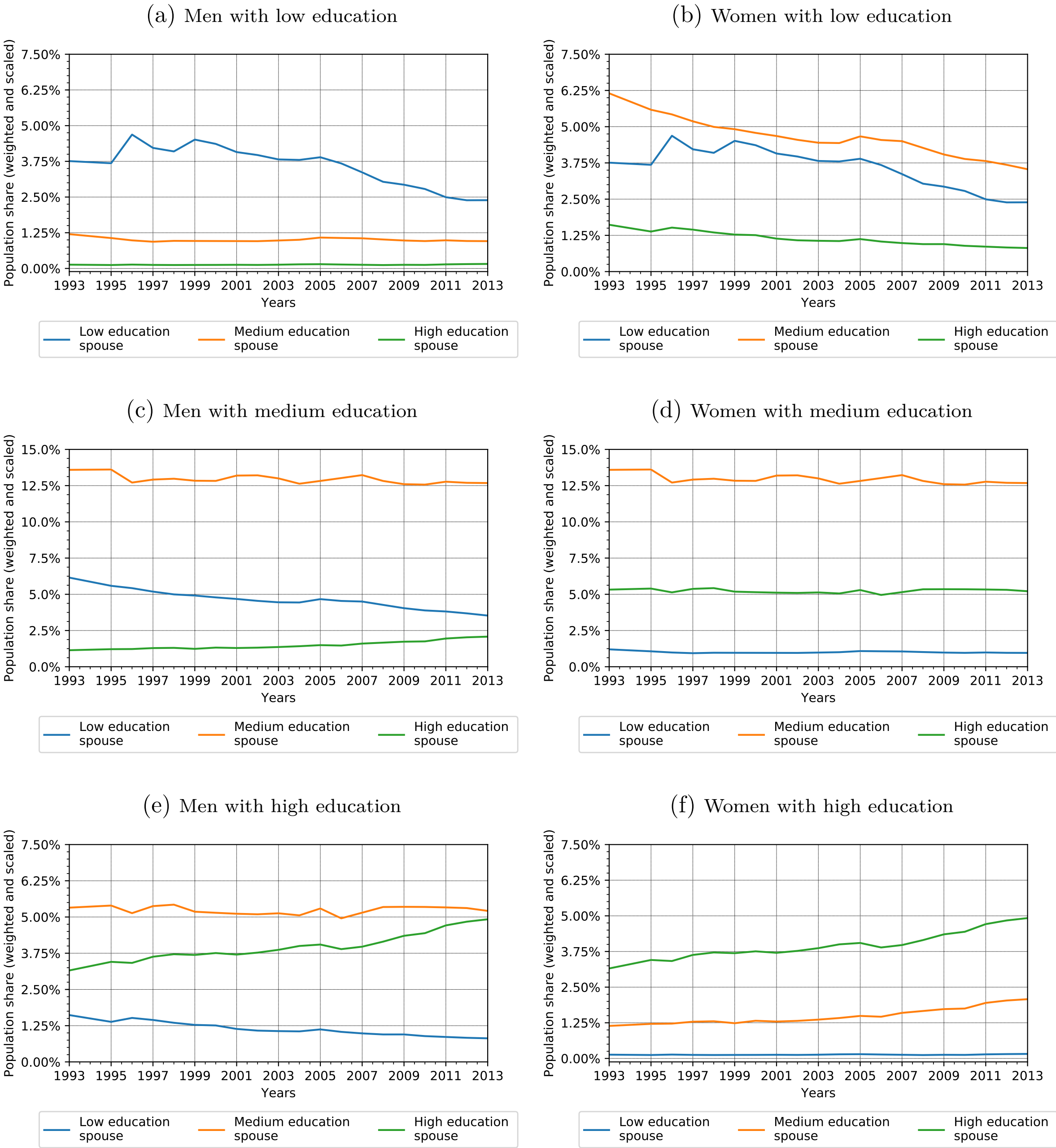
steadily from just above 0.4 in 1993 to almost 0.6 in 2013. Today, it is more common that partners are either both employed or unemployed. In stark contrast to this observation, it is striking to see that the correlation between spouses' income levels has been negative until well into the 2000s. It turns positive in 2007 and increases further after 2009. This finding shows that the classical role model with one bread-winning individual, typically the husband, is still very dominant in Germany, even despite the high degree of education-based sorting. When both spouses are employed, however, earnings differences must be large in order to be consistent with an almost zero correlation between spouses' income. This is due to the fact that many working wives have a weak labor market attachment, work part-time or in marginal employment. One explanation for this observation is the German system of joint taxation for married couples, which provides strong disincentives for the secondary earner to increase labor supply.[21]

Figure 2 shows the extent of marriage market sorting in Germany in an alternative way. We now focus on education-based sorting only and use our MC aggregated data to calculate weighted population shares of men and women in partnerships of all possible education combinations. To set this into perspective, the population shares of all individuals by gender and education are depicted in Appendix Figure A.1. For both men and women, the share of highly educated individuals has increased and the low education share has decreased. This trend is much more pronounced for women.

Each row in Figure 2 shows men (left Panel) and women (right Panel) for the same

[21]See Gustafsson (1992) for a comparative empirical study of joint taxation and female labor supply in Germany and Sweden. The author exploits the Swedish switch to individual taxation in 1971 to identify the dampening effect of the high marginal tax rates on female labor supply under joint taxation.

Figure 2: Partner's education by education and gender of married individuals



Note: Population shares are weighted and scaled using the MC sample weights. 100% on the y-axis corresponds to the full population, including married and unmarried individuals of both sexes. Source: Research Data Center of the Statistical Offices of the Länder and the Federal State, Microcensus, 1993-2013, own calculations.

education category. For these individuals, we plot the share of marriages with partners of each education category.[22] We see an increase of education-based sorting particularly for highly educated individuals. For men with university degrees (Panel 2e), the share of marriages with highly educated women has almost doubled to 5% in 2013. At the same time, the share of marriages with women of the lowest education category has

[22]Note that the lines representing marriages with partners of the same education level are identical by construction for men and women in each row (blue in the first, orange in the second, green in the third).

decreased significantly. The widening gap between the green and the blue line represents increasing homophily of highly-educated individuals. From the perspective of highly educated females (Panel 2f), the share of marriages with men of both high and medium education has increased. The medium education share starts increasing later and does not grow beyond 2%. Highly-educated women are almost never married to men from the lowest education group.

The increase of education-based sorting is not homogeneous across education groups. The first row of Figure 2 shows that the share of marriages among lowly educated individuals only increased somewhat in the beginning of our observation period but steadily decreased thereafter. The decline in the share of married low educated women across all groups of men is driven by the overall decrease in marriages of low educated women over the same time horizon. The same is true for men, albeit to a lesser extent. To show the increasing shares of singles, Figure A.2 in the Appendix breaks down the population shares of the gender-education groups by marital status. The share of marriages between men and women of medium education (Panels A.3c and A.3d) is the highest in our sample with more than 12.5% and it is relatively stable over time. Sorting in this group is prevalent but not increasing. For the women, the share is constant across all groups. Men of medium education, however, become less likely to be married to women with low education and more likely to be married to highly educated women.

Results from MDR Data

We run a series of survival regressions on our linked marriage and divorce data in order to understand how the probability of divorce depends on the age difference in the married couple. Ideally, we would like to control for education. Since we do not have this information available in the MDR data, we use the age difference to proxy for the heterogeneity within married couples. The correlation of the differences in education and age for married couples in the MC data is 0.66, indicating that spouses with small age differences also tend to be homogeneous in terms of education.[23] To control for age differences, we classify couples as follows: the baseline group has an age difference of less than two years. Further, we group couples with more than two and up to five years, more than five and up to ten, more than ten and up to fifteen, and more than fifteen years of age difference. We estimate a semi-parametric Cox proportional hazard model as well as two parametric regressions, assuming Weibull and exponential distributions.

The estimated hazard ratios in Table 1 are precisely estimated and very close across specifications. Unanimously, we find that the hazard ratio is increasing in the age difference relative to the base category of less than two years. Couples with an age difference between two and five years have a 7.4% higher hazard rate compared to the baseline. For

[23]This is calculated for couples with a maximum age difference of 11 years, representing more than 95% of couples in our data.

Table 1: Hazard ratio estimation results

Age difference	(1) Cox	(2) Weibull	(3) Exponential
2-5 years	1.075***	1.079***	1.074***
	(0.014)	(0.001)	(0.001)
5-10 years	1.197***	1.214***	1.193***
	(0.002)	(0.002)	(0.002)
10-15 years	1.298***	1.320***	1.291***
	(0.002)	(0.003)	(0.003)
>15 years	1.353***	1.370***	1.347***
	(0.004)	(0.004)	(0.004)
constant		0.0078***	0.0191***
		(0.000)	(0.000)
s (shape)		1.345***	
		(0.000)	
N	17166070	17166070	17166070

Note: Robust standard errors in parentheses. Hazard ratio relative to age difference < 2 years. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. s is the estimated shape parameter of the respective parametric distribution. Source: Research Data Center of the Statistical Offices of the Länder and the Federal State, Marriage and Divorce Registers, 1991/1995-2013, own calculations.

the remaining age-difference groups the hazard ratio relative to the baseline increases by 21.4%, 32%, and finally 37% for couples with the largest age differences of more than 15 years. We interpret these hazard ratios as strong evidence that—on average—the likelihood of a quick divorce increases when couples are very different in terms of age. Put differently, homogeneous couples have a higher probability of staying together for a long time.

Summary

We find an abundance of evidence for positive assortative matching on age, education, and employment status on the German marriage market. Only income was – probably due to the incentives caused by joint income taxation – negatively correlated before 2007 and became positively correlated after 2009. Still, homophily is prevalent and increasing. Particularly for education, however, we see that the tendency to sort is not uniform across groups. Sorting is increasing for the highly educated, rather constant for the middle group and even decreasing for men and women with low education. Additionally, increased sorting appears not just to be about matching with the right partner in the first place. The fact that hazard ratios increase in the age difference indicates that heterogeneous (i.e. non-sorted) couples, at least in terms of age, are on average more likely to divorce.

In the light of these empirical findings, the question remains *why* heterogeneous couples divorce more quickly. We suspect that the reasons for splitting up, economic and non-economic, must differ across couple types. This is where our structural model comes into play. It allows us to investigate this hypothesis by decomposing the flow of divorces into separations caused by idiosyncratic shocks and by economic reasons like a transition from employment to unemployment of one spouse. For this reason, we now turn to the labor market.

3.2 Wage Distributions

Following Goussé et al. (2017), we interpret individual labor productivity as the empirical counterpart of the heterogeneity of men and women in the model. We construct our measure of labor productivity from wage information in the SIAB data. We estimate the wage densities of men and women on the same domain in order to use them as the underlying type distributions when solving the structural model. In order to remove transitory components from wages and use both observable and unobservable determinants of individual labor productivity, we run a Mincerian wage regression including a person-fixed effect. Following Card et al. (2013), we regress log wages on a person-fixed effect, an unrestricted set of year dummies, and quadratic and cubic terms in age fully interacted with educational attainment:

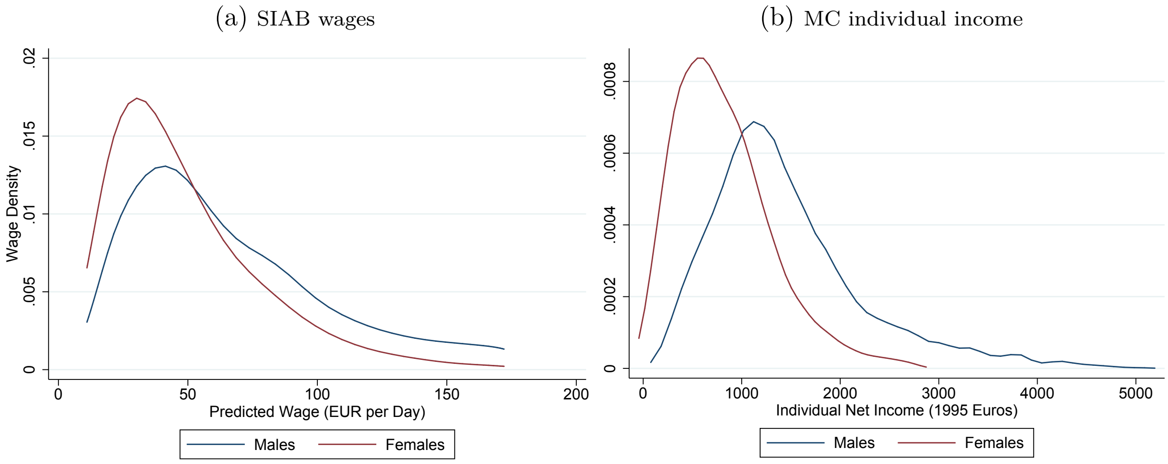
$$\ln w_{it} = x'_{it}\gamma + \phi_i + r_{it}. \tag{1}$$

$\ln w_{it}$ denotes the log real daily wage of a worker i in year t, x'_{it} includes the time-varying observable characteristics, ϕ_i is a worker-fixed effect, and r_{it} is the residual . The explanatory power of this wage regression (adjusted R^2 of 72%) is high, albeit below the Card et al. (2013) benchmark (about 90%). There are two reasons for this difference: we include men and women from both East and West Germany, whereas Card et al. (2013) focus on men in West Germany in a smaller age bracket. Additionally, using the universe of social security records, they can include firm-fixed effects. We are unable to consistently estimate firm-fixed effects using the SIAB sample.[24] Wage differences across firms, however, are not the subject of our study.

Based on the estimated contributions of both observable and unobservable character-

[24]See Andrews et al. (2008, 2012) on this topic.

Figure 3: Wage Distributions of Men and Women



Note: Kernel densities of wages based on SIAB data (left Panel, kernel: Epanechnikov, bandwidth: 5) and individual income based on MC data (right Panel, kernel: Gaussian, bandwidth: 100). MC data source: Research Data Center of the Statistical Offices of the Länder and the Federal State, Microcensus, 1993-2013, own calculations. SIAB source: Research Data Centre (FDZ) of the Federal Employment Agency at the Institute for Employment Research, SIAB SUF 7514, 1993-2013, own calculations.

istics we predict individual wages as follows:

$$\hat{w}_{it} = \exp(x'_{it}\hat{\gamma} + \hat{\phi}_i). \tag{2}$$

We effectively remove the estimated residuals. The standard deviations of predicted wages is 0.615 for men and 0.611 for women, so male wages are somewhat more dispersed.

Next, we run kernel density estimations for both men and women using all wage observations between the 1st and the 99th percentile. Figure 3 depicts the resulting distributions of the predicted wage for men and women on a common domain. We compare the densities estimated from SIAB data to kernel density estimations of individual income from MC data (for both married and single individuals). The distributions share the same qualitative features, even though the wage and income information in the two data sets are very different.[25] Nevertheless, both male wage distributions have a fatter tail, male mean and median wages/income (see blue lines) lie to the right of the female ones (see red lines).

3.3 Labor Market Transitions

Our final set of empirical results concerns the job-finding and separation rates of men and women conditional on education. In our structural model, these rates are an important element because we suspect that labor market transitions trigger a sizable share of divorces.

[25] The MC contains only categorical information on individual and household income, which we make comparable across waves.

Table 2: Labor market transition rates (%) by gender and education

Gender	Education	Job-Finding rate (UE)		Separation rate (EU)	
		Mean	STD	Mean	STD
All	-	4.579	0.158	0.639	0.083
Men	All	5.160	0.126	0.711	0.094
	Low	5.122	0.128	0.759	0.102
	Medium	5.689	0.328	0.453	0.033
	High	4.366	0.960	0.312	0.029
Women	All	3.880	0.260	0.552	0.067
	Low	3.654	0.272	0.565	0.069
	Medium	6.043	0.367	0.445	0.034
	High	5.773	1.019	0.450	0.048

Note: Mean and standard deviations of seasonally adjusted (X-13-ARIMA-SEATS) and HP-filtered ($\lambda = 900,000$) job-finding and separation rates by gender and education category. Rounded to three decimal places. SIAB source: Research Data Centre (FDZ) of the Federal Employment Agency at the Institute for Employment Research, SIAB SUF 7514, 1993-2013, own calculations.

We estimate monthly transition rates between different labor market states following Jung and Kuhn (2014), additionally conditioning on gender and education. After data cleaning and wage/education imputations (as described above), we subdivide the spells in our data into periods of employment, unemployment, and inactivity. These data are then transformed into monthly slices. To get the transition rates, one simply has to count, for instance, how many individuals are employed in a given month were unemployed in the previous month and then divide by the overall number of unemployed from the previous month. We compute the transition rates for unemployment to employment (UE) and employment to unemployment (EU) for the period of 1991-2014 and trim the series to 1993-2013. For one, to have a time frame similar to the MC data. Also, this allows for a burn-in of 24 months when computing flows. The resulting time series of labor market flows are highly seasonal and exhibit cyclical patterns. As we are seeking to connect an equilibrium search model to the data, we are (for now) not interested in the cyclical properties. We first apply the X-13-ARIMA-SEATS seasonal adjustment routine of the U.S. Census Bureau.[26] Afterwards, we use a Hodrick-Prescott filter with a penalty parameter of $\lambda = 900,000$ to remove the cyclical component from our monthly data. The statistics reported in the following are computed for the seasonally adjusted and filtered time series. For our analysis we take the yearly average of the monthly transition rates in order to make the data compatible with the MC-data.

Table 2 presents the means of the transition rate over the years 1993-2013. In the average month, 4.6% of unemployed workers find a job. The average job-finding rate is higher for men (5.2%) and lower for women (3.9%). There are sizable differences across education groups. Whereas low education women have the hardest time finding a job overall, medium and highly-educated women have higher job-finding rates than the corresponding men. Regarding separations, we note that on average men are more likely to separate in a given month (0.7%) than women (0.5%). This is not true, however, for all education groups. For both sexes, the overall level of the separation rate is mainly driven by low education individuals, who are still a sizable share of the German labor force (considering all age groups). Separation rates of better educated workers are lower and almost similar for men and women with medium education. Interestingly, however, highly-educated women have a 44% higher separation rate than similar men.

The monthly time series of job-finding and separation rates for men and women in all education groups are shown in Appendix Figure A.3. One insight from the transition rates' changes over time is worth keeping in mind for our structural empirical analysis in Section 5: women with high and medium education have the highest job-finding rates overall and they have increased significantly since the first half of the 2000s.

