

Liquidity Management, Leverage, and Monetary Policy

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February, 2018

Abstract

In this paper, I develop a dynamic macroeconomic model of financial intermediation in which short-term funding is subject to liquidity risk. In the model economy I develop, financial intermediaries provide both settlement services to households and financial intermediary services between households and non-financial firms. Because the provision of settlement services exposes them to random withdrawal shocks on their short-term liabilities, financial intermediaries demand bank reserves, which are liquid assets whose quantity supplied and return depend on monetary policy. I use this model economy to study the real effects of targeting the width of the corridor between the official lending and borrowing rates of bank reserves. I obtain that narrower interest-rate corridors of bank reserves increase liquidity ratios when financial intermediaries on aggregate are net lenders of reserves, while decrease liquidity ratios when the opposite happens. I also obtain that narrower interest-rate corridors always increase leverage multiples.

JEL Classification: E31, E32, E44, E52, E61, G01

*I thank Mark Aguiar, Markus Brunnermeier, Fiorella De Fiore, Wouter Den Haan, Oleg Itskhoki, Peter Karadi, Nobuhiro Kiyotaki, and Ezra Oberfield for useful comments and suggestions. Any remaining errors are my own.

1 Introduction

The provisions of settlement and financial intermediary services have traditionally been interrelated. At least since the Renaissance roots of banking in the 15th century, financial institutions at the centre of the settlement system have traditionally taken advantage of their key position to also intermediate funds and credit between borrowers and savers.¹ Notwithstanding, the literature has in general disregarded the macroeconomic consequences and the implications for the transmission mechanism of monetary policy of the joint provision of these two services.² The reason is mainly that the settlement and financial intermediary systems have functioned smoothly in developed economies during the postwar era.

This paper fills that gap in the literature. To such end, first, I develop a tractable model economy in which financial intermediaries jointly provide settlement services to households and financial intermediary services between households and non-financial firms. Then, I use the model economy to examine the dynamics of asset prices and macroeconomic aggregates, and to revisit the transmission mechanism of monetary policy. For the second purpose, I consider both conventional monetary policies that target the level of the inflation rate as well as unconventional monetary policies that set the quantity up to and the official rates at which the Monetary Authority lends and borrows bank reserves short-term.

The model economy I develop incorporates a tractable liquidity management problem into an otherwise standard macroeconomic model of financial intermediation. In the model economy, as standard in the literature, financial intermediaries are good at monitoring the activities of non-financial firms and, consequently, at intermediating funds from households (i.e., savers) to such firms (i.e., borrowers). Because of moral hazard considerations, nonetheless, financial intermediaries cannot intermediate further funds beyond a limited multiple of their net worth (Gertler and Kiyotaki 2010 and Gertler and Karadi 2011). The liquidity management problem arises from the combination of three key elements. The first is that to intermediate funds, financial intermediaries issue short-term and liquid liabilities (i.e., deposits), which households can use not only as a store of value but also as a mean of payment to settle the transactions they conduct among each other. As in Bianchi and Bigio (2016) and Piazzesi and Schneider (2016), the settlement of transactions with deposits at the household level gives rise to a random reshuffle of deposits at the intermediary level.

¹See Cipolla (1956) for a detailed description of the banking system in the Mediterranean World of the 5th to the 17th century. See Roberds (2008) and Kahn and Roberds (2009) for a comprehensive survey of the payment and settlement systems in the modern U.S. economy.

²Two notable exceptions are Bianchi and Bigio (2016) and Piazzesi and Schneider (2016).

The latter in fact represents from the point of view of individual financial intermediaries, idiosyncratic withdrawal shocks to deposits that may affect the cost of short-term funding and the total return on financial intermediation. The second element is that from time to time financial intermediaries are required by law to settle their net deposit flows, if any, immediately. The role of (and the justification for) this immediate settlement requirement is to curb counterparty risks in interbank markets, which in the model economy, is essential for curbing moral hazard problem across financial intermediaries and for safeguarding the proper functioning of credit markets in general. The last element is that bank reserves are the only asset saleable in short notice. This captures the notion that firms' financial claims in general are costly to liquidate.

These three elements together give bank reserves the special role of a hedge against withdrawal risks to deposit funding. Put it differently, they create a demand of financial intermediaries for liquid assets that only bank reserves can satisfy. Monetary policy can affect the opportunity cost of holding bank reserves (because bank reserves are nominal assets whose return negatively depends on the inflation rate) as well as the terms of borrowing and lending bank reserves ex-post (i.e., after withdrawal shocks realize) both bilaterally with the Monetary Authority and in the interbank market. It is indeed through these two channels that monetary policy can influence the liquidity and leverage decisions of financial intermediaries and the real course of the economy.

The paper's first main result is that narrow interest-rate corridors of bank reserves reinforce liquidity ratios and boost leverage multiples relative to their wide counterparts. The rationale is as follows. In interest-rate corridor systems, monetary policy adjusts the quantity supplied of bank reserves ex-post, so that the interbank rate (i.e., the benchmark rate at which bank reserves are traded in the interbank market) is the average of the official lending and borrowing rates of bank reserves. Relative to wide interest-rate corridors, their narrow counterparts improve the return on bank reserves that financial intermediaries obtain, reduce the cost of hedging against withdrawal shocks to deposits, and boost the total return on financial intermediation. Whether financial intermediaries increase their liquidity ratio in response to narrower interest-rate corridors depends on their aggregate trading position with respect to the Monetary Authority. When financial intermediaries on aggregate are net borrowers of bank reserves, which happens when the expected reserves return is low relative to the targeted inflation rate, narrower interest-rate corridors reduce the effective rate at which financial intermediaries borrow reserves, while keep the counterpart effective lending rate unchanged. This induces financial intermediaries to reduce their

liquidity ratios even more, further increasing the intermediaries aggregate net borrowing reserve position and the gap between their effective borrowing and lending rates, and so on. Exactly the opposite happens when financial intermediaries on aggregate are net lenders of bank reserves (and the expected reserves return is high relative to the targeted inflation rate). Independent of the intermediaries' aggregate net trading position of bank reserves, narrower interest-rate corridors boost the intermediaries' profitability, capacity to issue debt, and leverage multiples.

