The Nonlinear Effects of Fiscal Policy[*]

Pedro Brinca
Nova SBE, CEF-UP

Miguel Faria-e-Castro
FRB St. Louis

Miguel H. Ferreira
Nova SBE

Hans Holter
University of Oslo

September 20, 2018

Abstract

We argue that the fiscal multiplier of government purchases is *increasing* in the shock, in contrast to what is assumed in most of the literature: the fiscal multiplier is largest for large positive government spending shocks and smallest for large contractions in government spending. We empirically document this fact by analyzing two independent datasets and using two different empirical approaches. We find that a neoclassical, life-cycle, incomplete markets model calibrated to match key features of the US economy, including the distribution of wealth, can well explain this empirical finding. The mechanism works through the relationship between fiscal shocks, the distribution of wealth and the aggregate labor supply elasticity: liquidity constrained agents have less elastic labor supply responses to changes in future income. An increase (decrease) in government spending today acts as a negative (positive) shock to future income, as future wages will be lower (higher). A large increase (decrease) in government spending today will induce saving (borrowing) and move a larger fraction of the agents in the economy away from (towards) the borrowing limit.

VERY PRELIMINARY - DO NOT CIRCULATE

Keywords: Fiscal Multipliers, Nonlinearity, Asymmetry, Heterogeneous Agents
JEL Classification: E21; E62

1 Introduction

During the 2008-2009 financial crisis, many OECD countries adopted expansionary fiscal policies to stimulate economic activity. In many countries, this movement of fiscal expansions was promptly followed by a period of austerity measures aimed at reducing the size of the resulting high levels of government debt. This era of fiscal activism inspired the economic literature to revive the classical debate on the size of the fiscal multiplier and its determinants, such as the state of the economy, income and wealth inequality, demography, tax progressivity, the stage of development of a country, among others.[1]

However, most of the literature treats the effects of government interventions as being *linear*: contractionary and expansionary fiscal policies are assumed to have the same (symmetric) effects, and small and large shocks are assumed to have the same (linear) effects.[2] In this paper, we argue that fiscal multipliers from government spending shocks are *increasing* in the shock. Large contractions in government spending is associated with smaller multipliers whereas large expansions is associated with larger multipliers. We both verify this fact empirically and show that it holds true in a calibrated neoclassical, life-cycle model with incomplete markets and heterogeneous agents.

We begin our analysis by empirically documenting the sign and size dependence of fiscal multipliers: fiscal multipliers are increasin in the government expenditure shock. To arrive at this conclusion, we utilize two different datasets and empirical methodologies, based on two leading empirical papers in the area. First, we focus on Ramey and Zubairy (2018) who use quarterly data for the U.S. economy going back to 1889 and an identification scheme for government spending shocks that combines news about forthcoming variations in military

---

[1]See for example Auerbach and Gorodnichenko (2012), Ramey and Zubairy (2018), Brinca et al. (2016), Brinca et al. (2017), Hagedorn et al. (2016), Krueger et al. (2016), Basso and Rachedi (2017), Ferriere and Navarro (2014), Ilzetzki et al. (2013).

[2]Some notable recent exceptions include Barnichon and Matthes (2017) and Fotiou (2017), who study the asymmetry and nonlinear effects of fiscal policy from an empirical perspective. Barnichon and Matthes (2017) find that contractionary multipliers are larger than expansionary ones during periods of slack for the US. Fotiou (2017) uses a panel of countries to assess how different types of fiscal contractions (i.e. tax or expenditure based) can have nonlinear effects.

spending as in Ramey (2011b) and the identification assumptions of Blanchard and Perotti (2002). Using Jordà (2005)'s projection method and pooling observations across high and low unemployment periods, the authors find no evidence of a state dependent fiscal multiplier. We instead pool observations across periods with negative and positive fiscal shocks, and find evidence that the fiscal multiplier is quantitatively and statistically different across negative and positive shocks: the 1-year multiplier for positive shocks is 0.56, and for negative shocks it is 0.20.

Second, we use the fiscal consolidation episodes dataset from Alesina et al. (2015a), which comprises 16 OECD countries over the 1981-2014 period. Using a narrative approach based on Romer and Romer (2010) to identify exogenous fiscal consolidations, we find fiscal multipliers to be decreasing in the size of the consolidation (which is analogues to increasing in government spending).

Next, we rationalize these empirical findings in the context of a neoclassical, life-cycle, heterogeneous agents model with incomplete markets, similar to Brinca et al. (2016) and Brinca et al. (2017). The model is calibrated to match key features of the U.S. economy, such as the income and wealth distribution, hours worked, taxes, and social security. In our model, agents face uninunsurable labor income risk that induces precautionary savings behavior. The equilibrium features a positive mass of agents who are borrowing constrained: as is well known, the labor supply elasticity of these agents is lower and their work hours are less responsive to changes in aggregate variables such as factor prices.

We study how the economy responds to different changes in government spending, ranging from large contractions to large expansions. An increase in government spending, financed by debt, generates a negative future income effect. This effect is compounded by the crowding out of private capita and future tax increases. As the stock of capital falls, real wages also fall, reducing expected lifetime income. In addition taxes will be raised in the future to pay back debt. The negative shock to future income induces increased saving today this reduces the mass of agents at the borrowing constraint in the period following the

shock. Since unconstrained agents have a higher labor supply elasticity, aggregate labor supply reacts more to a larger fiscal policy shock, leading to larger fiscal multipliers. The logic is analogous for government spending contractions: this raises the mass of constrained households, reducing the sensitivity of labor supply to the fiscal shock, and therefore the fiscal multiplier. The larger the shock, the larger the changes in the distribution of wealth, which induces size dependence.[3]

The rest of the paper is organized as follows: Section 2 presents the empirical results on the non-linearity of fiscal multipliers. Section 3 argues that standard representative agent models can match the levels, but not the nonlinear patterns that we find in the data. Section 4 introduces the main quantitative model, and Section 5 describes the our calibration strategy. Section 7 presents the results from the quantitative model and Section 9 concludes.

2 Empirical Analysis

In this section, we use two different datasets and employ two different methodologies to provide evidence on the sign and size dependence of fiscal multipliers. We start by using the historical dataset of Ramey and Zubairy (2018) for the US and the Local Projection Method of Jordà (2005) to show that a positive government spending shock yields larger multipliers than a negative shock of the same magnitude. We then show that the fiscal multiplier depends not only on the sign but also on the size of the shock. Finally, we use that the dataset assembled by Alesina et al. (2015a) on consolidation episodes to corroborate our results, showing that larger consolidations yield smaller fiscal multipliers.

2.1 U.S. historical data

To compare the multipliers across positive and negative fiscal shocks, a sufficiently large span of observations for both types of shocks is needed. Using U.S. quarterly historical

---

[3]In related work, Athreya et al. (2017) study how redistributive policies can affect output due to heterogeneity in labor supply elasticities.

data addresses this problem, as it provides us with enough observations for both shocks[4]. Additionally, historical 20th century data spans many periods of expansion and recession, as well as different regimes for fiscal and monetary policy.

We employ the historical dataset constructed by Ramey and Zubairy (2018), which contains quarterly time series for the U.S. economy ranging from 1889 to 2015. The dataset includes real GDP, GDP deflator, government purchases, federal government receipts, population, unemployment rate, interest rates and defense news.[5]

To identify exogenous government spending shocks Ramey and Zubairy (2018) use two different approaches: 1) the defense news series proposed by Ramey (2011b), which consists of exogenous variations in government spending linked to political and military events that are identified using a narrative approach, and that are plausibly independent from the state of the economy; 2) shocks based on the identification hypothesis of Blanchard and Perotti (2002) that government spending does not react to changes in macroeconomic variables within the same quarter. Ramey and Zubairy (2018) argue that including both instruments simultaneously can bring advantages, as the Blanchard-Perotti shock is highly relevant in the short run (since it is the part of government spending not explained by lagged control variables), while defense news are more relevant in the long run (as news happen several quarters before the spending actually occurs).

Figure 1 plots the time series for both shocks. Large variations in the 1910s and 1940s reflect defense spending with the two world wars. The smaller variations throughout the rest of the sample mostly reflect Blanchard-Perotti shocks. The figure highlights that there is ample variation in this measure of exogenous spending shocks, both in terms of sign and size.

---

[4]255 observations for positive fiscal shocks and 249 observations for negative ones.

[5]For more details on the dataset see Ramey and Zubairy (2018)
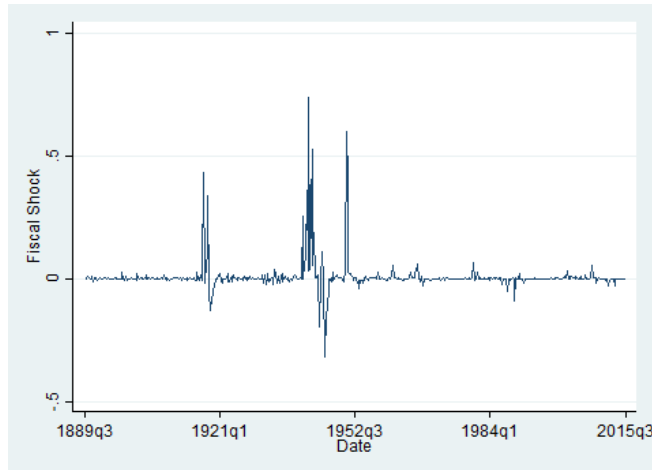
Figure 1: Fiscal shock as a percentage of real GDP

It is instructive to start with a non-parametric approach and look for signs of a nonlinear relationship between output and government spending in the data. Figure 2 shows the 1 quarter cumulative output response in the y-axis, and 1 quarter cumulative government spending in the x-axis, both normalized by trend GDP. The red line is a fitted quadratic polynomial: this line is increasing, which implies that the fiscal multiplier is *positive*; moreover, the line is *increasing*, suggesting that output increases by relatively more for larger shocks to government spending. This convexity arises from a positive quadratic term, which is both quantitatively large (0.49), but also statistically significant at the 1% level.[6]

---

[6]Appendix A.1 presents the same figure at the 4 and 8 quarter horizons.

Figure 2: 1 quarter cumulative real output in the y-axis and 1 quarter cumulative real government spending in the x-axis, both as percentage of trend GDP. The red line represents the quadratic fitted polynomial between the two variables. The correlation between output and the first order term of government spending is 0.44 (p-value 0.00) and with the second order term of government spending is 0.49 (p-value 0.00).

2.1.1 Testing for Sign-Dependence
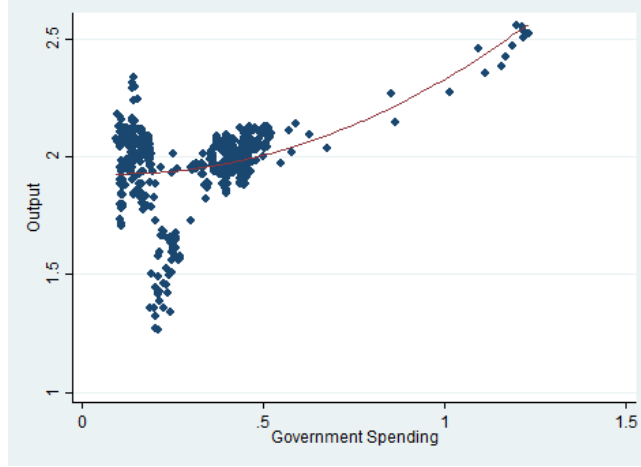
To formally test for potential asymmetries between positive and negative fiscal shocks, we use the same methodology as Ramey and Zubairy (2018), which is based on the Local Projection Method of Jordà (2005). This consists of estimating the following equation for different time horizons h

$$y_{t+h} = I_{t-1}[\alpha_{\text{pos},h} + \Psi_{\text{pos},h}(L)z_{t-1} + \beta_{\text{pos},h}\text{shock}_t]$$

$$+(1 - I_{t-1})[\alpha_{\text{neg},h} + \Psi_{\text{neg},h}(L)z_{t-1} + \beta_{\text{neg},h}\text{shock}_t] + \epsilon_{t+h}, \text{ for } h = 0, 1, 2, ... \quad (1)$$

where y is real GDP per capita divided by trend GDP, z is a vector of control variables, including real GDP per capita, government spending and tax revenues, all divided by trend GDP. z also includes the news variable to control for serial correlation. $\Psi_h(L)$ is a polynomial of order 4 in the lag operator and shock is the exogenous shock, which consists of the defense news variable and the Blanchard-Perotti spending shock. I is a dummy variable that is equal to one when shock ≥ 0.

Ramey and Zubairy (2018) follow a literature that highlights that in a dynamic environment, the multiplier should not be calculated as the peak of the output response to the initial government spending variation but rather as the integral of the output variation to the integral of the government spending variation.[7] This method has the advantage of measuring all the GDP gains in response to government spending variations in a given period. Ramey and Zubairy (2018) propose estimating the following instrumental variables specification that allows for the direct estimation of the integral multiplier,

$$\sum_{j=0}^{h} y_{t+j} = I_{t-1}[\delta_{\text{pos},h} + \phi_{\text{pos},h}(L)z_{t-1} + m_{\text{pos},h} \sum_{j=0}^{h} g_{t+j}]+ \tag{2}$$

$$(1 - I_{t-1})[\delta_{\text{neg},h} + \phi_{\text{neg},h}(L)z_{t-1} + m_{\text{neg},h} \sum_{j=0}^{h} g_{t+j}] + \epsilon_{t+h}, \text{ for } h = 0, 1, 2, ...$$

where shock$_t$ is used as an instrument to $\sum_{j=0}^{h} g_{t+j}$, which is the sum of the government spending from t to $t + h$. This way, $m_{\text{pos},h}$ and $m_{\text{neg},h}$ can be directly interpreted as the cumulative multiplier at horizon h for either regime (positive or negative shocks).

Estimation results for specification (2) are presented in Table 1; these results show that the two multipliers are quantitatively different, with the multiplier for positive fiscal shocks being larger than the multiplier for negative shocks. Ramey and Zubairy (2018) argue that the Blanchard-Perotti shocks may be anticipated, and this can raise concerns of instrument relevance. To test if the multipliers are also statistically different across positive and negative fiscal shocks, we use Anderson et al. (1949) (AR) statistics, which is robust to weak instruments. As it is possible to see in the last column in table 1 the instruments are not only quantitatively but also statistically different.

Barnichon and Matthes (2017) present results opposite to ours, with their estimates of the multiplier being larger for contractions than for expansions of government spending. This is related to different choices of methodology and instruments. First, Barnichon and

---

[7]See Mountford and Uhlig (2009), Uhlig (2010) and Fisher and Peters (2010).

	Linear	Negative shocks	Positive shocks	AR P-value
Impact	0.20	0.13	0.44	AR = 0.14
	(0.17)	(0.33)	(0.14)	
1 year cumulative multiplier	0.27	0.20	0.56	AR = 0.13
	(0.14)	(0.26)	(0.17)	
2 year cumulative multiplier	0.45	0.30	0.66	AR = 0.06
	(0.10)	(0.24)	(0.11)	
3 year cumulative multiplier	0.56	0.55	0.72	AR = 0.05
	(0.09)	(0.16)	(0.09)	
4 year cumulative multiplier	0.58	0.68	0.73	AR = 0.07
	(0.09)	(0.12)	(0.08)	

Table 1: Impact and cumulative multipliers for 1, 2, 3 and 4 year horizons for positive and negative fiscal shocks.

Matthes (2017) do not combine defense news and Blanchard-Perotti shocks. As Ramey and Zubairy (2018) argue, the defense news variable fails to capture short run dynamics, while the Blanchard-Perotti identification hypothesis fails to capture the long run dynamics, so it becomes important to use both instruments at the same time to accurately capture both short and long run dynamics. Second, when using the Blanchard-Perotti identification hypothesis, the authors deviate from what Ramey and Zubairy (2018) propose, by first estimating a VAR to identify the shock and then including the shock in the Local Projection regression, while also including lagged control variables. As Ramey and Zubairy (2018) highlight, the Blanchard-Perotti shock is identified as the part of government expenditure not explained by lagged control variables. Including these lagged control variables in a regression with current government spending is enough to correctly identify the Blanchard-Perotti shock.

2.1.2 Testing for Size-Dependence

The previous exercise shows that positive fiscal spending shocks generate larger output effects than negative ones, but is not sufficient to say anything about the effects of shocks with the same sign but different sizes. We proceed to investigate the size-dependence of the fiscal multiplier. We start by extending specification (2) with quadratic terms for both fiscal expansions and contractions

$$\sum_{j=0}^{h} y_{t+j} = I_{t-1}[\delta_{\text{pos},h} + \phi_{\text{pos},h}(L)z_{t-1} + m_{\text{pos},h}\sum_{j=0}^{h} g_{t+j} + m_{\text{2pos},h}\sum_{j=0}^{h} g_{t+j}^2]+ \quad (3)$$

$$(1 - I_{t-1})[\delta_{\text{neg},h} + \phi_{\text{neg},h}(L)z_{t-1} + m_{\text{neg},h}\sum_{j=0}^{h} g_{t+j} + m_{\text{2neg},h}\sum_{j=0}^{h} g_{t+j}^2] + \epsilon_{t+h}, \text{ for } h = 0, 1, 2, ...$$

If the effects of fiscal policy are size-dependent, coefficients $m_{\text{2pos},h}$ and $m_{\text{2neg},h}$ should be statistically different from zero. Table 2 reports the estimation results: in the short run, nonlinearities are stronger for fiscal expansions than for contractions, with the quadratic coefficient for fiscal expansions being statistically different from zero and indicating that the fiscal multiplier is largest for large expansions.

