Abstract
This paper develops a two-country model in which transmission of financial shocks arises despite a flexible exchange rate regime and substitutable financial assets, unlike usual results under these two conditions. This emerges from a search and matching environment between local banks and international financiers. This structure allows dissociating two types of financial shocks: (i) an asymmetric liquidity contraction implies a recession at home but an expansion abroad, nest- ing the standard monetary contraction open-macroeconomy literature result as a particular case; (ii) an asymmetric shock to banks’ capitalization cost, which does generate recessionary effects in both countries, under the same conditions.

JEL Classification: C78, E44, E51, F41-42, G01, G15
Keywords: international contagion, financial multiplier, financial crises, credit rationing, open economy macroeconomics, search and matching theory

1 Introduction
The Great Recession rapidly transmitted worldwide despite originating from an US-specific turmoil, the housing market downturn in 2006 and subsequent
subprime market collapse in 2007. Among developed economies, the eurozone was particularly affected, even though a flexible exchange rate with the dollar, advanced financial markets and a large internal market could have made it less vulnerable to such an external shock than other regions, such as emerging market economies, for instance.

This transmission is still surprisingly at odd with the theory. Indeed, there are two types of circumstances in which asymmetric financial shocks generate international contagion, i.e positive output comovements, in the open-macroeconomy literature. On the one hand, when the exchange rate regime is fixed, by tying the hands of monetary authorities, as in the early Mundell-Fleming open economy models. On the other hand, imposing ad hoc complementarity between home and foreign financial assets. This is because it increases the exposure of foreigners’ balance sheets to a decline in the value of domestic assets, by assumption (see Krugman, 2008, for instance). This wealth effect is able to offset the traditional (flexible) exchange rate channel that boosts foreign exports in case of an adverse financial shock, in standard two-country models (see Obstfeld and Rogoff, 1995).

However, while there is little doubt that financial complementarity is an important transmission mechanism from developed to emerging market economies, whose own financial markets are incomplete (Allen and Gale, 2000, Dooley and Hutchison, 2009), the evidence is very mixed as far as developed countries are concerned. Indeed, in 2008-2009, limited exposure to US subprime assets in particular, as well as a strong home bias in asset portfolio in general, have restricted the scope for these pure balance sheet effects (Rose and Spiegel, 2011, Kamin and DeMarco, 2012). As emphasized by Dedola and Lombardo (2012), financial integration makes developed countries’ financial assets rather substitutable to the US ones. This leaves room for alternative stories than the complementarity hypothesis to explain financial contagion within the floating exchange rate world.

1Consumption shocks may generate contagion even without complementarity effects (Pavlova and Rigobon, 2010), but the scope of this paper is limited to financial shocks. They are very different since they first and foremost affect the supply side of the economy, such as monetary contractions or credit squeezes impeding firms’ funding.
This paper builds a two-country model in which financial contagion arises, despite a flexible exchange rate system and substitutable financial assets. It uses the search and matching theory on several markets, including in the relationships between regional banks and international financiers who make arbitrages between investments across countries. This structure allows not only for shocks to capital or liquidity availability per se, but also for non-walrasian shocks to bank capitalization costs without scarcity of liquidity. Only the latter type generates financial contagion under the flexible exchange rate regime considered here.

The intuition is as follows. With a traditional liquidity scarcity shock (or negative monetary policy shock), the interest rate increases, the currency appreciates, and a recession spreads at home. Meanwhile, the symmetric second country benefits from devaluation to export more, at least up to the point where financial arbitrages are made. This is the standard ‘beggar-ty-neighbor effect’ of the open-macroeconomy literature, present for instance in Obstfeld and Rogoff (1995). This type of shocks can be mimicked in the search and matching environment considered here by increasing the search cost of international financiers, such that their participation to local banking activities decreases, and thereby liquidity becomes more scarce. On the contrary, a shock to the search cost of local banks reduces demand for international funds, hence puts downward pressure on interest rate and currency value at home, in spite of recessionary effects from lower banking activity at home. Hence, the traditional price competitiveness channel is weakened here compared to standard monetary contractions. Meanwhile, the substitutability between home and foreign financial assets helps propagate the crisis by equalizing external finance conditions worldwide. Financiers are free to make global arbitrage but would prefer to exit both markets in such a case, as real opportunities decrease in both zones. As a consequence, both home and foreign output decline following the asymmetric shock.

The search and matching à la Mortensen and Pissarides (1994) is adopted here in several markets, namely the financial markets between banks and international investors, the credit market between local entrepreneurs and banks, and the labor market between entrepreneurs and workers. This type
of multi-layer search and matching environment has been earlier adopted by Wasmer and Weil (2004) and Petrosky-Nadeau and Wasmer (2013, 2015), yet without the international financing stage which is at the core of international transmission here. This model structure provides elegant setups which can be solved analytically despite the interplay between several frictions. It also easily nests frictionless cases. Here, it also allows to dissociate a standard liquidity contraction from a search-specific shock to the cost of bank capitalization that is independent of the liquidity supply in the economy. One may easily accept that, because of some bad news for instance, banks may find it harder to raise funds even if savings are abundant at the aggregate level. The international contagion properties of such a shock can then be easily studied in this stylized setup, with very simple analytic results and comparative statics. Finally, such a setup makes the time dimension of creating relationships matter. Although financial markets are highly frictionless in normal times, they still experience disruption episodes, as during the Great Recession, after which an (endogenous) time spell is needed before a normal functioning is restored. This way, financial markets are potentially non-walrasian in the sense that asset price movements may not be sufficient for immediate market-clearing in distress times, though not necessarily inefficient in normal times.

The remainder of this paper is as follows. Section 2 develops the multi-layer search and matching model in autarky, and presents the steady-state equilibrium. Section 3 analyzes the effect of financial shocks in such a context. This is here done for the dynamic setup, with impulse response functions. However, the qualitative holds if we were doing simple comparative statics from steady-state. Regardless of the solution method, both types of financial shocks generate a recession, and other expected effects in a closed economy. Section 4 presents the two-country version and analyzes the spillovers effects of the same financial shocks (for now, at steady-state only). In particular, the ‘liquidity supply shock’ is found to generate standard negative output co-movement of monetary policy shocks, whereas the ‘bank capitalization cost’ shock implies positive one. Contagion thus emerge in spite of a floating exchange rate regime and fully integrated financial mar-
kets, contrary to the standard literature results, but highly reminiscent of the 2007-2009 episode. Section 5 opens discussions on (i) features of the 2007-2009 financial crisis captured by the model, (ii) the role of each market friction in driving the qualitative results of the model, (iii) the contribution vis-à-vis the literature. Finally, Section 6 concludes.

2 Model in autarky

2.1 A sequential search and matching process

The economy is composed of four types of infinitely-lived risk-neutral agents: financial investors, commercial banks, entrepreneurs, and workers. These agents interact in three (potentially) frictional markets: the financial market, the credit market and the labor market. It is assumed that a sequential double bargaining process takes place, with the timing of events given in Figure 1.

Figure 1: Timing of Events

In stage 0, commercial banks look for investors in the financial market in order to raise funds that will be used to lend to entrepreneurs later on. Financial investors are assumed to be endowed with capital but not to have the competence to make investments in real projects on their own, hence need intermediaries. On the other hand, commercial banks are supposed to be in need of capital at this stage, for instance because their leverage ratios

\footnote{Financial investors here can be thought of uninsured depositors or any kind of actors such as savings banks, hedge funds, foreign sovereign funds.}
are already maximal such that they have to find new capital before conceding new loans. One should note that this specification does not preclude matches at infinitely high rates if investment and commercial banking activities are integrated.

In stage 1, bankers look for a profitable loan opportunity among entrepreneurs who seek a credit to create a firm. Wasmer and Weil (2004) constructed a model in which such a credit market stage precedes a labor market stage, leading to a situation where frictions on both markets reinforce one another. Following them, I assume that entrepreneurs have no proper wealth ex ante and must find a credit before entering the worker recruitment stage. Acemoglu (2001) indeed documents that credit market frictions significantly constrain job creation for new firms, especially in Europe. Moreover, credit dependence of firms may be particularly relevant when a deep financial shock prevents even large firms from issuing equities as a perfectly substitutable fund-raising means.

In stage 2, entrepreneurs and workers search for one another, in the usual way. Finally, in stage 3, the newly created firm produces and pays back the banker, who in turn pays for the former investor’s services.

Assuming Stage 0 as the beginning of the sequential process aims at representing a constrained access to capital for the intermediaries. Hale and Santos (2010) have provided empirical evidence that, first, even US banks that do not rely on the bond market to fund their activities (but exclusively on deposits) have become exposed to the conditions in the bond market, and second, banks do pass debt market shocks to all their borrowers, whether these borrowers have themselves access to the bond market or not.

\[\text{Campello, Graham and Harvey (2010) found that 86\% of constrained US firms declared having restrained, canceled or postponed planned investment in attractive projects during the crisis of 2008, and so did almost half of unconstrained firms. Chodorow-Reich (2014) found that credit rationing accounted for between one-third and one-half of the employment decline at small and medium firms in the year following the Lehman bankruptcy.}\]

\[\text{Because the model is expressed in real terms, we can equally refer to the capital exchanged in these financial operations as liquidity. For simplicity, this is the only source of funds for the bankers. The investors can decide between keeping it idle or investing it in a durable financial relationship with the banker, as if they were buying a long-term financial asset.}\]
This suggests that a rarefaction of capital holders may affect real activity through credit rationing even when both banks and firms do have access to alternative sources of funds (for e.g deposits for the banks or direct finance for the firms). The present paper studies real effects of a freeze in these financial relationships via a significant rise of external capitalization costs to banks, possibly stemming from the inability of potential investors to assess their solvency. Note that if the process had started at the so-called Stage 1, an exogenous decrease in the liquidity access of commercial banks would be similar to a traditional monetary contraction and lead to the counterintuitive international comovements mentioned earlier.

Each market considered here is characterized by a ‘tightness’ that determines the frequency of matches and thus the duration of the search stage. **Stage 2** is the most standard, as a simplistic version of a labor market à la Mortensen and Pissarides (1994). There is a finite number $N_U$ of unemployed workers looking for a job and a finite number $N_V$ of open job vacancies. A constant returns-to-scale matching function $m_L(N_U, N_V)$ determines the flow of new firms as matches between one entrepreneur and one worker. The ratio of vacancies to unemployed workers defines the *labor market tightness*, $\theta \equiv N_V/N_U$, from which are inferred the instantaneous probabilities $q_L(\theta) = m_L(N_U, N_V)/N_V$ for an entrepreneur and $\theta q_L(\theta) = m_L(N_U, N_V)/N_U$ for a worker to get matched with one another on the labor market. As soon as the match occurs, production (Stage 3) starts.

**Stage 1** depicts the credit market similarly. A recent literature argues that information asymmetry between borrowers and lenders makes the creation of new credit relationships time- and effort-consuming. At each point of time, some bankers are screening credit applications from a pool of unmatched entrepreneurs who are seeking a bank willing to provide them with a credit line. Dell’Ariccia and Garibaldi (2005) and Craig and Haubrich (2013) have provided empirical evidence of a departure of gross from net credit flows which is persistent over time, that is, coexistent credit creation and credit destruction flows. Therefore, let us consider that the ratio of $N_E$ entrepreneurs looking for a bank to $N_C$ bankers seeking a desirable borrower
measures the credit market tightness, denoted $\phi$. A constant returns-to-scale technology, $m_C(N_E, N_C)$, increasing in both arguments, then determines the instantaneous probabilities $q_C(\phi) = m_C(N_E, N_C)/N_E$ for an entrepreneur to obtain a credit line and $\phi q_C(\phi) = m_C(N_E, N_C)/N_C$ for a banker to find a suitable entrepreneur (project). Also note that the mass $N_E$ of entrepreneurs who are, actively but unsuccessfully, looking for a credit line at each period in time can be thought of a measure of credit rationing, in the spirit of Stiglitz and Weiss (1981).