4 The Model

We extend the frictional marriage and divorce model by Goussé et al. (2017), which itself is based on Shimer and Smith (2000) and Jacquemet and Robin (2013), by incorporating that single and married men and women change their labor market status l. For simplicity and compatibility with our data sources, we consider only employment (indexed by e) and voluntary or involuntary unemployment (indexed by u), i.e., $l \in \{e, u\}$. Besides the time varying labor market status, individuals differ in their level of education and the associated wages (or wage distributions). We capture the time-invariant heterogeneity of men and women by the indices i for men and j for women. We will be more specific when the model is taken to the data.

[26] We use the R package "seasonal" developed by Sax (2017).

4.1 Preferences and home production

Individual utility depends on private consumption c, leisure h, and the public good q. The public good of a married couple depends on the time inputs of the spouses (d_i, d_j). We assume that these inputs are complements. The public good also depends on the types ij and on an idiosyncratic bliss shock z drawn from the cumulative probability distribution G, i.e., $q = zF^1_{ij}(d_i, d_j)$. A bliss shock arrives at the type specific rate δ_{ij}. The public good of a single is solely a function of the time input d_i, i.e., $q = F^0_i(d_i)$. Employment and unemployment enter the production of the public good indirectly by changing the time available for the production of the public good. The total time available for an individual is given by $T > 1$. We assume fixed working hours normalized to 1 for an employed worker. Thus, the time remaining for leisure h and household production d is given by,

$$d_i + h_i = T^l_i = \begin{cases} T - 1 & \text{for } l = e, \\ T & \text{for } l = u. \end{cases}$$

Working individuals will receive a wage w_i depending on their type. Unemployed individuals receive a replacement income bw_i if they are unemployed. Since working time cannot be adjusted at the intensive margin, private consumption of a single is given by

$$c_i = R^l_i = \begin{cases} w_i & \text{for } l = e, \\ bw_i & \text{for } l = u. \end{cases}$$

Following Goussé et al. (2017) we assume that households are subject to living cost C_{ij}, which is a function of the exogenous types ij. This fixed living cost is paid by the spouses through transfers, i.e., $t_i + t_j = C_{ij}$, which are determined by Nash-Bargaining. The respective private consumption of a spouse is given by

$$c_i = R^l_i - t_i = \begin{cases} w_i - t_i & \text{for } l = e, \\ bw_i - t_i & \text{for } l = u. \end{cases}$$

The flow utility of a single individual depends on the public good $F^0_i(d_i)$, consumption (equal income) $c_i = R^l_i$, and leisure $h_i = T^l_i - d_i$, i.e.,

$$u^l_i(d_i) = F^0_i(d_i)\left[R^l_i + T^l_i - d_i\right]. \tag{3}$$

The bliss variable z for singles is normalized to unity. The flow utility of a married individual depend on her own labor market status $l \in \{e, u\}$ and the labor market status of the partner $-l \in \{e,u\}$. It is assumed to have the following form,

$$u_i^{l,-l}(t_i,d_i,d_j|z) = zF_{ij}^1(d_i,d_j)\left[R_i^l + \frac{\iota}{2}\left(R_j^{-l}-R_i^l\right)^2 - t_i + T_i^l - d_i\right]. \tag{4}$$

The term $\frac{\iota}{2}\left(R_j^{-l}-R_i^l\right)^2$ takes into account the net income gain from joint taxation of couples. The utility depends on the time (d_i,d_j) devoted to public good production, the contribution t_i to the fixed cost of living, and the own and the partners labor market status l and $-l$ (via R_i^l, R_j^{-l} and T_i^l). Given the labor market status the time inputs to public good production $\{d_i,d_j\}$ are chosen to maximize joint surplus of the match and the transfers $\{t_i,t_j\}$ to ensure that each individual gets its respective fraction of the surplus.

4.2 Marriage formation and renegotiations

The present values of a marriage for a female (and the male respectively) depend on her own labor market status $l \in \{e,u\}$ and the labor market status of the partner $-l \in \{e,u\}$. We denote the flow utility of the married female for the optimal choices of $\{d_i,d_j,t_i,t_j\}$ by $u_j^{l,-l}$, where $(l,-l) \in \{(u,u),(u,e),(e,u),(e,e)\}$. The following Bellman equation,

$$\begin{aligned} rV_j^{l,-l} &= u_j^{l,-l} + \delta_{ij}\int\left[\max\left[V_j^l, V_j^{l,-l}(z')\right] - V_j^{l,-l}\right]dG(z') \\ &\quad + \tau_j(l)\left[\max\left[V_j^{l'}, V_j^{l',-l}\right] - V_j^{l,-l}\right] + \tau_i(-l)\left[\max\left[V_j^l, V_j^{l,-l'}\right] - V_j^{l,-l}\right], \end{aligned} \tag{5}$$

describes the present values of marriage. $\tau_j(l)$ denotes the exogenous transition rate from the current labor market status $l \in \{e,u\}$ into the labor market status $l' \neq l$ for an individual of type j. The last term in the Bellman equations describes the labor market transition of the partner of type i. Individuals do not make long-run commitments. If a labor market transition occurs or if a bliss shock occurs, both partners renegotiate their contributions (d_i,d_j) to the public good and the transfers (t_i,t_j) to finance the fixed living costs C_{ij}. If the outside option is higher than the surplus from remaining married, then the couple divorces. In the renegotiations $\{d_i,d_j,t_i,t_j\}$ are chosen such that the Nash-Product,

$$\left[V_j^{l,-l}-V_j^l\right]^{1-\beta}\left[V_i^{l,-l}-V_i^l\right]^{\beta}, \tag{6}$$

is maximized subject to the feasibility constraint $t_i + t_j = C_{ij}$ and the participation constraint,

$$V_j^{l,-l}-V_j^l \geq 0, \text{ and } V_i^{l,-l}-V_i^l \geq 0,$$

where V_i^l (V_j^l) is the outside option of the single male (female) individual. The present value of being a single female satisfies the Bellman equation,

$$rV_j^l = u_j^l + \lambda_{ij}\iiint\left[V_j^{l,-l'}(z') - V_j^l\right]W_{ij}^{ll}(z')\,dG(z')\,s(i,l')\,didl' + \tau_j(l)\left[V_j^{l'}-V_j^l\right]. \tag{7}$$

The maximized flow utility of a single is denoted by $u_j^l = \max_{d_i} u_i^l(d_i)$. λ_{ij} denotes the type specific meeting rate of a potential partner. A meeting only results in a marriage if the joint surplus is positive. The respective willingness to marry (or stay in the marriage) is denoted by the index $W_{ij}^{ll}(z)$. If a pair is willing to marry (stay together) then $W_{ij}^{ll}(z) = 1$, and zero otherwise. The willingness to marry depends on the types and the labor market status (the first l corresponds to the male's labor market status, the second to the female's) as well as the bliss shock z. We denote by α_{ij}^{ll} the probability that $W_{ij}^{ll}(z) = 1$.

The marriage surplus is defined as the gain from marriage for the female and the male of type ij and labor market status ll, where the first l corresponds to the male's labor market status, the second to the female's, i.e.,

$$S_{ij}^{ll} \equiv \left[V_i^{l,-l} - V_i^l\right] + \left[V_j^{l,-l} - V_j^l\right]. \tag{8}$$

Using the first order conditions for the transfers and the time devoted to public goods production $\{d_i, d_j, t_i, t_j\}$ – derived in Appendix B.1 – the surplus for any type ij and employment status ll is given by,

$$\begin{aligned} & \left(r+\delta_{ij}+\tau_i(l)+\tau_j(l')\right) S_{ij}^{ll'}(z) \\ = \;& u_{ij}^{ll'}(z) + \delta_{ij}\int \max\left[S_{ij}^{ll'}(z'),0\right] dG(z') \\ & +\tau_i(l)\max\left[S_{ij}^{l'l'}(z),0\right] + \tau_j(l')\max\left[S_{ij}^{ll}(z),0\right] \\ & -\lambda_{ij}(1-\beta)\iiint \max\left[S_{ij}^{ll''}(z'),0\right] dG(z')\, s(j,l'')\, dj dl'' \\ & -\lambda_{ij}\beta\iiint \max\left[S_{ij}^{l''l'}(z'),0\right] dG(z')\, s(i,l'')\, di dl'', \end{aligned} \tag{9}$$

where $u_{ij}^{ll}(z)$ denotes the maximized joint flow surplus of both partners, i.e.,

$$\begin{aligned} u_{ij}^{ll'}(z) &\equiv u_i^{l,-l'} + u_j^{l',-l} - u_i^l - u_j^{l'} \\ &= z\kappa_{ij}\left[W_{ij}+\psi_{ij}^{ll'}\right]^{\kappa} - \kappa_i\left[w_i+\psi_i^l\right] - \kappa_j\left[w_j+\psi_j^{l'}\right] \end{aligned} \tag{10}$$

with

$$\kappa = 1 + K_f^1 + K_m^1, \qquad \kappa_{ij} = Z_{ij}K, \qquad \kappa_i = \left(K_i^0\right)^{K_i^0}, \qquad \kappa_j = \left(K_j^0\right)^{K_j^0},$$
$$\psi_{ij}^{ll'} = -C_{ij} + T_i^l - D_i^1 + T_j^{l'} - D_j^1, \qquad \psi_i = T_i^l - D_i^0 - K_i^0, \qquad \psi_j = T_j^{l'} - D_j^0 - K_j^0,$$
$$W_{ij} = w_i + w_j + \iota\left(w_i - w_j\right)^2.$$

The maximized joint flow surplus $u_{ij}^{ll'}(z)$ is strictly increasing in z. This ensures that also the surplus functions are strictly increasing in z. The cutoff bliss values $z_{ji}^{ll'}$ for $ll' \in \{ee, ue, eu, uu\}$ are defined such that the surplus is equal to zero, i.e., $S_{ji}^{ll'}\left(z_{ji}^{ll'}\right) = 0$. Since $u_{ji}^{ll'}(z)$ is increasing in z it follows that $S_{ji}^{ll'}(z) > 0$ for $z > z_{ji}^{ll'}$. This allows us to write the probability $\alpha_{ij}^{ll'}$ that a couple of type ij and labor market status ll' is willing to marry upon meeting as,

$$\alpha_{ij}^{ll'} = \left(1 - G\left(z_{ij}^{ll'}\right)\right). \tag{11}$$

4.3 Steady state flows and measures

We denote by $m(i,j,l,l)$ the number of married couples of type ij and labor market status ll. The number of single males (females) of type i (j) and labor market status l is denoted by $s(i,l)$ $(s(j,l))$. The number of married couples of type ij and labor market status ll divorce, if a bliss shock reduces the bliss value below z_{ij}^{ll}, or change into another labor market status ll if one partner changes her/his employment status (at rate $\tau_i(l) + \tau_j(l)$). The inflow, i.e., the number of new marriages of type ij and labor market status ll formed, is given by $\lambda_{ij}\alpha_{ij}^{ll} s(i,l)\, s(j,l)$, where α_{ij}^{ll} denotes the probability that a couple of type ij and labor market status ll is willing to marry upon meeting. There are additional inflows into the group $m(i,j,l,l)$ from couples of labor market status $m(i,j,l',l)$ and $m(i,j,l,l')$. The probability that a couple stays together after a change of the labor market status from $l'l$ to ll is equal to 1 if $z_{ij}^{ll} \le z_{ij}^{l'l}$ and equal to $\alpha_{ij}^{ll}/\alpha_{ij}^{l'l} < 1$ if $z_{ij}^{ll} > z_{ij}^{l'l}$, i.e., equal to $\min\left[\left(\alpha_{ij}^{ll}/\alpha_{ij}^{l'l}\right), 1\right]$. We therefore get,

$$\begin{aligned} & \left[\delta_{ij}\left(1 - \alpha_{ij}^{ll}\right) + \left(\tau_i(l) + \tau_j(l)\right)\right] m(i,j,l,l) \\ = \; & \lambda_{ij}\alpha_{ij}^{ll} s(i,l)\, s(j,l) \\ & + \tau_i(l') \min\left[\left(\alpha_{ij}^{ll}/\alpha_{ij}^{l'l}\right), 1\right] m(i,j,l',l) \\ & + \tau_j(l') \min\left[\left(\alpha_{ij}^{ll}/\alpha_{ij}^{ll'}\right), 1\right] m(i,j,l,l'). \end{aligned} \tag{12}$$

Let us now consider the flow equations for the respective single groups. The outflow of a single male of type i with labor market status l is given by the rate at which the individual marries with single female of type j with labor market status l'', i.e., the rate $\lambda_{ij}\alpha_{ij}^{ll''} s(j,l'')$, plus the rate at which the single male changes her/his labor market status, i.e., the rate $\tau_i(l)$. The inflow is given by the rate at which the single males with the opposite labor market status l' change their status (at rate $\tau_i(l')$) plus the rate at which the respective marriages break up. This happens when a bliss shock occurs (at rate $\delta_{ij}\left(1 - \alpha_{ij}^{ll''}\right) m(i,j,l,l'')$) or when the married male of type i or the married female of type j changes the labor market status (at rates $\widetilde{\tau}_i(l') \max\left[1 - \left(\alpha_{ij}^{ll''}/\alpha_{ij}^{l'l''}\right), 0\right] \widetilde{m}(i,j,l',l'')$ or

$\widetilde{\tau}_j(l'') \max\left[1 - \left(\alpha_{ij}^{ll'}/\alpha_{ij}^{ll''}\right), 0\right] \widetilde{m}(i,j,l,l''))$. Formally,

$$
\begin{aligned}
& \left[\iint \lambda_{ij}\alpha_{ij}^{ll''} s(j,l'')\, djdl'' + \tau_i(l)\right] s(i,l) \\
= \;& \tau_i(l') s(i,l') + \iint \delta_{ij}\left(1-\alpha_{ij}^{ll''}\right) m(i,j,l,l'')\, djdl'' \\
& + \iint \widetilde{\tau}_i(l') \max\left[1-\left(\alpha_{ij}^{ll''}/\alpha_{ij}^{l'l''}\right), 0\right] \widetilde{m}(i,j,l',l'')\, djdl'' \\
& + \iint \widetilde{\tau}_j(l'') \max\left[1-\left(\alpha_{ij}^{ll'}/\alpha_{ij}^{ll''}\right), 0\right] \widetilde{m}(i,j,l,l'')\, djdl'',
\end{aligned}
\tag{13}
$$

because $\max\left[1-\left(\alpha_{ij}^{ll''}/\alpha_{ij}^{l'l''}\right), 0\right] = 1 - \min\left[\left(\alpha_{ij}^{ll''}/\alpha_{ij}^{l'l''}\right), 1\right]$. To get number of singles of a certain type and labor market status we can use the aggregate labor market transitions, e.g.,

$$\tau_i(l)s(i,l) + \tau_i(l)\iint m(i,j,l,l'')\, djdl'' = \tau_i(l')s(i,l') + \tau_i(l')\iint m(i,j,l',l'')\, djdl'',$$

and the market clearing conditions for the different types of males and females, e.g.,

$$
\begin{aligned}
n(i) &= s(i,l') + s(i,l) + \iiint m(i,j,l,l'')\, djdl''dl, \\
n(j) &= s(j,l') + s(j,l) + \iiint m(i,j,l,l'')\, didl''dl.
\end{aligned}
$$

Substituting and rearranging then implies the following formula for singles of type i and labor market status l,

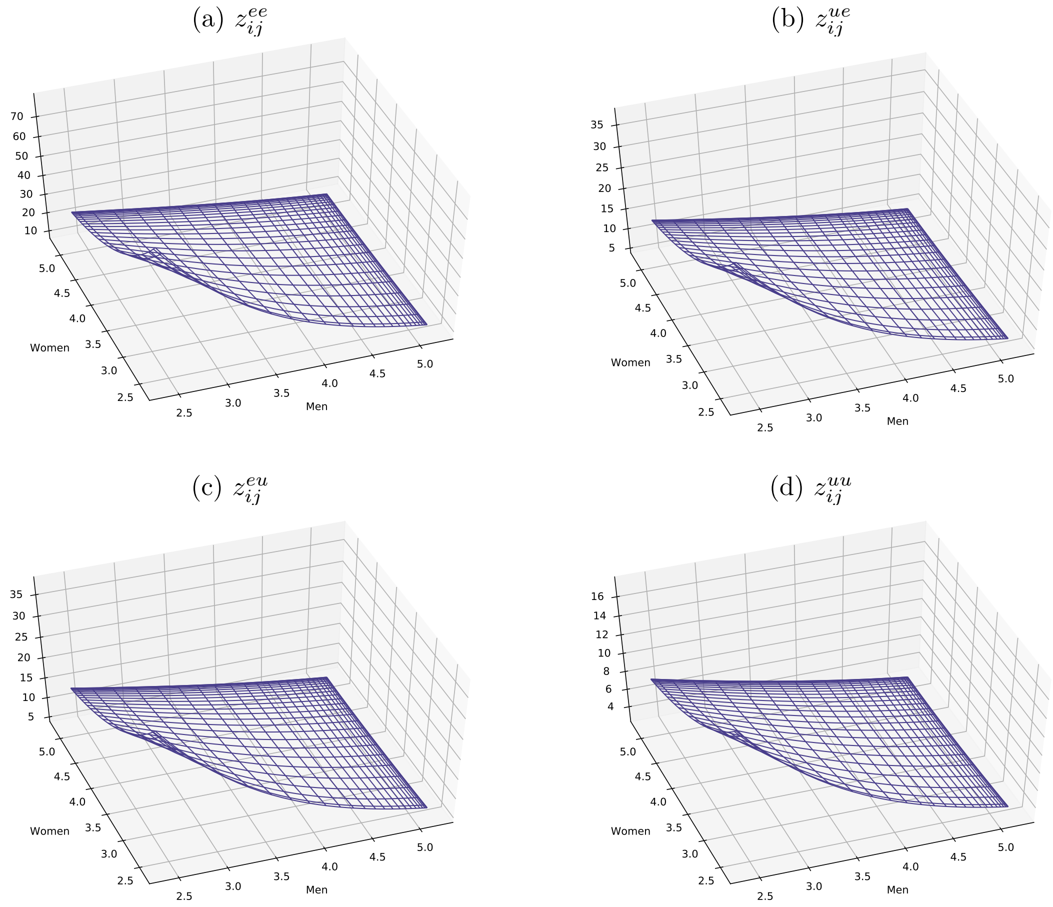
$$s(i,l) = \frac{\tau_i(l')}{\tau_i(l)+\tau_i(l')} n(i) - \iint m(i,j,l,l'')\, djdl''. \tag{14}$$

4.4 Equilibrium

The equilibrium is characterized by a set of surplus functions $S_{ij}^{ll}(z)$, cutoff bliss values z_{ij}^{ll}, and joint distributions of married couples $m(i,j,l,l)$ for each type ij and labor market status ll. We compute the equilibrium in the following way: Given a set of initial conditions, the cutoff bliss values z_{ij}^{ll} determine $\alpha_{ij}^{ll} \equiv \left(1 - G\left(z_{ij}^{ll}\right)\right)$. Given α_{ij}^{ll} we can use equations (12) and (14), i.e., a set of four equations for $m(i,j,l,l)$ for each $ll \in \{ee, ue, eu, uu\}$ and a set of two equations determining $s(i,l)$ and $s(j,l)$ for each $l \in \{e,u\}$, respectively, to compute $s(i,l)$ and $s(j,l)$. The number of singles $s(i,l)$ and $s(j,l)$ of type i (j) and labor market status l (l) determine the surplus functions $S_{ij}^{ll}(z)$ given by equation (9) for all types ij and labor market status ll. The bliss values z_{ij}^{ll} for all types ij and labor market status combinations ll are then pinned-down at a value such that the respective surplus is zero, i.e., $S_{ij}^{ll}\left(z_{ij}^{ll}\right) = 0$. The problem involves alternating between solving the two fixed-point systems of $S_{ij}^{ll}(z)$ and z_{ij}^{ll} until convergence. Appendix B.2 describes in detail how the fixed point system are solved computed numerically.