The paper's second main result is that eliminating the opportunity cost of holding bank reserves or the cost of using bank reserves to hedge against withdrawal risks is not optimal. The problem with such monetary policies is that they induce financial intermediaries to take too much leverage and, therefore, generate too strong feedback loops in financial markets and too wide fluctuations in asset prices and in financial and macroeconomic aggregates. Such monetary policies in general are dominated in terms of social welfare by countercyclical monetary policies that provide bank reserves at higher costs when financial intermediaries are better capitalized.

Related Literature This paper relates to a body of literature that studies the interactions between liquidity management and leverage decisions at the intermediary level and their implications for monetary policy. Following Bianchi and Bigio (2014) and Piazzesi and Schneider (2016), I introduce liquidity management considerations based on the dual role of store-of-value and mean-of-payment that deposits usually serve. A key difference with respect to Bianchi and Bigio (2014) is that I consider no fractionary reserve requirement on deposits, but a settlement requirement with reserves on net deposit outflows. A key difference with respect to Piazzesi and Schneider (2016) is that I work with a continuous-time dynamic stochastic general equilibrium (DSGE) framework, which is suitable for analyzing the dynamics of macroeconomic aggregates. Drechsler, Savov and Schnabl (2015a) also work with a standard continuous-time DSGE model, but they assume that financial intermediaries fully self-insure against liquidity risks. The main difference with respect to Brunnermeier and Sannikov (2016) is that they introduce liquidity considerations based on the store-of-value property of reserves.

On methodological grounds, my model economy builds upon the works of Gertler and Karadi (2011) and Gertler and Kiyotaki (2010); Brunnermeier and Sannikov (2014); Bianchi and Bigio (2014); and Van der Gucht (2017). I adapt the Gertler and Kiyotaki (2010) economy to continuous time, to account for the non-linear dynamics associated with occasionally constrained agents. To this end, I use a continuous-time framework similar to

Brunnermeier and Sannikov (2014) and Van der Ghote (2017). I borrow from Bianchi and Bigio (2014) the market structure of the market for reserves.

2 The Model

Time is continuous and runs forever. The model is a standard Gertler and Kiyotaki (2010) economy in which financial intermediaries face also a liquidity management problem. The model adapts the Gertler and Kiyotaki (2010) economy to continuous time using a framework that is similar to Brunnermeier and Sannikov (2014) and Van der Ghote (2017).

2.1 Agents

A continuum of identical households and of financial intermediaries populate the model economy. Households are composed of a continuum of family members of unit measure. Family members share the same preferences for consumption and the same discount rate, $\rho > 0$. There is perfect consumption insurance within the household meaning that all of its family members perfectly share their income to always consume the same amount of goods, $c_t \geq 0$.

Family members serve different occupations. A fraction $f \in (0, 1)$ of them are financiers and the remaining fraction are savers. Family members switch their occupation stochastically according to Poisson processes with arrival rate $\tilde{\rho} > 0$ for financiers, and arrival rate $\tilde{\rho}f/(1-f)$ for savers.

Financiers run the financial intermediaries and savers run the household. Specifically, financiers take the investment portfolio decisions of the financial intermediary company that they manage. Savers choose consumption c_t on behalf of all the family members that constitute the household. Savers also take the investment portfolio decisions of the household that they manage.

2.2 Assets & Production Technologies

Asset classes comprise physical capital, deposit contracts, reserves and government bonds.

Physical capital Physical capital is a real asset. Physical capital is the key input for producing a final consumption good. There is a linear production technology that

transforms physical capital k_t into final output flows y_t according to

$$y_t = ak_t$$

being $a \in \{a_h, a_f\}$ the productivity coefficient. The value of the productivity coefficient depends on who manages the physical capital. If financial intermediaries manage the k_t units of physical capital, $a = a_f$. Otherwise, if households manage them, $a = a_h < a_f$. The gap between a_f and a_h measures the comparative advantage that financial intermediaries have at managing physical capital relative to households. Intuitively, the gap $a_f - a_h$ captures the idea that financial intermediaries are good relative to households at allocating funds across non-financial firms; the latter being the agents ultimately involved in the production process of the final consumption good.³

Physical capital evolves stochastically according to

$$dk_t/k_t = [I(\iota_t) - \delta] dt + \sigma dZ_t \quad (1)$$

The process Z_t is a standard Wiener process defined on a filtered probability space (Ω, \mathcal{F}, P) . The term dZ_t is an aggregate shock that is common to all units of physical capital independently of who manages them. The term dZ_t can also be interpreted as an shock aggregate to the effective units of physical capital.⁴ The diffusion coefficient σ measures the strength at which the dZ_t shock affects the growth rate of physical capital.

Physical capital depreciates deterministically at the constant rate of δ . The depreciation rate δ is common to all units of physical capital as well. There is an internal investment technology that transforms $\iota_t k_t$ units of final output into physical capital at the rate of $I(\iota_t)$. The function $I(\iota_t)$ represents a standard investment technology with adjustment costs: $I(0) = 0$, $I'(\iota) > 0$, $I''(\iota) < 0$.

The total return on capital $dR_{a,t}$ depends on who manages the physical capital. Let q_t denote the price of capital in terms of the final consumption good.⁵ The total return $dR_{a,t}$ amounts to the sum of the net dividend yields and the capital gain/loss rate. Specifically,

$$dR_{a,t} \equiv \frac{a - \iota_t}{q_t} dt + \frac{d(q_t k_t)}{q_t k_t}$$

³See Van der Ghote (2017) for a model economy that explicitly incorporates lending/borrowing relationships between financial intermediaries/households and non-financial firms.