	$m_{\text{pos},h}$	$m_{\text{pos},h}$	$m_{\text{neg},h}$	$m_{\text{2neg},h}$
Impact	0.33	0.17	0.15	0.03
	(0.12)	(0.07)	(0.33)	(0.07)
1 year	0.27	0.14	0.32	-0.02
	(0.12)	(0.07)	(0.32)	(0.06)
2 years	0.08	0.13	0.32	-0.01
	(0.31)	(0.08)	(0.40)	(0.08)
3 years	-0.65	0.18	-0.44	0.17
	(0.63)	(0.09)	(0.76)	(0.13)
4 years	-1.36	0.21	-1.31	0.24
	(0.91)	(0.09)	(0.93)	(0.11)

Table 2: Linear and quadratic terms for 1, 2, 3 and 4 year horizons for positive and negative fiscal shocks.

Note that the inclusion of these quadratic terms means that $m_{i,h}$ can no longer be interpreted as a multiplier. Due to size dependence, there is no longer such thing as "the" fiscal multiplier. An estimate for the marginal fiscal multiplier can be obtained as $m_{i,h} + 2 \times m_{2i,h} \times G_h$ for $i = \text{pos, neg}$. Table 3 reports the multipliers for the average G in fiscal expansions and contractions, as well as the average G in a fiscal expansion plus one standard deviation, and the average G in a fiscal contraction minus one standard deviation. These estimated multipliers are, once again, larger for expansions than for contractions at short horizons. In the context of a fiscal expansion, raising G by one standard deviation, increases the cumulative multiplier by 19% on impact. During a fiscal contraction, reducing

G by one standard deviation only decreases the multiplier by 7%, on average.

	Average negative - st.dev.	Average negative	Average positive	Average positive + st.dev.
Impact mult.	0.15	0.16	0.40	0.47
1 year cum. mult.	0.32	0.30	0.40	0.57
2 year cum. mult.	0.32	0.31	0.29	0.57
3 year cum. mult.	-0.50	-0.09	-0.22	0.32
4 year cum. mult.	-1.45	-0.08	-0.71	0.07

Table 3: Impact and cumulative multipliers for 1, 2, 3 and 4 year horizons for positive and negative fiscal shocks.

An alternative way to illustrate this size-dependence is to calculate the fiscal multiplier for each observation in our sample, using the estimates in Table 2, and plot these estimates against the size of the respective fiscal shocks. This is done in Figure 3: asymmetry is very clear, as multiplier estimates for negative spending shocks are much lower than those associated with positive spending shocks (about 0.2 vs. 0.5 on average). Size dependence has an interesting pattern that reflects our earlier estimates: the slope is very small (but positive) for negative spending shocks, and much steeper for positive ones. The range of multipliers for positive shocks is $[0.26, 0.75]$, and $[0.16, 0.22]$ for negative ones.[8]
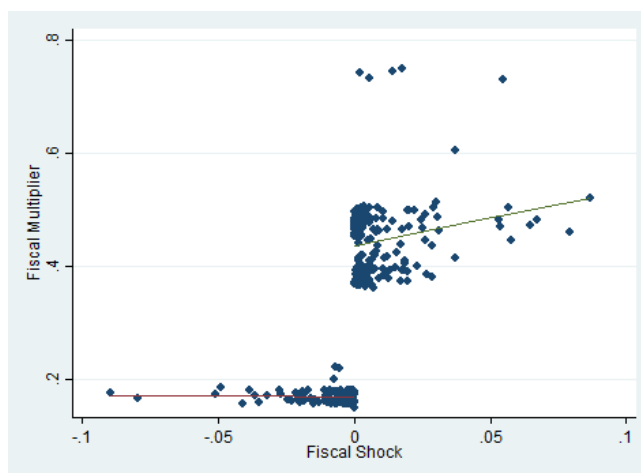

Figure 3: Impact multiplier vs Fiscal shock: On the y-axis we have the impact multiplier and on the x-axis we have the size of the fiscal shock. For negative shocks the correlation is -0.04 (P-value 0.56) while for positive shocks is 0.20 (P-value 0.02).

Alternatively to specification 3, we test the inclusion of a quadratic term in a linear

---

[8]Figures 20 - 22 in Appendix A.1 show the same relation between fiscal shocks and multipliers at the 1, 2, and 3-year horizons. While for those horizons the multiplier is always increasing with the shock, independently of the shock being positive of negative, the correlation with negative shocks is always weaker than with positive shocks.

specification similar to equation (2), without pooling observations across periods of fiscal expansions and contractions. The link between the sign and the size of the shock and the fiscal multiplier holds under this alternative approach. Results are robust to pooling observations above and below the median positive or the median negative shocks, with the fiscal multiplier being largest for large expansions and smallest for large contractions. All these robustness checks are reported in Appendix A.1 in Tables 11 and 12.

2.1.3 Robustness and Other Tests

Naturally, our results may be sensitive to the choice of specification and sample. To assuage these concerns, we perform several robustness checks, whose results can be found in Appendix A.1. In particular, we show that our results hold even when excluding World War 2, as well as when we only consider a post 1947 sample. We also show that our results are robust to including additional controls, or the number of lags for the controls.

We also test for nonlinear effects of fiscal policy on other macroeconomic variables: consumption and investment. There is a large literature on the effects of fiscal shocks on different components of private expenditure, i.e. Ramey (2012), Blanchard and Perotti (2002), Ramey (2011a). While there is a consensus in the literature that government spending crowds out investment, the effects on consumption are less consensual. We use the Federal Reserve Economic Data (FRED) series for nominal consumption and investment, starting in 1947 and estimate equation (3) with private consumption and private investment as left-hand side variables. Results for consumption and investment (Tables 15 and 17 in Appendix A.1 respectively) indicate that, at all horizons, the multipliers are consistently larger for fiscal expansions than contractions and that these multipliers are largest for large expansions and smallest for large contractions. Notice also that while the multiplier for consumption is positive on impact, becoming negative at the end of year 1, the multiplier for investment is always negative, which is consistent with the consensus on the literature.

We also test whether our results hold in a specification where we do not pool observa-

tions across fiscal expansion and contraction episodes, and simply include a quadratic term. These results (Tables 16 and 18 in Appendix A.1) are in line with the previous ones: the consumption multiplier is positive on impact and then becomes negative and investment multiplier always negative, with multipliers being increasing on the size of the shock.

2.2 IMF shocks

In this section we provide supporting evidence that the nonlinearities of the fiscal multiplier are not only related to the sign of the shock but also to the absolute variation. We now show that larger fiscal consolidations are associated with smaller multipliers using the Alesina et al. (2015a) annual dataset of fiscal consolidation episodes, which includes 16 OECD countries and ranges from 1981 to 2014.[9]

Alesina et al. (2015a) expand the original dataset of Devries et al. (2011) with exogenous fiscal consolidations episodes, known as IMF shocks. Devries et al. (2011) use the narrative approach of Romer and Romer (2010) to identify exogenous fiscal consolidations, i.e. consolidations driven uniquely by the desire to reduce budget deficits. The use of the narrative approach filters out all policy actions driven by the business cycle, guaranteeing that the identified consolidations are independent from the current state of the economy.

Besides expanding the dataset of Devries et al. (2011), Alesina et al. (2015a) use the methodological innovation introduced by Alesina et al. (2015b), who alert for the fact that a fiscal adjustment is rather a multi-year plan than an isolated change and consequently it results in policies that are both unexpected and others that are known in advance. Ignoring the link between both expected and unexpected policies may yield biased results.

The way Alesina et al. (2015a) define a fiscal consolidation is: expenditure deviations relative to expenditures if no policy had been adopted and expected revenue changes stemming from tax code revisions. Moreover, fiscal consolidations that were not implemented are not included in the dataset and so all fiscal consolidation episodes that are included are assumed

---

[9]The countries are: Australia, Austria, Belgium, Canada, Germany, Denmark, Spain, Finland, France, Japan, United Kingdom, US, Ireland, Italy, Portugal and Sweden.

to be fully credible.

We estimate the following specification,

$$\Delta y_{i,t} = \alpha_i + \beta_1 e_{i,t}^u + \beta_2 (e_{i,t}^u)^2 + \beta_3 e_{i,t}^a + \beta_4 (e_{i,t}^a)^2 \qquad (4)$$

where $\Delta y_{i,t}$ is the output growth rate in country i in year t, $e_{i,t}^u$ is unanticipated fiscal consolidation shocks and $e_{i,t}^a$ is anticipated fiscal consolidation shocks. We include squared terms to capture the nonlinear effects of fiscal shocks. We follow Alesina et al. (2015a) and estimate the equation using Seemingly Unrelated Regressions (SUR), imposing cross-country restrictions on the β coefficients.

Results are presented in table 4 and validate our hypothesis that the nonlinear effects of fiscal shocks are not only related to the sign of the shock, but also to the size. The coefficients associated with the linear terms of both announced and unexpected fiscal consolidations are negative, indicating that fiscal consolidations lead to a decrease in output. However, the coefficients of interest, β_2 and β_4, have a positive sign, meaning that the larger the consolidation, the smaller is the fiscal multiplier (even though only the coefficient associated with the squared term of announced fiscal consolidations is statistically significant). This coefficient is not only statistically significant but also economically meaningful, as an increase in one standard deviation of announced consolidations leads to a decrease of 80% in the fiscal multiplier.

Variables	Benchmark
β_1	-0.004**
	(0.002)
β_2	0.001
	(0.001)
β_3	-0.024***
	(0.002)
β_4	0.007***
	(0.001)
*** p<0.01, ** p<0.05, * p<0.1	

Table 4: Non-linear effects of fiscal consolidation shocks.

3   Fiscal Policy Nonlinearities in Representative Agent Environments

We are interested in understanding what mechanisms generate the nonlinearities and asymmetries that we empirically documented in the previous section. To do so, we proceed incrementally, and show that standard representative agent models are unable to generate the nonlinearities that we find in the data. Even adding standard ingredients that are known to amplify the effects of fiscal policy, such as nominal rigidities or adjustment costs of investment, is not enough to match the data.

3.1   Real Business Cycle Model

Set-up   We start with the textbook real business cycle model, where preferences of the representative agent are separable in consumption and labor, and the representative firm produces according to a Cobb-Douglas function that depends on capital and labor. The framework follows Cooley and Prescott (1995), and the details of the model are present in Appendix B.[10]

We augment the model with a government that engages in socially wasteful spending. The aggregate resource constraint can then be written as

$$C_t + K_t - (1 - \delta)K_{t-1} + G_t = z_t K_{t-1}^{\alpha} N_t^{1-\alpha}$$

where C_t is aggregate consumption, K_{t-1} is the current stock of capital, N_t is labor, and G_t is government spending. The mode of financing is irrelevant for allocations due to Ricardian Equivalence. The calibration is standard and can be found in Appendix B.

---

[10]The main deviations from the cited benchmark are: separable preferences in consumption and leisure, and no trend growth for TFP.

Fiscal Shock   We assume that government spending follows an AR(1) in logs

$$\log G_t = (1 - \rho_G) \log G_{SS} + \rho_G G_{t-1} + \varepsilon_t^G$$

where ρ_G is assumed to be 0.9 at a quarterly frequency, consistent with the estimates of Nakamura and Steinsson (2014) for military procurement spending.

Experiment   We consider a range of values for ε_t^G that correspond to changes from -10% to 10% of steady state government spending on impact. The resulting fiscal multipliers, at different horizons, are plotted in Figure 4. We adopt the standard definition of discounted integral multiplier that accounts for the cumulative effects of fiscal policy on output at a given horizon h,

$$\mathcal{M}_h = \frac{\sum_{i=0}^{h} \prod_{j=0}^{i} R_j^{-1} (Y_i - Y_{SS})}{\sum_{i=0}^{h} \prod_{j=0}^{i} R_j^{-1} (G_i - G_{SS})} \tag{5}$$

for $h = 0$, this becomes the traditional definition of the multiplier measured at impact.

The figure shows that the standard RBC model is not able to match the size of the fiscal multipliers in the data, as it is well known. Additionally, the standard model implies that the fiscal multiplier is roughly constant with the change in G: the model is not able to capture the nonlinearities or asymmetries that we find in the data. In fact, the model predicts the multiplier to be slightly *decreasing* with the change in G, violating the asymmetric pattern that we find. These results hold regardless of the horizon.

3.2   *New Keynesian Model*

One standard way of generating fiscal multipliers that more closely match those measured in the data is by providing a role for aggregate demand to affect economic activity, which can be achieved by including nominal rigidities. We augment the model to include quadratic costs of price adjustment for firms, which generates a Phillips Curve relating output and inflation, as well as a Taylor Rule for the Central Bank. Again, the model ingredients and calibration are standard, and can be found in Appendix B.
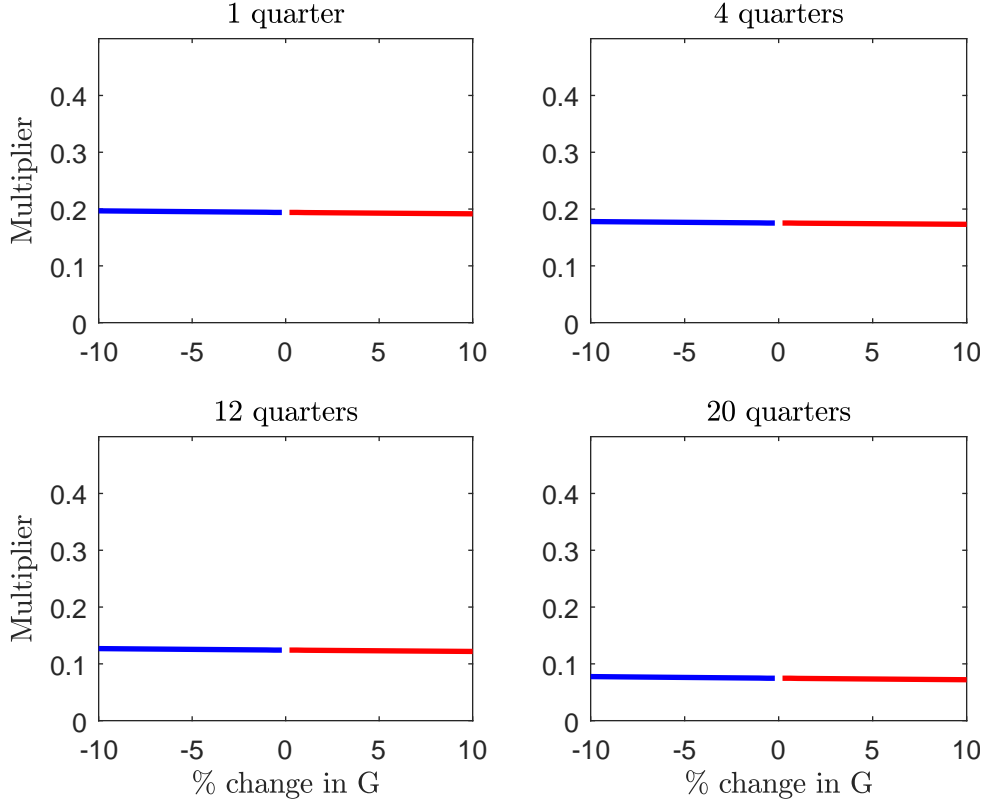
Figure 4: Representative Agent, RBC Model: fiscal multipliers as a function of the size of the variation in G, at different horizons. The blue line corresponds to G contractions, while the red line represents G expansions.

Figure 5 shows the outcome of the same experiment in the context of a New Keynesian model with investment: again, multipliers are low and do not vary with the size or sign of the shock in an economically meaningful way. For this particular example, we use a standard Volcker-Greenspan calibration for the Taylor Rule, which is known to produce relatively low multipliers.[11] It is well known that the level of the fiscal multiplier is very sensitive to the speficic parametrization of the Taylor Rule. What is important is that alternative parametrizations that raise the level of the fiscal multiplier, such as making the Central Bank less responsive to changes in inflation, do not alter the fact that the multiplier is roughly constant with respect to the sign and size of the shock to G_t.

---

[11]In particular, we assume a standard Taylor Rule with interest rate smoothing,

$$\log R_t = \rho_R \log R_{t-1} + (1 - \rho_R)[\log R_{SS} + \phi_\Pi(\log \Pi_t - \log \Pi_{SS}) + \phi_Y(\log Y_t - \log Y_{SS})]$$
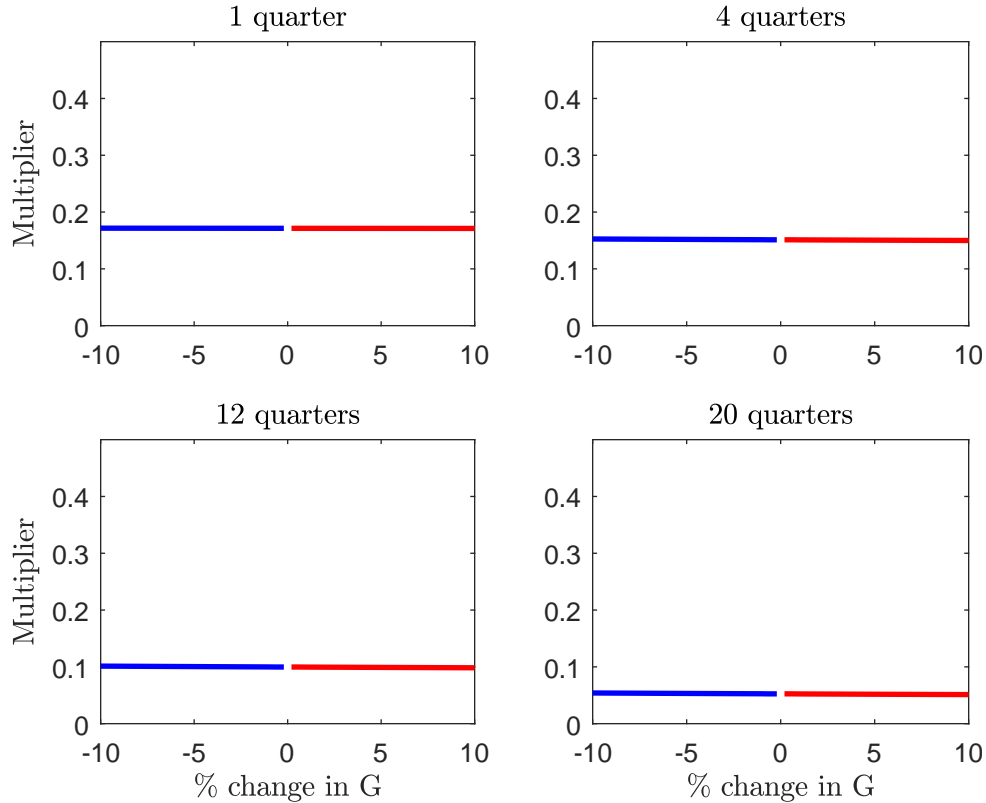
with $\rho_R = 0.80, \phi_\Pi = 1.50, \phi_Y = 0.50$.