Stage 0’s financial relationships are finally considered in a similar way. Financial markets may go from quasi-frictionless in normal times to situations of ‘freezes’ in which capital return adjustments do not suffice for a walrasian immediate clearing. The search approach easily capture these different degrees of frictions by making endogenous the duration before a transaction is realized.\footnote{Many financial markets are over-the-counter (OTC) markets, hence non walrasian, by nature. This is the case of the interbank market (Afonso and Lagos, 2015), as well as many markets for derivatives, bonds, and credit (Trejos and Wright, 2014).} Therefore, denoting $N_I$ the financiers seeking investment opportunities and $N_B$ commercial banks looking for capital, the ratio $\xi \equiv N_B/N_I$ defines a comparable measure of instantaneous financial market tightness. From a similar matching function $m_F(N_I, N_B)$ are derived the time-varying arrival rates $q_F(\xi) = m_F(N_I, N_B)/N_B$ at which a banker raises funds on the financial market and $\xi q_F(\xi) = m_F(N_I, N_B)/N_I$ at which an investor meets a suitable banker.

Note that, in each market, the matching rate decreases in the tightness on the demand side ($\partial q_L(\theta)/\partial \theta, \partial q_C(\phi)/\partial \phi$, and $\partial q_F(\xi)/\partial \xi < 0$), while the reverse holds on the supply side ($\partial \theta q_L(\theta)/\partial \theta, \partial \phi q_C(\phi)/\partial \phi$, and $\partial \xi q_F(\xi)/\partial \xi > 0$).

2.2 Individual problems

2.2.1 Entrepreneurs

Entrepreneurs’ decision entry takes place in Stage 1 since they have productive ideas but not the necessary wealth to start recruitment on their own and
thus rely on the credit market to open a job vacancy. Let us assume that searching for a loan involves a non-pecuniary flow cost (effort cost), denoted $c_E$. Once matched with a banker, with probability $q_C(\phi)$, an entrepreneur starts looking for a suitable worker in Stage 2. This in turn implies a recruitment flow cost $\gamma_L$. For simplicity, the capital that is transferred from the investor to the banker, and from the banker to the entrepreneur, corresponds exactly to this recruitment cost. In that sense, the financiers invest in the firm by paying for the expected cost endured before it operates. Once the match with a worker arrives, at rate $q_L(\theta)$, the firm enters in Stage 3, where is produces one unit of output per period, sold at price $p^6$. Out of this revenue, the firm must pay for the worker’s wage $w$, which is exogenous here for the sake of simplicity, and for a periodic return $\rho_C$ to the banker, which corresponds to a Nash bargaining rule given further below$^7$. Finally, there is a probability $s$ of separation in each period, in which case the worker becomes unemployed and the entrepreneur out of financing again.

Therefore, Bellman equations in Stages 1, 2, and 3, respectively are

\begin{align}
E_{1,t} &= -c_E + \beta[1 - q_C(\phi_t)]E_{1,t+1} + q_C(\phi_t)E_{2,t} \\
E_{2,t} &= \beta[1 - q_L(\theta_t)]E_{2,t+1} + q_L(\theta_t)E_{3,t} \\
E_{3,t} &= p_t - w - \rho_C,t + \beta(1 - s)E_{3,t+1}
\end{align}

Note that the timing of Bellman equations is such that transition to the next stage happens within the current period, whereas continuation in the same stage goes across periods. This is just for technical convenience when writing down the Nash bargaining rules to compare values of consecutive stages within the same time period. Of course, with a continuous time or

$^6$In autarky, this is the price of the only good produced in the economy, and therefore a numeraire. In the two-country version, this will be the price of the domestically produced good, in competition with the foreign good.

$^7$One could consider either a competitive wage or a bargained wage without impact on the qualitative results of the paper. It is kept exogenous here for tractability.
steady-state version of this model, this becomes irrelevant.

Free entry of entrepreneurs imposes that $E_{1,t} = 0\forall t$. This implies an equilibrium condition which, in steady-state, reads as

$$\frac{c_E}{q_C(\phi)} = \frac{q_L(\theta)}{r + q_L(\theta)} \frac{p - w - \rho_C}{r + s}$$ (4)

This says that entrepreneurs overall decide to start looking for a loan in order to launch a business if and only if the present-discounted expected cash flows in Stage 3 is equal to the present-discounted expected costs of searching in Stage 1.[8] Given the riskfree rate $r$ and the conditional probability $q_C(\phi)$ of transition from Stage 1 to Stage 2, $c_E$ is discounted by $\frac{1}{r + q_C(\phi)}$. Given the transition rates $q_L(\theta)$ from Stage 2 to Stage 3, and $s$ from Stages 3 to 1, entrepreneurs’ instantaneous payoff $(p - w - \rho_C)$ has to be discounted by $\frac{q_C(\phi)}{r + q_C(\phi)} \frac{q_L(\theta)}{r + q_L(\theta)} \frac{1}{r + s}$ (see Appendix for computational details). After rearranging the terms, the left-hand side is composed of the periodic search cost times the average duration of the credit search stage $1/q_C(\phi)$, while the right-hand side is present-discounted expected net gains given the riskfree rate $r$, the firm separation rate $s$, and the recruitment stage average duration as a function of the tightness $\theta$.

### 2.2.2 Bankers

Commercial banks enter the process one stage earlier than entrepreneurs since they have to raise funds from financiers before lending to entrepreneurs. Let us denote $c_{B,t}$ the non-pecuniary search cost that banks have to bear in each period of Stage 0 when looking for a financier. This may be interpreted as the effort required to gather proofs of solvency for instance, or more generally, as a non-pecuniary cost associated with capitalizing the bank. It is time-varying to allow for a shock, further described in Section 3. Once a financial match is concluded, the investor provides the banker with the capital that is necessary to look for a suitable entrepreneur ($\gamma_C$ per period during Stage 1) and the credit line to this entrepreneur ($\gamma_L$ per period during Stage 3).
period during Stage 2). During the production Stage 3, bankers receive $\rho_C$ from entrepreneurs, from which is extracted an instantaneous payout $\rho_F$ to the investor, determined further below.

Hence, the Bellman equations respectively are

$$B_{0,t} = -c_{B,t} + \beta[1 - q_F(\xi_t)]E_t B_{0,t+1} + q_F(\xi_t)B_{1,t}$$ (5)

$$B_{1,t} = \beta[1 - \phi_t q_C(\phi_t)]E_t B_{1,t+1} + \phi_t q_C(\phi_t) B_{2,t}$$ (6)

$$B_{2,t} = \beta[1 - q_L(\theta_t)]E_t B_{2,t+1} + q_L(\theta_t) B_{3,t}$$ (7)

$$B_{3,t} = \rho_C,t - \rho_F,t + \beta(1 - s)E_t B_{3,t+1}$$ (8)

At steady-state, free entry implies

$$\frac{c_B}{q_F(\xi)} = \frac{\phi q_C(\phi)}{r + \phi q_C(\phi)} \frac{q_L(\theta)}{r + q_L(\theta)} \frac{\rho_C - \rho_F}{r + s}$$ (9)

On the left hand side, expected costs of raising funds for bankers depend on the financial market tightness $\xi$ which gives the expected duration of Stage 0. On the right hand side, bankers’ output share ($\rho_C - \rho_F$) is discounted by the respective duration of the credit and labor search stages.

### 2.2.3 Financial investors

In Stage 0, financial investors are looking for a bank they consider able to turn their idle capital into a profitable long term investment opportunity. This implies a search cost $c_{I,t}$ per period of time. It allows for a shock, interpreted as a ‘liquidity supply shock’ since, everything else equal, it determines the participation of financiers, and therefore how much savings are poured into banking activities. After the match, investors bear the pecuniary flow costs $\gamma_C$ and $\gamma_L$ of the bank’s credit application screening (Stage 1) and of the entrepreneur recruitment (Stage 2). Finally, they earn $\rho_F$ in each period
of production (Stage 3), until a separation brings them back to Stage 0.

The Bellman equations are

\[ I_{0,t} = -c_{I,t} + \beta [1 - \xi q_F(\xi_t)] E_t I_{0,t+1} + \xi q_F(\xi_t) I_{1,t} \]  \hspace{1cm} (10)

\[ I_{1,t} = -\gamma_C + \beta [1 - \phi_C(\phi_t)] E_t I_{1,t+1} + \phi_C(\phi_t) I_{2,t} \]  \hspace{1cm} (11)

\[ I_{2,t} = -\gamma_L + \beta [1 - q_L(\theta_t)] E_t I_{2,t+1} + q_L(\theta_t) I_{3,t} \]  \hspace{1cm} (12)

\[ I_{3,t} = \rho_F t + \beta (1 - s) E_t I_{3,t+1} \]  \hspace{1cm} (13)

The steady-state equilibrium condition is thus

\[ \frac{c_I}{\xi q_F(\xi)} = \frac{-\gamma_C}{r + \phi_C(\phi)} + \frac{\phi_C(\phi)}{r + \phi_C(\phi)} \left( \frac{-\gamma_L}{r + q_L(\theta)} + \frac{q_L(\theta)}{r + q_L(\theta)} \frac{\rho_F}{r + s} \right) \]  \hspace{1cm} (14)

Forward-looking investors’ willingness to provide capital depends on the costs induced by search activities (\(\gamma_C\) and \(\gamma_L\) times the respective expected search durations) and the present-discounted output share \(\rho_F\) in stage 3.

2.2.4 Workers

Finally, workers intervene only at Stages 2 and 3 of the model, as they are either unemployed and searching for an entrepreneur or employed. Their Bellman equations are

\[ W_{2,t} = \beta [1 - \theta_L(\theta_t)] E_t W_{2,t+1} + \theta_L(\theta_t) W_{3,t} \]  \hspace{1cm} (15)

\[ W_{3,t} = w + \beta (1 - s) E_t W_{3,t+1} + s W_{2,t} \]  \hspace{1cm} (16)

This very simple structure makes the unemployed workers accept the job they encounter as long as the wage, \(w\) is sufficiently high, given the good price \(P\), the discount rate \(r\), and the separation rate \(s\), i.e. when \((w/P)/(r + s) > 0\).
This way, they are given a passive role in the model, consistent with the observation that frictional unemployment is rather minor in crisis times.

2.3 Surplus sharing

The two rates $\rho_C$ and $\rho_F$ that share the surplus between entrepreneurs, bankers, and investors, are determined by Nash bargaining rules à la Pissarides (2000). One can think of it as the personal interactions a borrower has to make with a lender in order to pin down the rate of return on capital that will be paid once the production starts, particularly relevant in the absence of collateral wealth.

The credit repayment rate $\rho_C$ from the firm to the banker thus maximizes the value of the match between the banker and the entrepreneur

$$\rho_C = \arg\max (B_{2,t} - B_{1,t})^{\delta_C} (E_{2,t} - E_{1,t})^{(1-\delta_C)}$$

where $B_1$ and $B_2$ (respectively $E_1$ and $E_2$) are the asset values of bankers (respectively entrepreneurs) in stages 1 and 2, according to the bargaining power $\delta_C$ of bankers relatively to entrepreneurs in the credit market.

Similarly, the flow $\rho_F$ from bankers to investors is given by

$$\rho_F = \arg\max (I_{1,t} - I_{0,t})^{\delta_F} (B_{1,t} - B_{0,t})^{(1-\delta_F)}$$

where $I_0$ and $I_1$ are the asset values of investors in stages 0 and 1, and where $\delta_F$ is the bargaining power of investors in the financial market.