⁴See Brunnermeier and Sannikov (2014) and di Tella (2015) for a detailed discussion.

⁵Physical capital is tradable. Physical capital is traded on the spot at the real price of q_t .

with $a \in \{a_h, a_f\}$. Net dividend yields are the ratio of net dividend returns $(a - \iota_t) k_t$ to the market value of capital holdings $q_t k_t$. Dividend returns are net of expenditures on internal investment. The capital gain/loss rate is the percentage change on the market value of capital holdings during the time interval dt .

The total return on capital that financial intermediaries earn is $dR_{a,t}$ evaluated at $a = a_f$ while that that households earn is also $dR_{a,t}$ but evaluated at $a = a_h$. Because the productivity gap $a_f - a_h$ is positive, the total return $dR_{f,t}$ that financial intermediaries obtain is higher than the total return $dR_{h,t}$ that households obtain.

We conjecture that the price of capital q_t follows an Ito process with drift process $u_{q,t}$ and diffusion process $\sigma_{q,t}$.⁶ Total returns on capital are locally risky and satisfy

$$dR_{a,t} = \left[\frac{a - \iota_t}{q_t} + I(\iota_t) - \delta + u_{q,t} + \sigma_{q,t}\sigma \right] dt + (\sigma_{q,t} + \sigma) dZ_t$$

Deposits Deposits are financial securities. Specifically, deposits are in zero-net supply; agents can issue as well as save on deposits. Deposits have a short-term maturity and a fixed payment structure: Deposits issued at time t mature at time $t + dt$ and yield a locally risk-free nominal interest rate of $i_t \geq 0$. Let p_t denote the nominal price of the final consumption good. The real return on deposit is then $i_t dt - dp_t/p_t$, being dp_t/p_t the inflation rate between time t and time $t + dt$.

Deposits serve the dual role of store-of-value and mean-of-payment. Besides being useful for borrowing and saving, deposits are also useful for settling transactions.⁷ We do not explicitly model the procedure through which depositors decide to use deposits to settle transactions. Instead, we introduce settlement decisions in reduced-form. Our approach is based on Bianchi and Bigio (2014) and Piazzesi and Schneider (2016).

Specifically, depositors settle transactions with deposits stochastically. The settlement of transactions with deposits generates in probability a situation in which net deposit positions across issuers become unbalanced (i.e. gross deposit positions differ before and after settlement for at least two issuers of deposits). For simplicity, we assume that depositors only reshuffle deposits across issuers: The aggregate stock of deposits therefore remains the same before and after settlement.

A Poisson process J_t dictates whether net deposit positions become unbalanced after settlement. The process J_t is exogenous and has an arrival rate of θ . When J_t does not

⁶The processes $\mu_{q,t}$ and $\sigma_{q,t}$ are endogenous objects to be determined in equilibrium.

⁷Only the agents who save on deposits, i.e. depositors, can use deposits to settle transactions.

jump, i.e. $J_t^+ - J_t = 0$,⁸ nothing relevant happens as net deposit positions remain perfectly balanced across issuers. When J_t jumps, i.e. $J_t^+ - J_t = 1$, net deposit positions become unbalanced and are whence reshuffled stochastically across issuers according to a continuous cumulative probability distribution F . The domain of the function $F(\omega)$ is the interval $(-\infty, 1]$. A negative realization $\omega < 0$ means that the corresponding issuer receives a share $-\omega$ of net deposit inflows. The share $-\omega$ is computed relative to the stock of deposits before settlement. A positive realization $\omega > 0$ has the same interpretation but for net deposit outflows. The function $F(\omega)$ satisfies that $\int \omega dF(\omega) = 0$, confirming that deposits are just re-allocated across issuers.⁹ We interpret a positive shock ω as a withdrawal shock on deposits.

Deposits are subject to an immediate settlement requirement. The settlement requirement obliges issuers to settle all of their net deposit outflows with reserves, immediately after they occur (i.e. immediately after J_t jumps) and immediately before time $t + dt$ arrives. The settlement requirement implies that withdrawal risks on deposits are potentially costly for issuers. Whether withdrawal risks are costly or not depends on the trading protocol of reserves and on the opportunity cost of holding reserves.¹⁰

Reserves Reserves are nominal assets in non-negative supply. A Monetary Authority issues and distributes reserves. The Monetary Authority pays no nominal interest rate on reserves and therefore the real interest rate of reserves equals the negative of the inflation rate. The Monetary Authority may nonetheless pay a positive rate of return on *excess reserves* (see below).¹¹

Reserves are tradable and are traded both in primary markets and in secondary markets at a real price of $1/p_t$. Reserves are liquid meaning that they are also traded between the moment at which J_t jumps, if any, and time $t + dt$. Reserves are the only liquid asset in the economy. Conditional on a jump $J_t^+ - J_t = 1$, the trading protocol of reserves, and the settlement protocol of net deposit outflows with reserves, are as follows.

⁸The Poisson processes J_t and J_t^+ count number of jumps. The process J_t counts the number of jumps until time t . The process J_t^+ counts the number of jumps until *immediately* after time t . Through the lens of discrete-time models, intuitively, J_t counts the number of jumps up to the morning of date t whereas J_t^+ counts the number of jumps up to the night of date t . Jumps, if any, occur at noon.

⁹The shock ω is independent of the size of the issuer.

¹⁰In our model economy, there is no fractionary reserve requirement on deposits. That is, issuers are not obliged to park a fraction of their deposits on reserves. Incorporating a fractionary reserve requirement is feasible (see Bianchi and Saki 2014 and Piazzesi and Schneider 2016). The model economy can also be accommodated to impose a settlement requirement on *gross* deposit outflows (rather than on *net* deposit outflows).

¹¹Introducing a positive nominal interest rate on reserves is feasible.