Figure 5: Representative Agent, New Keynesian Model: fiscal multipliers as a function of the size of the variation in G, at different horizons. The blue line corresponds to G contractions, while the red line represents G expansions.

3.3 Adjustment Costs of Investment

The reason why the standard RBC and New Keynesian models with capital are unable to generate large multipliers is primarily due to a very high sensitivity of investment to government spending shocks through movements in the real rate. As discussed, one way that New Keynesian models can partially address is by making the Central Bank, who sets the real rate, less responsive to output and inflation. Still, in order to generate multipliers of empirically plausible magnitudes, one would need to parametrize the Taylor Rule in such a way that is at odds with a multitude of empirical estimates (at least prior to 2007).

A direct way to address the excess volatility of investment is to introduce adjustment costs, which have become a standard feature of DSGE models. Adjustment costs of investment are able to generate empirically plausible fiscal multipliers while maintaining standard assumptions for monetary policy.

Figure 6 repeats the basline experiment by introducing adjustment costs of investment in

the New Keynesian specifications. It shows that, while raising multipliers, adjustment costs of investment are not sufficient to generate empirically plausible levels for the multipliers or for the nonlinearities. Importantly, however, they help generate the correct asymmetry: fiscal multipliers now become slightly increasing in the shock to G.
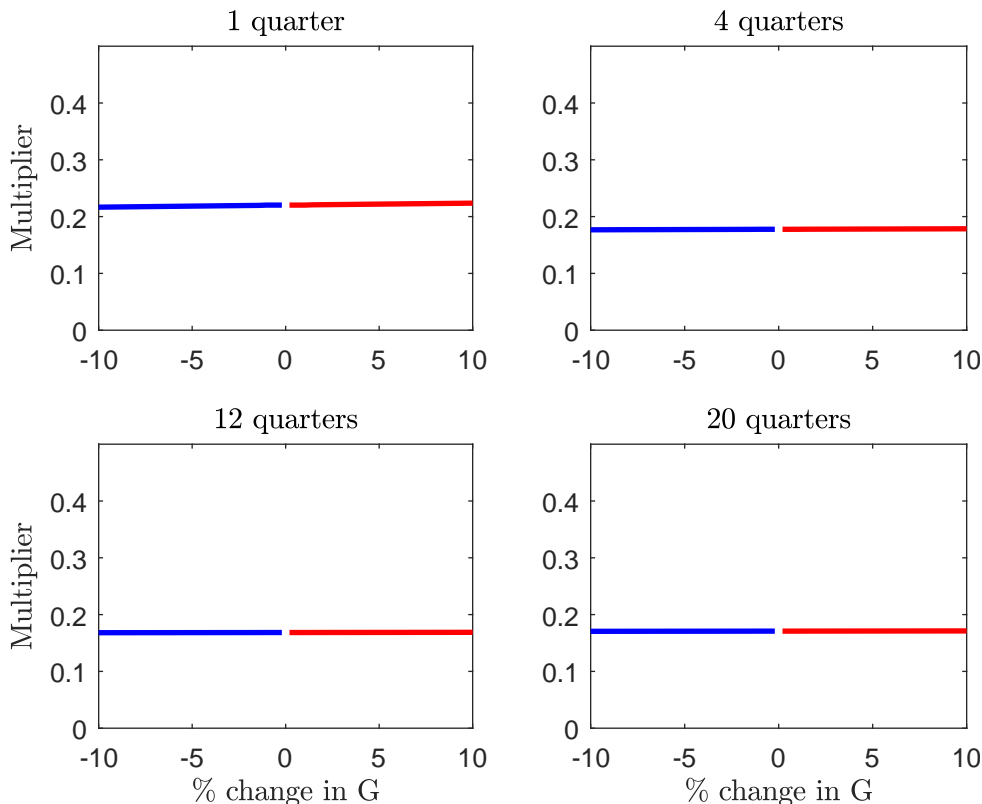
Figure 6: Representative Agent, New Keynesian Model with Adjustment Costs of Investment: fiscal multipliers as a function of the size of the variation in G, at different horizons. The blue line corresponds to G contractions, while the red line represents G expansions.

An increase in government spending affects the supply of the two factors of production with opposing effects: on one hand, real interests rise, which crowds out investment and causes the capital stock to fall; on the other hand, the negative income effect expands labor supply. Adjustment costs of investment dampen the sensitivity of investment to real rates, thereby curbing the first effect and raising fiscal multipliers. Still, none of this is sufficient to match either the levels or the patterns that are detected in the data.

Figure 7 shows that in the extreme case of infinite adjustment costs, so that capital is fixed throughout the experiment, the level of the multiplier can be raised to match the data,

but this is still not enough to generate any meaningful nonlinearities.
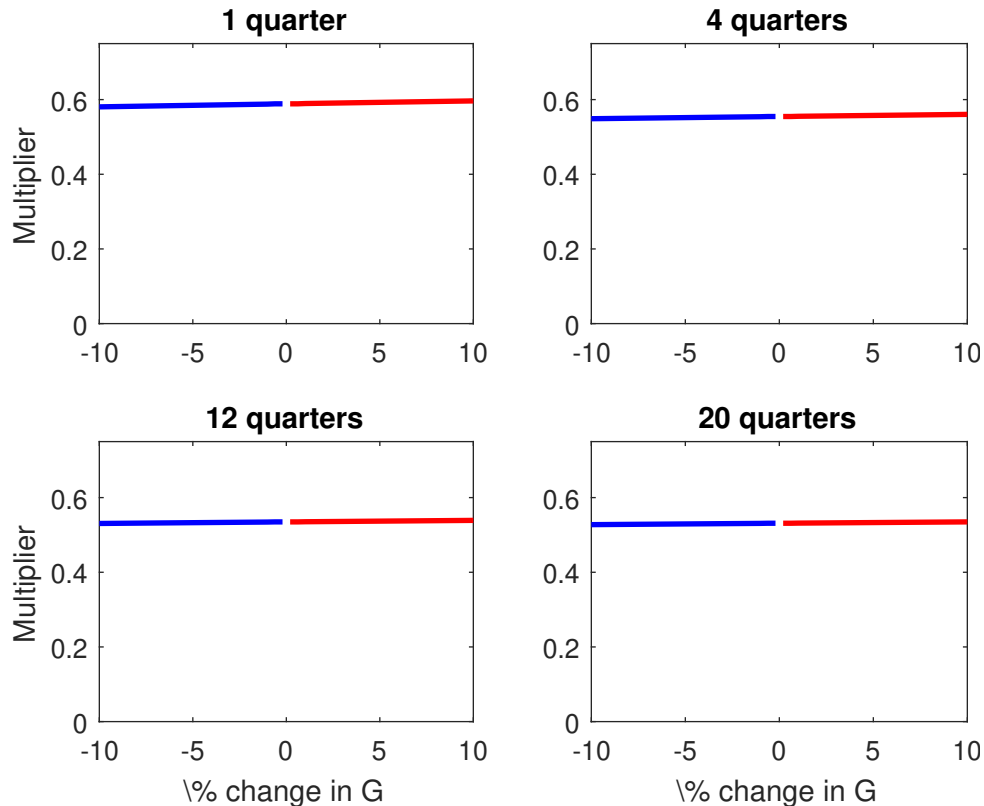
Figure 7: Representative Agent, New Keynesian Model with Infinite djustment Costs of Investment: fiscal multipliers as a function of the size of the variation in G, at different horizons. The blue line corresponds to G contractions, while the red line represents G expansions.

4   Heterogeneous Agents Model

In the previous section, we show that the empirical evidence suggests that: (i) fiscal expansions have larger effects on output than consolidations, this being true across time and space; and (ii) larger consolidations tend to have a smaller effect on output. In other words, the macroeconomic effects of fiscal shocks depend both on the *size* and *sign* of the shock. In this section, we present a quantitative model that allows us to rationalize these findings. The model follows closely Brinca et al. (2016) and Brinca et al. (2017).

Technology

The production sector is standard and neoclassical, with the representative firm having access to a Cobb-Douglas production function,

$$Y_t(K_t, L_t) = K_t^\alpha [L_t]^{1-\alpha} \tag{6}$$

where L_t is the labor input, measured in efficiency units, and K_t is the capital input. The law of motion for capital is

$$K_{t+1} = (1 - \delta)K_t + I_t \tag{7}$$

where δ is the capital depreciation rate and I_t is the gross investment. Firms choose labor and capital inputs each period in order to maximize profits,

$$\Pi_t = Y_t - w_t L_t - (r_t + \delta)K_t. \tag{8}$$

Under a competitive equilibrium, factor prices are paid their marginal products,

$$w_t = \partial Y_t / \partial L_t = (1 - \alpha) \left(\frac{K_t}{L_t} \right)^\alpha \tag{9}$$

$$r_t = \partial Y_t / \partial K_t - \delta = \alpha \left(\frac{L_t}{K_t} \right)^{1-\alpha} - \delta \tag{10}$$

Demographics

Our economy is populated by J overlapping generations of households. Peterman and Sager (2016) highlight the importance of having a life-cycle economic when assessing the effects of government debt. Households start their life at age 20, and retire at age 65, after which they face an age-dependent probability of dying, $\pi(j)$. They die with certainty at age 100. $j \in \{0.25, \dots, 81.0\}$ is the household's age (minus 19.75). A period in the model corresponds to 1 quarter, so households work for 180 quarters, equivalent to 45 years. We assume no population growth in our economy and normalize the size of each new cohort to

1. $\omega(j) = 1 - \pi(j)$ defines the age-dependent probability of surviving; applying the law of large numbers, this means that the mass of retired agents at any given period is equal to $\Omega_j = \prod_{q=65}^{q=J-1} \omega(q)$.

   Households also differ with respect to persistent idiosyncratic productivity shocks, asset holdings, discount factors that are uniformly distributed and can take three distinct values $\beta \in \{\beta_1, \beta_2, \beta_3\}$, and also in terms of ability, which is a permanent productivity factor randomly assigned at birth. Working age agents have to choose how much to work n, how much to consume c, and how much to save k to maximize their utility. Retired households make consumption and saving decisions and receive a retirement benefit Ψ_t.

   Stochastic survivability after retirement implies that a share of households leave unintended bequests Γ. We assume these bequests to be uniformly redistributed across the living households. We assume that retired households value these bequests in their utility in order to better match the data on retired household wealth.

Labor Income

The wage received by an household depends on three different factors that determine the number of labor efficiency units each household is endowed in each period: age j, permanent ability $a \sim N(0, \sigma_a^2)$, and an idiosyncratic productivity shock u, which follows an AR(1) process:

$$u' = \rho u + \epsilon, \quad \epsilon \sim N(0, \sigma_\epsilon^2) \tag{11}$$

Finally, labor income per hour worked depends on the wage rate per efficiency unit of labor w; this income is given by

$$w_i(j, a, u) = w e^{\gamma_1 j + \gamma_2 j^2 + \gamma_3 j^3 + a + u} \tag{12}$$

$\gamma_{1\iota}$, $\gamma_{2\iota}$ and $\gamma_{3\iota}$ are calibrated directly from the data to capture the age profile of wages.

Preferences

Household utility $U(c, n)$ is standard: time-additive, separable and of the isoelastic form for $n \in (0, 1]$,

$$U(c, n) = \frac{c^{1-\sigma}}{1 - \sigma} - \chi \frac{n^{1+\eta}}{1 + \eta} \tag{13}$$

The utility function for retired households includes an extra term, the utility from a bequest of size k,

$$D(k) = \varphi \log(k) \tag{14}$$

Government

The government runs a balanced budget social security system that operates independently from the main government budget constraint. Social security levies taxes on employees' gross labor income at rate τ_{ss} as well as on the representative firm at rate $\tilde{\tau}_{ss}$. The proceeds are used to pay retirement benefits, Ψ_t.

In the main government budget, revenues include flat rate taxes over consumption τ_c, and capital income τ_k. Labor income taxes follow a non-linear schedule as in Benabou (2002):

$$\tau(y) = 1 - \theta_0 y^{-\theta_1} \tag{15}$$

where θ_0 and θ_1 define the level and progressivity of the tax schedule, respectively, y is the pre-tax labor income and $y_a = [1 - \tau(y)]y$ is the after tax labor income. [12]

Tax revenues from consumption, capital, and labor income are used to finance public consumption of goods G_t, public debt interest expenses rB_t, and lump sum transfers g_t.

Denoting social security revenues by R^{ss} and the other tax revenues as R, the government

---

[12]See the appendix for a more detailed discussion of the properties of this tax function.

budget constraint is defined as

$$g \left(45 + \sum_{j \geq 65} \Omega_j \right) = R - G - rB, \tag{16}$$

$$\Psi \left(\sum_{j \geq 65} \Omega_j \right) = R^{ss}. \tag{17}$$

Recursive Formulation of the Household Problem

In a given period, a household is defined by her age j, asset position k, time discount factor $\beta \in \beta_1, \beta_2, \beta_3$, permanent ability a and persistent idiosyncratic productivity shock u. Given this, a working-age household chooses consumption c, work hours n, and future asset holdings k', to maximize the present discounted value of utility. The problem can be written recursively as:

$$V(k, \beta, a, u, j) = \max_{c, k', n} \left[U(c, n) + \beta \mathbb{E}_{u'} \left[V(k', \beta, a, u', j+1) \right] \right]$$

$$\text{s.t.:}$$

$$c(1 + \tau_c) + k' = (k + \Gamma)(1 + r(1 - \tau_k)) + g + Y^L$$

$$Y^L = \frac{nw(j, a, u)}{1 + \tilde{\tau}_{ss}} \left(1 - \tau_{ss} - \tau_l \left(\frac{nw(j, a, u)}{1 + \tilde{\tau}_{ss}} \right) \right)$$

$$n \in [0, 1], \quad k' \geq -b, \quad c > 0 \tag{18}$$

where Y^L is the household's labor income net of social security (both on the employee and on the employer) and labor income taxes. The problem of a retired household differs on three dimensions: age dependent probability of dying $\pi(j)$, bequest motive $D(k')$, and labor

income is replaced by retirement benefits. We can write it as

$$V(k, \beta, j) = \max_{c, k'} \left[U(c, n) + \beta(1 - \pi(j))V(k', \beta, j + 1) + \pi(j)D(k') \right]$$

s.t.:

$$c(1 + \tau_c) + k' = (k + \Gamma)(1 + r(1 - \tau_k)) + g + \Psi,$$

$$k' \geq 0, \quad c > 0 \tag{19}$$

Stationary Recursive Competitive Equilibrium

Let the distribution over the individual states be denoted as $\Phi(k, \beta, a, u, j)$. Then, we can define a stationary recursive competitive equilibrium as

1. Taking the factor prices and the initial conditions as given, the value function $V(k, \beta, a, u, j)$ and policy functions $c(k, \beta, a, u, j)$, $k'(k, \beta, a, u, j)$, and $n(k, \beta, a, u, j)$ solve the consumers' optimization problem.

2. Markets clear:

$$K + B = \int k d\Phi$$

$$L = \int (n(k, \beta, a, u, j)) \, d\Phi$$

$$\int c d\Phi + \delta K + G = K^\alpha L^{1-\alpha}$$

3. Factor prices are paid their marginal productivity:

$$w = (1 - \alpha)\left(\frac{K}{L}\right)^\alpha$$

$$r = \alpha\left(\frac{K}{L}\right)^{\alpha - 1} - \delta$$

4. The government budget balances:

$$g \int d\Phi + G + rB = \int \left(\tau_k r(k + \Gamma) + \tau_c c + n\tau_l \left(\frac{nw(a, u, j)}{1 + \tilde{\tau}_{ss}} \right) \right) d\Phi$$

5. The social security system budget balances:

$$\Psi \int_{j \geq 65} d\Phi = \frac{\tilde{\tau}_{ss} + \tau_{ss}}{1 + \tilde{\tau}_{ss}} \left(\int_{j < 65} nw d\Phi \right)$$

6. The assets of the dead are uniformly distributed among the living:

$$\Gamma \int \omega(j) d\Phi = \int (1 - \omega(j)) \, k d\Phi$$

Fiscal Experiment and Transition

The fiscal experiment that we analyze in this paper is a variation of the government spending (G), by a varying percentage of GDP, for 10 periods. We focus on deficit financing experiments and permanent changes in public debt, so an increase (decrease) in G is financed by a permanent increase (decrease) in public debt. After 10 periods, G returns to its initial level and the economy converges to a new stationary recursive competitive equilibrium featuring a different debt to GDP ratio.