4.5 Model Solution

Figure 4: z **cutoffs**

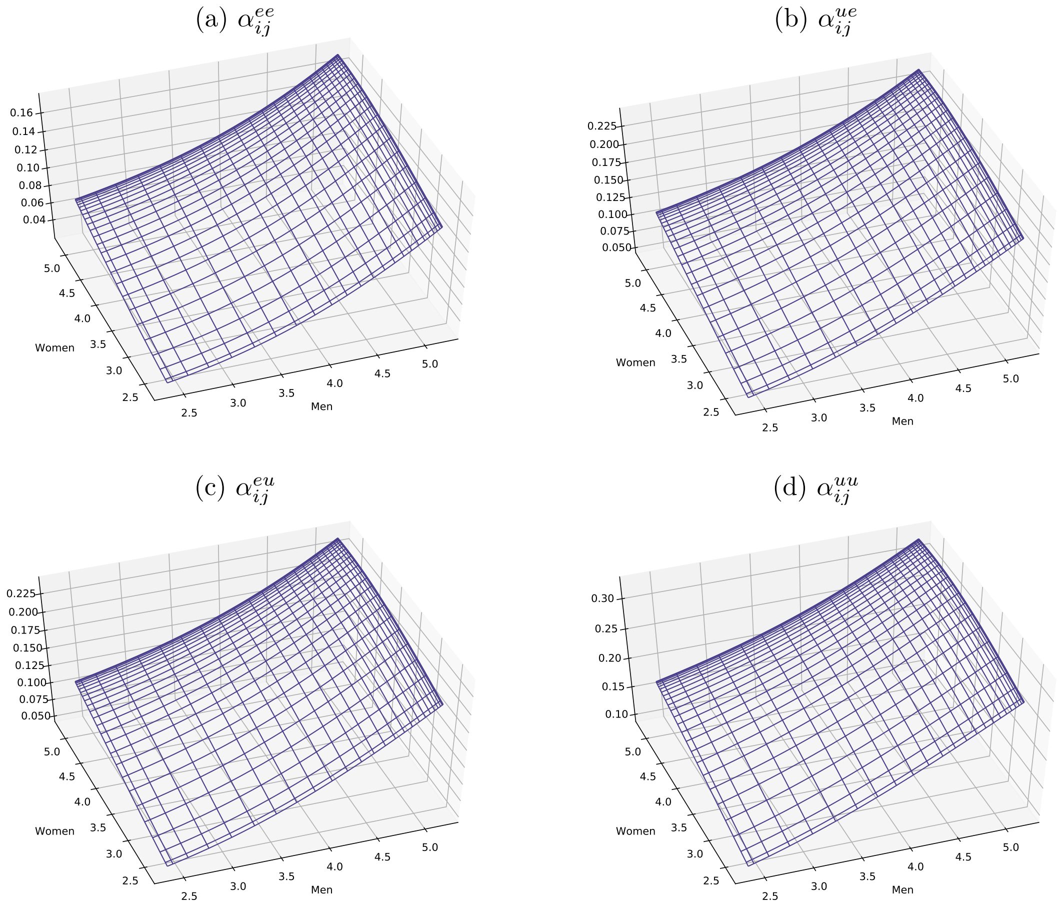


(a) z_{ij}^{ee} (b) z_{ij}^{ue} (c) z_{ij}^{eu} (d) z_{ij}^{uu}

We solve the model on a Chebyshev grid with 50×50 nodes. We use the empirical wage distribution functions estimated for men and women in Section 3.2 to set up the underlying distributions of men and women, $n(i)$ and n_j. Given a first parametrization (see Appendix B.3), the model's stationary equilibrium exhibits a number of interesting properties. Here, we focus on the distribution of married couples across type and employment status combinations because this is what we see in our data.

Figure 4 shows the minimum realizations of the bliss shocks a couple needs to draw upon meeting in order to form a marriage. This value is highest for the lowest types of men and women. The higher the types of the man and woman who are meeting the lower is the z they need to draw in order to have a positive marriage surplus. This pattern holds across all employment status combinations, the levels, however, are very different. The outside option of continued search in the marriage market is higher for employed individuals, hence the necessity to draw higher z values in order to compensate both parties. Meetings between unemployed men and women need the lowest z overall to result in wedlock.

Figure 5: Marriage probabilities α_{ij}^{ll}



(a) α_{ij}^{ee} (b) α_{ij}^{ue} (c) α_{ij}^{eu} (d) α_{ij}^{uu}

The model-generated marriage probabilities α_{ij}^{ll} are the mirror image of the z values. They are depicted in Figure 5. Again, the general pattern is the same across all employment status combinations, the marriage probability increased in both partners' types. Both high types and employment lead to lower marriage probabilities due to the better outside option that the engaging individuals need to be compensated for. Our structural estimation in the next Section will enable us to compare the model generated marriage probabilities to their empirical counterparts.

5 Structural Decomposition of Marriage and Divorce

We now take our structural model of the marriage market to the data. This connection allows us to go beyond the descriptive analysis in Section 3. We use our model to uncover the different contributing factors to matching and separation decisions at the individual level from the data. This allows us to decompose and explain the observed aggregate dynamics of marriage and divorce. We first highlight the relevant matching mechanisms in our model and then describe how to identify their relative importance from the data.

According to our model, matching in the marriage market has two components: The meeting rate, λ_{ij}, determines the likelihood of meeting a certain type of partner in the frictional marriage market. The ij dependence resembles the idea that individuals, depending on their type, have different probabilities to meet with heterogeneous members of the other sex. This is likely to occur, since many couples meet at education institutions or at the workplace. The second component of matching decisions are the acceptance probabilities, α_{ij}. Conditional on meeting, they capture the likelihood of wedlock. It differs across ij combinations because, according to our model, the option value of continued search for another partner may dominate forming the marriage. The willingness to marry α_{ij} and the meeting rate λ_{ij} may also differ across labor market statuses.

Regarding divorce, two things can happen: First, a negative update of the match-specific bliss shock z occurs, decreasing home production and flow utilities. This may drive the marriage surplus below zero and lead to a divorce. Second, as we have emphasized throughout, labor market transitions may trigger divorces. These "labor market divorces" may happen for two reasons: First, an employed spouse becomes unemployed. Depending on the combination of types in the couple, the drop in household income may outweigh the increase in home production (via the time input) and, hence, decrease utility flows and lead to divorce. Second, a previously unemployed spouse may find it optimal to divorce after finding a new job. Theoretically, the outside option of starting over in the marriage market as an employed person can dominate the option value of staying in the current match.

Looking at the data through the lens of our model, we now let the data decide which channels drive marriage and divorce in Germany and analyze how their respective contributions have evolved over time.

5.1 Meetings & Marriages

Using our three sources of micro data (MC, SIAB, MDR), we estimate the simultaneous flow equation system from our model, summarized in Equation (13), using variation across time and single/couple types. The details of our estimation procedure are included in Appendix C. In short, we construct the empirical counterparts of the (joint) distributions of singles and married couples from MC data and define them as follows: $\widetilde{s}_{it}^{l} = s\left(i, l\right)$ and

Table 3: Estimates of matching probability α_{ij}^{ll}

(a)

	$\hat{\alpha}_{ij}^{ee}$	j low	medium	high
	low	0.96	0.62	0.53
i	medium	0.99	0.72	1.00
	high	0.53	1.00	0.78

(b)

	$\hat{\alpha}_{ij}^{eu}$	j low	medium	high
	low	0.01	0.79	0.89
i	medium	0.88	1.00	1.00
	high	0.89	0.01	1.00

(c)

	$\hat{\alpha}_{ij}^{ue}$	j low	medium	high
	low	0.45	0.14	0.05
i	medium	0.20	1.00	1.00
	high	0.05	1.00	0.24

(d)

	$\hat{\alpha}_{ij}^{uu}$	j low	medium	high
	low	0.00	0.05	0.45
i	medium	0.06	1.00	0.07
	high	0.45	0.00	1.00

Note: Estimated marriage probabilities as derived from our model for men and women with three education categories (low, medium, high) and two employment categories (employed, unemployed). Source: Research Data Center of the Statistical Offices of the Länder and the Federal State, Microcensus, 1993-2013, own calculations. SIAB source: Research Data Centre (FDZ) of the Federal Employment Agency at the Institute for Employment Research, SIAB, 1993-2013, own calculations.

$\widetilde{m}_{ijt}^{ll} = m\left(i, j, l, l\right)$. The observed labor market transition rates $\tilde{\tau}_{it}^{l} = \tau_i\left(l\right)$ are the second data input. Due to the nature of our aggregate data, variation is limited and we need to discipline the estimation. We derive a large set of equality, inequality, and non-linear constraints from our model and impose them on the parameters to be estimated. In particular, our constraints guarantee that all estimated values of $\hat{\alpha}_{ij}^{ll}$, the estimated matching probabilities, lie in the unit interval. Given the flow equations and the constraints, we estimate a set of composite parameters using a non-linear least squares method.[27] It is then possible to back out the model parameters from the estimated composite parameters.

Table 3 shows our estimates of the four α_{ij}^{ll} matrices, one for each combination of the spouses' labor market status ($ll \in \{ee, ue, eu, uu\}$). Our descriptive analysis of education-based sorting in Section 3.1 has revealed that the tendency to sort is not uniform across education groups. Based on our model, we can now refine this statement by additionally taking into account the estimated matching probabilities across types and labor market statuses of the spouses.

In each Panel, the horizontal dimension of the table represents the female education types (j) and the vertical dimension the male types (i). In many cases, the restrictions we impose on the data are binding; we get matching probabilities of one. Meetings of two employed singles (Panel 3a) have the overall highest probabilities of ending in wedlock. These couples also contribute to marriage market sorting. Estimated probabilities are high on the main diagonal and low for the low/high and high/low combinations. Probabilities are one for medium/high and high/medium couples. Panel 3b contains marriage

[27] We rely on the excellent lmfit package for Python by Newville et al. (2014).

probabilities of employed men and unemployed women of all types. The matching probability of two low education individuals with this employment status combination is almost zero, the other two values on the main diagonal, however, are one. Hence, these combinations also sort positively. Interestingly, the combination of a high-type man and a medium-type woman has a very low estimated matching probability while in the inverse case, high-type women (unemployed) and medium-type men (employed) are very likely to match. Panel 3c shows the case of an unemployed man and an employed woman. For two highly-educated individuals, this labor market status combination is not likely to lead to marriage (24%). Marriage is ensured conditional on meeting, however, for the medium/medium, high/medium and medium/high type combinations, what is in line with positive sorting. Unions involving low type individuals have matching probabilities which are monotonically decreasing in the partner's education type. Finally, we show in Panel 3d that two unemployed singles who meet are ensured to mate if both partners have medium or high education. All other probabilities are very low, with the exception of the two high-low combinations. Both in Panels 3b and 3d the high matching probabilities for the high-low combinations can be rationalized with the prevalence of joint taxation in Germany, which creates incentives for negative sorting.

While the overall picture of a tendency to sort positively is confirmed across the alphas, our prior from the descriptive analysis can be updated. It is true that the tendency to sort varies across education types but there is also interesting variation across employment status combinations. While two employed individuals will get married with a very high probability in all education categories, dating couples with an unemployed man are very unlikely to match if one of the potential partners has low education. Unemployment of the woman, however, seems to matter much less, matching probabilities are high almost everywhere.

Two unemployed individuals are least likely to get married overall but they still contribute to positive sorting, the upper two main diagonal elements are one. Note also that the solution of our model depicted in Section 4.5 can match the overall picture in the data of higher matching probabilities for higher education types but it cannot reproduce the heterogeneity across education cells. More work is needed to calibrate the model and to map the finer wage grid on which the theoretical model is solved into the education categories we have as a proxy in our data.

Given the estimates for α_{ij}^{ll} and our constraints, we can go one step further and calculate the estimated number of meetings per month across partner types. The number of meetings per month for a given education-pair ij with labor market status combination ll is given by multiplying the mean of the respective single shares, $\bar{s}_i^l$ and $\bar{s}_j^l$, with the estimated $\hat{\lambda}_{ij}$ parameter. Comparing the number of meetings across education groups and labor market status allows us to analyze whether search in the marriage market is random or directed.

Table 4: Estimates of meeting rates λ_{ij}^{ll}

(a)

$\hat{\lambda}_{ij}^{ee}\bar{s}_i^e\bar{s}_j^e$		j low	medium	high
i	low	1.33e-04	5.50e-05	4.19e-04
	medium	8.51e-04	2.80e-03	1.90e-01
	high	5.46e-04	3.38e-02	8.24e-04

(b)

$\hat{\lambda}_{ij}^{eu}\bar{s}_i^e\bar{s}_j^u$		j low	medium	high
i	low	2.45e-02	2.44e-05	1.39e-04
	medium	1.01e-03	7.73e-04	9.55e-03
	high	1.59e-03	1.00e-04	7.29e-05

(c)

$\hat{\lambda}_{ij}^{ue}\bar{s}_i^u\bar{s}_j^e$		j low	medium	high
i	low	1.56e-04	6.44e-05	4.91e-04
	medium	4.58e-04	1.51e-03	2.83e-03
	high	2.53e-04	5.42e-04	3.84e-04

(d)

$\hat{\lambda}_{ij}^{uu}\bar{s}_i^u\bar{s}_j^u$		j low	medium	high
i	low	2.88e-02	2.86e-05	1.63e-04
	medium	5.42e-04	1.31e-04	5.19e-03
	high	7.37e-04	4.65e-05	5.94e-06

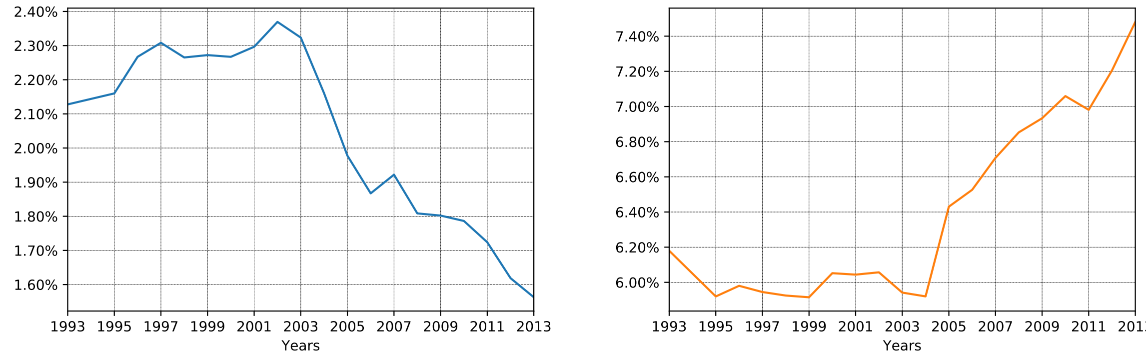
Note: Estimated meeting rates for men and women with three education categories (low, medium, high) and two employment categories (employed, unemployed), multiply by the respective share of single in the population. Source: Research Data Center of the Statistical Offices of the Länder and the Federal State, Microcensus, 1993-2013, own calculations. SIAB source: Research Data Centre (FDZ) of the Federal Employment Agency at the Institute for Employment Research, SIAB, 1993-2013, own calculations.

Table 4 presents the estimated number of meetings per month again across all marriage and labor market type combinations. We note that for employed singles (Panel 4a) the number of meetings are highest for medium and high type individuals, with the surprising exception that high type singles have a rather low number of meetings. One has to take into account, however, that both male and female singles with high education are rare in the marriage market. Panel 4b reveals that unemployed low type women have a high number of meetings with all types of employed men. The ranking of matching probabilities for the same cells, however, was opposite (recall Table 3). There are a lot of meetings between low-type men and low type women in the (eu) category but only a small fraction of them ends in marriage. Between low type women and medium/high type men, the number of meetings are lower by a factor of 10 but, conditional on meeting, marriage is very likely. This pattern seems to be consistent with random search of low type individuals in the marriage market, that is, search is not directed towards partner types with a high likelihood of marriage. For the high/medium type combinations, however, search appears to be directed as high values of $\hat{\alpha}_{ij}^{eu}$ coincide with a high number of meetings $\hat{\lambda}_{ij}^{eu}\bar{s}_i^e\bar{s}_j^u$. This pattern of random search of low type individuals and directed search of medium and high type individuals is repeated in Panel 4c. It seems to be a consistent feature of the German marriage market. Panel 4d, however, is a special case. For two unemployed individuals, the highest number of meetings (low/low) essentially never leads to marriage. Conversely unemployed high type singles are very unlikely to meet but very likely to marry.

5.2 Divorces

Figure 6: Divorce rate and share of idiosyncratic divorces

(a) Divorce rate (b) Share of "Labor Market Divorces"



Note: MC data source: Research Data Center of the Statistical Offices of the Länder and the Federal State, Microcensus, 1993-2013, own calculations. SIAB source: Research Data Centre (FDZ) of the Federal Employment Agency at the Institute for Employment Research, SIAB SUF 7514, 1993-2013, own calculations.