Trading and settlement protocols involve two stages. There is no discounting between stages. At the first stage, there is no trading but only settlement. Specifically, at the first stage, issuers have to settle the largest possible share of their net deposit outflows with their own reserves holdings. Let $\bar{\omega}_t \geq 0$ denote the liquidity ratio of issuers just before the jump $J_t^+ - J_t = 1$ occurs.¹² Following this first round of settlement, the reserves holdings that issuers end up with, as well as their liquidity needs, depend on the relative size of the withdrawal shock ω to that of the liquidity ratio $\bar{\omega}_t$. If $\omega > \bar{\omega}_t$, issuers lose all of their reserves holdings and end up with liquidity needs of $\omega - \bar{\omega}_t$ per unit of initial deposits. Issuers therefore will have to borrow later on $\omega - \bar{\omega}_t$ units of real reserves (per unit of initial deposits) to finish settling all of their net deposit outflows. If $\omega \in [0, \bar{\omega}_t]$, issuers lose a share $\omega/\bar{\omega}_t$ of their reserve holdings but end up without liquidity needs. Issuers indeed end up with *excess reserves* over settlement needs of $\bar{\omega}_t - \omega$ per unit of initial deposits, and can lend those excess reserves later on if they want to. If $\omega < 0$, issuers keep all of their reserves holdings and end up with liquidity excesses as well. Their excess reserves not only comprise their initial reserves holdings but also the reserves inflows that they receive jointly with the net deposit inflows $-\omega$. Reserves inflows per unit of deposits amount to $\Delta_t(\omega; \tilde{\omega}_t)$, with

$$\Delta_t(\omega; \tilde{\omega}_t) \equiv \frac{-\omega}{E_F[-\tilde{\omega}\mathbf{1}_{\tilde{\omega}<0}]} E_F[\tilde{\omega}\mathbf{1}_{\tilde{\omega}\in[0,\tilde{\omega}_t]} + \tilde{\omega}_t\mathbf{1}_{\tilde{\omega}>\tilde{\omega}_t}]$$

In this notation, $\hat{\omega}$ denotes the withdrawal shock of the other issuers in the economy and $\tilde{\omega}_t$ denotes their corresponding liquidity ratio.¹³ The expectation operator $E_F[\cdot]$ is defined relative to the cumulative probability distribution function of withdrawal shocks F . The first factor in the RHS is the share of the aggregate net deposit outflows that the issuers with withdrawal shock $\omega < 0$ receive. The second factor is the aggregate reserves holdings that the others issuers in the economy use to settle net deposit outflows.

At the second stage, after the first round of settlement concludes, a market for reserves opens. In the market for reserves, only issuers can trade; in particular, the Monetary Authority cannot trade. Issuers who have excess reserves can place lending orders to try to lend their reserves holdings in the market. The other issuers can place borrowing orders to try to borrow reserves. The market for reserves is a directed over-the-counter market

¹²The liquidity ratio is the ratio of the real reserves holdings to the quantity of deposits issued.

¹³We conjecture that all issuers set the same liquidity ratio of $\tilde{\omega}_t$. Of course, under our conjecture, $\bar{\omega}_t = \tilde{\omega}_t$.

(OTC) that has a similar structure to Bianchi and Bigio (2014).¹⁴ Besides trading in the market, issuers can also trade reserves with the Monetary Authority. Issuers who have excess reserves can park their reserves in the balance sheet of the Monetary Authority at a rate of return of $r_{e,t}$. Issuers who need reserves can borrow in the discount window at a rate of return of $r_{w,t}$. The Monetary Authority sets both the interest rate on excess reserves $r_{e,t}$ and the discount window rate $r_{w,t}$ unilaterally. The Monetary Authority commits to trade any quantity of reserves at those rates. For simplicity, we express $r_{e,t}$ and $r_{w,t}$ in real terms. We interpret the rates $r_{e,t}$ and $r_{w,t}$ as fee payments within time t (rather than as interest payment between different time periods). We restrict attention to the case in which $r_{w,t} > r_{e,t} \geq 0$, which implies that borrowing reserves at the discount window is costly and that parking reserves in the balance sheet of the Monetary Authority is weakly profitable.

The trading behavior of the Monetary Authority and the OTC market structure of the market for reserves determine the trading outcome of reserves. By trading outcome we mean the trading strategies of issuers; the terms of trade and payoffs that issuers obtain; and the market rate $r_{m,t}$.

The policy rates $r_{w,t}$ and $r_{e,t}$ represent the outside options to trading reserves in the market. The rate $r_{w,t}$ represents the outside option to borrowing reserves. The rate $r_{e,t}$ represents the outside option to lending reserves. The market for reserves admits a single rate of return $r_{m,t}$ (i.e. all trades at the market are executed at the same rate $r_{m,t}$) because bargaining takes place between reserve orders.¹⁵ Furthermore, because reserve orders Nash bargain about real transfers, the market rate $r_{m,t}$ solves

$$r_{m,t} = \arg \max_{\tilde{r} \in \mathbb{R}} (r_{w,t} - \tilde{r})^\xi (\tilde{r} - r_{e,t})^{1-\xi}$$

being $\xi \in [0, 1]$ the bargaining power of borrowing orders.

Issuers get better terms of trade in the market than with Monetary Authority because the market rate $r_{m,t}$ is a weighted average of the high discount window rate $r_{w,t}$ and the low

¹⁴Specifically, the OTC market for reserves is such that: (i) issuers can place a continuum of orders (but cannot place orders beyond their reserve needs or holdings); (ii) borrowing and lending orders are randomly matched among each other; (iii) bargain takes place between orders (not between the issuers who place the orders); (iv) orders bargain about real transfers according to Nash bargaining; and (v) orders do not bargain collectively (i.e. orders from a same issuer take as given the outcome of the other orders).