We define the equilibrium transition as follows. For a given level of initial capital stock, initial distribution of households and initial taxes, respectively K_0, Φ_0 and $\{\tau_l, \tau_c, \tau_k, \tau_{ss}, \tilde{\tau}_{ss}\}_{t=1}^{t=\infty}$, a competitive equilibrium is a sequence of individual functions for the household, $\{V_t, c_t, k_t', n_t\}_{t=1}^{t=\infty}$, of production plans for the firm, $\{K_t, L_t\}_{t=1}^{t=\infty}$, factor prices, $\{r_t, w_t\}_{t=1}^{t=\infty}$, government transfers $\{g_t, \Psi_t, G_t\}_{t=1}^{t=\infty}$, government debt, $\{B_t\}_{t=1}^{t=\infty}$, inheritance from the dead, $\{\Gamma_t\}_{t=1}^{t=\infty}$, and of measures $\{\Phi_t\}_{t=1}^{t=\infty}$, such that for all t:

1. For given factor prices and initial conditions, the value function $V(k, \beta, a, u, j)$ and the policy functions, $c(k, \beta, a, u, j)$, $k'(k, \beta, a, u, j)$, and $n(k, \beta, a, u, j)$ solve the consumers'

optimization problem.

2. Markets clear:

$$K_{t+1} + B_t = \int k_t d\Phi_t$$

$$L_t = \int \left(n_t(k_t, \beta, a, u, j) \right) d\Phi_t$$

$$\int c_t d\Phi_t + K_{t+1} + G_t = (1 - \delta)K_t + K^\alpha L^{1-\alpha}$$

3. The factor prices are paid their marginal productivity:

$$w_t = (1 - \alpha) \left(\frac{K_t}{L_t} \right)^\alpha$$

$$r_t = \alpha \left(\frac{K_t}{L_t} \right)^{\alpha - 1} - \delta$$

4. The government budget balances:

$$g_t \int d\Phi_t + G_t + r_t B_t = \int \left(\tau_k r_t(k_t + \Gamma_t) + \tau_c c_t + n_t \tau_l \left(\frac{n_t w_t(a, u, j)}{1 + \tilde{\tau}_{ss}} \right) \right) d\Phi_t + (B_{t+1} - B_t)$$

5. The social security system balances:

$$\Psi_t \int_{j \geq 65} d\Phi_t = \frac{\tilde{\tau}_{ss} + \tau_{ss}}{1 + \tilde{\tau}_{ss}} \left(\int_{j < 65} n_t w_t d\Phi_t \right)$$

6. The assets of the dead are uniformly distributed among the living:

$$\Gamma_t \int \omega(j) d\Phi_t = \int \left(1 - \omega(j) \right) k_t d\Phi_t$$

7. Aggregate law of motion:

$$\Phi_{t+1} = \Upsilon_t(\Phi_t)$$

5 Calibration

We calibrate the starting SRCE of our model to the U.S. economy. Some parameters are calibrated directly from empirical counterparts, while others are calibrated using Simulated Method of Moments (SMM) so that the model matches key features of the U.S. economy.

Wages

The wage profile through the life cycle (see equation 12) is calibrated directly from the data. We run the following regression, using data from the Luxembourg Income and Wealth Study.

$$\ln(w_i) = \ln(w) + \gamma_1 j + \gamma_2 j^2 + \gamma_3 j^3 + \varepsilon_i \tag{20}$$

where j is the age of individual i.

To estimate parameters ρ and σ_ϵ we use PSID yearly data and run equation 20. We then use the residuals of the equation to estimate both parameters for an yearly periodicity. To transform the parameters from yearly to quarterly we raise ρ to $\frac{1}{4}$ and divide σ_ϵ by 4. σ_a is among the parameters that are calibrated using SMM. The yearly variance of $\ln(w)$ is the corresponding moment.

Preferences

There has been a considerable debate in the literature on the value of the Frisch elasticity of labor supply, η, with estimates ranging from 0.5 to 2 or higher. We set it to 1 as in Brinca et al. (2016) and Brinca et al. (2017). The utility from bequests, disutility of work and the three discount factors (φ, χ, β_1, β_2, β_3) are among the parameters calibrated to match key moments in the data. The corresponding moments are the ratio of wealth owned by households in the age cohort 75-80 years old relative to an average household, share of hours worked and the three quartiles of the wealth distribution respectively.

Taxes and Social Security

As described before, to capture the progressivity of both the tax schedule and government transfers, we use the same labor income tax function as Benabou (2002) (equation 15). To estimate the parameters θ_0 and θ_1 of the U.S. economy we use OECD data on labor income tax and we estimate the equation for different family types. Then, we weight the value of each parameter by the weights of each family type on the overall population, so that we have a single parameter for the individual household in our model.

For the social security rates we assume no progressivity. Both social security tax rates, on behalf of the employer and on behalf of the employee, are set to 7.65%, using the value from the bracket covering most incomes. Finally, following Trabandt and Uhlig (2011), the consumption and capital tax rates are set respectively to 23.3% and 1.55%.

Parameters Calibrated Endogenously

As mentioned before, some parameters that do not have any direct empirical counterparts are calibrated using SMM. These parameters are φ, β_1, β_2, β_3, b, χ and σ_a, bequest motive, discount factors, borrowing limit, disutility from working and variance of ability respectively. The SMM is set so that it minimizes the following loss function:

$$L(\varphi, \beta_1, \beta_2, \beta_3, b, \chi, \sigma_a) = ||M_m - M_d|| \tag{21}$$

with M_m and M_d being the moments in the model and in the data respectively.

We have seven parameters to calibrate endogenously, so we need seven data moments to have a system that is exactly identified. The seven moments we select in the data are the the ratio of wealth owned by households in the age cohort 75-80 years old relative to an average household, share of hours worked, the three quartiles of the wealth distribution, the variance of log wages and capital to output ratio. Table 6 presents the calibrated parameters and Table 5 presents the calibration fit.

Table 5: Calibration Fit

Data Moment	Description	Source	Data Value	Model Value
75-80/all	Share of wealth households aged 75-80	LWS	1.51	1.51
K/Y	Capital-output ratio	PWT	12.292	12.292
$\text{Var}(\ln w)$	Yearly variance of log wages	LIS	0.509	0.509
$\bar{n}$	Fraction of hours worked	OECD	0.248	0.248
Q_{25}, Q_{50}, Q_{75}	Wealth Quartiles	LWS	-0.014, 0.004, 0.120	-0.012, -0.005, 0.123

Table 6: Parameters Calibrated Endogenously

Parameter	Value	Description
Preferences		
φ	20.62	Bequest utility
$\beta_1, \beta_2, \beta_3$	0.998, 0.985, 0.797	Discount factors
χ	8.1	Disutility of work
Technology		
b	1.81	Borrowing limit
σ_ϵ	0.619	Variance of ability

6 Intuition: Labor Supply and Credit Constraints

To build intuition on why credit constraints can generate asymmetric effects for fiscal policy shocks, we consider a simplified version of the model described in the previous section where agents are infinitely lived, taxes are lump-sum, and there is no social security and discount factor heterogeneity. Households solve a simplified problem given by

$$V(s_t^i) = \max_{c_t^i, k_{t+1}^i, n_t^i} \left[U(c_t^i, n_t^i) + \beta \mathbb{E}_t[V(s_{t+1}^i)|s_t^i] \right]$$

$$\text{s.t.}$$

$$c_t^i + k_{t+1}^i = k_t^i(1 + r_t) + w_t u_t^i n_t^i - T_t$$

$$k_{t+1}^i \geq -\underline{b}$$

$$s_t^i = (k_t^i, u_t^i)$$

where u_t^i is some idiosyncratic productivity shock.

In the standard neoclassical model the response of output to changes in government

purchases depends only on changes in factor employment: capital and labor. Since capital is fixed in the short-run, the impact multiplier is determined solely by changes in labor supply.[13] We start by decomposing the different channels through which a change in government spending can affect individual labor supply, given individual states s_t^i and the aggregate state is X_t,[14]

Proposition 6.1. *The (total) response of labor supply $n(s_t^i)$ to a change in current government consumption G_t is given by*

$$
\begin{aligned}
\frac{\mathrm{d}n_t^i}{\mathrm{d}G_t} =& [\alpha_1(s_t^i; X_t) + \alpha_2(s_t^i; X_t)\Lambda_1(s_t^i; X_t)(1 - \mathbb{1}_t^i)]\frac{\mathrm{d}w_t}{\mathrm{d}G_t} \\
&+ \alpha_2(s_t^i; X_t)[1 - (1 - \mathbb{1}_t^i)\Lambda_2(s_t^i; X_t)]\left(\frac{\mathrm{d}T_t}{\mathrm{d}G_t} - k_t^i\frac{\mathrm{d}r_t}{\mathrm{d}G_t}\right) \\
&+ \alpha_2(s_t^i; X_t)(1 - \mathbb{1}_t^i)\mathcal{F}(s_t^i; X_t)
\end{aligned}
$$

where $\mathbb{1}_t^i = 1$ if the individual is constrained at t and zero otherwise; $\alpha_1, \alpha_2, \Lambda_1, \Lambda_2 > 0$ are time-invariant functions of the individual's current states s_t^i and X_t; and $\mathcal{F}$ is a time-invariant function of future changes in wages, interest rates, and taxes.

The labor supply of constrained agents does not respond to future changes in factor prices and taxes contained in $\mathcal{F}$. Since $\alpha_2 > 0$, constrained agents react relatively *less* to changes in current wages, but relatively more to changes in the current non-labor component of income (taxes and interest rates). In other words, constrained agents display a lower labor supply elasticity with respect to both current and future wages. The relative labor supply response between constrained and unconstrained agents will then crucially depend on the mode of financing: balanced budget fiscal expansions, for example, should trigger a relatively larger response by constrained agents, as they involve changes in current taxes.

Proposition 6.2. *The effects of future changes in taxes and factor prices on individual labor*

---

[13]Athreya et al. (2017) provide an extensive analysis in the context of general equilibrium models with incomplete markets such as this one.

[14]All derivations are in the Appendix.

supply $\mathcal{F}(s_t^i; X_t)$ *can be written recursively as*

$$\mathcal{F}(s_t^i; X_t) = -\Lambda_3(s_t^i; X_t)[1 - \Lambda_2(s_{t+1}^i; X_{t+1})]\frac{\mathrm{d}w_{t+1}}{\mathrm{d}G_t}$$
$$+[\Lambda_4(s_t^i; X_t) + \Lambda_5(s_t^i; X_t)\Lambda_2(s_{t+1}^i; X_{t+1})k_{t+1}^i]\frac{\mathrm{d}r_{t+1}}{\mathrm{d}G_t}$$
$$+\Lambda_5(s_t^i; X_t)[1 - \Lambda_2(s_{t+1}^i; X_{t+1})]\frac{\mathrm{d}T_{t+1}}{\mathrm{d}G_t}$$
$$+\Lambda_5(s_t^i; X_t)\mathcal{F}(s_{t+1}^i; X_{t+1})$$

where

$$\Lambda_3(s_t^i; X_t)[1 - \Lambda_2(s_{t+1}^i; X_{t+1})] \geq 0$$
$$\Lambda_5(s_t^i; X_t)[1 - \Lambda_2(s_{t+1}^i; X_{t+1})] \geq 0$$
$$\Lambda_5(s_t^i; X_t) \geq 0$$

The above proposition shows that current labor supply responds positively to increases in future taxes (i.e. unconstrained agents observe the Ricardian Equivalence), and negatively to increases in future wages. Assuming that an increase (decrease) in government spending causes a standard crowding out (in) effect on investment, the capital stock falls (increases), and future wages are lower (higher). This means that the labor supply response on impact for unconstrained agents reflects not only the standard Ricardian effects but also the path of future wages. These two forces are absent from the labor supply response of constrained agents. Therefore, deficit financed fiscal expansions can potentially have a much larger effect on the labor supply for unconstrained agents, who internalize the present discounted value of the fiscal costs as well as of the fall in wages.

This differential response of labor supply between constrained and unconstrained agents is the key mechanism that drives our main results: regardless of the financing scheme, if an *increase* in government purchases changes the mass of constrained agents differently than a *decrease* in spending, we should observed asymmetric effects on aggregate labor supply and,

therefore, on output. We can show that the savings function for an individual agent is given by

$$\frac{\mathrm{d}k_{t+1}^i}{\mathrm{d}G_t} = \Lambda_1(s_t^i; X_t)\frac{\mathrm{d}w_t}{\mathrm{d}G_t} + \Lambda_2(s_t^i; X_t)\left(k_t^i\frac{\mathrm{d}r_t}{\mathrm{d}G_t} - \frac{\mathrm{d}T_t}{\mathrm{d}G_t}\right) + \mathcal{F}(s_t^i; X_t)$$

Not surprisingly, savings comove with $\mathcal{F}$: in particular, increases in future taxes or decreases in future wages induce agents to increase their savings. Faced with a deficit-financed fiscal expansion, agents close to their borrowing constraint are induced to save more and, therefore, move away from the constraint. This increases the mass of unconstrained agents and, therefore, the aggregate labor supply elasticity. On the other hand, faced with a decrease in spending, the positive wealth effect induces agents to save less and potentially hit the constraint. This increase in the mass of constrained agents means that labor supply will be much less response, and therefore output moves by less.

Balanced Budget Fiscal Policies The intuition described above applies to the case of deficit financing, when current changes in G are financed with public debt and future taxes. Alternatively, the government could finance current changes in G with contemporaneous changes in T, keeping B constant. This can potentially atenuate the effect as current tax increases induce a relatively larger labor supply response by constrained agents. Still, as long as the rise in spending (and taxes) is persistent, unconstrained agents still react to changes in current taxes (albeit by less), and additionally respond to future changes in taxes and wages.

7   Quantitative Results

In the context of our experiment, variations in public debt affect the stock of productive capital in the economy. An increase in public debt shifts away resources from the productive sector, driving down the marginal productivity of labor and causing labor supply to expand, resulting in an expansion of output today.

In this section, we use the calibrated model to show that the extent to which changes in spending affect output on impact depends on the size and sign of the shock G. This dependence arises from the way that changes in G affect the mass of agents who are at the constraint: when G increases (for example) the future tax liability as well as the decline in labor income cause a negative income effect that induces agents to work and save relatively more. As agents save more and borrow less, the mass of constrained agents falls, and these agents have a very low labor supply elasticity (i.e. they will tend to work the same amount of hours regardless of aggregate factors).

What we show in this section is that the fiscal multiplier will change with the size and the sign of the variation in G, with the multiplier increasing in the shock. To rationalize this first note that a negative variation of G will send agents to the borrowing constraint, as agents expect higher future wages, they will borrow more against their future income, causing more agents to be constrained. For a positive variation of G the opposite will happen. Agents will internalize the decrease in their life-time income, diminishing their borrowing and consequently taking agents away from the borrowing constraint. As the constrained agents are hand-to-mouth, they will adjust consumption by the variation of their current income and their labor supply elasticity is close to zero. So, the larger the share of constrained agents, the smaller the labor supply response, causing the multiplier to shrink.

In the case of small shocks, the variation in agents' life-time income will be small and the percentage of agents constrained will not change, causing the multiplier to be locally symmetric. Although, for large shocks, agents will change their savings/consumption decisions, leading to variations in the percentage of agents constrained. The larger the size of the shocks, the larger the variations in the percentage of agents constrained, causing the fiscal multiplier asymmetry to be stronger. It is well known in the fiscal policy literature that the effects of government spending shocks depend crucially on (i) the persistence of the shock (Nakamura and Steinsson, 2014) and (ii) the mode of financing, in the context of models like ours where the Ricardian Equivalence does not hold. We show that the mechanism we

propose survives different assumptions regarding persistence and public financing.

Permanent Fiscal Shocks

We start by considering the case of permanent increases (decreases) in G that are financed with temporary increases (decreases) in public debt, which are then paid for with permanent decreases (increases) in transfers, as these elicit the strongest (and more easily interpretable) effects. Figure 8 plots fiscal multipliers (impact) as a function of the size of the change in G: the fiscal multiplier is monotonically increasing in G. It is lower for fiscal contractions than for fiscal expansions, and is larger (smaller) for larger fiscal expansions (contractions).
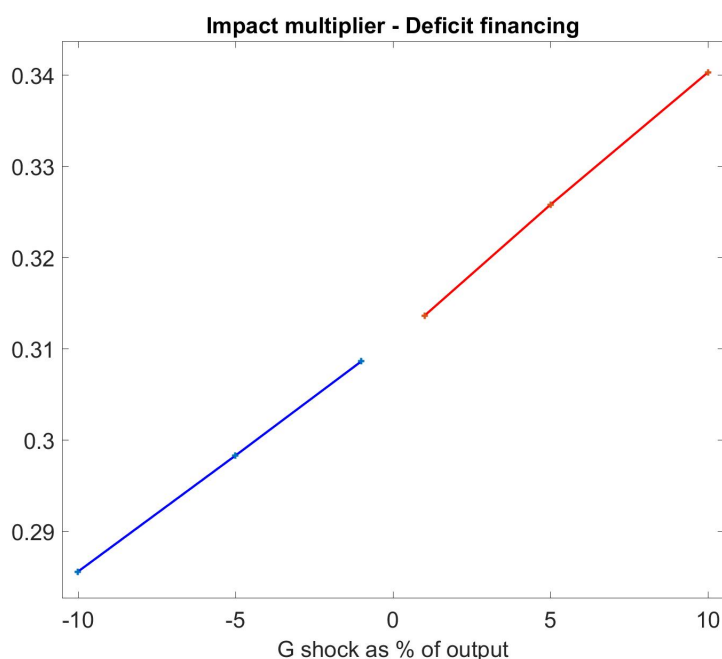
Figure 8: This figure plots the fiscal multiplier as a function of the size of the variation in G (as a % of GDP). The blue line corresponds to G contractions, while the red line represents G expansions.