Our model implies that the aggregate flow of divorces must be consistent with the following aggregated flow equation:

$$\begin{aligned} \widetilde{\Delta}_t \;=\; & \delta \iiiint \left(1-\alpha_{ij}^{l''l}\right) \widetilde{m}_{ijt}^{l''l} di dj dl'' dl \\ & + \iiiint \widetilde{\tau}_{it}^{l''} \max\left[1-\left(\alpha_{ij}^{l'l}/\alpha_{ij}^{l''l}\right), 0\right] \widetilde{m}_{ijt}^{l''l} di dj dl'' dl \\ & + \iiiint \widetilde{\tau}_{jt}^{l} \max\left[1-\left(\alpha_{ij}^{l''l'}/\alpha_{ij}^{l''l}\right), 0\right] \widetilde{m}_{ijt}^{l''l} di dj dl'' dl. \end{aligned} \tag{15}$$

$\widetilde{\Delta}_t$ is the aggregate number of divorces in the data. By plugging in our estimated $\hat{\alpha}_{ij}^{ll}$ matrices on the RHS of equation (15) we can decompose the divorce flow into the shares of divorces caused by idiosyncratic shocks (first term on the RHS), the share caused by male labor market transitions (second term on the RHS), and the share caused by female labor market transitions (third term on the RHS).

We are interest in the share of divorces induced by labor market transitions, that is, one spouse transferring from employment to unemployment or vice versa. For succinctness, we refer to them as "labor market divorces". We can further differentiate these divorces for our four types of couples by labor market status combination before the transition and the underlying heterogeneity of types (education).

As a first step, we look at the aggregate divorce rate, see the left Panel of Figure 6. The overall number of divorces has declined significantly during our period of observation, it fell from 7% to below 5%. The right Panel shows the share of labor market divorces in all separations. The majority of divorces is not triggered by labor market shocks. According to our theory, the "residual", between about 92% and 94% of all divorces, are triggered by an update of the bliss shock z. Remarkably, however, the share of labor market divorces has increased over time, against the overall trend of a declining divorce rate. The share was quite stable at around 6% until 2004 and started increasing rapidly thereafter, with a small correction in 2011, reaching almost 7.5% in 2013.

To understand which couple types contributed to the increasing share of labor market divorces, we now further differentiate it according to our gender-education-marriage-type cells. We look at married men and women in marriages with all four employment status combinations and across all 9 education types. The data for all cells can be found in Appendix A.3.

For males affected by a job loss, the share of separations has most drastically increased for couples in which both spouses are highly educated and employed. It increased from 0.12% to 0.25% of all divorces. This corresponds to roughly 10% of the overall increase we measure. Interestingly, this education combination never divorces upon job loss of the man when the woman is already unemployed. Conversely, a sizable number of divorces occur for similar couples with the only difference being that the husband has a lower education type (medium) than the woman (high). Male education seems to matter a lot for the survival probability of marriages. Other important contributions to divorces triggered by male job loss come from employed couples with low education males and low or medium educated females, respectively. This is true irrespectively of the female employment status. The share has not increased in the second half of our sample, however. Rather, they decreased towards the end.

We find that a male finding a job, so a transition from unemployment to employment, almost never triggers a divorce. The only exception: when both partners have medium education and the wife is already employed, the husband's new job leads to 0.33% of divorces in the beginning of our period and to 0.74% in the end. This is roughly a third of the overall increase of labor market divorces.

Let us now turn to female labor market transitions. Overall, the likelihood of a divorce is much lower when women lose their job as compared to men. Two groups of couples have sizable propensities to divorce upon female job loss, however: employed couples with two low education spouses and employed couples with a high education husband and a medium education wife. For the latter group, the share of divorces has increased from 0.19% to 0.30%, about 10% of the overall effect. The share of the low-education employed couple is also more than 0.10% but it has not increased over time.

Finally, we look at married females who exit from unemployment. There are four striking cases which, in combination, make up most of the time dynamics of labor market divorces we observe. First, highly-educated couples in which both spouses were unemployed before the woman finds a job are responsible for 2.10% of the overall number of divorces in 2013. This share has almost quintupled over time and alone accounts for more than the observed aggregate increase of labor market divorces. Second, the divorce share of unemployed couples with a low education husband and a highly educated wife has also increased significantly over time, it grew by almost 50%. Conversely, the share of the opposite couple type, high education man and low education woman who are both unemployed and divorce when the woman starts working is also sizable (0.32% in 2013) but decreasing over time. Third, now looking at couples where the man is already employed and the woman starts working, the share of divorces between, again, two highly educated spouses has almost doubled to 0.45% in 2013. Fourth, the share of the biggest contributor to labor market divorces in the beginning of our sample (almost 2%) has decreased significantly. Couples who share medium education and the woman joins the labor market in addition to a working husband, however, are still responsible for 1.31% of all divorces in 2013.

The analysis of the shares of labor market divorces reveals an interesting general picture: it is mostly the group of couples with two high or medium education spouses that drives labor market divorces, so the *sorted couples*. Some combinations of low education couples are also affected by labor market uncertainty but their share of divorces has decreased significantly over time.

Strikingly, in many cases these sorted high education couples divorce when the woman starts working. The increasing share of this kind of divorces can be connected an earlier observation we made based on the labor market flow data. The job-finding rate of high and medium education women has increased significantly in the second half of our sample, much more strongly than the respective transition rates of males.

Finally, recall Figure 1. We have observed that formerly increasing educational sorting has leveled off in the second half of the 2000s. Our structural decomposition of divorce flows has enabled us to make sense of this observation. If sorted couples show the tendency to react more strongly to both job-finding and separation shocks it would be natural to expect that the correlation between education values will not surpass a certain point.

6 Conclusions

This piece of research has connected the two-sided marriage market model of Goussé et al. (2017) to the labor market. The uncertainty that singles and married couples face regarding their labor market status is, as we show theoretically and empirically, an important driving force of matching decisions in the marriage market. Using three sources of German micro data, we document that the German marriage market is coined by positive sorting of in the marriage market based on education, income, and employment status. The trend towards more educational sorting, however, has stalled in recent years.

We perform a structural empirical analysis that allows us to back out key elements of our marriage market model from the data, specifically meeting rates and matching probabilities. We find that search patterns in the marriage market appear to be directed for highly educated individuals while single with low education search randomly. Based on our data and the estimated model parameters, we decompose the aggregate flow of divorces into the share induced by labor market transitions and by match-specific shocks. Transitions from employment to unemployment or vice versa make up only a fraction of all divorces. This fraction, however, shows an interesting dynamic. The share of labor market divorces has grown by more than 20% since the mid 2000s and most of the additional divorcées are highly educated and were married to highly educated individuals. Most of these marriages break up when a previously unemployed woman starts working, especially if the husband stays unemployed. In 2013, 5.3% of all divorces happened when a previously unemployed woman started to work. This percentage share equals 27,968 divorces. The case that the literature has previously analyzed, divorce upon male job loss, accounts only for a shrinking fraction of all divorces in Germany.

One possible explanation for the differential changes of different couple types in the overall number of labor market divorce relates to the booming German labor market in the second half of the 2000s. Many low education couples divorce for reasons of economic hardship and related stress in the relationship when they become unemployed. This divorce hazard may have been mitigated by the shrinking unemployment rate in Germany and the good general macroeconomic environment. High education couples who are the source of assortative matching in the marriage market, however, seem to divorce for other reasons. When a high education women starts working, for instance, this might change the balance of power and the resource sharing in a household. Due to favorable outside options of two employed persons, the option value of searching for a new partner in the marriage market might become dominant.

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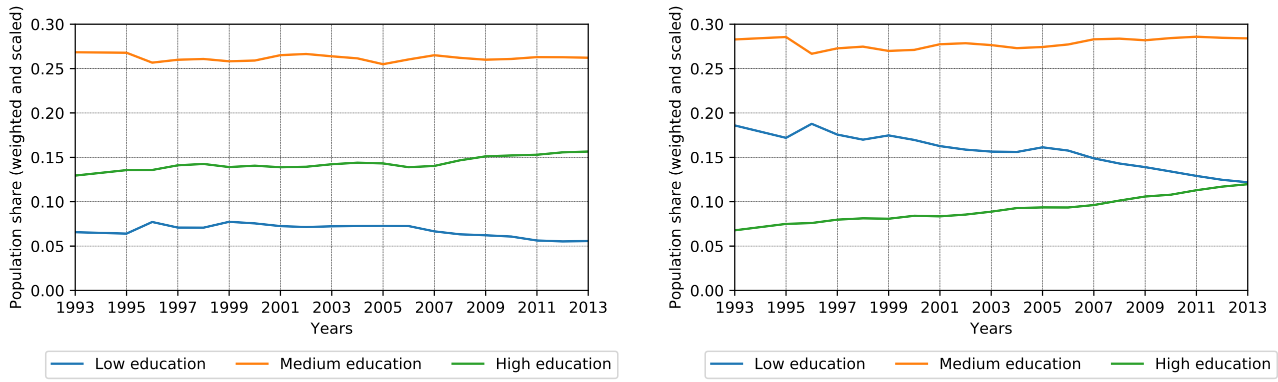
A Additional Results

A.1 MC Data

Figure A.1: Education by gender

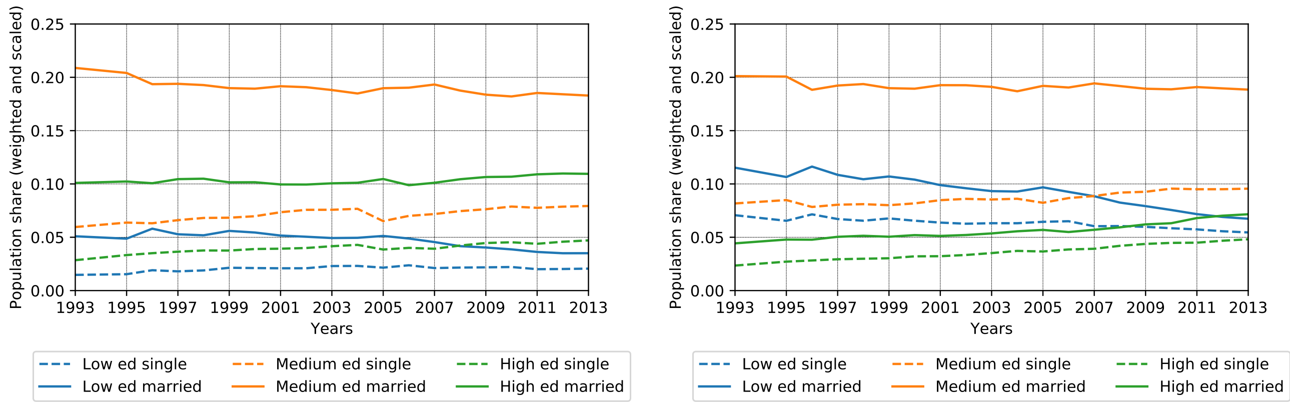
(a) Men (b) Women



Note: Population shares are weighted and scaled using the MC sample weights. 100% on the y-axis corresponds to the full population, including married and unmarried individuals of both sexes. MC source: Research Data Center of the Statistical Offices of the Länder and the Federal State, Microcensus, 1993-2013, own calculations.

Figure A.2: Education and marital status by gender

(a) Men (b) Women

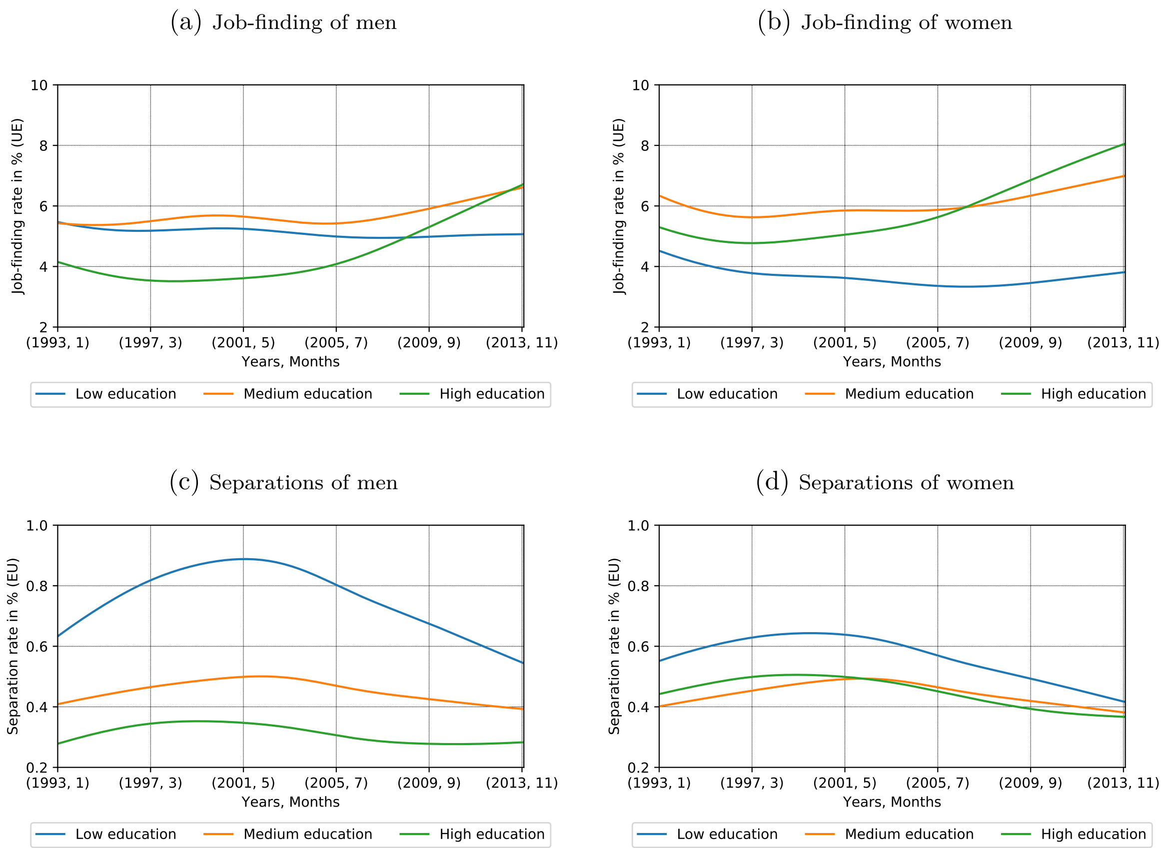


Note: Population shares are weighted and scaled using the MC sample weights. 100% on the y-axis corresponds to the full population, including married and unmarried individuals of both sexes. MC source: Research Data Center of the Statistical Offices of the Länder and the Federal State, Microcensus, 1993-2013, own calculations.

Figure A.1 depicts the respective population shares of men and women in each of three education categories: Panel (a) shows that for men the education distribution has not changed much between 1991 and 2013. The share of highly educated men increased somewhat and surpassed 15% in 2009. For women in Panel (b), the share of women with high education has increased much stronger, from 7.2% in 1991 to almost 12% in 2013. The share of low education women has decreased accordingly. For both men and women the share with medium education is roughly constant over time. Figure A.2 further breaks down the three education shares into married and single individuals.

A.2 SIAB Data

Figure A.3: Job-finding and separation rates



Note: seasonally adjusted (X-13-ARIMA-SEATS) and HP-filtered ($\lambda = 900,000$) monthly job-finding and separation rates by gender and education category. SIAB source: Research Data Centre (FDZ) of the Federal Employment Agency at the Institute for Employment Research, SIAB SUF 7514, 1993-2013, own calculations.

A.3 Shares Labor Market Transitions in Divorce Flows

The following four tables show the decomposition of the aggregate number of divorces into shares for all gender-education-marriage cells, using equation (15).

Table A.1: Contribution of male separations (EU) to the overall flow of divorces

	$\widetilde{m}^{ee}_{11t}\widetilde{\tau}^{e}_{1t}$	$\widetilde{m}^{ee}_{12t}\widetilde{\tau}^{e}_{1t}$	$\widetilde{m}^{ee}_{13t}\widetilde{\tau}^{e}_{1t}$	$\widetilde{m}^{ee}_{21t}\widetilde{\tau}^{e}_{2t}$	$\widetilde{m}^{ee}_{22t}\widetilde{\tau}^{e}_{2t}$	$\widetilde{m}^{ee}_{23t}\widetilde{\tau}^{e}_{2t}$	$\widetilde{m}^{ee}_{31t}\widetilde{\tau}^{e}_{3t}$	$\widetilde{m}^{ee}_{32t}\widetilde{\tau}^{e}_{3t}$	$\widetilde{m}^{ee}_{33t}\widetilde{\tau}^{e}_{3t}$	$\widetilde{m}^{eu}_{11t}\widetilde{\tau}^{e}_{1t}$	$\widetilde{m}^{eu}_{12t}\widetilde{\tau}^{e}_{1t}$	$\widetilde{m}^{eu}_{13t}\widetilde{\tau}^{e}_{1t}$	$\widetilde{m}^{eu}_{21t}\widetilde{\tau}^{e}_{2t}$	$\widetilde{m}^{eu}_{22t}\widetilde{\tau}^{e}_{2t}$	$\widetilde{m}^{eu}_{23t}\widetilde{\tau}^{e}_{2t}$	$\widetilde{m}^{eu}_{31t}\widetilde{\tau}^{e}_{3t}$	$\widetilde{m}^{eu}_{32t}\widetilde{\tau}^{e}_{3t}$	$\widetilde{m}^{eu}_{33t}\widetilde{\tau}^{e}_{3t}$
1993	0.10%	0.14%	0.04%	0.09%	0.00	0.00	0.02%	0.00	0.12%	0.16%	0.18%	0.03%	0.12%	0.00	0.11%	0.01%	0.06%	0.00
1995	0.10%	0.14%	0.04%	0.09%	0.00	0.00	0.02%	0.00	0.15%	0.18%	0.19%	0.02%	0.11%	0.00	0.11%	0.01%	0.06%	0.00
1996	0.14%	0.15%	0.05%	0.09%	0.00	0.00	0.02%	0.00	0.16%	0.22%	0.19%	0.03%	0.11%	0.00	0.11%	0.01%	0.06%	0.00
1997	0.13%	0.14%	0.05%	0.08%	0.00	0.00	0.02%	0.00	0.17%	0.20%	0.18%	0.03%	0.10%	0.00	0.11%	0.01%	0.06%	0.00
1998	0.12%	0.14%	0.05%	0.08%	0.00	0.00	0.02%	0.00	0.17%	0.21%	0.18%	0.03%	0.10%	0.00	0.11%	0.01%	0.06%	0.00
1999	0.16%	0.15%	0.05%	0.09%	0.00	0.00	0.02%	0.00	0.18%	0.22%	0.17%	0.02%	0.10%	0.00	0.11%	0.01%	0.06%	0.00
2000	0.17%	0.16%	0.05%	0.09%	0.00	0.00	0.02%	0.00	0.19%	0.22%	0.17%	0.02%	0.09%	0.00	0.11%	0.01%	0.06%	0.00
2001	0.17%	0.16%	0.05%	0.10%	0.00	0.00	0.02%	0.00	0.18%	0.21%	0.16%	0.02%	0.09%	0.00	0.11%	0.01%	0.06%	0.00
2002	0.16%	0.16%	0.04%	0.09%	0.00	0.00	0.02%	0.00	0.19%	0.20%	0.16%	0.02%	0.09%	0.00	0.11%	0.01%	0.06%	0.00
2003	0.14%	0.14%	0.04%	0.09%	0.00	0.00	0.02%	0.00	0.18%	0.18%	0.15%	0.02%	0.08%	0.00	0.10%	0.01%	0.05%	0.00
2004	0.13%	0.13%	0.04%	0.08%	0.00	0.00	0.01%	0.00	0.18%	0.17%	0.14%	0.02%	0.08%	0.00	0.10%	0.01%	0.05%	0.00
2005	0.13%	0.14%	0.04%	0.09%	0.00	0.00	0.02%	0.00	0.18%	0.18%	0.14%	0.02%	0.08%	0.00	0.10%	0.01%	0.05%	0.00
2006	0.13%	0.14%	0.04%	0.09%	0.00	0.00	0.01%	0.00	0.18%	0.17%	0.14%	0.02%	0.08%	0.00	0.10%	0.01%	0.05%	0.00
2007	0.11%	0.15%	0.03%	0.09%	0.00	0.00	0.01%	0.00	0.18%	0.15%	0.13%	0.02%	0.08%	0.00	0.10%	0.01%	0.05%	0.00
2008	0.10%	0.14%	0.03%	0.09%	0.00	0.00	0.01%	0.00	0.20%	0.14%	0.13%	0.01%	0.08%	0.00	0.09%	0.01%	0.05%	0.00
2009	0.10%	0.13%	0.03%	0.08%	0.00	0.00	0.01%	0.00	0.20%	0.12%	0.11%	0.01%	0.07%	0.00	0.09%	0.01%	0.04%	0.00
2010	0.09%	0.12%	0.03%	0.08%	0.00	0.00	0.01%	0.00	0.21%	0.11%	0.10%	0.01%	0.07%	0.00	0.08%	0.00%	0.04%	0.00
2011	0.08%	0.12%	0.03%	0.08%	0.00	0.00	0.01%	0.00	0.23%	0.10%	0.09%	0.01%	0.06%	0.00	0.08%	0.00%	0.04%	0.00
2012	0.07%	0.11%	0.03%	0.08%	0.00	0.00	0.01%	0.00	0.24%	0.09%	0.08%	0.01%	0.06%	0.00	0.08%	0.00%	0.04%	0.00
2013	0.07%	0.11%	0.03%	0.08%	0.00	0.00	0.01%	0.00	0.25%	0.09%	0.08%	0.01%	0.06%	0.00	0.07%	0.00%	0.04%	0.00