¹⁵See Afonso and Lagos (2012) and Bianchi and Bigio (2014) for a detailed discussion. The rationale is that value functions do not affect the outcome of the bargain if orders (rather than issuers) are the ones who bargain.

interest rate on excess reserves $r_{e,t}$. Therefore, issuers try to place as many successful orders of reserves in the market as possible. The terms of trade that issuers ultimately obtain are given by the effective rate of return for borrowing reserves $r_{b,t}$ and by the effective rate of return for lending reserves $r_{l,t}$. The effective rates $r_{b,t}$ and $r_{l,t}$ satisfy

$$r_{b,t} = r_{m,t} \min \left\{ \frac{\tilde{\omega}_t}{E_F [(\hat{\omega} - \tilde{\omega}_t) \mathbf{1}_{\hat{\omega} > \tilde{\omega}_t}]}, 1 \right\} + r_{w,t} \max \left\{ \frac{E_F [(\hat{\omega} - \tilde{\omega}_t) \mathbf{1}_{\hat{\omega} > \tilde{\omega}_t}] - \tilde{\omega}_t}{E_F [(\hat{\omega} - \tilde{\omega}_t) \mathbf{1}_{\hat{\omega} > \tilde{\omega}_t}]}, 0 \right\}$$

$$r_{l,t} = r_{m,t} \min \left\{ \frac{E_F [(\hat{\omega} - \tilde{\omega}_t) \mathbf{1}_{\hat{\omega} > \tilde{\omega}_t}]}{\tilde{\omega}_t}, 1 \right\} + r_{e,t} \max \left\{ \frac{\tilde{\omega}_t - E_F [(\hat{\omega} - \tilde{\omega}_t) \mathbf{1}_{\hat{\omega} > \tilde{\omega}_t}]}{\tilde{\omega}_t}, 0 \right\}$$

The effective rates $r_{b,t}$ and $r_{l,t}$ are a weighted average of the market rate $r_{m,t}$ and of the policy rates $\{r_{e,t}, r_{w,t}\}$. The weights in $r_{b,t}$ and $r_{l,t}$ reflect the likelihood of trading orders successfully in the market. Those likelihoods depend on the trading behavior of all the issuers in the economy, and therefore are taken as given by individual issuers. If issuers on aggregate have enough reserves holdings to meet all of their liquidity needs in the first round of settlement, i.e. $\tilde{\omega}_t > E_F [(\hat{\omega} - \tilde{\omega}_t) \mathbf{1}_{\hat{\omega} > \tilde{\omega}_t}]$, all borrowing orders trade successfully in the market and $r_{b,t} = r_{m,t}$. The effective lending rate $r_{l,t}$ is a weighted average of $r_{m,t}$ and $r_{e,t}$ because only the share $E_F [(\hat{\omega} - \tilde{\omega}_t) \mathbf{1}_{\hat{\omega} > \tilde{\omega}_t}] / \tilde{\omega}_t$ of lending orders trade successfully. In this case, issuers who borrow reserves obtain the best possible terms of trade given the policy rates $\{r_{e,t}, r_{w,t}\}$. Those who lend reserves obtain relatively poor terms of trade. If, alternatively, issuers on aggregate lack enough reserves holdings to meet all of their liquidity needs in the first round of settlement, i.e. $\tilde{\omega}_t < E_F [(\hat{\omega} - \tilde{\omega}_t) \mathbf{1}_{\hat{\omega} > \tilde{\omega}_t}]$, the opposite happens. Specifically, all lending orders trade successfully in the market and whence $r_{l,t} = r_{m,t}$. The effective borrowing rate $r_{b,t}$ is a weighted average of $r_{m,t}$ and $r_{w,t}$ because only the share $\tilde{\omega}_t / E_F [(\hat{\omega} - \tilde{\omega}_t) \mathbf{1}_{\hat{\omega} > \tilde{\omega}_t}]$ of borrowing orders trade successfully. In this second case, issuers who lend reserves obtain the best possible terms of trade while those who borrow reserves obtain relatively poor terms of trade.

A second and final round of settlement begins after the trade of reserves concludes. Payments related to lending/borrowing reserves in the market, and to lending/borrowing with/against the Monetary Authority, are executed simultaneously with this second round of settlement. We assume that after the second round of settlement concludes deposits are reshuffled back to the position prior to the jump $J_t^+ - J_t = 1$. We assume also that re-allocating deposits and reserves after at the second reshuffle is costless. These two assumptions simplify the analysis: Together, they imply that, at the beginning of time

$t + dt$, net deposit positions remain perfectly balanced across issuers, independently of whether J_t has jumped during the time interval dt or not.¹⁶

Government Bonds Government bonds are financial assets in perfectly elastic supply. Government bonds are short-term and pay a locally risk-free nominal interest rate. Government bonds are valid for settling transactions among depositors but not for settling net deposit outflows. The nominal interest rate on government bonds is therefore the same as the nominal deposit rate (government bonds and deposits are perfectly arbitrated). From now on, then, we refer to i_t as the nominal interest rate in the economy.

Government bonds matter only for the implementation of the nominal interest rate i_t . The Monetary Authority implements a nominal interest rate through open market operations: That is, the purchase of government bonds with reserves, and vice versa, at the prevailing market prices. Following Drechsler, Savov and Schnabl (2015a), we restrict attention to implementation mechanisms that generate a locally risk-free inflation rate

$$dp_t/p_t = \pi_t dt + 0 * dZ_t \quad (2)$$

with $\pi_t dt \equiv E_t [dp_t/p_t]$. A locally risk-free inflation rate is consistent with non-financial firms that adjust their nominal price sluggishly according to Calvo (1983) pricing.¹⁷ In subsection 2.6, we derive the law of motions for government bonds and for the aggregate stock of reserves that are consistent with a locally risk-free inflation rate.

We assume for simplicity that neither financial intermediaries nor households can take positions government bonds. We assume also that the real interest rate payments on government bonds are financed with lump-sum taxes on households.