Figures 9 and 10 help us understand the forces behind the mechanism. Figure 9 plots the percentage of constrained agents the period following the shock as a function of its size. As government spending increases, so does public debt, which crowds out capital. This permanent reduction in the capital stock lowers wages, and thus the life-time income for most agents in this economy. This reduction in life-time income leads to a decrease in borrowing, which then leads to less agents being credit constrained. Conversely, a fall in

spending leads to a reduction in public debt, which contributes to an increase in the capital stock and higher wages going forward. As life-time income rises, agents borrow more and the share of credit constrained agents increases. These changes in the mass of constrained agents affect the fiscal multiplier, since the labor supply of constrained agents is less elastic.
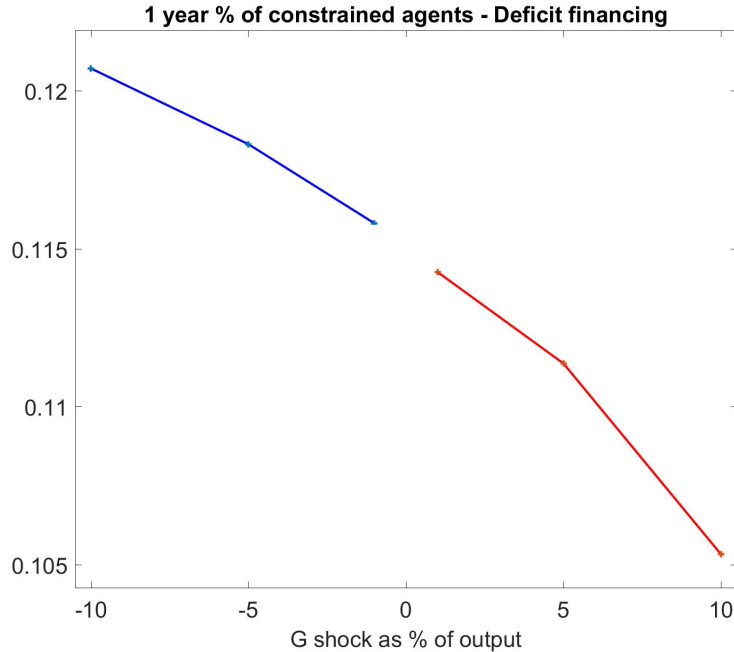

Figure 9: Government spending variation and percentage of constrained agents: In the X-axis we have the variation in G in percentage of GDP and in the Y-axis we have the percentage of credit constrained agents in the period following the shock. Blue line represents G contractions and red G expansions. The percentage of credit constrained agents is decreasing in the shock.

Figure 10 presents this relationship: the labor supply response of different types of agents as a function of the increase in G (left panel) or the decrease in G (right panel). A permanent increase in G reduces wages going forward; unconstrained agents (black line) react strongly to this fall in lifetime income and their labor supply expands relatively more in the period of the shock. Constrained agents (blue line), on the other hand, do not respond to changes in future income, and their labor supply is only affected by current conditions. As a consequence, it responds considerably less. Since more agents become unconstrained when G increases, the effective aggregate labor supply elasticity in the economy increases, leading to a larger output response to fiscal shocks. The same logic applies to fiscal contractions, represented in the
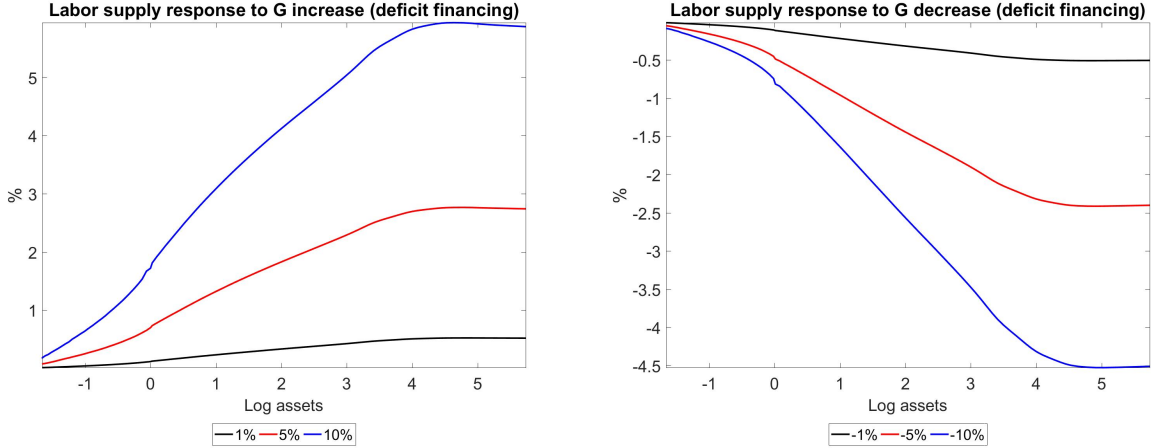
Figure 10: Government spending variation and relative labor supply response: this graph plots the labor supply response relative to the stationary steady state as a function of the initial level of assets for a permanent spending shock financed with deficits. The left panel corresponds to positive changes in G, while the right panel corresponds to negative changes in G.

right panel: in this case, there is an increase in lifetime income, which leads unconstrained agents to reduce their labor supply today. Constrained agents react only to current wages, and so their labor supply response is muted. Since more agents become constrained in response to the fiscal contraction, this attenuates the effect of fiscal shocks on GDP: output falls but not by as much as it would expand for an expansion of the same size.

Temporary Fiscal Shocks

We proceed to show that our mechanism is present even when the shock is not permanent, and when the change in G is temporary. We also consider two different financing regimes: (i) deficit financing, the temporary shock is absorbed by changes in public debt until a certain point in time, after which lump-sum transfers adjust to ensure that the economy returns to the initial (pre-shock) level of public debt, and (ii) balanced budget financing, in which taxes and transfers adjust in such a way to keep public debt constant during the entire transition.

Deficit Financing   As a benchmark, we consider a case where G changes to a different level G_{alt} for 10 periods, and then returns to its initial level. In Appendix E, we show that our results are robust to different assumptions regarding the persistence of the shock and the speed of decay towards its initial level (i.e. a standard autoregressive process). Figure

11 shows the multiplier as a function of the size of the shock for the case of deficit financing: the overall pattern of monotonicity is unchanged. The main difference are the magnitudes: since the shock is no longer permanent, it no longer causes a permanent decrease in wages, therefore leading to muted effects on lifetime income and resulting in smaller movements in aggregate labor supply on impact. The left panel plots the impact multipliers (measured the quarter right after the shock), while the right panel plots the one-year integral multipliers. The latter are necessarily smaller in magnitude, as the present discounted value of the fiscal shock becomes smaller as time passes, resulting in smaller movements of labor supply. The qualitative relationship between the multiplier and G is, however, preserved.
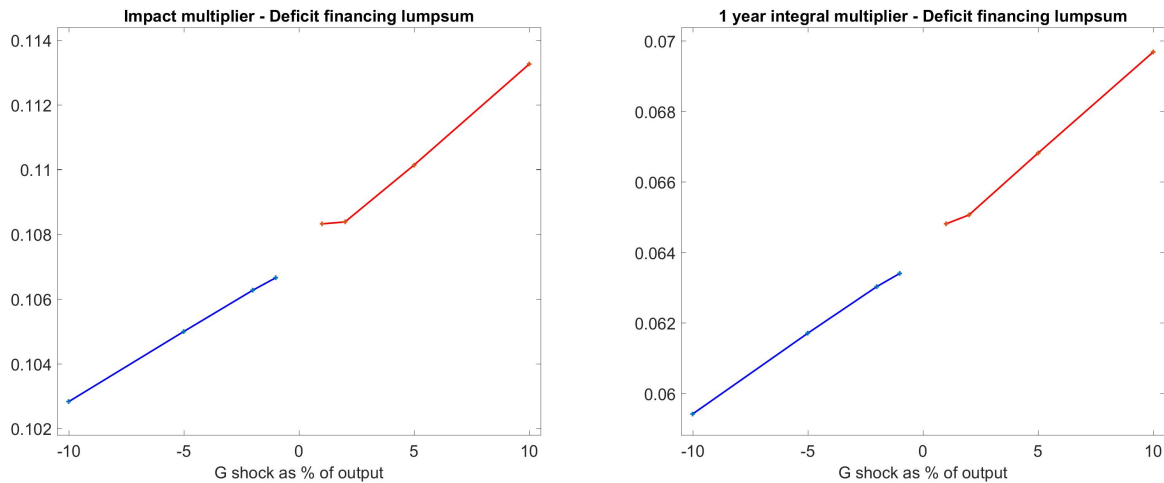
Figure 11: Fiscal multiplier as a function of G for a temporary 10-period shock, deficit financing. The left panel presents impact multipliers (one quarter after the shock), the right panel presents the 1-year integral multipliers.

Figure 12 illustrates the mechanism behind the effect, by plotting the relative change in labor supply across the asset distribution. The change in labor supply is monotonically increasing (decreasing) both in the level of assets and in the size of the shock (the line starts at the constraint). Positive (negative) fiscal shocks lead to an decrease (increase) of agents at the constraint, which contributes to a greater (lower) elasticity of labor supply with respect to the fiscal shock. The change in the share of agents at the constraint is shown in figure 13: the fiscal shock causes agents to move away from the constraint. Since the fiscal policy is financed with future taxes, and both the responses of the wage and interest

37

rates are backloaded, the labor supply of constrained agents responds much less than that of unconstrained ones.

Figure 12: (Relative) labor supply response to different changes in G over the asset distribution. Left panel plots the relative response to increases in G, right panel plots the relative response to decreases in G.

Figure 13: Percentage of agents at the borrowing constraint, deficit financing, one year after the shock, for different levels of the shock to G.

Balanced Budget   Figure 14 plots the same measures of the fiscal multiplier for the case where the government runs a balanced budget, and thus decreases (increases) transfers when G increases (decreases). The qualitative results are identical, but the sizes of the multipliers are much larger, and the mechanism is slightly different. This is because balanced budget

interventions affect the income of constrained agents contemporaneously, and these agents react very strongly to changes in their current income. On top of the mechanism that we propose, there is a more conventional one: when G increases, not only unconstrained agents react strongly due to changes in future wages, but also constrained agents now expand their labor supply due to rises in current taxes/decreases in current transfers.
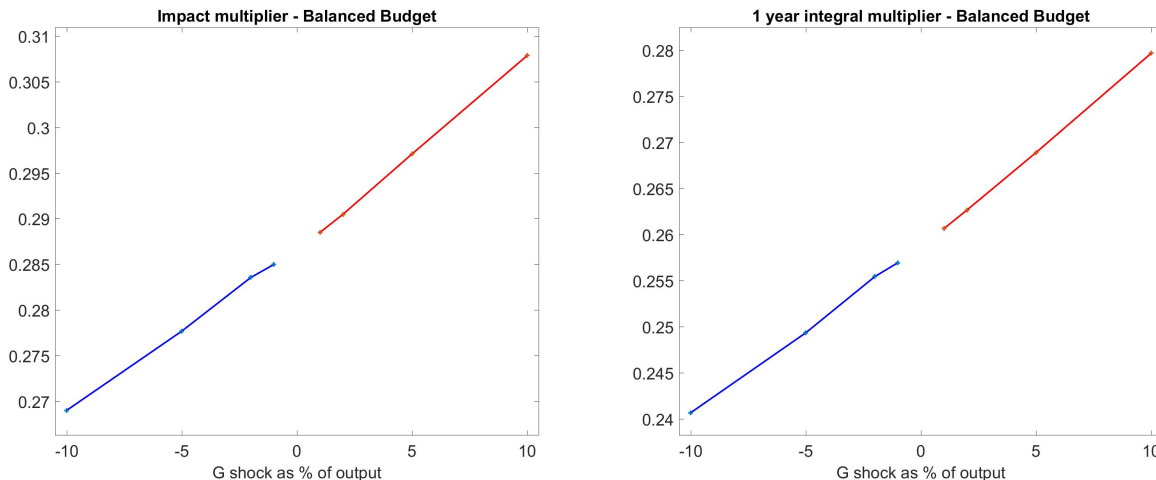
Figure 14: Fiscal multiplier as a function of G for a temporary 10-period shock, balanced budget financing. The left panel presents impact multipliers (one quarter after the shock), the right panel presents the 1-year integral multipliers.

The asymmetry is now driven by agents who are unconstrained but have negative wealth: since the fiscal shock is financed contemporaneously, constrained agents tend to react relatively more to positive fiscal shocks (for example) than unconstrained agents. Unconstrained agents with negative wealth, however, have a labor supply response that is very close to that of constrained agents. On top of that, their labor supply response is further affected by future increases in the interest rate (decreases, for decreases in G). This causes them to react relatively more than constrained agents. Therefore, to the extent that a positive (negative) shock to G causes constrained agents to become unconstrained (constrained), but still with negative wealth, the aggregate labor supply response is still asymmetric. Figure 15 plots the relative labor supply responses, which are hump shaped, are larger for unconstrained agents with negative wealth than for constrained ones (and then monotonically decreasing/increasing). Figure 16 shows that the percentage of constrained agents decreases

(increases) for positive (negative) shocks to G, while the percentage of unconstrained agents with negative wealth increases (decreases), which is the crucial force for explaining why the asymmetry is preserved.[15]
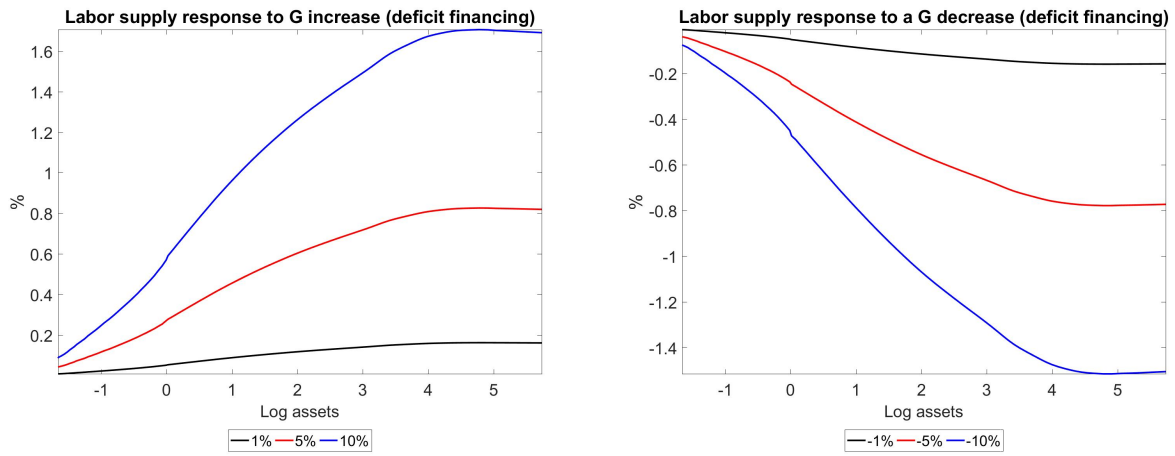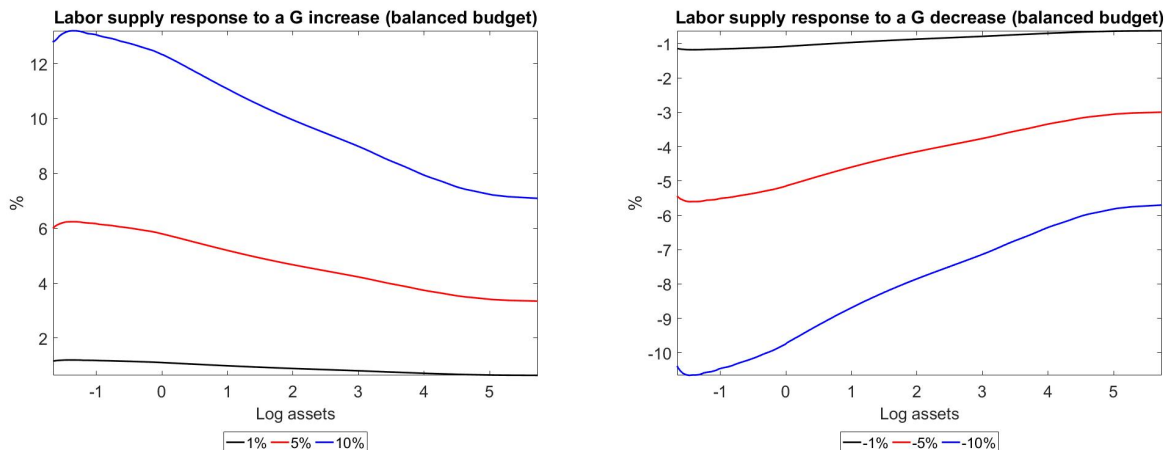

Figure 15: (Relative) labor supply response to different changes in G over the asset distribution, balanced budget. Left panel plots the relative response to increases in G, right panel plots the relative response to decreases in G.
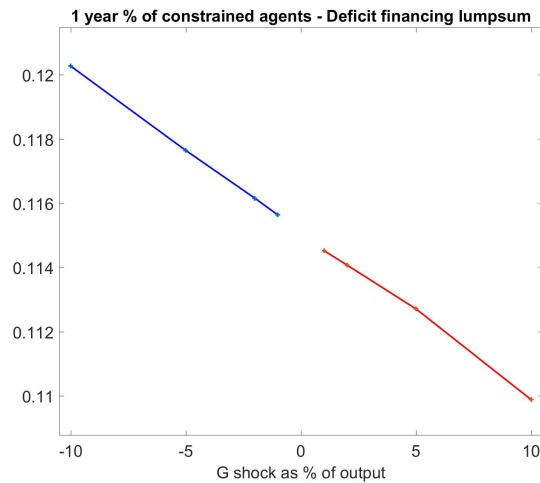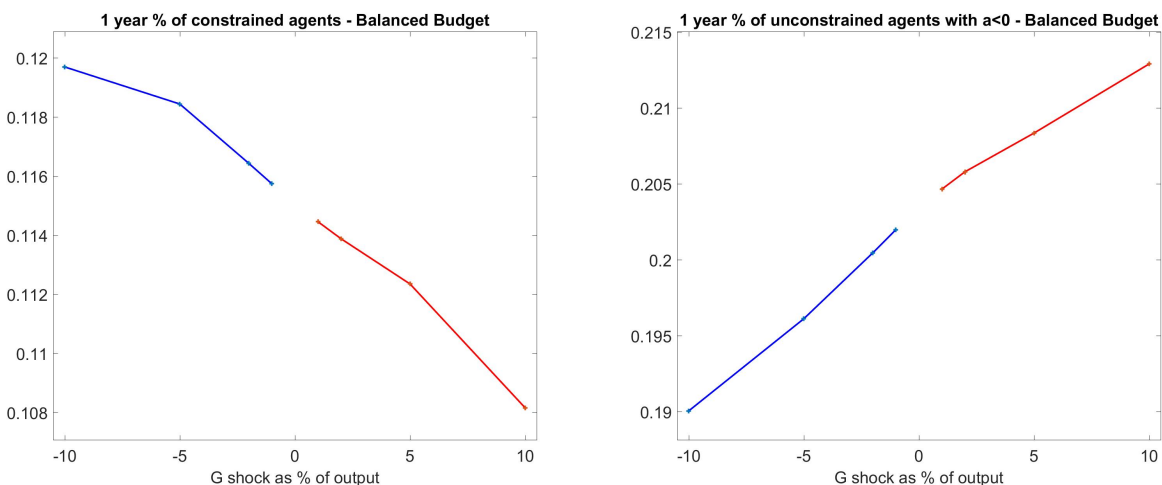

Figure 16: Percentage of agents at the borrowing constraint (left panel), and unconstrained with negative wealth (right panel), balanced budget, one year after the shock, for different levels of the shock to G.