Note: SIAB source: Research Data Centre (FDZ) of the Federal Employment Agency at the Institute for Employment Research, SIAB SUF 7514, 1993-2013, own calculations. MC source: Research Data Center of the Statistical Offices of the Länder and the Federal State, Microcensus, 1993-2013, own calculations.

Table A.2: Contribution of male job-findings (UE) to the overall flow of divorces

	$\widetilde{m}_{11t}^{ue}\widetilde{\tau}_{1t}^{u}$	$\widetilde{m}_{12t}^{ue}\widetilde{\tau}_{1t}^{u}$	$\widetilde{m}_{13t}^{ue}\widetilde{\tau}_{1t}^{u}$	$\widetilde{m}_{21t}^{ue}\widetilde{\tau}_{2t}^{u}$	$\widetilde{m}_{22t}^{ue}\widetilde{\tau}_{2t}^{u}$	$\widetilde{m}_{23t}^{ue}\widetilde{\tau}_{2t}^{u}$	$\widetilde{m}_{31t}^{ue}\widetilde{\tau}_{3t}^{u}$	$\widetilde{m}_{32t}^{ue}\widetilde{\tau}_{3t}^{u}$	$\widetilde{m}_{33t}^{ue}\widetilde{\tau}_{3t}^{u}$	$\widetilde{m}_{11t}^{uu}\widetilde{\tau}_{1t}^{u}$	$\widetilde{m}_{12t}^{uu}\widetilde{\tau}_{1t}^{u}$	$\widetilde{m}_{13t}^{uu}\widetilde{\tau}_{1t}^{u}$	$\widetilde{m}_{21t}^{uu}\widetilde{\tau}_{2t}^{u}$	$\widetilde{m}_{22t}^{uu}\widetilde{\tau}_{2t}^{u}$	$\widetilde{m}_{23t}^{uu}\widetilde{\tau}_{2t}^{u}$	$\widetilde{m}_{31t}^{uu}\widetilde{\tau}_{3t}^{u}$	$\widetilde{m}_{32t}^{uu}\widetilde{\tau}_{3t}^{u}$	$\widetilde{m}_{33t}^{uu}\widetilde{\tau}_{3t}^{u}$
1993	0.00	0.00	0.00	0.00	0.33%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1995	0.00	0.00	0.00	0.00	0.37%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1996	0.00	0.00	0.00	0.00	0.40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1997	0.00	0.00	0.00	0.00	0.43%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1998	0.00	0.00	0.00	0.00	0.46%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1999	0.00	0.00	0.00	0.00	0.48%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2000	0.00	0.00	0.00	0.00	0.49%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2001	0.00	0.00	0.00	0.00	0.53%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2002	0.00	0.00	0.00	0.00	0.55%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2003	0.00	0.00	0.00	0.00	0.56%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2004	0.00	0.00	0.00	0.00	0.55%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2005	0.00	0.00	0.00	0.00	0.58%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2006	0.00	0.00	0.00	0.00	0.60%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2007	0.00	0.00	0.00	0.00	0.61%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2008	0.00	0.00	0.00	0.00	0.61%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2009	0.00	0.00	0.00	0.00	0.66%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2010	0.00	0.00	0.00	0.00	0.68%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2011	0.00	0.00	0.00	0.00	0.67%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2012	0.00	0.00	0.00	0.00	0.68%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2013	0.00	0.00	0.00	0.00	0.74%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Note: SIAB source: Research Data Centre (FDZ) of the Federal Employment Agency at the Institute for Employment Research, SIAB SUF 7514, 1993-2013, own calculations. MC source: Research Data Center of the Statistical Offices of the Länder and the Federal State, Microcensus, 1993-2013, own calculations.

Table A.3: Contribution of female separations (EU) to the overall flow of divorces

	$\widetilde{m}^{ee}_{11t}\widetilde{\tau}^{e}_{1t}$	$\widetilde{m}^{ee}_{21t}\widetilde{\tau}^{e}_{1t}$	$\widetilde{m}^{ee}_{31t}\widetilde{\tau}^{e}_{1t}$	$\widetilde{m}^{ee}_{12t}\widetilde{\tau}^{e}_{2t}$	$\widetilde{m}^{ee}_{22t}\widetilde{\tau}^{e}_{2t}$	$\widetilde{m}^{ee}_{32t}\widetilde{\tau}^{e}_{2t}$	$\widetilde{m}^{ee}_{13t}\widetilde{\tau}^{e}_{3t}$	$\widetilde{m}^{ee}_{23t}\widetilde{\tau}^{e}_{3t}$	$\widetilde{m}^{ee}_{33t}\widetilde{\tau}^{e}_{3t}$	$\widetilde{m}^{ue}_{11t}\widetilde{\tau}^{e}_{1t}$	$\widetilde{m}^{ue}_{21t}\widetilde{\tau}^{e}_{1t}$	$\widetilde{m}^{ue}_{31t}\widetilde{\tau}^{e}_{1t}$	$\widetilde{m}^{ue}_{12t}\widetilde{\tau}^{e}_{2t}$	$\widetilde{m}^{ue}_{22t}\widetilde{\tau}^{e}_{2t}$	$\widetilde{m}^{ue}_{32t}\widetilde{\tau}^{e}_{2t}$	$\widetilde{m}^{ue}_{13t}\widetilde{\tau}^{e}_{3t}$	$\widetilde{m}^{ue}_{23t}\widetilde{\tau}^{e}_{3t}$	$\widetilde{m}^{ue}_{33t}\widetilde{\tau}^{e}_{3t}$
1993	0.16%	0.02%	0.00	0.00	0.00	0.19%	0.00	0.00	0.00	0.04%	0.02%	0.00	0.02%	0.00	0.02%	0.00	0.02%	0.00
1995	0.16%	0.02%	0.00	0.00	0.00	0.22%	0.00	0.00	0.00	0.04%	0.03%	0.00	0.02%	0.00	0.02%	0.00	0.02%	0.00
1996	0.21%	0.02%	0.00	0.00	0.00	0.22%	0.00	0.00	0.00	0.06%	0.03%	0.00	0.02%	0.00	0.02%	0.00	0.03%	0.00
1997	0.18%	0.02%	0.00	0.00	0.00	0.23%	0.00	0.00	0.00	0.06%	0.03%	0.00	0.02%	0.00	0.03%	0.00	0.03%	0.00
1998	0.17%	0.02%	0.00	0.00	0.00	0.24%	0.00	0.00	0.00	0.06%	0.03%	0.00	0.02%	0.00	0.03%	0.00	0.03%	0.00
1999	0.22%	0.02%	0.00	0.00	0.00	0.24%	0.00	0.00	0.00	0.07%	0.03%	0.00	0.02%	0.00	0.03%	0.00	0.03%	0.00
2000	0.23%	0.02%	0.00	0.00	0.00	0.26%	0.00	0.00	0.00	0.06%	0.03%	0.00	0.02%	0.00	0.03%	0.00	0.03%	0.00
2001	0.23%	0.02%	0.00	0.00	0.00	0.26%	0.00	0.00	0.00	0.06%	0.03%	0.00	0.02%	0.00	0.03%	0.00	0.03%	0.00
2002	0.21%	0.02%	0.00	0.00	0.00	0.26%	0.00	0.00	0.00	0.06%	0.03%	0.00	0.02%	0.00	0.03%	0.00	0.03%	0.00
2003	0.19%	0.01%	0.00	0.00	0.00	0.26%	0.00	0.00	0.00	0.06%	0.03%	0.00	0.02%	0.00	0.03%	0.00	0.03%	0.00
2004	0.17%	0.01%	0.00	0.00	0.00	0.25%	0.00	0.00	0.00	0.06%	0.03%	0.00	0.02%	0.00	0.04%	0.00	0.03%	0.00
2005	0.17%	0.01%	0.00	0.00	0.00	0.27%	0.00	0.00	0.00	0.06%	0.03%	0.00	0.02%	0.00	0.04%	0.00	0.03%	0.00
2006	0.17%	0.01%	0.00	0.00	0.00	0.26%	0.00	0.00	0.00	0.06%	0.03%	0.00	0.02%	0.00	0.04%	0.00	0.03%	0.00
2007	0.15%	0.02%	0.00	0.00	0.00	0.28%	0.00	0.00	0.00	0.05%	0.03%	0.00	0.02%	0.00	0.04%	0.00	0.03%	0.00
2008	0.14%	0.01%	0.00	0.00	0.00	0.30%	0.00	0.00	0.00	0.04%	0.03%	0.00	0.02%	0.00	0.04%	0.00	0.03%	0.00
2009	0.14%	0.01%	0.00	0.00	0.00	0.30%	0.00	0.00	0.00	0.04%	0.03%	0.00	0.02%	0.00	0.04%	0.00	0.03%	0.00
2010	0.12%	0.01%	0.00	0.00	0.00	0.30%	0.00	0.00	0.00	0.04%	0.02%	0.00	0.02%	0.00	0.04%	0.00	0.03%	0.00
2011	0.10%	0.01%	0.00	0.00	0.00	0.30%	0.00	0.00	0.00	0.03%	0.02%	0.00	0.02%	0.00	0.04%	0.00	0.03%	0.00
2012	0.10%	0.01%	0.00	0.00	0.00	0.31%	0.00	0.00	0.00	0.03%	0.02%	0.00	0.02%	0.00	0.04%	0.00	0.03%	0.00
2013	0.10%	0.01%	0.00	0.00	0.00	0.30%	0.00	0.00	0.00	0.03%	0.02%	0.00	0.02%	0.00	0.04%	0.00	0.03%	0.00

Note: SIAB source: Research Data Centre (FDZ) of the Federal Employment Agency at the Institute for Employment Research, SIAB SUF 7514, 1993-2013, own calculations. MC source: Research Data Center of the Statistical Offices of the Länder and the Federal State, Microcensus, 1993-2013, own calculations.

Table A.4: Contribution of female job-findings (UE) to the overall flow of divorces

	$\widetilde{m}^{eu}_{11t}\widetilde{\tau}^{u}_{1t}$	$\widetilde{m}^{eu}_{21t}\widetilde{\tau}^{u}_{1t}$	$\widetilde{m}^{eu}_{31t}\widetilde{\tau}^{u}_{1t}$	$\widetilde{m}^{eu}_{12t}\widetilde{\tau}^{u}_{2t}$	$\widetilde{m}^{eu}_{22t}\widetilde{\tau}^{u}_{2t}$	$\widetilde{m}^{eu}_{32t}\widetilde{\tau}^{u}_{2t}$	$\widetilde{m}^{eu}_{13t}\widetilde{\tau}^{u}_{3t}$	$\widetilde{m}^{eu}_{23t}\widetilde{\tau}^{u}_{3t}$	$\widetilde{m}^{eu}_{33t}\widetilde{\tau}^{u}_{3t}$	$\widetilde{m}^{uu}_{11t}\widetilde{\tau}^{u}_{1t}$	$\widetilde{m}^{uu}_{21t}\widetilde{\tau}^{u}_{1t}$	$\widetilde{m}^{uu}_{31t}\widetilde{\tau}^{u}_{1t}$	$\widetilde{m}^{uu}_{12t}\widetilde{\tau}^{u}_{2t}$	$\widetilde{m}^{uu}_{22t}\widetilde{\tau}^{u}_{2t}$	$\widetilde{m}^{uu}_{32t}\widetilde{\tau}^{u}_{2t}$	$\widetilde{m}^{uu}_{13t}\widetilde{\tau}^{u}_{3t}$	$\widetilde{m}^{uu}_{23t}\widetilde{\tau}^{u}_{3t}$	$\widetilde{m}^{uu}_{33t}\widetilde{\tau}^{u}_{3t}$
1993	0.00	0.00	0.14%	0.40%	1.98%	0.00	0.17%	0.00	0.25%	0.00	0.00	0.39%	0.00	0.02%	0.00	0.46%	0.00	0.38%
1995	0.00	0.00	0.11%	0.34%	1.77%	0.00	0.13%	0.00	0.24%	0.00	0.00	0.33%	0.00	0.02%	0.00	0.40%	0.00	0.45%
1996	0.00	0.00	0.11%	0.31%	1.57%	0.00	0.14%	0.00	0.22%	0.00	0.00	0.36%	0.00	0.02%	0.00	0.44%	0.00	0.47%
1997	0.00	0.00	0.10%	0.29%	1.54%	0.00	0.13%	0.00	0.24%	0.00	0.00	0.34%	0.00	0.02%	0.00	0.43%	0.00	0.53%
1998	0.00	0.00	0.09%	0.27%	1.52%	0.00	0.12%	0.00	0.25%	0.00	0.00	0.33%	0.00	0.02%	0.00	0.42%	0.00	0.58%
1999	0.00	0.00	0.08%	0.26%	1.44%	0.00	0.11%	0.00	0.24%	0.00	0.00	0.31%	0.00	0.02%	0.00	0.41%	0.00	0.56%
2000	0.00	0.00	0.08%	0.26%	1.45%	0.00	0.11%	0.00	0.25%	0.00	0.00	0.31%	0.00	0.02%	0.00	0.42%	0.00	0.62%
2001	0.00	0.00	0.07%	0.25%	1.44%	0.00	0.10%	0.00	0.26%	0.00	0.00	0.30%	0.00	0.03%	0.00	0.42%	0.00	0.66%
2002	0.00	0.00	0.07%	0.24%	1.43%	0.00	0.10%	0.00	0.26%	0.00	0.00	0.28%	0.00	0.03%	0.00	0.41%	0.00	0.73%
2003	0.00	0.00	0.06%	0.23%	1.34%	0.00	0.10%	0.00	0.27%	0.00	0.00	0.28%	0.00	0.03%	0.00	0.43%	0.00	0.79%
2004	0.00	0.00	0.06%	0.23%	1.31%	0.00	0.10%	0.00	0.28%	0.00	0.00	0.29%	0.00	0.03%	0.00	0.46%	0.00	0.87%
2005	0.00	0.00	0.06%	0.24%	1.38%	0.00	0.11%	0.00	0.29%	0.00	0.00	0.32%	0.00	0.03%	0.00	0.54%	0.00	1.04%
2006	0.00	0.00	0.06%	0.25%	1.43%	0.00	0.11%	0.00	0.29%	0.00	0.00	0.32%	0.00	0.03%	0.00	0.56%	0.00	1.09%
2007	0.00	0.00	0.06%	0.25%	1.47%	0.00	0.10%	0.00	0.31%	0.00	0.00	0.32%	0.00	0.03%	0.00	0.58%	0.00	1.24%
2008	0.00	0.00	0.05%	0.25%	1.42%	0.00	0.10%	0.00	0.34%	0.00	0.00	0.32%	0.00	0.03%	0.00	0.61%	0.00	1.42%
2009	0.00	0.00	0.06%	0.24%	1.35%	0.00	0.11%	0.00	0.36%	0.00	0.00	0.32%	0.00	0.03%	0.00	0.64%	0.00	1.55%
2010	0.00	0.00	0.05%	0.24%	1.35%	0.00	0.10%	0.00	0.38%	0.00	0.00	0.32%	0.00	0.03%	0.00	0.65%	0.00	1.69%
2011	0.00	0.00	0.05%	0.23%	1.32%	0.00	0.11%	0.00	0.40%	0.00	0.00	0.31%	0.00	0.03%	0.00	0.63%	0.00	1.75%
2012	0.00	0.00	0.05%	0.23%	1.32%	0.00	0.11%	0.00	0.42%	0.00	0.00	0.31%	0.00	0.04%	0.00	0.65%	0.00	1.93%
2013	0.00	0.00	0.05%	0.24%	1.31%	0.00	0.11%	0.00	0.45%	0.00	0.00	0.32%	0.00	0.04%	0.00	0.67%	0.00	2.10%

Note: SIAB source: Research Data Centre (FDZ) of the Federal Employment Agency at the Institute for Employment Research, SIAB SUF 7514, 1993-2013, own calculations. MC source: Research Data Center of the Statistical Offices of the Länder and the Federal State, Microcensus, 1993-2013, own calculations.