2.4 Equilibrium (autarky)

First, let us derive the equilibrium value of the financial market tightness. The Nash rule for $\rho_F$ above, together with zero-profit conditions ($I_0 = 0$ and $B_0 = 0$), determines $\bar{\xi}$ (upperbars standing for equilibrium values henceforth)

\[\bar{\xi} = \frac{\rho_F}{\delta_F} \frac{B_0}{I_0} \]

Indeed, business cycle fluctuations are mostly related to involuntary rather than frictional unemployment. Michaillat (2012) found that the frictional part of unemployment largely accounts for total unemployment when the rate is close to 5% in the US. However, it falls to less than 2% when the rate goes to 9%, i.e becomes minor in bad times.
as

\[
\bar{\xi} = \frac{1 - \delta_F c_I}{\delta_F c_B} \tag{17}
\]

(see Appendix for details). In equilibrium, the financial tightness increases in the flow cost borne by investors while looking for a commercial bank: *ceteris paribus*, a higher \(c_I\) makes it less profitable for investors to enter the financial market, thus increasing the relative number of bankers to investors willing to trade, \(N_{BI}/N_I\), which precisely defines the financial market tightness. Inversely, a higher \(c_B\) decreases the tightness by reducing the relative number of bankers entering the market. The financial market tightness also increases in bankers’ bargaining power \((1 - \delta_F)\) relatively to investors’ \(\delta_F\) as bankers expect a greater share of the surplus when their bargaining power rises, everything else equal.

Recursively, the Nash rule for \(\rho_C\) above with the free-entry condition for entrepreneurs \((E_1 = 0)\) gives the equilibrium credit market tightness \(\bar{\phi}\) as

\[
\bar{\phi} = \frac{1 - \delta_C c_B}{\delta_C c_E q_F(\xi)} \tag{18}
\]

This expression similarly says that the relative number of entrepreneurs to banks increases in entrepreneurs’ bargaining power \((1 - \delta_C)\) in the credit market. On the contrary, the credit market tightness slackens when bankers’ bargaining power \(\delta_C\) goes up — as bankers’ share of output generated on credit lines increases — or when their access to external capital, captured by \(q_F(\xi)\), improves. While \(c_B\) tends to increase the credit market tightness as it makes it costlier for banks to look for external finance, the screening costs \(\gamma_C\) have no direct effect as those are ultimately borne by the financiers. Finally, note that when the discount rate is nil, the credit market becomes frictionless as capitalized bankers have no better option than participating, whatever the speed at which they find an entrepreneur or the duration of the match. On the contrary, when \(r > 0\), a banker decides to enter the credit market if the time spent in stages 1 and 2 is valuable enough to outweigh
the discounting effect of the risk-free rate.\footnote{Similarly in the labor literature, an unemployed worker is willing to search for a job if he/she knows that the wage is higher than his/her income flow while unemployed.}

Finally, the steady-state unemployment rate $\bar{u}$ equalizes flows into unemployment $s(1 - u)$ and flows out of unemployment $\theta q_L(\theta) u$, that is

$$\bar{u} = \frac{s}{\theta q_L(\theta) + s} \quad (19)$$

**Definition 1.** The autarkic economy is characterized by the system of equations (1)–(6) in six unknowns $\{\theta, \phi, \xi, \rho_C, \rho_F, u\}$. In particular, replacing $\bar{\xi}$ from (4) and $\bar{\phi}$ from (5) in (1)–(3) recursively gives the equilibrium repayment rates, $\bar{\rho}_C$ and $\bar{\rho}_F$, as well as the equilibrium labor tightness $\bar{\theta}$ as the solution of

$$\left(\frac{c_B}{q_F(\xi)} + \frac{c_I}{\xi q_F(\xi)} + \frac{\gamma_C}{r + \phi q_C(\phi)}\right) r + \bar{\phi} q_C(\bar{\phi}) q_C(\bar{\phi}) \frac{c_E}{q_C(\bar{\phi})} = \frac{q_L(\theta)}{r + q_L(\theta)} p - w - \frac{\gamma_L}{r + q_L(\theta)}$$

from which the steady-state unemployment rate $\bar{u}$ is then determined by (6).

Let us represent this equilibrium in the $(\theta, \xi)$ space in Figure 2. In order to depict the three representative agents in this two-dimensional space, equations (1) and (2) are put together by isolating $\rho_C$ so as to obtain a joint equilibrium condition for bankers and entrepreneurs (constrained agents) as

$$\left(\frac{c_B}{q_F(\xi)} \frac{r + \bar{\phi} q_C(\bar{\phi})}{\bar{\phi} q_C(\bar{\phi})} + \frac{c_E}{q_C(\bar{\phi})}\right) \frac{r + q_L(\theta)}{q_L(\theta)} = \frac{p - w - \rho_F}{r + s}$$

This is the $BE$ curve in Figure 2. It is downward-sloping since, for a given credit market tightness, the tighter the labor market (high $\theta$), and thus the longer the recruitment stage, the slacker the capital access must be (low $\xi$) for capital-constrained agents to remain on the same condition. Conversely, a relatively short duration of stage 2 while looking for a suitable worker offsets long fund-raising stages. The upward-sloping $II$ curve stands for financial investors' condition (3), which reflects the fact that financiers bring less capital to the banking sector (\xi is high) when the labor market is tight.
This is because a large $\xi$ increases financiers’ instantaneous matching rate in stage 0, as required when stage 2 is time-consuming in order to remain on the same condition. The intersection point A depicts the closed economy equilibrium described in Definition 1 above. This representation will be used to give the intuition of the mechanism with comparative statics of shocks in Section 4.

Figure 2: Initial equilibrium in autarky

3 Effect of financial shocks (autarky)

Before turning to the two-country version of the model, it is useful to describe the effects of financial shocks in the closed-economy version. Let us consider

(i) a shock to $c_{I,t}$, which is interpreted as a liquidity supply shock, since this parameter pins down the capital inflows to the banking sector, and thereby the real sector of the economy, everything else equal.

(ii) a shock to $c_{B,t}$, interpreted as a bank capitalization cost, since it is the cost that commercial banks have to bear in the fund-raising stage, independently from the level of liquidity (idle capital) in the economy.

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11 A higher financial market tightness always means a higher matching probability for investors in equilibrium assuming that the level of information asymmetry about the participants or participants’ creditworthiness is unaltered in the long run.

12 One could think of it as a confidence shock about the solvency of the banking sector.
These shocks both follow an AR1 process as

\[ x_t = \alpha x_{t-1} + (1 - \alpha) x + \varepsilon \]

where \( \alpha \) denotes the persistence and \( \varepsilon \) a normally distributed disturbance of mean 0 and variance 1, for each shock \( x = c_I, c_B \).

### 3.1 Effect of a liquidity supply shock

An increase in \( c_I \) makes investors less willing to participate the financial market. Therefore, the financial market tightness, \( \xi \), defined as the ratio of the number of bankers over investors in Stage 0, increases. This decreases due to a change in the degree of heterogeneity among banks or the degree of information asymmetry at the expense of liquidity holders for instance. These features are not modelled here but help to interpret the shock.

![Figure 3: Impulse response function to an asymmetric liquidity supply shock (1% increase in \( c_I \), persistence 0.9).]
the matching rate of bankers, which are thus less numerous in Stage 1 for a
given number of entrepreneurs, i.e the credit market tightness $\phi$ increases.
Consequently, also less entrepreneurs can match such that the labor market
tightness $\theta$ decreases and unemployment $u$ goes up. Nevertheless, the varia-
tions in $\phi$ and $\theta$ are much more moderate than the change in $\xi$ since bankers
and entrepreneurs can also exit the market rather than continuing to search
as conditions become less favorable. Finally, the repayment rates $\rho_F$ and $\rho_C$
decrease to compensate bankers entrepreneurs for lower outside options as
their matching rates decrease.

3.2 Effect of a bank capitalization cost shock

A rise in $c_B$ now limits bankers’ participation in the search process such
that the financial market tightness, $\xi$, now decreases. Yet, as in the previ-
ous case, the number of bankers going to Stage 1 is lower than before the shock, increasing the credit market tightness \( \phi \) here again. A smaller pool of financed entrepreneurs in turn decreases the labor market tightness \( \theta \) and increases unemployment \( u \). The same forces as in the previous case affect bargaining such that the repayment rates \( \rho_F \) and \( \rho_C \) also decrease here.

Overall, in the closed economy, both financial shocks are therefore very similar to one another. They reduce the channeling of funds from financiers to entrepreneurs and thereby increase unemployment. The only key difference is the change in the financial market tightness, and therefore the probability at which a bank gets funds, \( q_F(\xi) \), or at which a financier gets matched, \( \xi q_F(\xi) \). This single feature will generate critically different propagation properties in the two-country version, despite all other similarities between the two financial shocks.

4 International spillovers of financial shocks

4.1 Two-country equilibrium

Let us now consider two economies characterized by the multiple search process described above. The final good produced by firms in each country is consumed internationally, without any trade cost, for simplicity. Each operating firm (in Stage 3) produces one unit of the good corresponding to its location every period as long as Stage 3 lasts (i.e until separation). The respective parts of its output to be sold at home and abroad are determined by the prices in each location, \( p_{i,j,t} \) for any good \( i \) in country \( j \) at time \( t \), with \( i,j = (h,f) \) for Home and Foreign henceforth, and the exchange rate (see below).

It is assumed that goods and investors’ capital are freely mobile across countries, however the other agents are country-specific. For entrepreneurs, country-specificity can be justified by the inertia in production relocation decisions following unexpected financial shocks. A somewhat less restrictive

\[^{13}\text{Also for simplicity, each final good is consumed in both countries in the same proportions, but home bias or perfectly substitutable goods (i.e a unique final good) would leave the qualitative results unchanged here.}\]
assumption could be to let entrepreneurs migrate across countries conditional on paying for some sunk costs — for changing their specialization for example — but I do not make this outside option explicit here for simplicity. Workers, whether employed or not, are supposed immobile as largely observed between large economic areas — think of US and EU — as compared to labor mobility between countries within each of these areas. This allows to abstract from a comparable outside option for workers, and to normalize to one each national working population so as to obtain deviations in terms of unemployment rates.

Commercial banks are also local here. Recall that each ‘bank’ consists of a single credit line, which intermediates capital between (integrated) financial markets and a small and local entrepreneur. In this respect, there are understood as local bank agencies that belongs to the country where the producer is. Both theoretical and empirical literatures have widely documented that geographical distance between lenders and (potential) borrowers indeed affect loan decisions. There are two mechanisms through which distance matters (Agarwal and Hauswald, 2010). On the one hand, transportation costs hinder matching between remote credit market participants: a potential borrower has to spend time and effort to personally interact with loan officers or to look for a suitable loan (because of product differentiation) while banks endure costs in assessing loan applicants or in monitoring loans that both increase with physical distance (Sussman and Zeira, 1995). On the other hand banks’ capacity to collect critical information about expected returns and probabilities of default of potential borrowers is enhanced by proximity, thus encouraging banks to concentrate on a limited geographical area to benefit from the monopoly power created by this informational advantage (Hauswald and Marquez, 2006). This results in spatial price discrimination and geographical credit rationing, empirically supported at a micro level (Degryse and Ongena, 2005; Agarwal and Hauswald, 2010) and within a medium size country (Casolaro and Mistrulli, 2008). Generalizing

Here again, the model could be extended to large intermediaries partly operating abroad without changing the qualitative predictions but this additional outside option would be at the expense of tractability.
this conclusion at a two-country level is not straightforward, yet at least two arguments support it. First, if transaction costs and informational advantages are decisive channels in quite limited areas, they are probably even more important when additional differences, in regulation for instance, further hinder the collection of private information or the detection of credit delinquency. Second, both banks (credit lines) and firms are small ones and new ones, two characteristics for which the aforementioned channels are particularly strong in this literature.