Empirically we establish that the non-linearities are much stronger for positive shocks than for negative ones, with the difference being larger at shorter horizons. Model results

---

[15]Using the notation of the previous section, it is possible to show that $k_t^i < 0 \Rightarrow \mathcal{F}_t^i > 0$. The future decrease in wages, rise in taxes, and rise in interest rates affects unconstrained agents with negative wealth more than any other type of agents for the case of balanced budget fiscal shocks, motivating the large response of labor supply.

indicate that, in an heterogeneous agents model, financing matters, with a balanced budget experiment yielding much larger multipliers and stronger non-linearities than deficit financing. If positive and negative shocks follow a different fiscal rule empirically, this would explain the asymmetry found. To test this hypothesis we estimate equation (2) now with tax revenue in percentage of potential GDP as the dependent variable. $m_{pos,h}$ and $m_{neg,h}$ can now be interpreted as the revenue multiplier: how much tax revenue changes in response to each type of shock. Results in Figure 17 indicate the revenue multiplier is larger for positive fiscal shocks during the first ten periods, which help explain the fact that multiplier is invariant to the size of the negative shocks on impact.
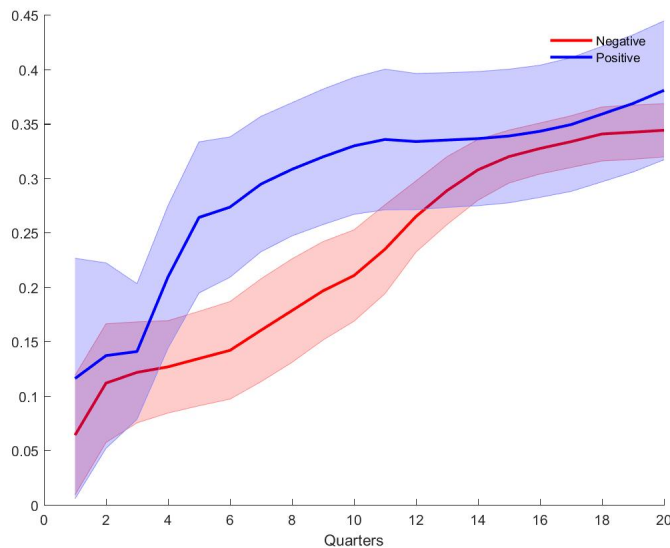
Figure 17: Tax multiplier:

8   Aggregate Demand Externalities: HANK

Our explanation for the asymmetry and monotonicity of the multiplier relies on an negative relationship between fiscal expenditure shocks and the path of future income (especially wages and taxes, the relevant income components for agents close to the constraint): when government expenditure increases, the stock of capital falls along with wages. This leads to more agents being unconstrained, and unconstrained agents expand their labor supply by

41

more. In this section, we explore whether the mechanism is robust to the introduction of aggregate demand externalities through the form of nominal price rigidities. In principle, aggregate demand externalities can moderate our result as they lead real wages to fall by less in response to any shock to autonomous component of aggregate demand (such as government spending). If these externalities are large enough, they can even overturn our result: if aggregate demand externalities are strong enough, real wages can *increase* in response to positive government spending shocks.[16]

TBC

9   Conclusion

In this paper, we provide empirical evidence on the non-linear effects of fiscal policy, both in terms of sign and size, using three different datasets and methodologies. We start by using historical data for the U.S. and Jorda's Projection Method to illustrate that positive variations of government spending yield larger multipliers than negative ones. The last piece of empirical evidence consists on a dataset considering only fiscal consolidations exogenous to the economic cycle, and we show that the larger the consolidation the smaller the multiplier.

To explain the empirical results we develop a life-cycle, overlapping generations model with heterogeneous agents and uninsurable idiosyncratic risk. The model is calibrated to match key characteristics of the U.S. economy. Running simulations of positive and negative variations in government spending we match the empirical results. Moreover, we find that it is the impact that the fiscal shock has on the percentage of agents that are credit constrained that drives the result. Positive variations in government spending will result in a smaller share of constrained agents. As government spending is financed through deficit, life-time income will decrease, making agents decrease their borrowing and consequently yielding a

---

[16]It is well known that nominal rigidities tend to increase the size of fiscal multipliers through aggregate demand externalities (see for example Woodford (2011) for a review). More recently, other papers have studied the nonlinearity of fiscal multipliers in the context of models with nominal rigidities: Hagedorn et al. (2016) and Faria-e-Castro (2017) for example.

smaller share of constrained agents. In contrast, negative variations in government spending will be used to pay off debt, increasing wages and resulting in more agents borrowing against their future income, yielding a larger share of constrained agents. The impact on the share of credit constrained agents is essential to determine the fiscal multiplier as these agents have a labor supply less elastic, so, the larger the share of constrained the smaller the labor supply response and consequently the smaller the multiplier.

References

Alesina, A., Barbiero, O., Favero, C., Giavazzi, F., and Paradisi, M. (2015a). Austerity in 2009–13. *Economic Policy*, 30(83):383–437.

Alesina, A., Favero, C., and Giavazzi, F. (2015b). The output effect of fiscal consolidation plans. *Journal of International Economics*, 96:S19–S42.

Anderson, T. W., Rubin, H., et al. (1949). Estimation of the parameters of a single equation in a complete system of stochastic equations. *The Annals of Mathematical Statistics*, 20(1):46–63.

Athreya, K., Owens, A., and Schwartzman, F. (2017). Does redistribution increase output? the centrality of labor supply. *Quantitative Economics*, 8(3):761–808.

Auerbach, A. J. and Gorodnichenko, Y. (2012). Measuring the output responses to fiscal policy. *American Economic Journal: Economic Policy*, 4(2):1–27.

Barnichon, R. and Matthes, C. (2017). Understanding the size of the government spending multiplier: It's in the sign. Working Paper 17-15, Federal Reserve Bank of Richmond.

Basso, H. and Rachedi, O. (2017). The young, the old and the government: Demographics and fiscal multipliers. Mimeo.

Benabou, R. (2002). Tax and education policy in a heterogeneous agent economy: What levels of redistribution maximize growth and efficiency? *Econometrica*, 70:481–517.

Blanchard, O. and Perotti, R. (2002). An empirical characterization of the dynamic effects of changes in government spending and taxes on output. *the Quarterly Journal of economics*, 117(4):1329–1368.

Brinca, P., Ferreira, M. H., Franco, F., Holter, H. A., and Malafry, L. (2017). Fiscal consolidation programs and income inequality.

Brinca, P., Holter, H. A., Krusell, P., and Malafry, L. (2016). Fiscal multipliers in the 21st century. *Journal of Monetary Economics*, 77:53–69.

Cooley, T. F. and Prescott, E. C. (1995). *Frontiers of Business Cycle Research*, chapter 1, pages 1–38. Princeton University Press.

Devries, P., Guajardo, J., Leigh, D., and Pescatori, A. (2011). A new action-based dataset of fiscal consolidation.

Faria-e-Castro, M. (2017). Fiscal multipliers and financial crises. Unpublished manuscript, Federal Reserve Bank of St. Louis.

Ferriere, A. and Navarro, G. (2014). The heterogeneous effects of government spending: It's all about taxes.

Fisher, J. D. and Peters, R. (2010). Using stock returns to identify government spending shocks. *The Economic Journal*, 120(544):414–436.

Fotiou, A. (2017). Non-linearities and fiscal policy. Job Market Paper, European University Institute.

Hagedorn, M., Manovskii, I., and Mitman, K. (2016). The fiscal multiplier. Technical report, mimeo, University of Oslo.

Ilzetzki, E., Mendoza, E. G., and Vegh, C. A. (2013). How big (small?) are fiscal multipliers? *Journal of Monetary Economics*, 60(2):239–254.

Jordà, Ò. (2005). Estimation and inference of impulse responses by local projections. *American economic review*, pages 161–182.

Krueger, D., Mitman, K., and Perri, F. (2016). Macroeconomics and household heterogeneity. *Handbook of Macroeconomics*, 2:843–921.

Mountford, A. and Uhlig, H. (2009). What are the effects of fiscal policy shocks? *Journal of applied econometrics*, 24(6):960–992.

Nakamura, E. and Steinsson, J. (2014). Fiscal stimulus in a monetary union: Evidence from us regions. *American Economic Review*, 104(3):753–92.

Peterman, W. and Sager, E. (2016). Optimal public debt with life cycles motives. Technical report, Working Paper.

Ramey, V. A. (2011a). Can government purchases stimulate the economy? *Journal of Economic Literature*, 49(3):673–85.

Ramey, V. A. (2011b). Identifying government spending shocks: it's all in the timing. *The Quarterly Journal of Economics*, 126(1):1–50.

Ramey, V. A. (2012). Government spending and private activity. Technical report, National Bureau of Economic Research.

Ramey, V. A. and Zubairy, S. (2018). Government spending multipliers in good times and in bad: evidence from us historical data. Technical Report 2.

Romer, C. D. and Romer, D. H. (2010). The macroeconomic effects of tax changes: estimates based on a new measure of fiscal shocks. *The American Economic Review*, 100(3):763–801.

Trabandt, M. and Uhlig, H. (2011). The laffer curve revisited. *Journal of Monetary Economics*, 58(4):305–327.

Uhlig, H. (2010). Some fiscal calculus. *The American Economic Review*, 100(2):30–34.

Woodford, M. (2011). Simple analytics of the government expenditure multiplier. *American Economic Journal: Macroeconomics*, 3(1):1–35.

A   Additional empirical evidence

A.1   U.S. historical data
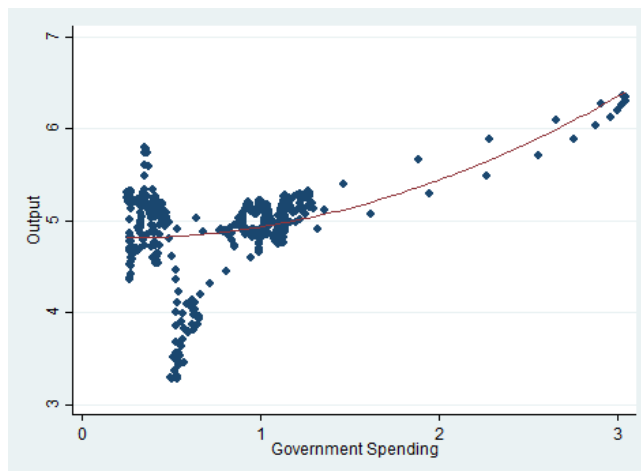
Figure 18: 1 year cumulative real output in the y-axis and 1 year cumulative real government spending in the x-axis, both as percentage of trend GDP. The red line represents the quadratic fitted polynomial between the two variables. The correlation between output and the first order term of government spending is 0.45 (p-value 0.00) and with the second order term of government spending is 0.50 (p-value 0.00).
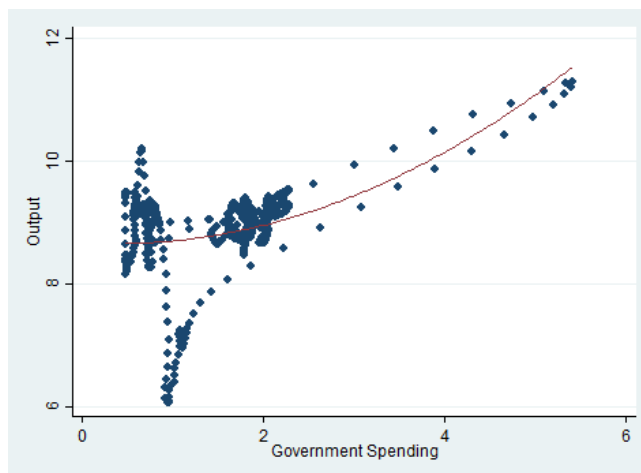
Figure 19: 2 years cumulative real output in the y-axis and 2 years cumulative real government spending in the x-axis, both as percentage of trend GDP. The red line represents the quadratic fitted polynomial between the two variables. The correlation between output and the first order term of government spending is 0.45 (p-value 0.00) and with the second order term of government spending is 0.50 (p-value 0.00).
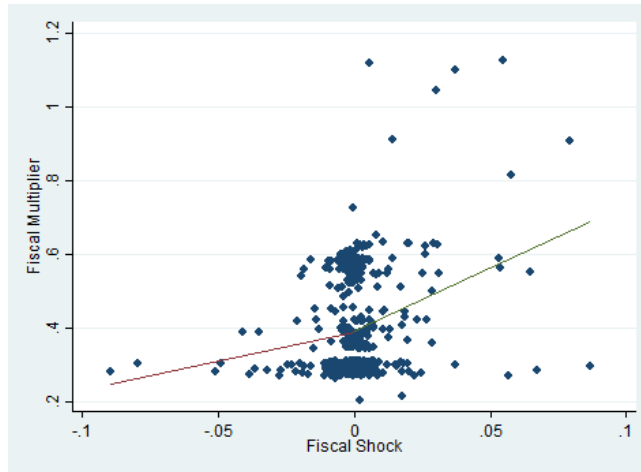
Figure 20: 1 year cumulative multiplier vs Fiscal shock: On the y-axis we have the 1 year cumulative multiplier and on the x-axis we have the size of the fiscal shock. For negative shocks the correlation is 0.14 (P-value 0.03) while for positive shocks is 0.28 (P-value 0.00).
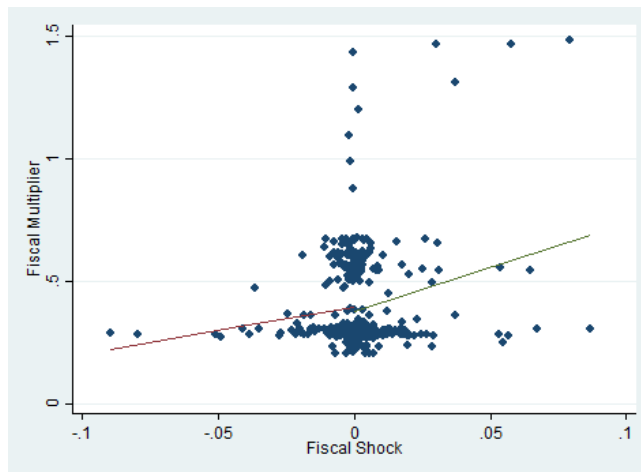
Figure 21: 2 year cumulative multiplier vs Fiscal shock: On the y-axis we have the 2 year cumulative multiplier and on the x-axis we have the size of the fiscal shock. For negative shocks the correlation is 0.12 (P-value 0.07) while for positive shocks is 0.25 (P-value 0.00).

Figure 22: 3 year cumulative multiplier vs Fiscal shock: On the y-axis we have the 3 year cumulative multiplier and on the x-axis we have the size of the fiscal shock. For negative shocks the correlation is 0.08 (P-value 0.19) while for positive shocks is 0.15 (P-value 0.03).

	Linear	Negative shocks	Positive shocks	AR P-value
Impact	0.22	0.18	0.41	AR = 0.34
	(0.15)	(0.33)	(0.14)	
1 year cumulative multiplier	0.28	0.30	0.54	AR = 0.26
	(0.13)	(0.21)	(0.17)	
2 year cumulative multiplier	0.45	0.42	0.65	AR = 0.13
	(0.09)	(0.17)	(0.12)	
3 year cumulative multiplier	0.54	0.61	0.73	AR = 0.10
	(0.08)	(0.12)	(0.11)	
4 year cumulative multiplier	0.56	0.70	0.72	AR = 0.09
	(0.09)	(0.09)	(0.11)	

Table 7: Impact and cumulative multipliers for 1, 2, 3 and 4 year horizons for positive and negative fiscal shocks in a specification without controlling for tax revenue.

	Linear	Negative shocks	Positive shocks	AR P-value
Impact	0.32	0.03	0.59	AR = 0.12
	(0.12)	(0.23)	(0.12)	
1 year cumulative multiplier	0.35	0.11	0.66	AR = 0.18
	(0.11)	(0.21)	(0.14)	
2 year cumulative multiplier	0.52	0.29	0.70	AR = 0.21
	(0.11)	(0.23)	(0.08)	
3 year cumulative multiplier	0.60	0.56	0.72	AR = 0.21
	(0.10)	(0.16)	(0.07)	
4 year cumulative multiplier	0.61	0.65	0.70	AR = 0.20
	(0.10)	(0.11)	(0.07)	

Table 8: Impact and cumulative multipliers for 1, 2, 3 and 4 year horizons for positive and negative fiscal shocks, considering 8 lags for the control variables.