2.3 Financial Intermediaries' Portfolio Problem

Based on the spread $dR_{f,t} - dR_{h,t} > 0$, we conjecture that financial intermediaries borrow to take levered positions on physical capital and that households lend. Let $n_{f,t} \geq 0$ denote the net worth of financial intermediaries. The capital positions that financial intermediaries take $k_{f,t}$ therefore satisfies $q_t k_{f,t} > n_{f,t}$ generically.¹⁸

The investment portfolio decisions of financial intermediaries comprise $\{k_{f,t}, m_{f,t}/p_t; \iota_t\}$.

¹⁶The two simplifying assumptions are consistent with idea that withdrawal shocks on deposits are mere funding/liquidity shocks to financial intermediaries.

¹⁷See Van der Ghote (2017) for a formal proof and a comprehensive discussion.

¹⁸The relationship $q_t k_{f,t} = n_{f,t}$ holds only when financial intermediaries own all of the wealth in the economy.

The process $m_{f,t}/p_t \geq 0$ is the real position that financial intermediaries take on reserves. The investment portfolio decisions of financial intermediaries dictate the evolution of net worth $n_{f,t}$ accordingly

$$\begin{aligned} dn_{f,t} = & dR_{f,t} q_t k_{f,t} - \pi_t m_{f,t}/p_t dt - (i_t - \pi_t) (q_t k_{f,t} + m_{f,t}/p_t - n_{f,t}) dt \\ & - \tau_{1,t} n_{f,t} dt + [R_t(\omega; \bar{\omega}_t) * (q_t k_{f,t} + m_{f,t}/p_t - n_{f,t}) + \tau_{2,t}(\omega) n_{f,t}] * (J_t^+ - J_t) \end{aligned} \quad (3)$$

The first line in the RHS describes the real return of the investment portfolio. The first line assumes that withdrawal shocks on deposits do not materialize, namely, that $J_t^+ - J_t = 0$. The quantity $q_t k_{f,t} + m_{f,t}/p_t - n_{f,t}$ is the amount of deposits that financial intermediaries issue. The first term in the second line of the RHS is a tax rate on financial intermediaries. The tax rate $\tau_{1,t}$ is proportional on net worth.

The second term in the second line of the RHS describes the gains/losses in net worth $n_{f,t}$ associated with withdrawal risks on deposits. The function $R_t(\omega; \bar{\omega}_t)$ measures the returns per unit of initial deposits from trading reserves ex-post. Ex-post means following the realizations of the jump $J_t^+ - J_t = 1$ and of the withdrawal shocks on deposits ω . The return $R_t(\omega; \bar{\omega}_t)$ is (see subsection 2.2)

$$R_t(\omega; \bar{\omega}_t) \equiv r_{l,t} [\bar{\omega}_t + \Delta_t(\omega; \tilde{\omega}_t)] \mathbf{1}_{\omega < 0} + r_{l,t} (\bar{\omega}_t - \omega) \mathbf{1}_{\omega \in [0, \bar{\omega}_t]} - r_{b,t} (\omega - \bar{\omega}_t) \mathbf{1}_{\omega > \bar{\omega}_t} \quad (4)$$

The liquidity ratio of financial intermediaries $\bar{\omega}_t$ is

$$\bar{\omega}_t \equiv \frac{m_{f,t}/p_t}{q_t k_{f,t} + m_{f,t}/p_t - n_{f,t}} \quad (5)$$

The total return from trading reserves ex-post is the product between $R_t(\omega; \bar{\omega}_t)$ and the quantity of deposits $q_t k_{f,t} + m_{f,t}/p_t - n_{f,t}$. The process $\tau_{2,t}(\omega) n_{f,t}$ denotes a transfer that financial intermediaries receive ex-post. The transfers $\tau_{2,t}(\omega)$ are proportional to net worth and contingent upon the realization of the idiosyncratic withdrawal shock ω .

Financial intermediaries take the tax rate $\tau_{1,t}$ and the transfers $\tau_{2,t}(\omega)$ as given. The policy schedule $\{\tau_{1,t}, \tau_{2,t}(\omega)\}$ serves only a technical purpose. The purpose of the transfers $\tau_{2,t}(\omega)$ is to guarantee that endogenous variables do not jump along with J_t .¹⁹ The purpose of the tax rate $\tau_{1,t}$ is to guarantee that the policy schedule $\{\tau_{1,t}, \tau_{2,t}(\omega)\}$ is self-financing. In

¹⁹From a technical point of view, the analysis is simpler when endogenous variables do not jump. In the conclusion, we discuss the main economic implications that may follow if endogenous variables were allowed to jump along with J_t .

subsection 2.6, we derive the policy schedule $\{\tau_{1,t}, \tau_{2,t}(\omega)\}$ that satisfies the aforementioned two properties. Intuitively, endogenous variables do not jump if the transfers $\tau_{2,t}(\omega)$ perfectly insure financial intermediaries against withdrawal risks on deposits. In such case, the net worth $n_{f,t}$ does not jump along with J_t . Because the jump $J_t^+ - J_t$ affects directly the evolution of $n_{f,t}$ only, none any other endogenous variable jumps along with J_t either.

Financial intermediaries are subject to a moral hazard problem. The moral hazard problem is similar to Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). At every point in time, financial intermediaries have to choose between diverting funds and behaving properly. If they divert funds, financial intermediaries can walk away with a fraction $1/\lambda$ of their capital holdings $k_{f,t}$ at the penalty of losing their franchise.²⁰ The value of diverting funds is therefore $q_t k_{f,t}/\lambda$. If they behave properly, financial intermediaries stay on business, and obtain the franchise value of their financial intermediary company V_t . The value of behaving properly is therefore V_t .