Finally, capital is perfectly mobile across countries, and so are the investors holding it. Investors are free to look for an intermediary in the country where their intertemporal value, denoted $I$, is the highest, given the search costs and transition rates they face in each location. Therefore, a no-arbitrage condition implies that $I_{0_{j,t}} = I_{0_{t}}$ for $j = h, f$ at all times. This implies that investors choose to locate their assets in the country where their matching rate is the highest, i.e. where the financial market is the tightest (since $\partial q_{F}(\xi)/\partial \xi > 0$), everything else equal. Equilibrium is thus characterized by a unique financial tightness at the world level, that is, an integrated financial market and external finance conditions equalized worldwide.

Given these international relationships, the balance of payments and the subsequent expression for the exchange rate are as follows. The current account of the home country expressed in domestic currency is standard as

$$CA_{t} \equiv C_{h_{f,t}}, S_{t}, p_{h_{f,t}} - C_{f_{h,t}}, p_{f_{h,t}}$$

where $C_{i_{j,t}}$ denotes the level of consumption of good $i$ in country $j$ at time $t$, and where $S$ is the floating exchange rate defined as the price of the domestic currency in terms of the foreign currency. The law of one price is assumed to always hold, such that the price of the each good at home depends on its price abroad times the exchange rate: $p_{i_{h,t}} = S_{t} p_{i_{f,t}}$.

As financial investors already matched with a banker cannot immediately withdraw their capital after a shock, the definition of the financial account comes down to the inter-country difference in new investor-banker relation-
ships, i.e. the difference of financial match flows at home and abroad

\[ FA_t \equiv m_F(N_{It}, N_{Bh,t}) - m_F(N_{It}, N_{Bf,t}) \]

Re-expressed in terms of matching rates (by definition, \( \xi_i q_F(\xi_i) \equiv \frac{m_F(N_{It}, N_{Bi})}{N_{It}} \)),
the inter-country financial market tightness differential replaces the traditional interest rate differential in driving international capital flows

\[ FA_t = N_{It} \xi_{h,t} q_F(\xi_{h,t}) - N_{It} \xi_{f,t} q_F(\xi_{f,t}) \]

Note that, even if financial flows are driven by the financial tightness differential, unmatched investors will not necessarily immediately pour their capital into the foreign market following a shock at home as general equilibrium effects will also imply a rarefaction of suitable bankers in the foreign country, and therefore similar movements of the financial tightness abroad. Scarcity of real business opportunities and internal credit frictions are magnified everywhere by the inertia in creating new financial relationships.

The balance of payments identity \( CA + FA = 0 \) finally gives the expression for the exchange rate as follows (dropping time subscripts)

\[ S = \frac{p_{fh} C_{fh} - N_{It} \xi_{h} q_F(\xi_{h}) + N_{It} \xi_{f} q_F(\xi_{f})}{p_{hf} C_{hf}} \]  

Unsurprisingly, the domestic currency appreciates (\( S \) decreases) with exports and relative financial opportunities to capital holders at home, while it depreciates with imports and relative financial advantages abroad.

**Definition 2.** The two-country general equilibrium is characterized by adding to the previous set of unknowns for both countries \{\( \theta_h, \theta_f, \phi_h, \phi_f, \xi_h, \xi_f, \rho_{Ch_h}, \rho_{Cf_f}, \rho_{Fh}, \rho_{Ff}, u_h, u_f \}\) the levels of home and foreign consumption in each country \( \{C_{h_h}, C_{f_h}, C_{h_f}, C_{f_f}\} \), the corresponding prices \( \{p_{h_h}, p_{f_h}, p_{h_f}, p_{f_f}\} \), and the exchange rate \( \{S\} \). Two first order conditions from consumers’ optimization problem, the two laws of one price, two aggregate constraints on goods, and two aggregate resource constraints (standard equations relegated in Appendix) are added to the set of equations, as well
as the balance of payments (8).

This two-country equilibrium is depicted in Figure 3. The same $BE$ and $II$ curves as of Figure 2 are represented symmetrically for both countries here. In addition, the balance of payments $BP$ horizontal line reflects financial market integration through which capital flows take advantage of all opportunities.  

Figure 5: World initial equilibrium

4.2 Effect of an asymmetric liquidity supply shock

Let us first consider, both graphically and analytically, the effects of an asymmetric liquidity supply shock, i.e. an exogenous rise in $c_I$ in the home country.

As depicted in Figure 4, the shock first displaces domestic liquidity holders' equilibrium condition $II_h^A$ leftwards. Indeed, the zero-profit condition

\[ \theta_f \rightarrow \hat{\theta}_f^A \quad \text{Home country} \]
\[ \theta_h \rightarrow \hat{\theta}_h^A \quad \text{Foreign country} \]

Figure 4: Effect of an asymmetric liquidity supply shock

16The perfect international mobility of financial investors assumed here for analytic and graphical simplicity ensures that financial market tightnesses, $\xi_h$ and $\xi_f$, are instantaneously equalized between countries. With any slightly upward-sloping $BP$ curve, the qualitative results of the model will remain even if it may quantitatively reduce the size of financial transmission. A steep $BP$ curve would correspond to high levels of control on capital flows which are more likely in emerging economies and thus not the purpose of this paper.
makes it preferable for some financiers to exit the market as $c_I$ rises. This increases the financial tightness as the relative number of bankers to financiers goes up along the $BE_h^A$ curve, displacing the domestic equilibrium from point A to point B. But since this makes it more time-consuming for bankers to raise funds, it also creates a ‘contagion effect’, typical of the search and matching models, making some bankers better off by exiting the financial market too, therefore leading to point C at home. Furthermore, feedback effects come from the international interactions. As the domestic currency appreciates at point C (above the balance of payments $BP$ line), a disadvantage from terms of trade appears for entrepreneurs in the home country, displacing again the $BE_h$ curve leftwards. The mirror depreciation abroad boosts the foreign economic activity, displacing $BE_f$ from points A to D. However, this expansionary effect is mitigated until point E by the reduced demand for foreign goods from home consumers (negative second-round effects due to home recession). Symmetrically, the net positive wealth effect abroad tends to dampen the fall in net exports at home (from D to E). The final equilibrium is point E where home and foreign financial tightnesses are equalized again, thanks to the no-arbitrage financial condition.

Figure 6: World effect of a domestic liquidity supply shock
The theoretical framework developed here also allows to derive simple analytic results to corroborate the above picture. Loglinearizing the expressions for the three equilibrium tightnesses, \( \bar{\xi} \), \( \bar{\phi} \), and \( \bar{\theta} \), the responses to variations in \( c_I \) are calculated and given in Table 1 below. A hatted variable stands for its loglinear deviation from steady-state (\( \hat{x} = \frac{x - \bar{x}}{\bar{x}} \)), while \( \eta_L \), \( \eta_C \), and \( \eta_F \) \( \in \) (0, 1) stand for the equilibrium elasticity of each matching function, respectively defined as 
\[ \eta_L \equiv -\frac{q'_L(\bar{\theta})\bar{\theta}}{q_L(\bar{\theta})} \] (labor market),
\[ \eta_C \equiv -\frac{q'_C(\bar{\phi})\bar{\phi}}{q_C(\bar{\phi})} \] (credit market), and
\[ \eta_F \equiv -\frac{q'_F(\bar{\xi})\bar{\xi}}{q_F(\bar{\xi})} \] (financial market). See Appendix for more details.

<table>
<thead>
<tr>
<th>( \partial \hat{\xi}<em>j / \partial \hat{c}</em>{I_h} )</th>
<th>( \eta_F )</th>
<th>( \partial \hat{\phi}<em>j / \partial \hat{c}</em>{I_h} )</th>
<th>( \eta_F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{q_L(\bar{\theta})}{\gamma_L \eta_L} \left{ -\frac{\xi_{B_h} \eta_F}{q_F(\xi_h)} - \frac{\xi_{C_h} \eta_F}{q_C(\phi_h)} + \frac{\xi_{E_h} \eta_C \eta_F}{\phi_{C_h}(\phi_h)} (1 - \bar{N}_I)(1 - \eta_F) \right} )</td>
<td></td>
<td>( \frac{q_L(\bar{\theta})}{\gamma_L \eta_L} (1 - \bar{N}_I)(1 - \eta_F) )</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Home and foreign elasticities to local liquidity supply shocks

Comparing the sign of the responses in the different markets (\( \partial \hat{\xi}_j / \partial \hat{c}_{I_h} \), \( \partial \hat{\phi}_j / \partial \hat{c}_{I_h} \), and \( \partial \hat{\theta}_j / \partial \hat{c}_{I_h} \)) between the two countries \( j \), of particular interest and confirms the graphical effects described above. Also, loglinearizing (6) as
\[ \hat{u} \approx -(1 - \eta_L)(1 - \bar{u})\hat{\theta}, \]
clearly shows that the unemployment rate in one country is a negative function of the labor market tightness of that particular country.

Therefore, an asymmetric negative liquidity supply shock is found to produce the effects described in the following proposition.

**Proposition 1.** A liquidity supply shock at home (i.e, a rise in \( c_I \))

(i) decreases the liquidity worldwide. Because of the exit of some investors and perfect arbitrage, financial markets tighten (\( \xi \) goes up) everywhere;

(ii) decreases the number of credit relationships worldwide, though less
than proportionally. Less capitalized banks are able to finance entrepreneurs, so the credit market tightness ($\phi$) has to rise in both countries;

(iii) creates a recession at home but an expansion abroad. The sign of the labor market response, and thus of output, is opposite from one country to the other because of the traditional terms of trade channel.

Overall, this liquidity shock illustrates that the model developed here is able to nest as a particular case the standard open macroeconomy prediction: a domestic monetary expansion (as for a positive shock) creates an expansion at home and a recession abroad within a flexible exchange rate system. Although this might sound puzzling in light of the recent financial crisis, this prediction has been a consensus from the two-country frameworks, from the early Mundell-Fleming model to Obstfeld-Rogoff (1999) notably. Indeed, it does describe quite well competitive devaluations or ‘beggar-thy-neighbor’ monetary policy episodes, characterized by domestic short-term booms at the expense of the trading partners. This probably helps to understand why the literature on international contagion within a floating exchange rate system has for a long time not integrated more sophisticated financial channels, and vice-versa, why the credit constraints literature has focused on closed economy or monetary union settings. See the discussion Section for more references.

Interestingly, this negative output co-movement result is preserved in modern two-country models, as long as the exchange rate remains flexible and no financial market incompleteness (financial asset complementarity) is imposed, in spite of the presence of financial multipliers. This is the case

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17 This is assuming that the sum of all terms in the southwestern cell of Table 1 is negative of course, see Appendix for a discussion.
18 Since there is a one-to-one relationship between employment and output in the model, a negative labor tightness co-movement is also associated with a negative unemployment and output co-movement across countries in that case.
19 A decrease in $c_I$ here would produce the traditional effects describing ‘currency wars’, which have been frequent in developed countries during the Great Depression or more recently in emerging countries in the aftermath of the Great Recession (Brazil, 2010, for example).
here where the co-existence of financial and credit market frictions generates a ‘financial multiplier’ that magnifies the impact of the rarefaction of liquidity holders via longer fund-raising stage durations for credit intermediaries, as derived formally in Wasmer and Weil (2004).\textsuperscript{20} Such a multiplier or accelerator alters the magnitude of the effects but not their sign, hence the qualitative effect is unchanged. Similarly, various assumptions such as a strong home bias in portfolio or any wage setting mechanism would affect the size but not the sign of the responses. Thus, the spillover effects of pure liquidity supply shocks are therefore similar to the ones of traditional monetary shocks, that is, negative output co-movements.