	Linear	Negative shocks	Positive shocks	AR P-value
Impact	-0.07	-0.66	0.54	AR = 0.15
	(0.12)	(0.22)	(0.20)	
1 year cumulative multiplier	-0.00	-0.45	0.69	AR = 0.16
	(0.12)	(0.36)	(0.30)	
2 year cumulative multiplier	0.19	-0.13	0.87	AR = 0.26
	(0.15)	(0.30)	(0.24)	
3 year cumulative multiplier	0.22	-0.12	0.92	AR = 0.24
	(0.20)	(0.33)	(0.27)	
4 year cumulative multiplier	0.06	-0.34	0.68	AR = 0.15
	(0.23)	(0.39)	(0.28)	

Table 9: Impact and cumulative multipliers for 1, 2, 3 and 4 year horizons for positive and negative fiscal shocks, omitting the world war II period.

	Linear	Negative shocks	Positive shocks	AR P-value
Impact	0.60	0.71	0.99	AR = 0.40
	(0.34)	(0.40)	(0.37)	
1 year cumulative multiplier	0.33	0.52	0.68	AR = 0.66
	(0.31)	(0.52)	(0.40)	
2 year cumulative multiplier	0.35	0.35	0.54	AR = 0.76
	(0.25)	(0.49)	(0.36)	
3 year cumulative multiplier	0.57	0.20	0.78	AR = 0.80
	(0.24)	(0.63)	(0.35)	
4 year cumulative multiplier	0.60	0.09	0.92	AR = 0.74
	(0.27)	(0.70)	(0.37)	

Table 10: Impact and cumulative multipliers for 1, 2, 3 and 4 year horizons for positive and negative fiscal shocks, considering only the post 1948 period.

	Linear	Quadratic term
Impact	0.13	0.06
	(0.16)	(0.04)
1 year	0.14	0.04
	(0.19)	(0.04)
2 years	0.14	0.06
	(0.29)	(0.05)
3 years	-0.29	0.011
	(0.50)	(0.06)
4 years	-0.91	0.015
	(0.72)	(0.007)

Table 11: Linear and quadratic term for 1, 2, 3 and 4 year horizons for fiscal shocks.

	Average - st.dev.	Average	Average + st.dev.
Impact	0.14	0.17	0.19
1 year cum. mult.	0.18	0.21	0.25
2 year cum. mult.	0.23	0.32	0.42
3 year cum. mult.	-0.05	0.20	0.46
4 year cum. mult.	-0.46	-0.02	0.41

Table 12: Impact and cumulative multipliers for 1, 2, 3 and 4 year horizons for average G, one standard deviation above and below, for the specification with a quadratic term.

	Linear	Below threshold	Above threshold	AR P-value
Impact multiplier	0.20	0.22	0.12	AR = 0.17
	(0.17)	(0.17)	(0.59)	
1 year cumulative multiplier	0.27	0.28	0.39	AR = 0.19
	(0.14)	(0.15)	(0.59)	
2 year cumulative multiplier	0.45	0.46	0.55	AR = 0.16
	(0.10)	(0.11)	(0.31)	
3 year cumulative multiplier	0.56	0.56	0.63	AR = 0.15
	(0.09)	(0.09)	(0.16)	
4 year cumulative multiplier	0.58	0.59	0.32	AR = 0.22
	(0.09)	(0.10)	(0.13)	

Table 13: Impact and cumulative multipliers for 1, 2, 3 and 4 year horizons for fiscal shocks above and below the median negative shock.

	Linear	Below threshold	Above threshold	AR P-value
Impact multiplier	0.20	0.12	0.44	AR = 0.22
	(0.17)	(0.35)	(0.15)	
1 year cumulative multiplier	0.27	0.19	0.54	AR = 0.18
	(0.14)	(0.29)	(0.19)	
2 year cumulative multiplier	0.45	0.28	0.66	AR = 0.12
	(0.10)	(0.28)	(0.11)	
3 year cumulative multiplier	0.56	0.46	0.72	AR = 0.12
	(0.09)	(0.21)	(0.08)	
4 year cumulative multiplier	0.58	0.60	0.71	AR = 0.18
	(0.09)	(0.16)	(0.07)	

Table 14: Impact and cumulative multipliers for 1, 2, 3 and 4 year horizons for fiscal shocks above and below the median positive shock.

	Average negative - st.dev.	Average negative	Average positive	Average positive + st.dev.
Impact mult.	-0.07	0.09	0.13	0.26
1 year cum. mult.	-0.36	-0.21	-0.29	-0.12
2 year cum. mult.	-0.89	-0.62	-0.72	-0.26
3 year cum. mult.	-2.12	-1.36	-0.76	-0.18
4 year cum. mult.	-4.55	-2.80	-0.59	-0.17

Table 15: Consumption impact and cumulative multipliers for 1, 2, 3 and 4 year horizons for positive and negative fiscal shocks.

	Average - st.dev.	Average	Average + st.dev.
Impact	0.11	0.21	0.31
1 year cum. mult.	-0.20	-0.12	-0.03
2 year cum. mult.	-0.54	-0.31	-0.08
3 year cum. mult.	-0.48	-0.23	0.02
4 year cum. mult.	-0.42	-0.22	-0.01

Table 16: Consumption impact and cumulative multipliers for 1, 2, 3 and 4 year horizons for average G, one standard deviation above and below, for the specification with a quadratic term.

	Average negative - st.dev.	Average negative	Average positive	Average positive + st.dev.
Impact mult.	-1.87	-1.12	-0.47	-0.39
1 year cum. mult.	-3.40	-2.09	0.09	-0.31
2 year cum. mult.	-5.05	-3.19	-0.58	-0.49
3 year cum. mult.	-8.46	-5.26	-1.84	-0.47
4 year cum. mult.	-15.50	-9.34	-3.93	-0.72

Table 17: Investment impact and cumulative multipliers for 1, 2, 3 and 4 year horizons for positive and negative fiscal shocks.

	Average - st.dev.	Average	Average + st.dev.
Impact	-0.82	-0.61	-0.39
1 year cum. mult.	-0.73	-0.59	-0.44
2 year cum. mult.	-1.06	-0.65	-0.25
3 year cum. mult.	-2.35	-1.01	0.33
4 year cum. mult.	-4.79	-1.90	0.99

Table 18: Investment impact and cumulative multipliers for 1, 2, 3 and 4 year horizons for average G, one standard deviation above and below, for the specification with a quadratic term.

B Details on Representative Agent Models

B.1 Real Business Cycle Model

Set-up and Equilibrium The set-up follows closely that of Cooley and Prescott (1995).
A representative household solves

$$\max_{\{C_t,N_t,K_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\nu}}{1+\nu} \right\}$$

s.t.

$$C_t + K_t + B_t = (1-\tau)w_t N_t + (1+r_t^k)K_{t-1} + R_t B_{t-1} - T_t$$

where C_t is consumption, N_t are hours worked, K_t is capital, w_t is the real wage, r_t^k is the
rate of return on capital, B_t are holdings of public debt, R_t is the return on public debt,
and T_t is a lump-sum tax/transfer from the government. The optimality conditions for the
household are standard

$$1 = \mathbb{E}_t \beta \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} (1+r_{t+1}^k)$$

$$1 = \mathbb{E}_t \beta \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} R_{t+1}$$

$$\chi C_t^{\sigma} N_t^{\nu} = w_t(1-\tau)$$

The representative firm hires capital and labor in spot markets,

$$\max_{K_{t-1},N_t} z_t K_{t-1}^{\alpha} N_t^{1-\alpha} - w_t N_t - (r_t^k + \delta)K_{t-1}$$

This yields the standard factor choice first-order conditions,

$$w_t = (1 - \alpha)z_t \left(\frac{K_{t-1}}{N_t}\right)^\alpha$$

$$r_t^k + \delta = \alpha z_t \left(\frac{N_t}{K_{t-1}}\right)^{1-\alpha}$$

Finally, the government's budget constraint is

$$G_t + R_t B_{t-1} = B_t + \tau w_t N_t + T_t$$

Due to Ricardian Equivalence, the specific fiscal rule is irrelevant for the value of the fiscal multiplier. The aggregate resource constraint is

$$C_t + K_t + G_t = z_t K_{t-1}^\alpha N_t^{1-\alpha} + (1 - \delta)K_{t-1}$$

Calibration We try to map the calibration of our baseline neoclassical heterogeneous agents model to the representative agent specification as closely as possible. The discount factor is chosen to yield an equilibrium real rate of 1.1% quarterly, $\beta = 0.9891$. Disutility of labor is $\chi = 8.1$; the coefficient of relative risk aversion is $\sigma = 1.2$; the Frisch elasticity of labor supply is $\nu = 1$; the depreciation rate is $\delta = 0.015$; the capital share is $\alpha = 1/3$. G_{SS} and B_{SS} are chosen to be 20% and 43% of GDP at steady state, respectively.

B.2   New Keynesian Model

We extend the basic RBC model with investment with the standard New Keynesian ingredients. We assume that production is now done by two sectors: a perfectly competitive final goods sector that produces final goods by aggregating a continuum of intermediate varieties

in Dixit-Stiglitz fashion. These firms solve a problem of the type

$$\max_{Y_t(i)} P_t \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_t(i) Y_t(i) \mathrm{d}i$$

This generate a demand curve for each variety

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\varepsilon} Y_t$$

where ε is the elasticity of substitution across varieties. Intermediate goods producers are monopolistic competitors and hire labor and capital in spot markets. Let $P_t(i)$ denote the price of intermediate variety sold by firm i. These firms face quadratic costs of adjusting their prices a la Rotemberg. The adjustment costs of price setting for firm i are given by

$$\Xi_t(i) = \frac{\xi}{2} Y_t \left[\frac{P_t(i)}{P_{t-1}(i)} \frac{1}{\Pi} - 1 \right]^2$$

For simplicity, we assume that these costs scale with total output, and it is free to adjust prices to keep track with trend inflation Π.

The firm's value in nominal terms is

$$P_t V_t[P_{t-1}(i); X_t] = \max_{P_t(i), Y_t(i), K_t(i), L_t(i)} P_t(i) Y_t(i) - P_t w_t L_t(i) - P_t(r_t + \delta) K_t(i) - P_t \Xi_t(i)$$
$$+ \mathbb{E}_t \frac{\Lambda_{t,t+1}}{\Pi_{t+1}} P_{t+1} V_{t+1}[P_t(i); X_{t+1}]$$

subject to the demand curve for variety i and the production function

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\varepsilon} Y_t$$
$$Y_t(i) = A_t K_t(i)^{\alpha} L_t(i)^{1-\alpha}$$

where $\frac{\Lambda_{t,t+1}}{\Pi_{t+1}}$ is the relevant stochastic discount factor for discounting the firm's payoffs,

adjusted for inflation. The firm's problem can be split into a static cost-minimization component and a dynmic price-setting one. The static problem yields the standard condition for cost minimization,

$$\frac{w_t}{r_t + \delta} = \frac{1 - \alpha}{\alpha} \frac{K_t(i)}{L_t(i)} \tag{22}$$

Combining this condition with the production function allows us to express total costs as a function of output and factor prices only,

$$
\begin{aligned}
TC_t(i) &= w_t L_t(i) + (r_t + \delta) K_t(i) \\
&= w_t \frac{Y_t(i)}{A_t \left[\frac{w_t}{r_t + \delta} \frac{\alpha}{1 - \alpha} \right]^\alpha} + (r_t + \delta) \frac{w_t}{r_t + \delta} \frac{\alpha}{1 - \alpha} \frac{Y_t(i)}{A_t \left[\frac{w_t}{r_t + \delta} \frac{\alpha}{1 - \alpha} \right]^\alpha} \\
&= \left(\frac{w_t}{1 - \alpha} \right)^{1 - \alpha} \left(\frac{r_t + \delta}{\alpha} \right)^\alpha \frac{Y_t(i)}{A_t}
\end{aligned}
$$

This is useful to now solve the firm's dynamic problem, just in terms of price and output choices

$$V_t[P_{t-1}(i); X_t] = \max_{P_t(i), Y_t(i)} \frac{P_t(i)}{P_t} Y_t(i) - TC_t(i) - \Xi_t(i) + \mathbb{E}_t \Lambda_{t,t+1} V_{t+1}[P_t(i); X_{t+1}]$$

subject to the demand function $Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\varepsilon} Y_t$. We can furthermore replace $Y_t(i)$ for the demand function and solve for $P_t(i)$ only. The FOC is then

$$-(\varepsilon - 1) P_t(i)^{-\varepsilon} P_t^{\varepsilon - 1} Y_t + \varepsilon MC_t P_t(i)^{-\varepsilon - 1} P_t^\varepsilon Y_t - \xi Y_t \left[\frac{P_t(i)}{P_{t-1}(i)\Pi} - 1 \right] \frac{1}{P_{t-1}(i)\Pi}$$

$$+ \mathbb{E}_t \Lambda_{t,t+1} \xi Y_{t+1} \left[\frac{P_{t+1}(i)}{P_t(i)\Pi} - 1 \right] \frac{P_{t+1}(i)}{P_t(i)^2 \Pi} = 0$$

where marginal costs are

$$MC_t \equiv \frac{\partial TC_t(i)}{\partial Y_t(i)} = \left(\frac{w_t}{1 - \alpha} \right)^{1 - \alpha} \left(\frac{r_t + \delta}{\alpha} \right)^\alpha \frac{1}{A_t}$$

We now invoke the symmetric equilibrium assumption to obtain the New Keynesian Phillips

Curve

$$[(\varepsilon - 1) - \varepsilon MC_t] + \xi \left[\frac{\Pi_t}{\Pi} - 1\right]\frac{\Pi_t}{\Pi} = \mathbb{E}_t \Lambda_{t,t+1}\xi\frac{Y_{t+1}}{Y_t}\left[\frac{\Pi_{t+1}}{\Pi} - 1\right]\frac{\Pi_{t+1}}{\Pi}$$

The Central Bank sets the nominal interest using a Taylor Rule,

$$R_t = R\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\Pi}\left(\frac{Y_t}{Y}\right)^{\phi_Y}$$

where R is some target rate, and (Π, Y) are output and inflation benchmarks. The real interest rate is determined via the Fisher Equation,

$$1 + r_t = \frac{R_t}{\Pi_t}$$

We assume that government debt pays a real return, and that all intermediate firm profits are rebated to the representative household.

Calibration We calibrate all common parameters to the same values as in the RBC model. For the New Keynesian parameters, we use standard values: menu costs are set so that firms change their prices once every three quarters $\eta = 58.10$; the elasticity of substitution across varieties is $\varepsilon = 6$; the Taylor Rule parameters are $\rho_R = 0.80, \phi_\Pi = 1.50, \phi_Y = 0.5$.

B.3 *Investment Adjustment Costs*

We introduce quadratic adjustment costs of investment of the type

$$\frac{\Phi}{2}K_{t-1}\left(\frac{I_t}{K_{t-1}} - \delta\right)^2$$

This changes the first-order condition for K_t, for the representative household

$$1 + \Phi\left(\frac{K_t}{K_{t-1}}\right) = \beta\mathbb{E}_t\left(\frac{C_t}{C_{t+1}}\right)^\sigma\left\{1 + r_{t+1}^k + \frac{\Phi}{2}\left[\left(\frac{K_{t+1}}{K_t}\right)^2 - 1\right]\right\}$$

Calibration We choose a standard quarterly value of $\Phi = 12.5$.