B Theoretical appendix

B.1 Surplus function

To obtain the suplus function, consider first the gain from marriage for a female of type j with labor market status l. i.e.,

$$
\begin{aligned}
& \left(r+\delta_{ij}+\tau_j(l)+\tau_i(-l)\right)\left[V_j^{l,-l}-V_j^l\right] \\
= \; & u_j^{l,-l}+\delta_{ij}\int \max\left[V_j^{l,-l}(z')-V_j^l,0\right]dG(z') \\
& +\tau_j(l)\max\left[V_j^{l',-l}-V_j^{l'},0\right]+\tau_i(-l)\max\left[V_j^{l,-l'}-V_j^l,0\right] \\
& -rV_j^l+\tau_j(l)\left[V_j^{l'}-V_j^l\right] \\
= \; & u_j^{l,-l}+\delta_{ij}\int \max\left[V_j^{l,-l}(z')-V_j^l,0\right]dG(z') \\
& +\tau_j(l)\max\left[V_j^{l',-l}-V_j^{l'},0\right]+\tau_i(-l)\max\left[V_j^{l,-l'}-V_j^l,0\right] \\
& -u_j^l-\lambda_{ij}\iiint\left[V_j^{l,-l'}(z')-V_j^l\right]W_{ij}^{ll}(z')\,dG(z')\,s(i,l')\,didl',
\end{aligned}
\tag{16}
$$

where the second equality follows from substituting rV_j^l using equation (7). The gain from marriage for the male partner is defined respectively. Take the per period utility functon $u_i^l(t_i,d_i,d_j|z)$ for couples as defined in equation (4)

$$
\begin{aligned}
u_i^{l,-l}(t_i,d_i,d_j|z) &= zF_{ij}^1(d_i,d_j)\left[R_i^l+\frac{\iota}{2}\left(R_j^{-l}-R_i^l\right)^2-t_i+T_i^l-d_i\right], \\
u_j^{l,-l}(t_j,d_j,d_i|z) &= zF_{ij}^1(d_i,d_j)\left[R_j^l+\frac{\iota}{2}\left(R_j^l-R_i^{-l}\right)^2-C_{ij}+t_i+T_j^l-d_j\right],
\end{aligned}
$$

where we used $t_i+t_j=C_{ij}$ to substitute t_j. Maximizing the Nash-Product (6) with respect to t_i impies,

$$
\begin{aligned}
\frac{1-\beta}{V_j^{l,-l}-V_j^l}\frac{\partial u_j^l(t_j,d_j,d_i|z)}{\partial t_i} &= \frac{\beta}{V_i^{l,-l}-V_i^l}\frac{\partial u_i^l(t_i,d_i,d_j|z)}{\partial t_i}, \\
(1-\beta)\left[V_i^{l,-l}-V_i^l\right] &= \beta\left[V_j^{l,-l}-V_j^l\right].
\end{aligned}
$$

Using the definition of the surplus in equation (8) allows us to write $\left[V_i^{l,-l}-V_i^l\right]=\beta S_{ij}^{ll}$ and $\left[V_j^{l,-l}-V_j^l\right]=(1-\beta)S_{ij}^{ll}$. Using the gain from marriage for a female of type j in equation (16) and the respective equation for a male of type i allows us to write the surplus for any employment status ll as stated in equation (9).

The optimal time input into the public good's production follows from differentiating the Nash-Product (6), i.e.,

$$
\begin{aligned}
\frac{1-\beta}{V_f^{ll}-V_j^l}\frac{\partial u_j^l\left(t_j,d_j,d_i|z\right)}{\partial d_i} &= -\frac{\beta}{V_m^{ll}-V_i^l}\frac{\partial u_j^l\left(t_j,d_j,d_i|z\right)}{\partial d_i} \\
z\frac{\partial F_{ij}^1\left(d_i,d_j\right)}{\partial d_i}\left[R_i^l+R_j^l+\iota\left(R_j^l-R_i^l\right)^2-C_{ij}+T_i^l-d_i+T_j^l-d_j\right] &= zF_{ij}^1\left(d_i,d_j\right)
\end{aligned}
$$

where the second line follows from taking into account that $(1-\beta)\left[V_m^{ll}-V_i^l\right]=\beta\left[V_f^{ll}-V_j^l\right]$. Under the assumption that the public good production functions are Stone-Geary, i.e.,

$$
F_{ij}^1\left(d_i,d_j\right)=Z_{ij}\left(d_i-D_i^1\right)^{K_i}\left(d_j-D_j^1\right)^{K_j},
$$

with $0<K_i+K_j<1$, we get,

$$
\frac{\partial F_{ij}^1\left(d_i,d_j\right)}{\partial d_i}=\frac{K_i}{d_i-D_i^1}F_{ij}^1\left(d_i,d_j\right).
$$

The optimal time inputs for the female and the male are given by,

$$
\begin{aligned}
\left(d_i-D_i\right) &= \frac{K_i}{1+K_j+K_i}\left[R_i^l+R_j^l+\iota\left(R_j^l-R_i^l\right)^2-C_{ij}+T_i^l-D_i^1+T_j^l-D_j^1\right], \\
\left(d_j-D_j\right) &= \frac{K_j}{1+K_j+K_i}\left[R_i^l+R_j^l+\iota\left(R_j^l-R_i^l\right)^2-C_{ij}+T_i^l-D_i^1+T_j^l-D_j^1\right].
\end{aligned}
$$

In equilibrium the public good is therefore given by,

$$
\begin{aligned}
F_{ij}^1\left(d_i,d_j\right) &= Z_{ij}\left(1+K_j+K_i\right)K\left[R_i^l+R_j^l+\iota\left(R_j^l-R_i^l\right)^2-C_{ij}+T_i^l-D_i^1+T_j^l-D_j^1\right]^{K_j+K_i} \\
\text{where } K &= \frac{\left(K_i\right)^{K_i}\left(K_j\right)^{K_j}}{\left(1+K_j+K_i\right)^{1+K_j+K_i}}.
\end{aligned}
$$

Maximized joint flow utility in a marriage is hence given by,

$$
u_i^{l,-l}+u_j^{l,-l}=zZ_{ij}K\left[R_i^l+R_j^l+\iota\left(R_j^l-R_i^l\right)^2-C_{ij}+T_i^l-D_i^1+T_j^l-D_j^1\right]^{1+K_j+K_i}.
$$

Given the flow utility function for singles in equation (3), and the public good production function $F_i^0\left(d_i\right)=\left(d_i-D_i^0\right)^{K_i^0}$ the optimal time input for the public good is hence given by $d_i-D_i^0=K_i^0$ and the maximized flow utility for a single male of type i by

$$
u_i^l=\left(K_i^0\right)^{K_i^0}\left[R_i^l+T_i^l-D_i^0-K_i^0\right].
$$

The maximized joint flow utility in a marriage $u_i^{l,-l}+u_j^{l,-l}$ and the maximized flow utility for the respective singles u_i^l and u_j^l give the maximized joint flow surplus of both partners in equation (10).

B.2 Computation of the fixed point

The first step to determine the surplus functions $S^{ll}_{ij}(z)$ and the cutoff values z^{ll}_{ij} is to compute integrated surpluses $\overline{S}^{l'l}_{z^{ll}_{ij}}$, where the subindex z^{ll}_{ij} indicates the support over which the surplus is integrated, i.e.,

$$\overline{S}^{l'l}_{z^{ll}_{ij}} \equiv \int_{z^{ll}_{ij}}^{\infty} S^{l'l}_{ij}(z'')\, dG(z'') .$$

Integrating the surplus functions (9) for the different labor market status combinations $\{ll, l'l, ll', l'l'\}$ over the support under consideration, i.e., support $\left[z^{ll}_{ij}, \infty\right]$ if the considered cutoff value is z^{ll}_{ij}, gives the following fixed-point equation for each labor market status combinations $\{ll, l'l, ll', l'l'\}$ given z^{ll}_{ij}, i.e.,

$$\begin{aligned}
(r+\delta_{ij}+\tau_i(l)+\tau_j(l))\,\overline{S}^{ll}_{z^{ll}_{ij}} \quad = \quad & \kappa_{ij}\left[W_{ij}+\psi^{ll}_{ij}\right]^{\kappa}\Phi\left(\frac{\sigma^2-\ln z^{ll}_{ij}}{\sigma}\right)e^{\frac{1}{2}\sigma^2} \\
& +\left(\delta_{ij}\overline{S}^{ll}_{z^{ll}_{ij}}-\widehat{\Theta}^{ll}_{ij}\right)\left[1-\Phi\left(\frac{\ln z^{ll}_{ij}}{\sigma}\right)\right] \\
& +\tau_i(l)\,\eta^{(ll,l'l)}_{ij}\overline{S}^{l'l}_{z^{ll}_{ij}}+\tau_i(l)\left(1-\eta^{(ll,l'l)}_{ij}\right)\overline{S}^{l'l}_{z^{l'l}_{ij}} \\
& +\tau_j(l)\,\eta^{(ll,ll')}_{ij}\overline{S}^{ll'}_{z^{ll}_{ij}}+\tau_j(l)\left(1-\eta^{(ll,ll')}_{ij}\right)\overline{S}^{ll'}_{z^{ll'}_{ij}},
\end{aligned}$$

$$\begin{aligned}
(r+\delta_{ij}+\tau_i(l')+\tau_j(l))\,\overline{S}^{l'l}_{z^{ll}_{ij}} \quad = \quad & \kappa_{ij}\left[W_{ij}+\psi^{l'l}_{ij}\right]^{\kappa}\Phi\left(\frac{\sigma^2-\ln z^{ll}_{ij}}{\sigma}\right)e^{\frac{1}{2}\sigma^2} \\
& +\left(\delta_{ij}\overline{S}^{l'l}_{z^{l'l}_{ij}}-\widehat{\Theta}^{l'l}_{ij}\right)\left[1-\Phi\left(\frac{\ln z^{ll}_{ij}}{\sigma}\right)\right] \\
& +\tau_i(l')\,\overline{S}^{ll}_{z^{ll}_{ij}} \\
& +\tau_j(l)\,\eta^{(ll,l'l')}_{ij}\overline{S}^{l'l'}_{z^{ll}_{ij}}+\tau_j(l)\left(1-\eta^{(ll,l'l')}_{ij}\right)\overline{S}^{l'l'}_{z^{l'l'}_{ij}},
\end{aligned}$$

$$\begin{aligned}
(r+\delta_{ij}+\tau_i(l)+\tau_j(l'))\,\overline{S}^{ll'}_{z^{ll}_{ij}} \quad = \quad & \kappa_{ij}\left[W_{ij}+\psi^{ll'}_{ij}\right]^{\kappa}\Phi\left(\frac{\sigma^2-\ln z^{ll}_{ij}}{\sigma}\right)e^{\frac{1}{2}\sigma^2} \\
& +\left(\delta_{ij}\overline{S}^{ll'}_{z^{ll'}_{ij}}-\widehat{\Theta}^{ll'}_{ij}\right)\left[1-\Phi\left(\frac{\ln z^{ll}_{ij}}{\sigma}\right)\right] \\
& +\tau_i(l)\,\eta^{(ll,l'l')}_{ij}\overline{S}^{l'l'}_{z^{ll}_{ij}}+\tau_i(l)\left(1-\eta^{(ll,l'l')}_{ij}\right)\overline{S}^{l'l'}_{z^{l'l'}_{ij}} \\
& +\tau_j(l')\,\overline{S}^{ll}_{z^{ll}_{ij}},
\end{aligned}$$

$$
\begin{aligned}
\left(r+\delta_{ij}+\tau_i\left(l'\right)+\tau_j\left(l'\right)\right)\overline{S}^{l'l'}_{z^{ll}_{ij}} \;\;=\;\; & \kappa_{ij}\left[W_{ij}+\psi^{l'l'}_{ij}\right]^{\kappa}\Phi\left(\frac{\sigma^2-\ln z^{ll}_{ij}}{\sigma}\right)e^{\frac{1}{2}\sigma^2} \\
& +\left(\delta_{ij}\overline{S}^{l'l'}_{z^{l'l'}_{ij}}-\widehat{\Theta}^{l'l'}_{ij}\right)\left[1-\Phi\left(\frac{\ln z^{ll}_{ij}}{\sigma}\right)\right] \\
& +\tau_i\left(l'\right)\eta^{(ll,ll')}_{ij}\overline{S}^{ll'}_{z^{ll}_{ij}}+\tau_i\left(l'\right)\left(1-\eta^{(ll,ll')}_{ij}\right)\overline{S}^{ll'}_{z^{ll'}_{ij}} \\
& +\tau_j\left(l'\right)\eta^{(ll,l'l)}_{ij}\overline{S}^{l'l}_{z^{ll}_{ij}}+\tau_j\left(l'\right)\left(1-\eta^{(ll,l'l)}_{ij}\right)\overline{S}^{l'l}_{z^{l'l}_{ij}},
\end{aligned}
$$

where

$$
\eta^{(ll,l'l)}_{ij}=\begin{cases}0 & \text{if } z^{ll}_{ij}\leq z^{l'l}_{ij},\\ 1 & \text{if } z^{ll}_{ij}>z^{l'l}_{ij},\end{cases}
$$

and

$$
\begin{aligned}
\widehat{\Theta}^{ll}_{ij} &= \kappa_i\left[w_i+\psi^{l}_{i}\right]+\kappa_j\left[w_j+\psi^{l}_{j}\right] \\
&+ \lambda_{ij}\left(1-\beta\right)\iint\overline{S}^{ll''}_{z^{ll''}_{ij}}s\left(j,l''\right)djdl''+\lambda_{ij}\beta\iint\overline{S}^{l''l}_{z^{l''l}_{ij}}s\left(i,l''\right)didl'', \\
\widehat{\Theta}^{l'l}_{ij} &= \kappa_i\left[w_i+\psi^{l'}_{i}\right]+\kappa_j\left[w_j+\psi^{l}_{j}\right] \\
&+ \lambda_{ij}\left(1-\beta\right)\iint\overline{S}^{l'l''}_{z^{l'l''}_{ij}}s\left(j,l''\right)djdl''+\lambda_{ij}\beta\iint\overline{S}^{l''l}_{z^{l''l}_{ij}}s\left(i,l''\right)didl'', \\
\widehat{\Theta}^{ll'}_{ij} &= \kappa_i\left[w_i+\psi^{l}_{i}\right]+\kappa_j\left[w_j+\psi^{l'}_{j}\right] \\
&+ \lambda_{ij}\left(1-\beta\right)\iint\overline{S}^{ll''}_{z^{ll''}_{ij}}s\left(j,l''\right)djdl''+\lambda_{ij}\beta\iint\overline{S}^{l''l'}_{z^{l''l'}_{ij}}s\left(i,l''\right)didl'', \\
\widehat{\Theta}^{l'l'}_{ij} &= \kappa_i\left[w_i+\psi^{l'}_{i}\right]+\kappa_j\left[w_j+\psi^{l'}_{j}\right] \\
&+ \lambda_{ij}\left(1-\beta\right)\iint\overline{S}^{l'l''}_{z^{l'l''}_{ij}}s\left(j,l''\right)djdl''+\lambda_{ij}\beta\iint\overline{S}^{l''l'}_{z^{l''l'}_{ij}}s\left(i,l''\right)didl''.
\end{aligned}
$$

These equations have to be solved simultaneously given a set of (initial) cutoff values $\left\{z^{ee}_{ij},z^{ue}_{ij},z^{eu}_{ij},z^{uu}_{ij}\right\}$. The values $\overline{S}^{l'l}_{z^{ll}_{ij}},\overline{S}^{ll'}_{z^{ll}_{ij}}$, and $\overline{S}^{l'l'}_{z^{ll}_{ij}}$ for each $z^{ll}_{ij}\in\left\{z^{ee}_{ij},z^{ue}_{ij},z^{eu}_{ij},z^{uu}_{ij}\right\}$ are not needed for further analysis. They are only required to find the fixed-points $\overline{S}^{ll}_{z^{ll}_{ij}}$ for each labor market status $ll\in\{ee,eu,ue,uu\}$. Given the fixed-points $\overline{S}^{ll}_{z^{ll}_{ij}}$ for each labor market status ll, we can use the following equation system based on the surplus function given in equation (9) to find the z_{ij}^{ll} associated with each labor market status ll, i.e.,

$$\begin{aligned}
0 &= z_{ij}^{ll}\kappa_{ij}\left[W_{ij}+\psi_{ij}^{ll}\right]^{\kappa}+\delta_{ij}\overline{S}_{z_{ij}^{ll}}^{ll}-\widehat{\Theta}_{ij}^{ll} \\
&\quad +\tau_i\left(l\right)\max\left[S_{ij}^{l'l}\left(z_{ij}^{ll}\right),0\right] \\
&\quad +\tau_j\left(l\right)\max\left[S_{ij}^{ll'}\left(z_{ij}^{ll}\right),0\right], \\
\left(r+\delta_{ij}+\tau_i\left(l'\right)+\tau_j\left(l\right)\right)S_{ij}^{l'l}\left(z_{ij}^{ll}\right) &= z_{ij}^{ll}\kappa_{ij}\left[W_{ij}+\psi_{ij}^{l'l}\right]^{\kappa}+\delta_{ij}\overline{S}_{z_{ij}^{l'l}}^{l'l}-\widehat{\Theta}_{ij}^{l'l} \\
&\quad +\tau_j\left(l\right)\max\left[S_{ij}^{l'l'}\left(z_{ij}^{ll}\right),0\right], \\
\left(r+\delta_{ij}+\tau_i\left(l\right)+\tau_j\left(l'\right)\right)S_{ij}^{ll'}\left(z_{ij}^{ll}\right) &= z_{ij}^{ll}\kappa_{ij}\left[W_{ij}+\psi_{ij}^{ll'}\right]^{\kappa}+\delta_{ij}\overline{S}_{z_{ij}^{ll'}}^{ll'}-\widehat{\Theta}_{ij}^{ll'} \\
&\quad +\tau_i\left(l\right)\max\left[S_{ij}^{l'l'}\left(z_{ij}^{ll}\right),0\right], \\
\left(r+\delta_{ij}+\tau_i\left(l'\right)+\tau_j\left(l'\right)\right)S_{ij}^{l'l'}\left(z_{ij}^{ll}\right) &= z_{ij}^{ll}\kappa_{ij}\left[W_{ij}+\psi_{ij}^{l'l'}\right]^{\kappa}+\delta_{ij}\overline{S}_{z_{ij}^{l'l'}}^{l'l'}-\widehat{\Theta}_{ij}^{l'l'} \\
&\quad +\tau_i\left(l'\right)\max\left[S_{ij}^{ll'}\left(z_{ij}^{ll}\right),0\right] \\
&\quad +\tau_j\left(l'\right)\max\left[S_{ij}^{l'l}\left(z_{ij}^{ll}\right),0\right],
\end{aligned}$$

where the zero in the first equation follows from $S_{ij}^{ll}\left(z_{ij}^{ll}\right)=0$. Again the values $S_{ij}^{l'l}\left(z_{ij}^{ll}\right)$, $S_{ij}^{ll'}\left(z_{ij}^{ll}\right)$, and $S_{ij}^{l'l'}\left(z_{ij}^{ll}\right)$ for each $z_{ij}^{ll}\in\left\{z_{ij}^{ee},z_{ij}^{ue},z_{ij}^{eu},z_{ij}^{uu}\right\}$ are not needed for further analysis. Iterating between the two equation systems while updating the (joint) distributions of married individuals as well as singles in every iteration using Equations (12) and (14) determines the fixed-point of the system for $\overline{S}_{z_{ij}^{ll}}^{ll}$ and z_{ij}^{ll} and each combination of labor market statuses $ll\in\{ee,eu,ue,uu\}$. In practice, a Python implementation of the model converges pretty fast, in less than one minute on a Chebyshev grid with 50×50 nodes.