4.3 Effect of an asymmetric shock bank cost shock

Now, let us consider an alternative financial shock making it more costly for banks to raise funds from international capital holders, namely a rise in banks’ search cost $c_B$. This shock is still of a financial nature as it directly impacts the financial Stage 0, but it does affect the solvency of the banking sector while keeping unchanged the overall liquidity supply in the economy.\textsuperscript{21} Thanks to the non-walrasian search and matching structure considered here, interest rate adjustments will not suffice to clear the market immediately and international contagion, i.e positive output co-movement across countries, will ensue.

Just as in the previous case, let us describe the comparative statics first and the analytical results then. As depicted in Figure 5, an increase in banks’ search cost (a rise in $c_B$) first shifts the $BE_h$ curve to the left because

\textsuperscript{20}In the case of labor and credit search frictions in Wasmer and Weil (2004)’s closed economy.

\textsuperscript{21}One can imagine that bank capitalization is suddenly costlier – without any change in the amount of idle liquidity in the economy – because of higher (real or perceived) heterogeneity in the solvency of the banking sector, requiring either that banks make a sustained effort to gather the proofs of their creditworthiness to investors and/or that they bear a higher opportunity cost in a climate of mistrust due to prohibitive information asymmetry to investors. A formal description of the underlying informational structure would allow to determine the drivers of the rise in $c_B$, but is well beyond the scope of this paper. In reduced form, the shock is very close to an exogenous rise in spreads, as frequent in macro nowadays, but with an effect on the extensive margin here as it makes some banks exit the market.
the zero-profit condition directly implies that the number of banks present in the financial market decreases. Moreover, anticipating fewer credit providers, some entrepreneurs leave the credit market too, participating in the displacement of the $BE_h$ curve. In turn, the number of investors in the financial market also diminishes as it becomes more difficult (time-consuming) to find a suitable bank, slightly displacing the $II$ curve to the left (search congestion effects). At point C here, and contrary to the previous case, the domestic currency now depreciates. This favors exports for the rare but yet newly created firms (i.e the entrepreneurs successfully matched with the remaining banks). Abroad, the symmetric currency appreciation and the lower demand from domestic consumers in contrast deteriorates the current account. This is the price competitiveness channel playing in opposite direction compared to the previous shock. In addition, there is financial arbitrage channel. Financiers are completely free to move to the foreign country where the banks have not been impacted by the initial shock. However, the size of the surplus is lower there as the economy enters a recession. Indeed, having fewer competitive entrepreneurs abroad makes it longer for foreign banks to find an appropriate match and the Nash bargaining less favorable. The global investors thus do not necessarily relocate their (idle) capital from the first to the second economy but just exit more. This worsens the fund raising and thus deepens the recession in both countries, until the final equilibrium point D.

**Proposition 2.** A bank capitalization cost shock (a rise in $c_B$) at home

(i) increases the available liquidity ($\xi$ goes down) worldwide. More banks than investors exit in both countries, making it more difficult for the same quantity of aggregate capital to be channeled to local credit markets.

(ii) decreases the number of credit relationships worldwide, though less than proportionally. Less capitalized banks are able to finance entrepreneurs, so the credit market tightness ($\phi$) has to rise in both countries;
(iii) decreases employment and output (as $\theta$ falls) in both countries.\footnote{Again, assuming that the sum of all terms in the southwestern cell of Table 2 is negative so as to stay consistent with Proposition 1.}

Overall, unlike in the case of the previous shock, the shock to bank capitalization cost here creates a positive real co-movement across countries despite an identical initial setup and the fact that the shock is still local ($c_B$ rises only in the home country). The worldwide downturn actually tempers the recession at home by transmitting them partly to the other zone. In addition, there is a slackening of the (global) financial tightness from point A to point D, confirming that the global downturn is not due to a lack of liquidity at the aggregate level – and thus differs from the standard monetary contraction – but rather to some change in the cost efficiency of the financial market, that may reflect uncertainty about the situation of banks or the real sector.

In Table 2, the first row confirms that the financial tightness is lower, and thus that the reason why banks have less opportunities to raise funds is not liquidity scarcity \textit{per se} but larger frictions on the financial market. The second row reflects credit rationing worldwide as in the case of liquidity scarcity.

Figure 7: World effect of a domestic rise in bank fund-raising costs
Table 2: Home and foreign elasticities to a rise in bank fund-raising costs

<table>
<thead>
<tr>
<th></th>
<th>Home country</th>
<th>Foreign country</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\partial \hat{\xi}<em>j / \partial \hat{c}</em>{Bh}$</td>
<td>$-1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$\partial \hat{\phi}<em>j / \partial \hat{c}</em>{Bh}$</td>
<td>$1 - \eta_F$</td>
<td>$1 - \eta_F$</td>
</tr>
<tr>
<td>$\partial \hat{\theta}<em>j / \partial \hat{c}</em>{Bh}$</td>
<td>$\frac{q_L(\theta)(1-\eta_F)}{\gamma_L \eta_L} \left{ \frac{-\hat{\varepsilon}<em>{Bh}}{q_F(\xi_h)} - \frac{\hat{\varepsilon}</em>{Ih}}{q_F(\xi_h)} \right}$</td>
<td>$\frac{-q_L(\theta)(1-\eta_F)}{\gamma_L \eta_L} (1 - \bar{N}_I)$</td>
</tr>
</tbody>
</table>

supply shocks (the magnitude only differs by the elasticity of the financial market to both types of shocks). Finally, home and foreign labor tightnesses (and thus outputs) are here positively correlated and interpreted as financial contagion.

4.4 Quantitative exercise

This subsection estimates the magnitude of international spillovers resulting from both types of financial shocks. Since most of the financial parameters considered in this new approach lack of empirical counterparts, they are chosen so that steady-state values are plausible, while discussion about micro measures of missing data could constitute further research. The calibration of labor markets is standard and kept as simple as possible here.

The matching functions are supposed to be Cobb-Douglas as

$$m_F(N_B, N_I) = \mu_F N_B^{\eta_F} N_I^{1-\eta_F}$$

$$m_C(N_E, N_C) = \mu_C N_E^{\eta_C} N_C^{1-\eta_C}$$

$$m_L(N_U, N_V) = \mu_L N_U^{\eta_L} N_V^{1-\eta_L}$$

23 Afonso and Lagos (2015) use Fedwire data to calibrate a search version of the US interbank market; however, aggregate capital transfers from international investors to country-specific commercial banks, through both capitalization and debt, is not available to our knowledge.

24 For specific discussions about the quantitative performance of search and matching models of labor markets for macroeconomic analysis, see Yashiv (2009) or Cardullo (2010).
where $\mu_F$, $\mu_C$, and $\mu_L$, stand for matching efficiency measures in the financial market, credit market, and labor market, respectively. On the financial and credit markets, let us normalize this efficiency parameter to unity, let us consider that the elasticities of matching functions to the tightness, $\eta_C$ and $\eta_L$ are 0.5, and that the bargaining powers of investors and bankers in the Nash rules, $\delta_C$ and $\delta_F$, are also equal to 0.5. On the labor market, the tightness elasticity $\eta_L$ is set up to 0.66 and so does the share of the surplus earned by workers at equilibrium, so as to be consistent with the Hosios rule. The matching efficiency on the labor market is allowed to vary between 1.1 and 1.5, a range around Shimer (2005)’s 1.355 value. The quarterly separation rate is 0.1 and the riskfree rate is 0.05.

Let us first note that one can technically obtain realistic labor market estimations from moderate financial and credit frictions. Let set $c_I = c_B = 0.1$ so that the equilibrium financial market tightness in (4) is equal to 1. With an entrepreneurs’ non pecuniary cost $c_E$ at 0.005, the equilibrium credit market tightness is then also equal to 1 by (5). Finally, the flow cost $\gamma_C$ of bankers screening credit applications is equally set to 0.1. With a flow cost $\gamma_L$ of job vacancies at 1.5, it results by (7) and (6) that the predicted unemployment rate ranges from 4.68% to 7.26% according to the labor market structural efficiency parameter. These are particularly close to the pre-crisis rates in the US and in the euro area, respectively at 4.8% and 7.2% in February, 2008; however bank fund-raising was relatively easy before the crisis.

Let us thus rather consider a more realistic initial situation in which unemployment rates are of similar magnitude but in a context where banks find liquidity at very high rates, whereas entrepreneurs are indeed moderately credit constrained. In other words, the steady-state is re-parameterized in order to make a distinction between the credit market, where the information about entrepreneurs’ creditworthiness is not immediately available to bankers, on the one hand and the financial market, where banker-investor relationships are essentially frictionless in normal times, on the other hand. Assuming that investors’ and bankers’ bargaining powers on the financial market are now $\delta_F = 0.995$ and $(1 - \delta_F) = 0.005$ respectively, with un-
changed values for the search costs \((c_I = c_B = \gamma_C = 0.1)\), it results from (4) that bankers now raise funds much more quickly as the Poisson rate at which they match with a financier \((q_F(\xi))\) is now 14 times larger. This reflects the existence of large excess liquidity in the pre-crisis equilibrium. Still targeting a credit tightness of 1 now implies that entrepreneurs’ flow cost must equal 0.00035 from (5). Therefore, with \(\gamma = 0.5\), the initial unemployment rates are now evaluated at 4.94\% and 7.66\% (when \(\mu_L = 1.5\) and \(\mu_L = 1.1\) respectively), that is, quite close to the previous numbers. This verifies that the model is able to reproduce frictionless financial markets in normal times, which may be a more realistic initial equilibrium.

Turning now to the effect of different financial shocks, let us keep the average labor market efficiency \((\mu_L = 1.355)\), that is an initial unemployment rate at 5.72\%, in order to remain consistent with the symmetry of the model. It turns out, first, that the elasticity of the labor market tightnesses to liquidity supply shocks (corresponding to the expressions in row 3 of Table 1) are \(-0.82\) at home and \(+0.19\) abroad. In line with the qualitative analysis, this confirms the literature negative co-movements between domestic and foreign responses, in strike opposition with financial conglomeration episodes. In the case of shocks to the cost of banking capitalization, the elasticities (Table 2, row 3) respectively become \(-0.44\) and \(-0.19\), that is indeed contagion\(^{25}\). In terms of unemployment rate, the corresponding figure would be \(+0.0083\) at home and \(+0.0036\) abroad. One should carefully note that these numbers cannot be given a direct interpretation since there is no data equivalence for the parameters considered here but rather give a feel of the magnitude of the contagion as the relative unemployment deviation is 2.3 times bigger in the home country.

\(^{25}\)Both financial shocks have the same negative real effect at the world level \((-0.63)\).
5 Discussions

5.1 An illustration: Stylized facts from the 2007-2009 financial crisis

The model predictions fit particularly well the 2007-2009 financial crisis in terms of international contagion, in particular from the US to other large developed countries with a flexible exchange rate vis-à-vis the dollar, such as the euro area. The stylized facts documented here are very well known, yet they emphasize the relevance of the model predictions. In particular, those differ qualitatively from the literature in demonstrating that positive real co-movements can emerge from an asymmetric financial shock, instead of negative co-movements in standard two-country models. Therefore, the sign and the relative magnitudes of the effects in the different countries are of main interest here.

Originated in the US in 2006, the subprime crisis started to affect credit markets more widely from the end of July 2007 when Bear Sterns liquidated two hedge funds which had invested in some mortgage-backed securities. This is the point in time that can be captured by the financial shock denoted $c_B$ in the model, as perceived credit riskiness increased dramatically from that moment. Figure 6 shows that the daily TED spread, calculated as the difference between the 3-Month LIBOR rate and the 3-Month US Treasury Bill, rose extremely rapidly from its long term 0.5% to close to 2.5%.