C Derivations for Section 5

The solution to the household problem is standard, where λ_t^i is the Lagrange multiplier on the borrowing constraint

$$(c_t^i)^\sigma (n_t^i)^\eta = u_t^i w_t$$

$$(c_t^i)^{-\sigma} = \beta \mathbb{E}_t (1 + r_{t+1}) c_{t+1}^i)^{-\sigma} + \lambda_t^i$$

$$c_t^i + k_{t+1}^i = k_t^i (1 + r_t) + w_t u_t^i n_t^i - T_t$$

$$k_{t+1}^i \geq -\underline{b} \perp \lambda_t^i \geq 0$$

Combining the labor supply FOC with the budget constraint allows us to derive the response of labor supply to a change in G_t,

$$\frac{\mathrm{d}n_t^i}{\mathrm{d}G_t} = \alpha_1(s_t^i; X_t) \frac{\mathrm{d}w_t}{\mathrm{d}G_t} + \alpha_2(s_t^i; X_t) \left[(1 - \mathbb{1}_t^i) \frac{\mathrm{d}k_{t+1}^i}{\mathrm{d}G_t} + \frac{\mathrm{d}T_t}{\mathrm{d}G_t} - k_t^i \frac{\mathrm{d}r_t}{\mathrm{d}G_t} \right] \tag{23}$$

where

$$\alpha_1(s_t^i; X_t) = \frac{\frac{n_t^i}{\eta w_t}}{1 + \frac{\sigma}{\eta} \frac{w_t n_t^i u_t^i}{c_t^i}} \left(1 - \sigma \frac{w_t n_t^i u_t^i}{c_t^i} \right)$$

$$\alpha_2(s_t^i; X_t) = \frac{\frac{n_t^i}{\eta w_t}}{1 + \frac{\sigma}{\eta} \frac{w_t n_t^i u_t^i}{c_t^i}} \sigma \frac{w_t}{c_t^i}$$

For constrained agents $\frac{\mathrm{d}k_{t+1}^i}{\mathrm{d}G_t} = 0$, but not for unconstrained ones. To determine the response of the savings policy to changes in G_t, we can combine the Euler equation with the budget constraint

$$\frac{\mathrm{d}k_{t+1}^i}{\mathrm{d}G_t} = \Lambda_1(s_t^i; X_t) \frac{\mathrm{d}w_t}{\mathrm{d}G_t} + \Lambda_2(s_t^i; X_t) \left(k_t^i \frac{\mathrm{d}r_t}{\mathrm{d}G_t} - \frac{\mathrm{d}T_t}{\mathrm{d}G_t} \right)$$
$$- \mathbb{E}_t \Lambda_3(s_t^i; X_t) \frac{\mathrm{d}w_{t+1}}{\mathrm{d}G_t} + \mathbb{E}_t \Lambda_4(s_t^i; X_t) \frac{\mathrm{d}r_{t+1}}{\mathrm{d}G_t} + \mathbb{E}_t \Lambda_5(s_t^i; X_t) \left(\frac{\mathrm{d}T_{t+1}}{\mathrm{d}G_t} + \frac{\mathrm{d}k_{t+2}^i}{\mathrm{d}G_t} \right)$$

with

$$\kappa(s_t^i; X_t) = \left[\frac{1 - \alpha_2(s_t^i; X_t) u_t^i w_t}{c_t^i} + \mathbb{E}_t \beta \left(\frac{c_t^i}{c_{t+1}^i} \right)^{\sigma} (1 + r_{t+1})^2 \frac{1 - \alpha_2(s_{t+1}^i; X_{t+1}) u_{t+1}^i w_{t+1}}{c_{t+1}^i} \right]$$

$$\Lambda_1(s_t^i; X_t) = \kappa(s_t^i; X_t)^{-1} \frac{u_t^i(\alpha_1(s_t^i; X_t) w_t + n_t^i)}{c_t^i}$$

$$\Lambda_2(s_t^i; X_t) = \kappa(s_t^i; X_t)^{-1} \frac{1 - \alpha_2(s_t^i; X_t) u_t^i w_t}{c_t^i}$$

$$\Lambda_3(s_t^i; X_t) = \kappa(s_t^i; X_t)^{-1} \beta \left(\frac{c_t^i}{c_{t+1}^i} \right)^{\sigma} (1 + r_{t+1}) \frac{u_{t+1}^i(\alpha_1(s_{t+1}^i; X_{t+1}) w_{t+1} + n_{t+1}^i)}{c_{t+1}^i}$$

$$\Lambda_4(s_t^i; X_t) = \kappa(s_t^i; X_t)^{-1} \beta \left(\frac{c_t^i}{c_{t+1}^i} \right)^{\sigma} \left[1/\sigma - (1 + r_{t+1}) k_{t+1}^i \frac{1 - \alpha_2(s_{t+1}^i; X_{t+1}) u_{t+1}^i w_{t+1}}{c_{t+1}^i} \right]$$

$$\Lambda_5(s_t^i; X_t) = \kappa(s_t^i; X_t)^{-1} \beta \left(\frac{c_t^i}{c_{t+1}^i} \right)^{\sigma} (1 + r_{t+1}) \frac{1 - \alpha_2(s_{t+1}^i; X_{t+1}) u_{t+1}^i w_{t+1}}{c_{t+1}^i}$$

We can rewrite this expression as

$$\frac{dk_{t+1}^i}{dG_t} = \Lambda_1(s_t^i; X_t) \frac{dw_t}{dG_t} + \Lambda_2(s_t^i; X_t) \left(k_t^i \frac{dr_t}{dG_t} - \frac{dT_t}{dG_t} \right) + \mathcal{F}(s_t^i; X_t) \tag{24}$$

where $\mathcal{F}(s_t^i; X_t)$ takes into account all changes in future factor prices and taxes. Combining 23 with 24 yields the expression in Proposition 2.1:

$$\begin{aligned}
\frac{dn_t^i}{dG_t} =& [\alpha_1(s_t^i) + \alpha_2(s_t^i)\Lambda_1(s_t^i)(1 - \mathbb{1}_t^i)] \frac{dw_t}{dG_t} \\
&+ \alpha_2(s_t^i)[1 - (1 - \mathbb{1}_t^i)\Lambda_2(s_t^i)] \left(\frac{dT_t}{dG_t} - k_t^i \frac{dr_t}{dG_t} \right) \\
&+ \alpha_2(s_t^i)(1 - \mathbb{1}_t^i)\mathcal{F}(s_t^i; X_t)
\end{aligned}$$

The comparative statics are immediate from signing $\alpha_1, \alpha_2, \Lambda_1, \Lambda_2$.

We can write $\mathcal{F}(s_t^i; X_t)$ recursively as

$$\mathcal{F}(s_t^i; X_t) = - \overbrace{\Lambda_3(s_t^i; X_t)[1 - \Lambda_2(s_{t+1}^i; X_{t+1})]}^{\geq 0} \frac{\mathrm{d}w_{t+1}}{\mathrm{d}G_t}$$

$$+ [\Lambda_4(s_t^i; X_t) + \Lambda_5(s_t^i; X_t)\Lambda_2(s_{t+1}^i; X_{t+1})k_{t+1}^i] \frac{\mathrm{d}r_{t+1}}{\mathrm{d}G_t}$$

$$+ \overbrace{\Lambda_5(s_t^i; X_t)[1 - \Lambda_2(s_{t+1}^i; X_{t+1})]}^{\geq 0} \frac{\mathrm{d}T_{t+1}}{\mathrm{d}G_t}$$

$$+ \Lambda_5(s_t^i; X_t)\mathcal{F}(s_{t+1}^i; X_{t+1})$$

It is possible to show that

$$k_{t+1}^i < 0 \Rightarrow [\Lambda_4(s_t^i; X_t) + \Lambda_5(s_t^i; X_t)\Lambda_2(s_{t+1}^i; X_{t+1})k_{t+1}^i] > 0$$

In which case we can show that $\mathcal{F}(s_t^i; X_t) \geq 0$ for

$$\frac{\mathrm{d}r_{t+j}}{\mathrm{d}G_t} \geq 0 \;,\; \frac{\mathrm{d}w_{t+j}}{\mathrm{d}G_t} \leq 0 \;,\text{and}\; \frac{\mathrm{d}T_{t+j}}{\mathrm{d}G_t} \geq 0, \forall j \geq 0$$

C.1 Derivation of the Firm's Problem with Nominal Rigidities

In the baseline model, firms are not responsible for the investment decision, they simply choose factors statically. This means that it is possible to split the firm's problem into a static one (factor choice/cost minimization) and a dynamic one (price setting/output choice). It is easier to set up the firm's problem recursively. The firm's value in nominal terms is

$$P_t V_t[P_{t-1}(i); X_t] = \max_{P_t(i), Y_t(i), K_t(i), L_t(i)} P_t(i)Y_t(i) - P_t w_t L_t(i) - P_t(r_t + \delta)K_t(i) - P_t \Xi_t(i)$$

$$+ \mathbb{E}_t \frac{\Lambda_{t,t+1}}{\Pi_{t+1}} P_{t+1} V_{t+1}[P_t(i); X_{t+1}]$$

subject to the demand curve for variety i and the production function

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\varepsilon} Y_t$$

$$Y_t(i) = A_t K_t(i)^\alpha L_t(i)^{1-\alpha}$$

where $\frac{\Lambda_{t,t+1}}{\Pi_{t+1}}$ is the relevant stochastic discount factor for discounting the firm's payoffs, adjusted by inflation (as we are discounting nominal payoffs). We first solve the static problem, which is to minimize costs given a desired level of production. This solves the factor choice problem and generates the expression for total costs,

$$\min_{L_t(i), K_t(i)} w_t L_t(i) + (r_t + \delta) K_t(i)$$

subject to $Y_t(i) \leq A_t K_t(i)^\alpha L_t(i)^{1-\alpha}$. Letting λ_t be the Lagrange multiplier on the production constraint, the FOC's are

$$w_t = \lambda_t A_t (1 - \alpha) K_t(i)^\alpha L_t(i)^{-\alpha}$$

$$r_t + \delta = \lambda_t A_t \alpha K_t(i)^{\alpha-1} L_t(i)^{1-\alpha}$$

We can combine them to generate the standard condition for cost minimization,

$$\frac{w_t}{r_t + \delta} = \frac{1 - \alpha}{\alpha} \frac{K_t(i)}{L_t(i)} \tag{25}$$

From this condition, we can derive an expression for total costs that is useful to solve the dynamic problem. Notice that it allows us to write the optimal choice of capital as a function of labor and factor prices, $K_t(i) = \frac{w_t}{r_t+\delta}\frac{\alpha}{1-\alpha} L_t(i)$. We can replace for capital in the production

function to write

$$Y_t(i) = A_t \left[\frac{w_t}{r_t + \delta} \frac{\alpha}{1-\alpha} L_t(i) \right]^\alpha L_t(i)^{1-\alpha} = A_t \left[\frac{w_t}{r_t + \delta} \frac{\alpha}{1-\alpha} \right]^\alpha L_t(i)$$

This means that we can write

$$L_t(i) = \frac{Y_t(i)}{A_t \left[\frac{w_t}{r_t+\delta} \frac{\alpha}{1-\alpha} \right]^\alpha}$$

$$K_t(i) = \frac{w_t}{r_t + \delta} \frac{\alpha}{1 - \alpha} \frac{Y_t(i)}{A_t \left[\frac{w_t}{r_t+\delta} \frac{\alpha}{1-\alpha} \right]^\alpha}$$

Thus allowing us to write total (real) costs as a function of desired production $Y_t(i)$ and factor prices,

$$TC_t(i) = w_t L_t(i) + (r_t + \delta) K_t(i)$$

$$= w_t \frac{Y_t(i)}{A_t \left[\frac{w_t}{r_t+\delta} \frac{\alpha}{1-\alpha} \right]^\alpha} + (r_t + \delta) \frac{w_t}{r_t + \delta} \frac{\alpha}{1 - \alpha} \frac{Y_t(i)}{A_t \left[\frac{w_t}{r_t+\delta} \frac{\alpha}{1-\alpha} \right]^\alpha}$$

$$= \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{r_t + \delta}{\alpha} \right)^\alpha \frac{Y_t(i)}{A_t}$$

We can now replace this expression in the firm's value function and solve a problem just in terms of total prices and output choices (now expressed in real terms),

$$V_t[P_{t-1}(i); X_t] = \max_{P_t(i), Y_t(i)} \frac{P_t(i)}{P_t} Y_t(i) - TC_t(i) - \Xi_t(i) + \mathbb{E}_t \Lambda_{t,t+1} V_{t+1}[P_t(i); X_{t+1}]$$

subject to the demand function $Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\varepsilon} Y_t$. We can furthermore replace $Y_t(i)$ for

the demand function and solve for $P_t(i)$ only. The FOC is then

$$- (\varepsilon - 1)P_t(i)^{-\varepsilon}P_t^{\varepsilon-1}Y_t + \varepsilon MC_t P_t(i)^{-\varepsilon-1}P_t^{\varepsilon}Y_t - \xi Y_t \left[\frac{P_t(i)}{P_{t-1}(i)\Pi} - 1\right]\frac{1}{P_{t-1}(i)\Pi}$$

$$+ \mathbb{E}_t\Lambda_{t,t+1}\xi Y_{t+1}\left[\frac{P_{t+1}(i)}{P_t(i)\Pi} - 1\right]\frac{P_{t+1}(i)}{P_t(i)^2\Pi} = 0$$

where marginal costs are

$$MC_t \equiv \frac{\partial TC_t(i)}{\partial Y_t(i)} = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t + \delta}{\alpha}\right)^{\alpha}\frac{1}{A_t}$$

While this equation seems complicated, we now make a simplifying assumption: except for $P_{t-1}(i)$, all firms are otherwise identical as their factor choice is static. If all firms start with the same chosen price level, their choice for a current price level will also be identical.

Aggregate Supply Equilibrium Conditions   The set of equilibrium conditions is then

$$\frac{r_t + \delta}{w_t} = \frac{\alpha}{1-\alpha}\frac{L_t}{K_t}$$

$$MC_t = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t + \delta}{\alpha}\right)^{\alpha}\frac{1}{A_t}$$

$$[(\varepsilon - 1) - \varepsilon MC_t] + \xi\left(\frac{\Pi_t}{\Pi} - 1\right)\frac{\Pi_t}{\Pi} = \mathbb{E}_t\Lambda_{t,t+1}\frac{Y_{t+1}}{Y_t}\xi\left(\frac{\Pi_{t+1}}{\Pi} - 1\right)\frac{\Pi_{t+1}}{\Pi}$$

D Parameters

Parameter	Value	Description	Source
Preferences			
η	1	Inverse Frisch Elasticity	Trabandt and Uhlig (2011)
σ	1.2	Risk aversion parameter	Literature
Technology			
α	0.33	Capital share of output	Literature
δ	0.015	Capital depreciation rate	Literature
ρ	0.761	$u' = \rho u + \epsilon, \quad \epsilon \sim N(0, \sigma_\epsilon^2)$	PSID 1968-1997
σ_ϵ	0.211	Variance of risk	PSID 1968-1997
Taxes			
θ_1	0.888	Income tax level	OECD
θ_2	0.137	Income tax progressivity	OECD
τ_c	0.047	Consumption tax	Trabandt and Uhlig (2011)
τ_k	0.364	Capital tax	Trabandt and Uhlig (2011)
$\tilde{\tau}_s s$	0.078	Social security tax:employer	OECD 2001-2007
$\tau_s s$	0.077	Social security tax: employee	OECD 2001-2007
Income profile parameters			
γ_1	0.265	Wage profile	LIS survey
γ_2	-0.005	Wage profile	LIS survey
γ_3	3.6E-05	Wage profile	LIS survey
Macro ratios			
B/Y	1.714	Debt to GDP ratio	

E Robustness: shock persistence
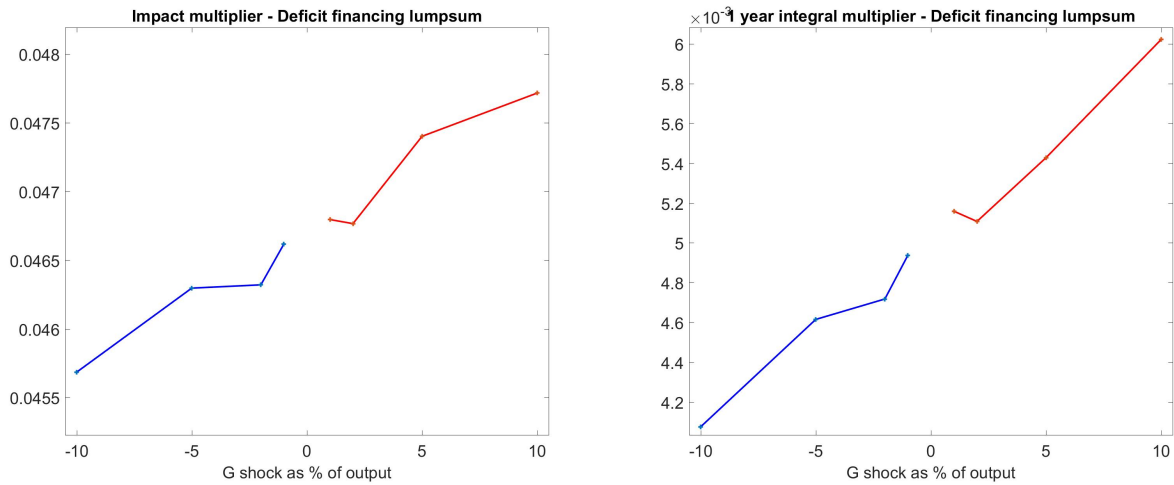
E.1 *Persistence*= 0.75

Figure 23: Fiscal multiplier as a function of G for a temporary 10-period shock with persistence of 0.75, deficit financing. The left panel presents impact multipliers (one quarter after the shock), the right panel presents the 1-year integral multipliers.
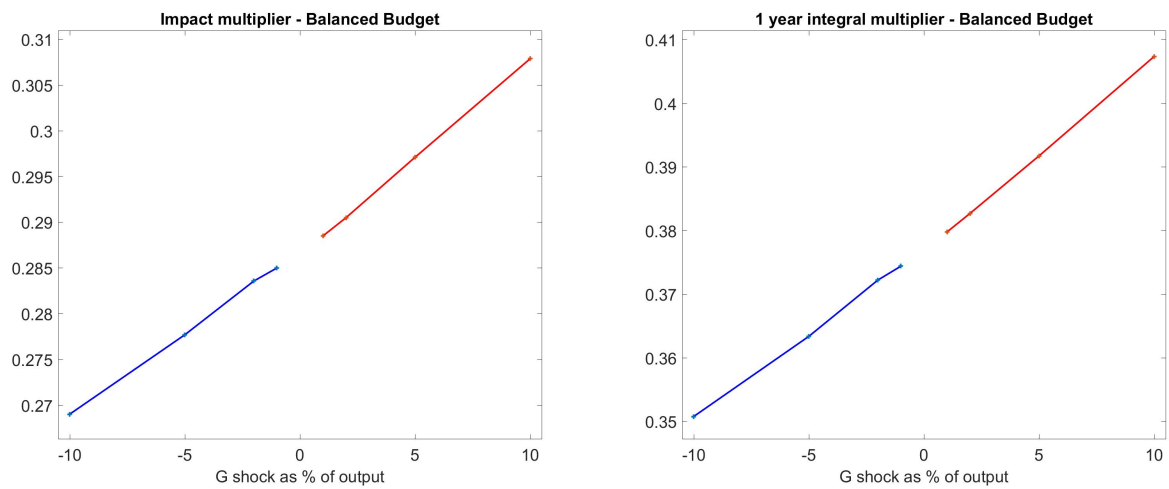
Figure 24: Fiscal multiplier as a function of G for a temporary 10-period shock with persistence of 0.75, balanced budget. The left panel presents impact multipliers (one quarter after the shock), the right panel presents the 1-year integral multipliers.