B.3 Calibration

Table A.5: **Parameter values for the calibration of the marriage market model**

Parameter	Symbol	Value	Source
Discount rate	r	0.05	-
Women's bargaining power	β	0.7	-
Value of nonmarket activity	b	0.8	-
Joint taxation	ι	0.1	-
Meeting rate	λ	8.5	-
Bliss shock updates	δ	0.1	-
Mean of z distribution	$\mu(z)$	1.65.	-
Standard deviation of z distribution	$\sigma(z)$	4.67.	-
Male job-finding rate	$\tau_i(u)$	5.16	SIAB data
Female job-finding rate	$\tau_j(u)$	3.88	SIAB data
Male separation rate	$\tau_i(e)$	0.71	SIAB data
Female separation rate	$\tau_j(u)$	0.60	SIAB data
Home production, single	K_f^0	0.02	Goussé et al. (2017) estimate
Home production, single	K_m^0	0.00	Goussé et al. (2017) estimate
Home production, married	K_f^1	0.02	Mean of Goussé et al. (2017) estimate
Home production, married	K_m^1	0.01	Mean of Goussé et al. (2017) estimate
Home production, married	D_f^1	0.06	Mean of Goussé et al. (2017) estimate
Home production, married	D_m^1	0.06	Mean of Goussé et al. (2017) estimate

C Structural Estimation

The probabilities $\alpha_{ij}^{ll'}$ that a meeting of a type i male and type j female with labor market status l and l' leads to a marriage is estimated with a constrained linear equation system using the flow equations of $m(i,j,l,l')$ in equation (12). To simplify the notation below we define the data of period t as follows, $\widetilde{\tau}_{it}^{l} = \tau_i(l)$, $\widetilde{m}_{ijt}^{ll} = m(i,j,l,l)$ and $\widetilde{s}_{it}^{l} = s(i,l)$. The system of four equations, one for each labor market status $ll' \in \{ee, eu, ue, uu\}$ can be written in the following matrix notation, $\mathbf{y}_{ijt} = \mathbf{Z}_{ijt}\mathbf{b}_{ij} + \epsilon_{ijt}$. Given the data on labor market transition rates for males and females, $\widetilde{\tau}_{it}^{l}$ and $\widetilde{\tau}_{jt}^{l}$, and the number of singles and married couples, $\widetilde{s}_{it}^{l}$,$\widetilde{s}_{jt}^{l}$ and $\widetilde{m}_{ijt}^{ll}$, the LHS of the equation system is given by the vector,

$$\mathbf{y}_{ijt} = \begin{pmatrix} y_{ijt}^{1} \\ y_{ijt}^{2} \\ y_{ijt}^{3} \\ y_{ijt}^{4} \end{pmatrix} = \begin{pmatrix} \left(\widetilde{\tau}_{it}^{e} + \widetilde{\tau}_{jt}^{e}\right)\widetilde{m}_{ijt}^{ee} \\ \left(\widetilde{\tau}_{it}^{u} + \widetilde{\tau}_{jt}^{e}\right)\widetilde{m}_{ijt}^{ue} \\ \left(\widetilde{\tau}_{it}^{e} + \widetilde{\tau}_{jt}^{u}\right)\widetilde{m}_{ijt}^{eu} \\ \left(\widetilde{\tau}_{it}^{u} + \widetilde{\tau}_{jt}^{u}\right)\widetilde{m}_{ijt}^{uu} \end{pmatrix}.$$

The RHS is given by

$$\mathbf{Z}_{ijt} = \begin{pmatrix} \mathbf{Z}_{ijt}^{1} & 0 & 0 & 0 \\ 0 & \mathbf{Z}_{ijt}^{2} & 0 & 0 \\ 0 & 0 & \mathbf{Z}_{ijt}^{3} & 0 \\ 0 & 0 & 0 & \mathbf{Z}_{ijt}^{4} \end{pmatrix}$$

with

$$\begin{aligned}
\mathbf{Z}_{ijt}^{1} &= \begin{pmatrix} z_{ijt}^{11} & z_{ijt}^{12} & z_{ijt}^{13} & z_{ijt}^{14} \end{pmatrix} = \begin{pmatrix} \widetilde{\tau}_{it}^{u}\widetilde{m}_{ijt}^{ue} & \widetilde{\tau}_{jt}^{u}\widetilde{m}_{ijt}^{eu} & \widetilde{s}_{it}^{e}\widetilde{s}_{jt}^{e} & \widetilde{m}_{ijt}^{ee} \end{pmatrix} \\
\mathbf{Z}_{ijt}^{2} &= \begin{pmatrix} z_{ijt}^{21} & z_{ijt}^{22} & z_{ijt}^{23} & z_{ijt}^{24} \end{pmatrix} = \begin{pmatrix} \widetilde{\tau}_{it}^{e}\widetilde{m}_{ijt}^{ee} & \widetilde{\tau}_{jt}^{u}\widetilde{m}_{ijt}^{uu} & \widetilde{s}_{it}^{u}\widetilde{s}_{jt}^{e} & \widetilde{m}_{ijt}^{ue} \end{pmatrix} \\
\mathbf{Z}_{ijt}^{3} &= \begin{pmatrix} z_{ijt}^{31} & z_{ijt}^{32} & z_{ijt}^{33} & z_{ijt}^{34} \end{pmatrix} = \begin{pmatrix} \widetilde{\tau}_{it}^{u}\widetilde{m}_{ijt}^{uu} & \widetilde{\tau}_{jt}^{e}\widetilde{m}_{ijt}^{ee} & \widetilde{s}_{it}^{e}\widetilde{s}_{jt}^{u} & \widetilde{m}_{ijt}^{eu} \end{pmatrix} \\
\mathbf{Z}_{ijt}^{4} &= \begin{pmatrix} z_{ijt}^{21} & z_{ijt}^{22} & z_{ijt}^{23} & z_{ijt}^{24} \end{pmatrix} = \begin{pmatrix} \widetilde{\tau}_{it}^{e}\widetilde{m}_{ijt}^{eu} & \widetilde{\tau}_{jt}^{e}\widetilde{m}_{ijt}^{ue} & \widetilde{s}_{it}^{u}\widetilde{s}_{jt}^{u} & \widetilde{m}_{ijt}^{uu} \end{pmatrix}
\end{aligned}$$

and the coefficient vector,

$$\mathbf{b}_{ij} = \begin{pmatrix} \beta_{ij}^{1\,\prime} \\ \beta_{ij}^{2\,\prime} \\ \beta_{ij}^{3\,\prime} \\ \beta_{ij}^{4\,\prime} \end{pmatrix}$$

with

$$
\begin{aligned}
\beta_{ij}^{1} &= \begin{pmatrix} \beta_{ij}^{11} & \beta_{ij}^{12} & \beta_{ij}^{13} & \beta_{ij}^{14} \end{pmatrix} \\
&= \begin{pmatrix} \min\left[\left(\alpha_{ij}^{ee}/\alpha_{ij}^{ue}\right),1\right] & \min\left[\left(\alpha_{ij}^{ee}/\alpha_{ij}^{eu}\right),1\right] & \lambda_{ij}\alpha_{ij}^{ee} & -\delta_{ij}\left(1-\alpha_{ij}^{ee}\right) \end{pmatrix} \\
\beta_{ij}^{2} &= \begin{pmatrix} \beta_{ij}^{21} & \beta_{ij}^{22} & \beta_{ij}^{23} & \beta_{ij}^{24} \end{pmatrix} \\
&= \begin{pmatrix} \min\left[\left(\alpha_{ij}^{ue}/\alpha_{ij}^{ee}\right),1\right] & \min\left[\left(\alpha_{ij}^{ue}/\alpha_{ij}^{uu}\right),1\right] & \lambda_{ij}\alpha_{ij}^{ue} & -\delta_{ij}\left(1-\alpha_{ij}^{ue}\right) \end{pmatrix} \\
\beta_{ij}^{3} &= \begin{pmatrix} \beta_{ij}^{31} & \beta_{ij}^{32} & \beta_{ij}^{33} & \beta_{ij}^{34} \end{pmatrix} \\
&= \begin{pmatrix} \min\left[\left(\alpha_{ij}^{eu}/\alpha_{ij}^{uu}\right),1\right] & \min\left[\left(\alpha_{ij}^{eu}/\alpha_{ij}^{ee}\right),1\right] & \lambda_{ij}\alpha_{ij}^{eu} & -\delta_{ij}\left(1-\alpha_{ij}^{eu}\right) \end{pmatrix} \\
\beta_{ij}^{4} &= \begin{pmatrix} \beta_{ij}^{41} & \beta_{ij}^{42} & \beta_{ij}^{43} & \beta_{ij}^{44} \end{pmatrix} \\
&= \begin{pmatrix} \min\left[\left(\alpha_{ij}^{uu}/\alpha_{ij}^{eu}\right),1\right] & \min\left[\left(\alpha_{ij}^{uu}/\alpha_{ij}^{ue}\right),1\right] & \lambda_{ij}\alpha_{ij}^{uu} & -\delta_{ij}\left(1-\alpha_{ij}^{uu}\right) \end{pmatrix}
\end{aligned}
$$

The coefficient matrix implies the following sets of constraints- which ensure $\alpha_{ij}^{ll} \leq 1$,

$$
\begin{aligned}
&1 \geq \beta_{ij}^{11} > 0, \quad 1 \geq \beta_{ij}^{12} > 0, \quad \beta_{ij}^{13} > 0, \quad \beta_{ij}^{14} < 0, \\
&1 \geq \beta_{ij}^{21} > 0, \quad 1 \geq \beta_{ij}^{22} > 0, \quad \beta_{ij}^{23} > 0, \quad \beta_{ij}^{24} < 0, \\
&1 \geq \beta_{ij}^{31} > 0, \quad 1 \geq \beta_{ij}^{32} > 0, \quad \beta_{ij}^{33} > 0, \quad \beta_{ij}^{34} < 0, \\
&1 \geq \beta_{ij}^{41} > 0, \quad 1 \geq \beta_{ij}^{42} > 0, \quad \beta_{ij}^{43} > 0, \quad \beta_{ij}^{44} < 0,
\end{aligned}
$$

$$
\begin{aligned}
\beta_{ij}^{11} &= \frac{\beta_{ij}^{13}}{\beta_{ij}^{23}} \text{ if } \frac{\beta_{ij}^{14}}{\beta_{ij}^{24}} > 1 \text{ and } \beta_{ij}^{11} = 1 \text{ otherwise,} \\
\beta_{ij}^{21} &= \frac{\beta_{ij}^{23}}{\beta_{ij}^{13}} \text{ if } \frac{\beta_{ij}^{24}}{\beta_{ij}^{14}} > 1 \text{ and } \beta_{ij}^{21} = 1 \text{ otherwise,} \\
\beta_{ij}^{31} &= \frac{\beta_{ij}^{33}}{\beta_{ij}^{43}} \text{ if } \frac{\beta_{ij}^{34}}{\beta_{ij}^{44}} > 1 \text{ and } \beta_{ij}^{31} = 1 \text{ otherwise,} \\
\beta_{ij}^{41} &= \frac{\beta_{ij}^{43}}{\beta_{ij}^{33}} \text{ if } \frac{\beta_{ij}^{44}}{\beta_{ij}^{34}} > 1 \text{ and } \beta_{ij}^{41} = 1 \text{ otherwise,} \\
\beta_{ij}^{12} &= \frac{\beta_{ij}^{13}}{\beta_{ij}^{33}} \text{ if } \frac{\beta_{ij}^{14}}{\beta_{ij}^{34}} > 1 \text{ and } \beta_{ij}^{12} = 1 \text{ otherwise,} \\
\beta_{ij}^{22} &= \frac{\beta_{ij}^{23}}{\beta_{ij}^{43}} \text{ if } \frac{\beta_{ij}^{24}}{\beta_{ij}^{44}} > 1 \text{ and } \beta_{ij}^{22} = 1 \text{ otherwise,} \\
\beta_{ij}^{32} &= \frac{\beta_{ij}^{33}}{\beta_{ij}^{13}} \text{ if } \frac{\beta_{ij}^{34}}{\beta_{ij}^{14}} > 1 \text{ and } \beta_{ij}^{32} = 1 \text{ otherwise,} \\
\beta_{ij}^{42} &= \frac{\beta_{ij}^{43}}{\beta_{ij}^{23}} \text{ if } \frac{\beta_{ij}^{44}}{\beta_{ij}^{24}} > 1 \text{ and } \beta_{ij}^{42} = 1 \text{ otherwise,}
\end{aligned}
$$

$$
\begin{aligned}
&\beta_{ij}^{14} \text{ is free,} \\
&\beta_{ij}^{24} \text{ is free,} \\
\beta_{ij}^{34} &= \frac{\beta_{ij}^{14}\beta_{ij}^{23} - \beta_{ij}^{24}\beta_{ij}^{13}}{\beta_{ij}^{23} - \beta_{ij}^{13}} - \frac{\beta_{ij}^{24} - \beta_{ij}^{14}}{\beta_{ij}^{23} - \beta_{ij}^{13}}\beta_{ij}^{33}, \\
\beta_{ij}^{44} &= \frac{\beta_{ij}^{14}\beta_{ij}^{23} - \beta_{ij}^{24}\beta_{ij}^{13}}{\beta_{ij}^{23} - \beta_{ij}^{13}} - \frac{\beta_{ij}^{24} - \beta_{ij}^{14}}{\beta_{ij}^{23} - \beta_{ij}^{13}}\beta_{ij}^{43}. \\
\lambda_{ij} &= \frac{\beta_{ij}^{14}\beta_{ij}^{23} - \beta_{ij}^{24}\beta_{ij}^{13}}{\beta_{ij}^{24} - \beta_{ij}^{14}} \\
-\delta_{ij} &= \frac{\beta_{ij}^{14}\beta_{ij}^{23} - \beta_{ij}^{24}\beta_{ij}^{13}}{\beta_{ij}^{23} - \beta_{ij}^{13}}
\end{aligned}
$$

Or alternatively,

$$
\begin{aligned}
&\beta_{ij}^{24} \text{ is free,} \\
&\beta_{ij}^{34} \text{ is free,} \\
\beta_{ij}^{14} &= \frac{\beta_{ij}^{24}\beta_{ij}^{33} - \beta_{ij}^{34}\beta_{ij}^{23}}{\beta_{ij}^{33} - \beta_{ij}^{23}} - \frac{\beta_{ij}^{24} - \beta_{ij}^{34}}{\beta_{ij}^{33} - \beta_{ij}^{23}}\beta_{ij}^{13}, \\
\beta_{ij}^{44} &= \frac{\beta_{ij}^{24}\beta_{ij}^{33} - \beta_{ij}^{34}\beta_{ij}^{23}}{\beta_{ij}^{33} - \beta_{ij}^{23}} - \frac{\beta_{ij}^{24} - \beta_{ij}^{34}}{\beta_{ij}^{33} - \beta_{ij}^{23}}\beta_{ij}^{43}, \\
\lambda_{ij} &= \frac{\beta_{ij}^{24}\beta_{ij}^{33} - \beta_{ij}^{34}\beta_{ij}^{23}}{\beta_{ij}^{24} - \beta_{ij}^{34}}, \\
-\delta_{ij} &= \frac{\beta_{ij}^{24}\beta_{ij}^{33} - \beta_{ij}^{34}\beta_{ij}^{23}}{\beta_{ij}^{33} - \beta_{ij}^{23}},
\end{aligned}
$$

or

$$
\begin{aligned}
&\beta_{ij}^{34} \text{ is free,} \\
&\beta_{ij}^{44} \text{ is free,} \\
\beta_{ij}^{14} &= \frac{\beta_{ij}^{34}\beta_{ij}^{43} - \beta_{ij}^{44}\beta_{ij}^{33}}{\beta_{ij}^{43} - \beta_{ij}^{33}} - \frac{\beta_{ij}^{34} - \beta_{ij}^{44}}{\beta_{ij}^{43} - \beta_{ij}^{33}}\beta_{ij}^{13} \\
\beta_{ij}^{24} &= \frac{\beta_{ij}^{34}\beta_{ij}^{43} - \beta_{ij}^{44}\beta_{ij}^{33}}{\beta_{ij}^{43} - \beta_{ij}^{33}} - \frac{\beta_{ij}^{34} - \beta_{ij}^{44}}{\beta_{ij}^{43} - \beta_{ij}^{33}}\beta_{ij}^{23} \\
\lambda_{ij} &= \frac{\beta_{ij}^{34}\beta_{ij}^{43} - \beta_{ij}^{44}\beta_{ij}^{33}}{\beta_{ij}^{34} - \beta_{ij}^{44}} \\
-\delta_{ij} &= \frac{\beta_{ij}^{34}\beta_{ij}^{43} - \beta_{ij}^{44}\beta_{ij}^{33}}{\beta_{ij}^{43} - \beta_{ij}^{33}}
\end{aligned}
$$

The α_{ij}^{ll} are obtained by the following equations,

$$\begin{aligned}
\alpha_{ij}^{ee} &= \frac{\beta_{ij}^{24}-\beta_{ij}^{14}}{\beta_{ij}^{24}-\beta_{ij}^{14}\beta_{ij}^{21}} \text{ if } \beta_{ij}^{11}=1 \text{ and } \beta_{ij}^{11}\frac{\beta_{ij}^{14}-\beta_{ij}^{24}}{\beta_{ij}^{14}-\beta_{ij}^{24}\beta_{ij}^{11}} \text{ otherwise,}\\
\alpha_{ij}^{ue} &= \frac{\beta_{ij}^{14}-\beta_{ij}^{24}}{\beta_{ij}^{14}-\beta_{ij}^{24}\beta_{ij}^{11}} \text{ if } \beta_{ij}^{21}=1 \text{ and } \beta_{ij}^{21}\frac{\beta_{ij}^{24}-\beta_{ij}^{14}}{\beta_{ij}^{24}-\beta_{ij}^{14}\beta_{ij}^{21}} \text{ otherwise,}\\
\alpha_{ij}^{eu} &= \frac{\beta_{ij}^{44}-\beta_{ij}^{34}}{\beta_{ij}^{44}-\beta_{ij}^{34}\beta_{ij}^{41}} \text{ if } \beta_{ij}^{31}=1 \text{ and } \beta_{ij}^{31}\frac{\beta_{ij}^{34}-\beta_{ij}^{44}}{\beta_{ij}^{34}-\beta_{ij}^{44}\beta_{ij}^{31}} \text{ otherwise,}\\
\alpha_{ij}^{uu} &= \frac{\beta_{ij}^{34}-\beta_{ij}^{44}}{\beta_{ij}^{34}-\beta_{ij}^{44}\beta_{ij}^{31}} \text{ if } \beta_{ij}^{41}=1 \text{ and } \beta_{ij}^{41}\frac{\beta_{ij}^{44}-\beta_{ij}^{34}}{\beta_{ij}^{44}-\beta_{ij}^{34}\beta_{ij}^{41}} \text{ otherwise,}
\end{aligned}$$

Given α_{ij}^{ll} one can obtain the λ_{ij} and δ_{ij} from the third and fourth row.