The model predicts that such a financial shock is associated with an appreciation of the foreign currency. As one can observe on Figure 7, the appreciation of the euro vis-à-vis the dollar clearly accelerated from the end of July 2007, then going from 1.36 to 1.6 a few months later. It is only one year later, and well after the beginning of the recession in Europe, that the euro started to plunge.\footnote{This subsequent decline has been continuing for several years and might be due to other structural difficulties in the zone which are not due to the immediate contagion effect described in this model.} Moreover, this increase in the nominal exchange rate has not translated into major changes in prices in the euro area, where the inflation rate was close to zero, therefore implying a similar pattern for...
the real exchange rate. Interestingly, similar movements have been observed in other developed countries such as Japan, with an appreciation of the yen up to April 2009, and in a less extent the UK up to the beginning of 2008.

This change in the terms of trade is particularly important because the price-competitiveness channel is playing here in the opposite direction from the standard monetary contraction in two-country models. Indeed, the appreciation of the foreign currency should affect trade balances abroad more than at home, even though the recession is sharper at home than abroad because of the initial wealth effect. The volume of exports of goods and services fell by 8.8% in the US and by 12.7% in the euro area from 2008 to 2009 (Figure 8). The unemployment rate has more than doubled in the US, from 4.6% in August 2007 to 10% in October 2009, while it increased by one third, from 7.5% to 10.1% over the same period (Figure 9). These numbers confirm that the relative variation in the real economy has been more severe.
in the home country than abroad, even though the trade channel mostly hit the foreign country.

Finally, note that the bulk of the effects on the real economy are a bit delayed compared to the financial shock and exchange rate variations. Yet, leading indicators of the real economy changed precisely at the very moment of the shock, i.e in the summer 2007. This is for instance the case of the OECD’s “Composite Leading Indicator” that fell as soon as July 2007 and continuously so up to the beginning of 2009. The US and euro area patterns are quite similar, yet again with a plunge relatively more pronounced in the US (Figure 10).

Overall, the very tractable theoretical framework presented in this paper is thus able to account for some qualitative aspects of the 2007-2009 crisis that differ from mainstream two-country models with flexible exchange rates.
Figure 10: Exports of goods and services, annually, Volume index, Index 100 in 2010. Source: OECD.

Figure 11: Harmonized Unemployment Rate (HUR), monthly. Source: OECD.
5.2 The specific role of each market friction

Besides creating an accelerator effect, as shown by Wasmer and Weil (2004), one may wonder what is the role of the different search and matching frictions here. First, as for the financial market, the tightness offers a very tractable way of handling symmetrically supply-side ($c_I$) and demand-side ($c_B$) shocks. This makes comparison in terms of transmission channels particularly transparent and convenient, without excluding the case in which the market is frictionless as $q_F(\xi)$ goes to infinity. An alternative to the $c_I$ shock could be to introduce money in the model; however this would have no real effect as long as prices are fully flexible and would require the New Keynesian like rigidities in order to generate the same results. On the contrary, it is unlikely that adding some type of liquidity constraints to the
banking sector in a DSGE model would suffice to mimic the $c_B$ shock. This is because the time during which bank fund-raising is made endogenously slower is crucial. Although this time dimension could be formally accounted for without a ‘searching’ process, with information acquisition for instance, it would be at the expense of analytical simplicity, and once again make the comparison with the supply shock not so straightforward.\footnote{See for instance Caballero and Simsek (2009) for a model where banks have to acquire information about their trading partners in the financial network through an auditing process.}

Second, the credit friction is a way to capture easily both credit constraints to entrepreneurs and difficulties that banks may encounter to find viable projects. The equilibrium labor market tightness is lower, and the unemployment level higher, when entrepreneurs are credit-constrained, as observed empirically.\footnote{See Acemoglu (2001) or Chodorow-Reich (2014) for instance. Wasmer and Weil (2004) show that the credit friction makes the labor tightness deviate from the Pissarides equilibrium.} As already mentioned, it is also compatible with the fact that there exist permanent credit creation and credit destruction flows over time. In addition, making this friction local further captures the idea that, because of potential moral hazard effects, new entrepreneurs cannot directly access external finance but have to address intermediaries that would be able to reduce the informational gap. As an alternative to ex post monitoring costs \textit{à la} Townsend, search costs stand for ex ante screening costs and allow sharing output with a simple Nash rule.

Finally, the labor market friction does not play a central role, and could easily be abandoned, but has two practical advantages here. On one hand, it allows to express the results in terms of employment outcomes instead of assuming that output is directly proportional to credit rationing. On the other hand, it actually simplifies the analytical results when assuming that workers both have an exogenous wage and are the only agents to consume. This does not seem a very restrictive assumption when studying the qualitative impact of financial shocks. Indeed, the sign of the effects would remain if all agent types were consuming part or totality of their respective (endogenous) income. However, this would create additional wealth effects through
individual income variations and not only from the fact that matched and
unmatched agents have different utility levels. Thus, the tractable option is
preferred.

5.3 Literature

This paper contributes to three separate bodies of literature. The first
one deals with the macroeconomic role of financial multipliers by which
credit constrained firms overreact to a change in borrowing conditions from
commercial banks. This paper has accounted for such a mechanism, al-
though modeled in a different manner than in Bernanke and Blinder (1989),
Bernanke, Gertler and Gilchrist (1999), or Kiyotaki and Moore (1997). Rather,
credit frictions adopted here follow the tractable closed-economy formaliza-
As shown in the case of liquidity supply shocks, this credit multiplier (or fric-
tion) has been proved not sufficient to solve the puzzle of contagion within
a floating exchange rate system when simply extended to an open-economy
framework.

Second, this paper finds that letting room for non-walrasian situations in
financial markets considerably alters the open-economy channels. The major
recent two-country models have indeed reached the same conclusions than
the early Mundell-Fleming-Dornbusch frameworks as far as monetary shocks
are concerned. This is because they roughly ignore financial relationships,
and particularly the fact that information asymmetry and agent heterogene-
yty can make non-walrasian frictions arise when financial distress appears.
Boivin, Kiley and Mishkin (2010) have reminded that “the core channels
of policy transmission (...) have remained steady from early policy-oriented
models to modern DSGE models”, and added that the exchange rate channel
was the sole neoclassical channel resulting from the openness of the economy.

For instance, the Obstfeld and Rogoff (1995)’s model, well-known for
having provided the Keynesian analysis with microeconomic foundations in a
two-country model, predicts that monetary expansions in one country imply
negative co-movements between home and foreign outputs. In their own
words, following a unilateral increase in home money supply, “the world real interest rate falls and world demand rises, but because the domestic currency depreciates, some world demand is shifted toward home products at foreign producers’ expense.” Here, I show that this “expenditure-switching channel” (substitution effect) can play in opposite directions depending on the financial shock considered. Meanwhile, I preserve a wealth effect through interest rate variations, although it primarily affects the cost of capital rather than the level of consumption as it would do in the new open-macroeconomics literature.

Later improvements of the Obstfeld-Rogoff monopolistic competition framework have not changed the co-movement predictions. Betts and Devereux (2000a, 2000b) included local price stickiness to depart from the law of one price and confirmed the negative output co-movement in case of monetary shocks in the presence of pricing-to-market. Even more surprisingly, in one-area estimated models used until very recently by the Federal Reserve and the European Central Bank, monetary contractions lead to significant and persistent nominal and real appreciations of the domestic currency (for e.g. Eichenbaum and Evans (1995) for the US, Smets and Wouters (2003) for the euro area). Since this type of shocks has been the only way to account for financial disturbance in these models, it would imply that large foreign countries benefit from the recession at home, which appears somewhat counterintuitive in light of the recent events.

The few two-country papers with more sophisticated monetary mechanisms have so far kept a fixed exchange rate regime and thus eluded the contagion puzzle discussed here. For instance, a financial multiplier is at work in Gilchrist, Hairault and Kempf (2002) but in the context of a monetary union; Devereux and Yetman (2010) have studied the international transmission of shocks when investors are highly leveraged but without questioning

\[ \text{Obstfeld and Rogoff (1995) precise that, in terms of welfare, this does not necessarily mean ‘beggar-thy-neighbor’ effects because foreigners “enjoy more leisure, improved terms of trade, and consumption higher than income” when output falls. However, it seems likely that, for a large and prolonged disruption, individuals perceive more disutility from reduced consumption and potential unemployment than utility from leisure. Thus I do not comment welfare implications but focus on the output co-movements puzzle here.} \]
the exchange rate regime.

Finally, aside from macro papers, a literature on financial contagion \textit{per se} has attempted to account for the complexity of modern financial interrelations but relying on the incompleteness of financial markets in the countries to which crises are transmitted. This approach is relevant for studying shocks from developed to emerging market economies, from the famous paper by Allen and Gale (2000) — underlying the claims that banking systems have on one another due to regional incompleteness of financial markets as observable in Asia or in the US in the late nineteenth century — to sudden stops in capital flows (Calvo, Izquierdo and Mejía, 2004) and the current evidence about recoupling movements with US financial circumstances for large and prolonged US financial distress (Dooley and Hutchison, 2009). Some recent papers have followed this approach in the context of the last financial contagion in order to highlight the weakening of effects due to the international trade-based mechanism but still poorly account for the transmission channels across developed economies. In the partial equilibrium model by Krugman (2008) notably, highly leveraged institutions hold both domestic and foreign assets, and this cross-holding is the main propagation channel. Nevertheless, as far as developed countries are concerned, it is more likely that domestic and foreign financial assets are \textit{substitutes} rather than \textit{complements}, and that the equalization of external finance premia across countries is instead the source of international propagation.

Indeed, Dedola and Lombardo (2012) thus developed a two-country general equilibrium model, where “financial and real interdependence can be very strong even with minimal balance sheet exposure to foreign risky assets, if asset markets are integrated across the board”. Yet, they also need a minimum level of asset cross-holdings in order to propagate the financial disruption. I have thus chosen a different approach here, assuming that leveraged banks issue equities on perfectly integrated financial markets, in order to prove that there is room for international contagion without relying on cross-holdings effects. Home and foreign financial assets are perfect substitutes here in the sense that the distributions of yields and risks are comparable, which is likely to be the case between the financial assets of the
US and of the euro area taken as a whole. Note that this is compatible with the evidence of a home bias in equity or bond portfolio, as suggested by Coeurdacier, Kollman, and Martin (2010) for instance, since it leaves the quantity of home versus foreign assets held undetermined.

On methodological grounds, this paper considers the application of search and matching frictions to financial markets. Several ways to formalize them have emerged, including on the money market (Kiyotaki and Wright, 1993), the interbank market (Afonso and Lagos, 2015), or financial trading (Duffie, Gârleanu and Pedersen, 2005). The approach adopted here is yet a distinct one, early developed by Dell’Ariccia and Garibaldi (1998) and later on by Wasmer and Weil (2004), that remains very close to the search and matching setup à la Mortensen and Pissarides (1994). It seems quite relevant as far as the market for commercial loans is concerned, as entrepreneurs and bankers clearly define the supply and demand sides on the market just as workers and firms on the labor market (and unlike the monetary market where the trading agents may be similar ex ante). Some empirical evidence indeed supports this approach as mentioned earlier (Dell’Ariccia and Garibaldi, 2005; Craig and Haubrich, 2013). To the best of my knowledge, this theoretical framework has not been previously considered to solve open macroeconomy puzzles as proposed here.