The moral hazard problem gives rise to the incentive-compatible constraint

$$q_t k_{f,t} \leq \lambda V_t \tag{6}$$

The incentive-compatible constraint guarantees that financial intermediaries never divert funds: Intuitively, if financial intermediaries were to divert funds, creditors would not lend in the first place. The incentive-compatible constraint guarantees also that deposits are de facto non-defaultable. Non-defaultable deposits are consistent with a notion of using deposits for settling transactions.

The objective of financial intermediaries is to maximize the present discounted value of their dividend payouts. Financial intermediaries pay out dividends either when they divert funds or when they retire. In either case, financial intermediaries transfer back to the household their entire net worth. Financial intermediaries retire when the financier in office switches her occupation. When they retire, financial intermediaries close their business after the settlement of net deposit outflows concludes. Financial intermediaries that retire are replaced with new financial intermediaries, that start with an initial endowment of $\kappa/\tilde{\rho}$ shares of the aggregate capital stock.

Financial intermediaries discount future dividend payouts with the stochastic discount

²⁰We assume for simplicity that financial intermediaries cannot walk away with reserves. Relaxing such assumption is feasible. See Gertler and Kiyotaki (2010) for a model economy in which safe assets in general also have a pledgeability ratio below 1.

factor (SDF) of the household, i.e. Λ_t ,²¹ weighted by the survival density function of the financier in office (financial intermediaries never divert funds). We conjecture that the SDF of the household Λ_t follows an Ito process (see subsection 2.4).

Financial intermediaries solve the portfolio problem

$$V_t \equiv \max_{k_{f,t}, m_{f,t}; u_t \geq 0} E_t \int_t^\infty \tilde{\rho} e^{-\tilde{\rho}(s-t)} \frac{\Lambda_s}{\Lambda_t} n_{f,s} ds \quad (7)$$

s.t. : $n_{f,t} \geq 0$, (3), (4), (5), (6)

Solving the Portfolio Problem We conjecture that the franchise value of financial intermediaries is linear on net worth

$$V_t = v_t n_{f,t}$$

with $v_t \geq 1$.²² A linear guess works here because the portfolio problem (7) is linear. Intuitively, problem (7) is linear because the objective function of financial intermediaries is linear, and because financial intermediaries take assets returns and prices as given (constraint functions are therefore linear as well). The process v_t is the Tobin's Q of financial intermediaries: Equivalently, v_t measures the marginal/average value of wealth in the financial intermediary sector. The process v_t is never below 1 because the Tobin's Q of an hypothetical financial intermediary that can invest only in deposits is always equal to 1.²³ We conjecture that v_t follows an Ito process.

Let $\phi_{.,t}$ denote the portfolio shares of financial intermediaries. Let $\phi_{k,t} \equiv q_t k_{f,t} / n_{f,t}$ denote the portfolio share in physical capital and let $\phi_{m,t} \equiv (m_{f,t} / p_t) / n_{f,t}$ denote the portfolio share in reserves. The incentive-compatible constraint reduces to the standard financing constraint

$$\phi_{k,t} \leq \lambda v_t$$

The leverage multiple amounts to $\phi_{k,t} + \phi_{m,t}$. The portfolio shares satisfy that $\phi_{k,t} > 1$ generically and that $\phi_{m,t} \geq 0$. The liquidity ratio is the same for all financial intermediaries. Specifically,

$$\bar{\omega}_t = \frac{\phi_{m,t}}{\phi_{k,t} + \phi_{m,t} - 1}$$

²¹The process Λ_t is an endogenous process to be determined in equilibrium.

²²The process v_t is an endogenous object to be determined in equilibrium.

²³See subsections 2.4 and 2.6. The reason is that households are indifferent on the margin between consumption and deposits.

Proposition 1 *The optimality conditions in the portfolio problem of financial intermediaries are four:*

1. *The internal investment condition*

$$I'(\iota_t) = 1/q_t$$

2. *The liquidity ratio condition*

$$-i_t dt + E_F [R_{m,t}(\omega; \bar{\omega}_t)] \theta dt \leq 0$$

with equality if $\bar{\omega}_t > 0$.

3. *The asset pricing condition on capital*

$$E_t [dR_{f,t}] + E_F [R_{k,t}(\omega; \bar{\omega}_t)] \theta dt - (i_t - \pi_t) dt + Cov_t [d\Lambda_t/\Lambda_t + dv_t/v_t, dR_{f,t}] \geq 0$$

with equality if $\phi_{k,t} < \lambda v_t$.

4. *The asset pricing condition on the Tobin's Q*

$$\tilde{E}_t [dR_{n,t}] + \frac{\tilde{\rho}}{v_t} dt + E_t [dv_t/v_t] - \tilde{\rho} dt + Cov_t [d\Lambda_t/\Lambda_t, dv_t/v_t] = 0$$

with

$$\tilde{E}_t [dR_{n,t}] \equiv \underbrace{E_t [dn_{f,t}/n_{f,t}] - (i_t - \pi_t) dt + Cov_t [d\Lambda_t/\Lambda_t + dv_t/v_t, dn_{f,t}/n_{f,t}]}_{\text{risk-adjusted excess return on equity}}$$

Proof.²⁴ ■

The internal investment condition is the same as in Brunnermeier and Sannikov (2014). The internal investment condition implies that ι_t is positively related with the price of capital q_t . Intuitively, when the price of capital is higher, financial intermediaries invest more, because physical capital is more valuable. The internal investment rate solves a completely static problem: the internal investment rate ι_t only depends on the spot price q_t .