The aggregate number of marriages is according to our theory (compare the outflow from singlehood in equation (13)) given by,

$$\widetilde{\Lambda}_t = \xi\left(\widetilde{s}_{mt}\right)^{-\frac{1}{2}}\left(\widetilde{s}_{ft}\right)^{-\frac{1}{2}}\iiiint \alpha_{ij}^{l''l}\widetilde{s}_{it}^{l''}\widetilde{s}_{jt}^{l}didjdl''dl,$$

the aggregate number of divorces (compare the inflow into singlehood in equation (13)) by

$$\begin{aligned}
\widetilde{\Delta}_t &= \delta\iiiint\left(1-\alpha_{ij}^{l''l}\right)\widetilde{m}_{ijt}^{l''l}didjdl''dl\\
&+\iiiint \widetilde{\tau}_{it}^{l''}\max\left[1-\left(\alpha_{ij}^{l'l}/\alpha_{ij}^{l''l}\right),0\right]\widetilde{m}_{ijt}^{l''l}didjdl''dl\\
&+\iiiint \widetilde{\tau}_{jt}^{l}\max\left[1-\left(\alpha_{ij}^{l''l'}/\alpha_{ij}^{l''l}\right),0\right]\widetilde{m}_{ijt}^{l''l}didjdl''dl.
\end{aligned}$$

C.0.1 Dynamic version

$$\begin{aligned}
\frac{\widetilde{m}_{ijt+1}^{ll}-\widetilde{m}_{ijt}^{ll}}{\widetilde{m}_{ijt}^{ll}} &= -\delta_{ij}\left(1-\alpha_{ij}^{ll}\right)-\left(\widetilde{\tau}_{it}^{l}+\widetilde{\tau}_{jt}^{l}\right)+\lambda_{ij}\alpha_{ij}^{ll}\frac{\widetilde{s}_{it}^{l}\widetilde{s}_{jt}^{l}}{\widetilde{m}_{ijt}^{ll}}\\
&+\min\left[\left(\alpha_{ij}^{ll}/\alpha_{ij}^{l'l}\right),1\right]\widetilde{\tau}_{it}^{l'}\frac{\widetilde{m}_{ijt}^{l'l}}{\widetilde{m}_{ijt}^{ll}}+\min\left[\left(\alpha_{ij}^{ll}/\alpha_{ij}^{ll'}\right),1\right]\widetilde{\tau}_{jt}^{l'}\frac{\widetilde{m}_{ijt}^{ll'}}{\widetilde{m}_{ijt}^{ll}}. \quad (17)
\end{aligned}$$

$$\mathbf{y}_{ijt}=\begin{pmatrix} y_{ijt}^{1}\\ y_{ijt}^{2}\\ y_{ijt}^{3}\\ y_{ijt}^{4}\end{pmatrix}=\begin{pmatrix}\frac{\widetilde{m}_{ijt+1}^{ee}-\widetilde{m}_{ijt}^{ee}}{\widetilde{m}_{ijt}^{ee}}\\ \frac{\widetilde{m}_{ijt+1}^{ue}-\widetilde{m}_{ijt}^{ue}}{\widetilde{m}_{ijt}^{ue}}\\ \frac{\widetilde{m}_{ijt+1}^{eu}-\widetilde{m}_{ijt}^{eu}}{\widetilde{m}_{ijt}^{eu}}\\ \frac{\widetilde{m}_{ijt+1}^{uu}-\widetilde{m}_{ijt}^{uu}}{\widetilde{m}_{ijt}^{uu}}\end{pmatrix}.$$

and

$$
\begin{aligned}
\mathbf{Z}^1_{ijt} &= \begin{pmatrix} z^{11}_{ijt} & z^{12}_{ijt} & z^{13}_{ijt} & z^{14}_{ijt} & z^{15}_{ijt} \end{pmatrix} = \begin{pmatrix} 1 & \left(\widetilde{\tau}^e_{it} + \widetilde{\tau}^e_{jt}\right) & \frac{\widetilde{s}^e_{it}\widetilde{s}^e_{jt}}{\widetilde{m}^{ee}_{ijt}} & \widetilde{\tau}^u_{it}\frac{\widetilde{m}^{ue}_{ijt}}{\widetilde{m}^{ee}_{ijt}} & \widetilde{\tau}^u_{jt}\frac{\widetilde{m}^{eu}_{ijt}}{\widetilde{m}^{ee}_{ijt}} \end{pmatrix} \\
\mathbf{Z}^2_{ijt} &= \begin{pmatrix} z^{21}_{ijt} & z^{22}_{ijt} & z^{23}_{ijt} & z^{24}_{ijt} & z^{25}_{ijt} \end{pmatrix} = \begin{pmatrix} 1 & \left(\widetilde{\tau}^u_{it} + \widetilde{\tau}^e_{jt}\right) & \frac{\widetilde{s}^u_{it}\widetilde{s}^e_{jt}}{\widetilde{m}^{ue}_{ijt}} & \widetilde{\tau}^e_{it}\frac{\widetilde{m}^{ee}_{ijt}}{\widetilde{m}^{ue}_{ijt}} & \widetilde{\tau}^u_{jt}\frac{\widetilde{m}^{uu}_{ijt}}{\widetilde{m}^{ue}_{ijt}} \end{pmatrix} \\
\mathbf{Z}^3_{ijt} &= \begin{pmatrix} z^{31}_{ijt} & z^{32}_{ijt} & z^{33}_{ijt} & z^{34}_{ijt} & z^{35}_{ijt} \end{pmatrix} = \begin{pmatrix} 1 & \left(\widetilde{\tau}^e_{it} + \widetilde{\tau}^u_{jt}\right) & \frac{\widetilde{s}^e_{it}\widetilde{s}^u_{jt}}{\widetilde{m}^{eu}_{ijt}} & \widetilde{\tau}^u_{it}\frac{\widetilde{m}^{uu}_{ijt}}{\widetilde{m}^{eu}_{ijt}} & \widetilde{\tau}^e_{jt}\frac{\widetilde{m}^{ee}_{ijt}}{\widetilde{m}^{eu}_{ijt}} \end{pmatrix} \\
\mathbf{Z}^4_{ijt} &= \begin{pmatrix} z^{41}_{ijt} & z^{42}_{ijt} & z^{43}_{ijt} & z^{44}_{ijt} & z^{45}_{ijt} \end{pmatrix} = \begin{pmatrix} 1 & \left(\widetilde{\tau}^u_{it} + \widetilde{\tau}^u_{jt}\right) & \frac{\widetilde{s}^u_{it}\widetilde{s}^u_{jt}}{\widetilde{m}^{uu}_{ijt}} & \widetilde{\tau}^e_{it}\frac{\widetilde{m}^{eu}_{ijt}}{\widetilde{m}^{uu}_{ijt}} & \widetilde{\tau}^e_{jt}\frac{\widetilde{m}^{ue}_{ijt}}{\widetilde{m}^{uu}_{ijt}} \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\beta^1_{ij} &= \begin{pmatrix} \beta^{11}_{ij} & \beta^{12}_{ij} & \beta^{13}_{ij} & \beta^{14}_{ij} & \beta^{15}_{ij} \end{pmatrix} \\
&= \begin{pmatrix} -\delta_{ij}\left(1-\alpha^{ee}_{ij}\right) & 1 & \lambda_{ij}\alpha^{ee}_{ij} & \min\left[\left(\alpha^{ee}_{ij}/\alpha^{ue}_{ij}\right), 1\right] & \min\left[\left(\alpha^{ee}_{ij}/\alpha^{eu}_{ij}\right), 1\right] \end{pmatrix} \\
\beta^2_{ij} &= \begin{pmatrix} \beta^{21}_{ij} & \beta^{22}_{ij} & \beta^{23}_{ij} & \beta^{24}_{ij} & \beta^{25}_{ij} \end{pmatrix} \\
&= \begin{pmatrix} -\delta_{ij}\left(1-\alpha^{ue}_{ij}\right) & 1 & \lambda_{ij}\alpha^{ue}_{ij} & \min\left[\left(\alpha^{ue}_{ij}/\alpha^{ee}_{ij}\right), 1\right] & \min\left[\left(\alpha^{ue}_{ij}/\alpha^{uu}_{ij}\right), 1\right] \end{pmatrix} \\
\beta^3_{ij} &= \begin{pmatrix} \beta^{31}_{ij} & \beta^{32}_{ij} & \beta^{33}_{ij} & \beta^{34}_{ij} & \beta^{35}_{ij} \end{pmatrix} \\
&= \begin{pmatrix} -\delta_{ij}\left(1-\alpha^{eu}_{ij}\right) & 1 & \lambda_{ij}\alpha^{eu}_{ij} & \min\left[\left(\alpha^{eu}_{ij}/\alpha^{uu}_{ij}\right), 1\right] & \min\left[\left(\alpha^{eu}_{ij}/\alpha^{ee}_{ij}\right), 1\right] \end{pmatrix} \\
\beta^4_{ij} &= \begin{pmatrix} \beta^{41}_{ij} & \beta^{42}_{ij} & \beta^{43}_{ij} & \beta^{44}_{ij} & \beta^{45}_{ij} \end{pmatrix} \\
&= \begin{pmatrix} -\delta_{ij}\left(1-\alpha^{uu}_{ij}\right) & 1 & \lambda_{ij}\alpha^{uu}_{ij} & \min\left[\left(\alpha^{uu}_{ij}/\alpha^{eu}_{ij}\right), 1\right] & \min\left[\left(\alpha^{uu}_{ij}/\alpha^{ue}_{ij}\right), 1\right] \end{pmatrix}
\end{aligned}
$$

$$
\begin{gathered}
\beta^{11}_{ij} < 0, \quad \beta^{12}_{ij} = 1, \quad \beta^{13}_{ij} > 0, \\
\beta^{21}_{ij} < 0, \quad \beta^{22}_{ij} = 1, \quad \beta^{23}_{ij} > 0, \\
\beta^{31}_{ij} < 0, \quad \beta^{32}_{ij} = 1, \quad \beta^{33}_{ij} > 0, \\
\beta^{41}_{ij} < 0, \quad \beta^{42}_{ij} = 1, \quad \beta^{43}_{ij} > 0, \\
\beta^{13}_{ij}, \beta^{23}_{ij}, \beta^{33}_{ij}, \beta^{43}_{ij} \text{ are free,}
\end{gathered}
$$

$$
\begin{aligned}
& \beta_{ij}^{21} \text{ is free,} \\
& \beta_{ij}^{31} \text{ is free,} \\
\beta_{ij}^{11} &= \frac{\beta_{ij}^{33}\beta_{ij}^{21} - \beta_{ij}^{23}\beta_{ij}^{31} - \beta_{ij}^{13}\beta_{ij}^{21} + \beta_{ij}^{13}\beta_{ij}^{31}}{\beta_{ij}^{33} - \beta_{ij}^{23}}, \\
\beta_{ij}^{41} &= \frac{\beta_{ij}^{33}\beta_{ij}^{21} - \beta_{ij}^{23}\beta_{ij}^{31} - \beta_{ij}^{43}\beta_{ij}^{21} + \beta_{ij}^{43}\beta_{ij}^{31}}{\beta_{ij}^{33} - \beta_{ij}^{23}}, \\
\lambda_{ij} &= \frac{\beta_{ij}^{33}\beta_{ij}^{21} - \beta_{ij}^{23}\beta_{ij}^{31}}{\beta_{ij}^{21} - \beta_{ij}^{31}}, \\
-\delta_{ij} &= \frac{\beta_{ij}^{33}\beta_{ij}^{21} - \beta_{ij}^{23}\beta_{ij}^{31}}{\beta_{ij}^{33} - \beta_{ij}^{23}},
\end{aligned}
$$

$$
\begin{aligned}
\beta_{ij}^{14} &= \frac{\beta_{ij}^{13}}{\beta_{ij}^{23}} \text{ if } \frac{\beta_{ij}^{11}}{\beta_{ij}^{21}} > 1 > \frac{\beta_{ij}^{13}}{\beta_{ij}^{23}},\ \beta_{ij}^{14} = 1 \text{ if } \frac{\beta_{ij}^{11}}{\beta_{ij}^{21}} < 1 < \frac{\beta_{ij}^{13}}{\beta_{ij}^{23}}, \text{ otherwise } 1 \geq \beta_{ij}^{14} > 0, \\
\beta_{ij}^{15} &= \frac{\beta_{ij}^{13}}{\beta_{ij}^{33}} \text{ if } \frac{\beta_{ij}^{11}}{\beta_{ij}^{31}} > 1 > \frac{\beta_{ij}^{13}}{\beta_{ij}^{33}},\ \beta_{ij}^{15} = 1 \text{ if } \frac{\beta_{ij}^{11}}{\beta_{ij}^{31}} < 1 < \frac{\beta_{ij}^{13}}{\beta_{ij}^{33}}, \text{ otherwise } 1 \geq \beta_{ij}^{15} > 0, \\
\beta_{ij}^{24} &= \frac{\beta_{ij}^{23}}{\beta_{ij}^{13}} \text{ if } \frac{\beta_{ij}^{21}}{\beta_{ij}^{11}} > 1 > \frac{\beta_{ij}^{23}}{\beta_{ij}^{13}},\ \beta_{ij}^{24} = 1 \text{ if } \frac{\beta_{ij}^{21}}{\beta_{ij}^{11}} < 1 < \frac{\beta_{ij}^{23}}{\beta_{ij}^{13}}, \text{ otherwise } 1 \geq \beta_{ij}^{24} > 0, \\
\beta_{ij}^{24} &= \frac{\beta_{ij}^{23}}{\beta_{ij}^{43}} \text{ if } \frac{\beta_{ij}^{21}}{\beta_{ij}^{41}} > 1 > \frac{\beta_{ij}^{23}}{\beta_{ij}^{43}},\ \beta_{ij}^{25} = 1 \text{ if } \frac{\beta_{ij}^{21}}{\beta_{ij}^{41}} < 1 < \frac{\beta_{ij}^{23}}{\beta_{ij}^{43}}, \text{ otherwise } 1 \geq \beta_{ij}^{25} > 0, \\
\beta_{ij}^{34} &= \frac{\beta_{ij}^{33}}{\beta_{ij}^{43}} \text{ if } \frac{\beta_{ij}^{31}}{\beta_{ij}^{41}} > 1 > \frac{\beta_{ij}^{33}}{\beta_{ij}^{43}},\ \beta_{ij}^{34} = 1 \text{ if } \frac{\beta_{ij}^{31}}{\beta_{ij}^{41}} < 1 < \frac{\beta_{ij}^{33}}{\beta_{ij}^{43}}, \text{ otherwise } 1 \geq \beta_{ij}^{34} > 0, \\
\beta_{ij}^{35} &= \frac{\beta_{ij}^{33}}{\beta_{ij}^{13}} \text{ if } \frac{\beta_{ij}^{31}}{\beta_{ij}^{11}} > 1 > \frac{\beta_{ij}^{33}}{\beta_{ij}^{13}},\ \beta_{ij}^{35} = 1 \text{ if } \frac{\beta_{ij}^{31}}{\beta_{ij}^{11}} < 1 < \frac{\beta_{ij}^{33}}{\beta_{ij}^{13}}, \text{ otherwise } 1 \geq \beta_{ij}^{35} > 0, \\
\beta_{ij}^{44} &= \frac{\beta_{ij}^{43}}{\beta_{ij}^{33}} \text{ if } \frac{\beta_{ij}^{41}}{\beta_{ij}^{31}} > 1 > \frac{\beta_{ij}^{43}}{\beta_{ij}^{33}},\ \beta_{ij}^{44} = 1 \text{ if } \frac{\beta_{ij}^{41}}{\beta_{ij}^{31}} < 1 < \frac{\beta_{ij}^{43}}{\beta_{ij}^{33}}, \text{ otherwise } 1 \geq \beta_{ij}^{44} > 0, \\
\beta_{ij}^{45} &= \frac{\beta_{ij}^{43}}{\beta_{ij}^{23}} \text{ if } \frac{\beta_{ij}^{41}}{\beta_{ij}^{21}} > 1 > \frac{\beta_{ij}^{43}}{\beta_{ij}^{23}},\ \beta_{ij}^{45} = 1 \text{ if } \frac{\beta_{ij}^{41}}{\beta_{ij}^{21}} < 1 < \frac{\beta_{ij}^{43}}{\beta_{ij}^{23}}, \text{ otherwise } 1 \geq \beta_{ij}^{45} > 0,
\end{aligned}
$$