Three major advantages of this approach have been revealed here. First, it allows to keep a two-country general equilibrium model particularly tractable and to provide analytical results, which is often hard to achieve when both home and foreign variables are endogenized. Second, it allows to account for the fact that creating new relationships between investors and banks, as well as between banks and credit borrowers, after a sudden disruption may be time-consuming. This implies a period of time in which the reinforcement between financial, credit and labor market frictions is economically painful. Third, it permits to depart from traditional monetary shocks and represent shocks of a different nature, for which interest rate adjustments cannot im-

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30. Of course, heterogeneity within the euro area is large in this respect, and partly responsible for the more recent sovereign debt crisis, but clearly beyond the scope of this paper.
mediately clear the market. Considering alternative financial shocks sheds some light on the link between financial markets frictions, financial integration, and the exchange rate regime, with critical implications on international contagion.

6 Conclusion

This paper develops a tractable multi-frictional model where an asymmetric financial shock can be transmitted between large economic areas despite financial market integration and a floating exchange rate regime. This results in positive output co-movements, which are absent from the open macroeconomy literature under the same conditions.

As a particular case, the model is able to nest a well-known result from the two-country literature — from the early Mundell-Fleming models to DSGE recently used by Central Banks —, namely a negative correlation between home and foreign outputs following asymmetric liquidity supply shocks, even in the presence of financial accelerators. This way, it may explain why new open macroeconomic models have for a long time disregarded the literature on financial frictions or alternative financial transmission mechanisms, and vice versa.

Besides, the model also gives room for another type of financial shocks which do generate international contagion, in an otherwise identical setup. In the search and matching structure adopted here, such a shock can be interpreted as an increase in the cost of capitalization of banks that hit in spite of available liquidity in the economy. The non-walrasian approach also accounts for any degree of financial market friction, from quasi-perfect efficiency in normal times to freezes in distressed times, in the sense that price adjustments do not suffice for immediate market-clearing.

Further research could consider several improvements or sophistications of this simple framework. First, the model could be inserted in a fully dynamic setup whereby the resources of financial investors would not be exogenous endowments but driven by saving decisions of the different agents. This would relax somehow the entrepreneurs’ credit constraint but can also
magnify the impact and the persistence of unexpected financial shocks by reducing the willingness to save and invest in risky assets in distressed times. Second, it would be interesting to allow for more than one-to-one relationships and study size effects on different agent types, whether firms or financiers. Crossed-holdings of financial assets between countries would also make the model more realistic and reinforce the contagion. However, these potential modifications are likely to change the magnitude but not the sign of the effects presented here.

Finally, monetary policy implications would be of main interest. The situation studied here is roughly the one that prevails in the absence of special interventions, and Central Banks are not given a proper role. Introducing an interbank market along with the frictional financial market would both diversify liquidity access to banks and confer a more realistic role to monetary authorities. In a two-country framework, positive externalities could then emerge and replace the standard ‘beggar-thy-neighbor’ monetary policy stance.

References


31 The Central bank could be considered as one of the liquidity providers through quantitative easing operations. In that case, it is likely that the uniqueness of Central Banks in each domestic economy would make search frictions disappear and thus directly improve welfare.


7 Mathematical appendix

7.1 Optimal consumptions and the price index

Utility is derived from consumption

\[ U = \mathbb{E}_0 \int_{t=0}^{\infty} \beta^t C_{i,t} \]

where \( C_{i,t} \) denotes the individual consumption level in country \( i \) (\( i = h, f \)) at time \( t \), \( \mathbb{E}_0 \) is the mathematical expectation conditional on information available at time 0, and \( 0 < \beta = (1 + r)^{-1} < 1 \) is the common discount factor. Utility is linear in consumption in order to analyze specific financial transmission channels independently of risk aversion effects. The consumption
level $C_{i,t}$ is a Dixit-Stiglitz composite index of home and foreign goods

$$C_{i,t} = [\alpha^{\lambda} C_{h_i,t}^{\lambda-1} + (1-\alpha)^{\lambda} C_{f_i,t}^{\lambda-1}]^{\frac{1}{\lambda}}$$

where $C_{j,t}$ stands for the consumption level of good $j$ ($j = h, f$) in country $i$ ($i = h, f$) at time $t$, and $\lambda$ is the intra-temporal elasticity of substitution across the two goods. Note that the results hold for any particular value for this degree of substitution between home and foreign aggregate outputs.

Workers’ budget constraint in the home country is simply $w = p_{h} C_{h} + p_{f} C_{f}$, where $w$ is the wage. For tractability, it is assumed that only workers consume here but all agents could consume with quite similar results. The intra-temporal first-order conditions in the home country are therefore

$$(C_{h_i}) : \alpha^{\lambda} C_{h_i}^{\lambda-1} + (1-\alpha)^{\lambda} C_{f_i}^{\lambda-1} = \Lambda P_{h}$$

$$(C_{f_i}) : (1-\alpha)^{\lambda} C_{f_i}^{\lambda-1} + (1-\alpha)^{\lambda} C_{f_i}^{\lambda-1} = \Lambda P_{f}$$

$$(\Lambda) : w = p_{h} C_{h} + p_{f} C_{f}$$

where $\Lambda$ is the Lagrangian multiplier. Solving gives

$$\bar{C}_{h} = \frac{\alpha w(p_{h})^{-\lambda}}{\alpha(p_{h})^{1-\lambda} + (1-\alpha)(p_{f})^{1-\lambda}} \quad \text{and} \quad \bar{C}_{f} = \frac{(1-\alpha)w(p_{f})^{-\lambda}}{\alpha(p_{h})^{1-\lambda} + (1-\alpha)(p_{f})^{1-\lambda}}$$

The Consumption-Based Price Index is defined as the least expenditure that buys a unit of the consumption index on which period utility depends (Obstfeld and Rogoff, 1996). It is computed first by substituting the optimal consumption levels in the initial utility function, and then by replacing the instantaneous income $w$ by the index, denoted $P$, while $C$ is equalized to 1 as $P$ is the minimum expenditure per single unit of consumption

$$\left[\alpha^{\lambda} \frac{\alpha P_{h}(p_{h})^{-\lambda}}{\alpha p_{h}^{1-\lambda} + (1-\alpha)(p_{f})^{1-\lambda}}\right]^{\frac{\lambda-1}{\lambda}} + (1-\alpha)^{\lambda} \left(\frac{(1-\alpha)P_{f}(p_{f})^{-\lambda}}{\alpha p_{h}^{1-\lambda} + (1-\alpha)(p_{f})^{1-\lambda}}\right)^{\frac{\lambda-1}{\lambda}} = 1$$

Rearranging gives the solution for $P$ in the home country

$$P_{h} = [\alpha p_{h}^{1-\lambda} + (1-\alpha)(p_{f})^{1-\lambda}]^{\frac{1}{\lambda-\alpha}}$$
The foreign price index expressed in domestic currency, $S_t P_{ft,t}$, is constructed similarly but does not need to equal $P_{ht,t}$ as preferences parameters ($\alpha$ and $\lambda$) are allowed to differ from one country to another. Further note that the price index is taken as given by a particular consumer since markets for final goods are competitive, but is endogenous at the aggregate level.

7.2 Individual behaviors and autarkic equilibrium

The worker-consumer problem is given here for general equilibrium understanding but the international propagation mechanisms are primarily driven by interactions between investors, bankers and entrepreneurs in the simplified sequential representation. In each period, workers are either unemployed and earn no revenue (in stage 2) or working for a given wage $w$ that allows for consumption (in stage 3). When an unemployed worker encounters an entrepreneur whose job offer matches his or her characteristics, he or she can either reject the offer and wait for a new job opportunity or accept the offer and earn $w$ until an adverse shock arrives. Worker-entrepreneur relationships end at the exogenous separation rate $s$. Hence, the optimal stochastic value function $W_{i,t}$ of an unemployed worker of country $i$ at time $t$ satisfies the following recursive problem

$$W_{i,t}(\theta_{i,t}, S_t) = \max_{\text{accept, reject}} \left\{ \max_{C_{hi,t}, C_{fi,t}} \left\{ U_*(\cdot) + \beta(1-s)W_{3i,t+dt} + \beta s W_{2i,t+dt} \right\}; \right.$$ 

$$\beta [1 - \theta_{i,t}q_L(\theta_{i,t})] W_{2i,t+dt} + \beta \theta_{i,t}q_L(\theta_{i,t}) W_{3i,t+dt} \right\}$$

\[\text{(*) s.t. } w_i = p_{bh,t}C_{hi,t} + S_t p_{ff,t}C_{fi,t}\]

where $W_2$ and $W_3$ are the value functions of workers in the respective stages 2 and 3 of the process described above, and where $p_{ji,t}$ is the price of good $j$ in country $i$ and expressed in country $i$ currency at time $t$.

The consumption index obtained above (Appendix A) allows re-expressing
the individual budget constraint as \( w_{i,t} = P_{i,t}C_{i,t} \), where \( C_{i,t} \) is the optimal consumption basket in country \( i \) at date \( t \). Therefore, dropping time and country subscripts, the simplified Bellman equations for a worker in the successive stages of the sequential process are

\[
W_2' = \theta q_L(\theta)(W_3 - W_2) \\
W_3' = \frac{w}{P} + s(W_2 - W_3)
\]

A similar problem for entrepreneurs gives the following Bellman equations

\[
E_1' = -c_E + q_C(\phi)(E_2 - E_1) \\
E_2' = -\gamma_L + \gamma_L + q_L(\theta)(E_3 - E_2) \\
E_3' = p - w - \rho_C + s(E_4 - E_3)
\]

with \( E_1, E_2, E_3 \) the respective intertemporal values of entrepreneurs in stages 1, 2 and 3, \( c_E \) the search cost in stage 1, and \( \gamma_L \) the search cost in the recruitment stage (offset by the amount borrowed from the bank).

Similarly, for the commercial banks,

\[
B_0' = -c_B + q_F(\xi)(B_1 - B_0) \\
B_1' = -\gamma_C + \gamma_C + \phi q_C(\phi)(B_2 - B_1) \\
B_2' = -\gamma_L + \gamma_L + q_L(\theta)(B_3 - B_2) \\
B_3' = \rho_C - \rho_F + s(B_4 - B_3)
\]

where \( c_B \) and \( \gamma_C \) stand for bankers' search costs in stage 0 and stage 1 respectively, and where \( \gamma_L \) is offset by the capital provided by the investor.

Similarly, for the financial investors, with \( c_I \) their search cost in stage 0,

\[
I_0' = -c_I + \xi q_F(\xi)(I_1 - I_0) \\
I_1' = -\gamma_C + \phi q_C(\phi)(I_2 - I_1)
\]

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\[ rI_2 = -\gamma_L + q_L(\theta)(I_3 - I_2) \]
\[ rI_3 = \rho_F + s(I_4 - I_3) \]

Free entry implies that, in equilibrium, \( E_1 = 0, B_0 = 0, \) and \( I_0 = 0. \) The first Bellman equation for each agent therefore gives their respective backward value one stage after entering the process as follows:

For entrepreneurs \( E_2 = \frac{cE}{qC(\phi)}; \)

For bankers \( B_1 = \frac{cB}{qF(\xi)}; \)

For investors \( I_1 = \frac{cI}{\xi qF(\xi)}. \)

Free exit \( (E_4 = 0, B_4 = 0, \) and \( I_4 = 0) \) similarly gives the value in stage 3 from the last Bellman equation in each group. Forward values for stages 1 and 2 are then obtained recursively as

For entrepreneurs \( E_3 = \frac{p - w - \rho_C}{r + s}, \quad E_2 = \frac{q_L(\theta)}{r + q_L(\theta)}E_3; \)

For bankers \( B_3 = \frac{\rho C - \rho F}{r + s}, \quad B_2 = \frac{q_L(\theta)}{r + q_L(\theta)}B_3, \quad B_1 = \frac{\phi qC(\phi)}{r + \phi qC(\phi)}B_2; \)

For investors \( I_3 = \frac{\rho F}{r + s}, \quad I_2 = \frac{-\gamma_L + q_L(\theta)I_3}{r + q_L(\theta)}, \quad I_1 = \frac{-\gamma C + \phi qC(\phi)I_2}{r + \phi qC(\phi)}; \)

Equalizing the backward and forward values for each agent finally gives the respective equilibrium conditions (1) to (3).

Alternatively, these equilibrium conditions could have been obtained from the fact that free entry implies a zero-profit condition which equalizes expected present-discounted costs and gains in equilibrium for each agent type. For instance, the following expression must hold for entrepreneurs

\[
E_0(T_1)\left\{\int_0^{T_1} (-cE)e^{-rt} dt + E_{R_1}(T_2)\left[\int_{T_1}^{T_2} 0e^{-rt} dt + E_{R_2}(T_3)\int_{T_2}^{T_3} (p - w - \rho C)e^{-rt} dt\right]\right\} = 0
\]
where $T_1$, $T_2$, and $T_3$ follow Poisson processes, and is thus rewritten as

$$c_E \int_0^t \int_t^\infty q_c(\phi)e^{-q_c(\phi)T_1}dT_1e^{-r}dt = (p-w-\rho_C)\int_0^\infty \int_{T_1}^\infty \int_t^\infty se^{-s(T_3-T_2)}dT_3 \times e^{-r}dt q_L(\theta)e^{-q_L(\theta)(T_2-T_1)}dT_2 q_c(\phi)e^{-q_c(\phi)(T_1-t)}dT_1$$

Solving yields \( \frac{c_E}{r+q_C(\phi)} = \frac{p-w-\rho_C}{r+q_C(\phi)} \frac{q_L(\theta)}{r+q_L(\theta)} \frac{q_c(\phi)}{r+q_c(\phi)} \) which simplifies as (1).

Finally, the Nash bargaining rule for the repayment $\rho_F$, $(1-\delta_F)(I_1-I_0) = \delta_F(B_1 - B_0)$, together with the backward values for $B_1$ and $I_1$ then gives the equilibrium financial market tightness as

$$\xi = \frac{1-\delta_F}{\delta_F} \frac{c_I}{c_B}$$

Recursively, the second Nash bargaining rule for the repayment $\rho_C$, $(1-\delta_C)(B_2 - B_1) = \delta_C(E_2 - E_1)$, together with the values of the agents at the time they meet and the previous value for $\xi$, gives the equilibrium credit market tightness as

$$\phi = \frac{1-\delta_C}{\delta_C} \frac{c_B}{c_E q_F(\xi)}$$

Solving (1) to (5) gives the equilibrium labor market tightness $\bar{\theta}$ in (7).

The equilibrium market tightnesses (4), (5), and (7) are loglinearized as

$$\hat{\xi} = \hat{c}_I - \hat{c}_B$$

$$\hat{\phi} = (1-\eta_F)\hat{c}_B + \eta_F\hat{c}_I - \hat{c}_E$$

$$\hat{\theta} \approx \frac{q_L(\bar{\theta})}{\eta_L \gamma_L} \left\{ \frac{\rho \bar{p}}{s} - \left[ (1-\eta_F)\hat{c}_B + \eta_F\hat{c}_I \right] \hat{\kappa} - \frac{\hat{c}_E(1-\eta_C)}{\phi q_C(\phi)} \left[ \frac{\hat{c}_C}{\phi q_C(\phi)} + \frac{\hat{c}_E}{q_C(\phi)} \right] \right\} - \frac{\hat{c}_L}{\eta_L}$$

where $\hat{\kappa} \approx \frac{\hat{c}_B}{q_F(\xi)} + \frac{\hat{c}_I}{\xi q_F(\xi)} + \frac{\eta_F \hat{c}_E}{q_C(\phi)} - \frac{(1-\eta_C)\gamma_C}{\phi q_C(\phi)}$ and with $\bar{\tau} = 0$

where a hatted variable denotes the loglinear deviation from its steady-state value ($\hat{x} = \frac{x-x_s}{x_s}$), and where $\eta_L, \eta_C, \eta_F \in (0,1)$ are the respective matching function elasticities at equilibrium ($\eta_L \equiv -q_L'(\bar{\theta})/q_L(\bar{\theta})$, $\eta_C \equiv -q_C'(\bar{\phi})/q_C(\bar{\phi})$, and $\eta_F \equiv -q_F'(\bar{\xi})/q_F(\bar{\xi})$). Loglinearizing (6) further gives the unemployment rate response as $\hat{u} \approx -(1-\eta_L)(1-\bar{u})\hat{\theta}$. Note that $\hat{\kappa}$
is assumed positive with plausible values of the parameters henceforth so that negative financial shocks realistically raise the unemployment rate in the closed economy.

7.3 International setup and financial spillovers

7.3.1 Aggregate setup and financial spillovers

Each firm of country \( i \) produces one unit of the good in which the economy is specialized \((i = h, f)\) and maximizes profits by determining the optimal division of this output unit between domestic sales \( C_{ih,t} \) and exports \( C_{if,t} \), taking prices \( p_{ih,t} \) and \( p_{if,t} \) and the exchange rate \( S_t \) as given.

In the two-country case, the equilibrium condition (1) for entrepreneurs in country \( i \) (expressed in domestic currency) is thus rewritten as

\[
\frac{c_{Ei}}{qC(\phi_i)} = \frac{qL(\theta_i)}{r + qL(\theta_i)} \frac{p_{ih}C_{ih} + Sp_{if}C_{if} - w_i - \rho C_i}{r + s_i}
\]

With a labor force normalized to one and one unit produced per firm, the instantaneous output of country \( i \) is merely its contemporaneous employment rate \((1 - u_{i,t})\). It gives four aggregate constraints on goods as

\[(1 - u_{i,t}) = C_{ih,t} + C_{if,t}, \quad i = h, f\]

In each period the country-specific income is either devoted to the pecuniary costs induced by search activities or consumed in the home and foreign goods. Assuming for simplicity that output and search costs are constant through time, the resource constraints expressed in domestic currency are

\[p_{ih,t}C_{ih,t} + S_t p_{if,t} C_{if,t} - \gamma C N_{Ci,t} - \gamma L N_{Ei,t} = p_{ih,t} C_{ih,i,t} + S_t p_{if,t} C_{if,t}\]

where the equilibrium values of \( N_C \) and \( N_E \) are respectively obtained when flows of bankers and entrepreneurs into and out of the market are equalized

\[(1 - N_{Ci})s_i = \phi_s qC(\phi_i) N_{Ci} \quad \text{and} \quad (1 - N_{Ei})s_i = qL(\theta_i) N_{Ei}\]
Similarly, the ratio of unmatched global financial investors at equilibrium is

\[ \tilde{N}_f = \frac{\tilde{s}_h + \tilde{s}_f}{\tilde{s}_h + \tilde{s}_f + \xi_h q_F(\xi_h) + \xi_f q_F(\xi_f)} \]

and roughly captures the amount of global excess liquidity at time \( t \).

### 7.3.2 Impact of financial shocks (two-country case)

Just as in the closed economy case, solving loglinear (open-economy) versions of equations (1)–(3), replacing \( \hat{\xi}_i = \hat{c}_I - \hat{c}_B \) and \( \hat{\phi}_i = (1 - \eta_F)\hat{c}_B + \eta_F \hat{c}_I \), and further simplifying \( \hat{\gamma}_{c} = \hat{\gamma}_{L} = \hat{\eta} = \hat{s}_f = \hat{s} = \hat{r} = 0 \), give the following expression for the domestic labor market tightness

\[ \hat{\theta}_h \approx \frac{q_L(\hat{\theta}_h)}{\eta_L \gamma_h} \left\{ \bar{p}_{h}\bar{C}_{h}\bar{h}(\bar{p}_{h} + \bar{C}_{h}) + \bar{S} \bar{p}_{f} \bar{C}_{f}\bar{h}(\bar{S} + \bar{p}_{f} + \bar{C}_{f}) - [(1 - \eta_F)\hat{c}_B + \eta_F \hat{c}_I] \bar{\kappa}_h \right\} \]

where \( \bar{\kappa}_h = \tilde{s}_h \left[ \frac{\tilde{e}_{B_h}}{q_F(\xi_h)} + \frac{\tilde{e}_{I_h}}{\xi_h q_F(\xi_h)} + \frac{\eta_c \tilde{e}_{B_h}}{q_C(\phi_h)} - \frac{(1 - \eta_c)\tilde{e}_{C_h}}{\phi_h q_C(\phi_h)} \right] \)

Then, loglinearizing the expression for the exchange rate (8) and given that \( \hat{\xi}_h q_F(\xi_h)\tilde{N}_f = (1 - \tilde{N}_f)\tilde{s}_h \), we have

\[ \bar{S} \bar{p}_{f} \bar{C}_{f}\bar{h}(\bar{S} + \bar{p}_{f} + \bar{C}_{f}) - \tilde{s}_h(1 - \tilde{N}_f)(1 - \eta_F)(\hat{\xi}_h - \hat{\xi}_f) \]

Substituting into the previous equation thus gives

\[ \hat{\theta}_h \approx \frac{q_L(\hat{\theta}_h)}{\eta_L \gamma_h} \left\{ \bar{p}_{h}\bar{C}_{h}\bar{h}(\bar{p}_{h} + \bar{C}_{h}) - \tilde{s}_h(1 - \tilde{N}_f)(1 - \eta_F)(\hat{\xi}_h - \hat{\xi}_f) + \bar{p}_{f} \bar{C}_{f}\bar{h}(\bar{p}_{f} + \bar{C}_{f}) - [(1 - \eta_F)\hat{c}_B + \eta_F \hat{c}_I] \bar{\kappa}_h \right\} \]

Finally, loglinearizing consumers’ budget constraint as

\[ \bar{p}_{h}\bar{C}_{h}\bar{h}(\bar{p}_{h} + \bar{C}_{h}) + \bar{p}_{f} \bar{C}_{f}\bar{h}(\bar{p}_{f} + \bar{C}_{f}) = \tilde{w} \tilde{\bar{w}} \]

and given that \( \tilde{w} = \hat{\xi}_f \), the labor market tightness simplifies to

\[ 33 \text{This holds as workers’ wage is exogenous in the model. Besides composition changes due to exchange rate variations following negative financial shocks (} c_I \text{ and } c_B \text{), consumption is thus reduced via greater unemployment (direct effect). As a second round effect, the change in firms’ profits will translate into prices rather than wage cuts here.} \]
\[ \hat{\theta}_h \approx -\frac{q_L(\hat{\theta}_h)}{\eta_L \gamma L_h} \left\{ \left[ (1 - \eta_F)\hat{\epsilon}_{B_h} + \eta_F \hat{\epsilon}_{I_h} \right] \tilde{\epsilon}_h + \tilde{s}_h (1 - \tilde{N}_I)(1 - \eta_F)(\hat{\epsilon}_{I_h} - \hat{\epsilon}_{B_h} - \hat{\epsilon}_{I_f} + \hat{\epsilon}_{B_f}) \right\} \]

where the first member in curly brackets is the direct financial transmission channel while the second is the expenditure-switching channel resulting from real exchange rate variations. The home and foreign labor tightness responses to asymmetric liquidity supply shocks are therefore respectively given by \( \partial \hat{\theta}_h / \partial \hat{\epsilon}_{I_h} \) and \( \partial \hat{\theta}_h / \partial \hat{\epsilon}_{I_f} \) (subsection 3.2), while responses to asymmetric shocks to bank capitalization cost are \( \partial \hat{\theta}_h / \partial \hat{\epsilon}_{B_h} \) and \( \partial \hat{\theta}_h / \partial \hat{\epsilon}_{B_f} \) (3.3).