The liquidity ratio condition weights the opportunity cost against the marginal benefits of holding reserves. The opportunity cost of holding reserves is the sum of the foregone real

²⁴See the Appendix A.

interest rate on deposits and the inflation rate: By the Fisher equation, it equals the nominal interest rate $i_t dt$. The marginal benefits of holding reserves are $E_F [R_{m,t}(\omega; \bar{\omega}_t)] \theta dt$, with

$$R_{m,t}(\omega; \bar{\omega}_t) \equiv R_t(\omega; \bar{\omega}_t) + [r_{l,t} \mathbf{1}_{\omega < \bar{\omega}_t} + r_{b,t} \mathbf{1}_{\omega > \bar{\omega}_t}] (1 - \bar{\omega}_t)$$

The factor θdt weights the marginal benefits by the likelihood of a jump during the time interval dt . The expectation operator $E_F [\cdot]$ weights the marginal benefits by the probability distribution F of the withdrawal shocks on deposits. The function $R_{m,t}(\omega; \bar{\omega}_t)$ is the partial derivative of the total return $R_t(\omega; \bar{\omega}_t) * (\phi_{k,t} + \phi_{m,t} - 1)$ with respect to $\phi_{m,t}$. The function $R_{m,t}(\omega; \bar{\omega}_t)$ reads as follows.

Marginally increasing the portfolio share in reserves $\phi_{m,t}$ brings a direct and an indirect effect. The direct effect is the consequence of raising more deposits: Increasing $\phi_{m,t}$ requires raising more deposits because financial intermediaries have already exhausted all of their net worth in capital positions (i.e. $\phi_{k,t} > 1$). The first term in the RHS accounts for the direct effect. The indirect effect is the consequence of increasing the liquidity ratio $\bar{\omega}_t$. Increasing the liquidity ratio boosts the return $R_t(\omega; \bar{\omega}_t)$ because it alleviates liquidity needs as well as improves liquidity excesses ex-post. For instance, if $\omega > \bar{\omega}_t$, and financial intermediaries happen to be borrowers of reserves, increasing the liquidity ratio saves $r_{b,t}$ per unit of deposits. Similarly, if $\omega < \bar{\omega}_t$, and financial intermediaries happen to be lenders of reserves, increasing the liquidity ratio yields an additional rate of return of $r_{l,t}$ per unit of deposits. The second term in the RHS accounts for indirect effect. The factor $1 - \bar{\omega}_t$ weights by the partial effect that $\phi_{m,t}$ has on $\bar{\omega}_t$ and by the deposits position $\phi_{k,t} + \phi_{m,t} - 1$.

The marginal benefits from holding and whence trading reserves ex-post $R_{m,t}(\omega; \bar{\omega}_t)$ is always non-negative. Furthermore, the marginal benefits $R_{m,t}(\omega; \bar{\omega}_t)$ are fully determined by the liquidity ratio. The liquidity ratio condition therefore implies that financial intermediaries set a positive liquidity ratio $\bar{\omega}_t$, if any, to match the expected marginal benefits from trading reserves ex-post to the opportunity cost of holding reserves. If $i_t dt > E_F [R_{m,t}(\omega; \bar{\omega}_t)] \theta dt$ for all $\bar{\omega}_t \in [0, 1]$, financial intermediaries prefer to set a null liquidity ratio $\bar{\omega}_t = 0$ and to hold no reserves.

The asset pricing conditions on capital and on the Tobin's Q are similar to Van der Gote (2017). The only difference with respect to them is the term $E_F [R_{k,t}(\omega; \bar{\omega}_t)] \theta dt$.

The asset pricing condition on capital is similar as well to the optimality conditions in the consumption-based asset capital model (C-CAPM). The LHS is the risk-adjusted excess return on capital over deposits that financial intermediaries earn. The first three

terms add up to the excess return on capital. The second term is unusual and results from the withdrawal risks on deposits. The function $R_{k,t}(\omega; \bar{\omega}_t)$ is the partial derivative of the total return $R_t(\omega; \bar{\omega}_t) * (\phi_{k,t} + \phi_{m,t} - 1)$ with respect to $\phi_{k,t}$. Specifically,

$$R_{k,t}(\omega; \bar{\omega}_t) \equiv R_t(\omega; \bar{\omega}_t) - [r_{l,t} \mathbf{1}_{\omega < \bar{\omega}_t} + r_{b,t} \mathbf{1}_{\omega > \bar{\omega}_t}] \bar{\omega}_t$$

The function $R_{k,t}(\omega; \bar{\omega}_t)$ has a similar interpretation to $R_{m,t}(\omega; \bar{\omega}_t)$ but for marginal increases on $\phi_{k,t}$. The term $E_F [R_{k,t}(\omega; \bar{\omega}_t)] \theta dt$ measures the extent up to which marginally increasing $\phi_{k,t}$ affects the expected returns from trading reserves ex-post. The function $R_{k,t}(\omega; \bar{\omega}_t)$ depends only on the liquidity ratio and is strictly increasing on $\bar{\omega}_t$. The second property in turn implies that higher liquidity ratios boost the excess return that financial intermediaries earn on capital. Intuitively, excess returns are higher when $\bar{\omega}_t$ is higher because financial intermediaries are less exposed to withdrawal risks.

The last term in the LHS, i.e. the covariance $Cov_t [d\Lambda_t/\Lambda_t + dv_t/v_t, dR_{f,t}]$, measures the compensation for holding capital risk that financial intermediaries demand. The comovement between dv_t/v_t and $dR_{f,t}$ matters for valuing capital risk because financial intermediaries are subject to financing constraints: Financial intermediaries are particularly concerned with the co-movement between their marginal value of wealth and the return on their capital investments.

The asset pricing condition on capital describes the preference relationship of financial intermediaries between physical capital and deposits. If the risk-adjusted excess return that they obtain on capital is positive, financial intermediaries strictly prefer capital to deposits, and take levered positions on capital until hitting their financing constraint. Otherwise, financial intermediaries are indifferent between capital and deposits, and are willing to take any position on physical capital and on deposits.

The asset pricing condition on the Tobin's Q describes the behavior of the marginal value of wealth of financial intermediaries. The first term in the LHS is the risk-adjusted excess return on equity over deposits that financial intermediaries earn. The term $\tilde{E}_t [dR_{n,t}]$ measures the expected profits that financial intermediaries earn per unit of net worth: Notice that $\tilde{E}_t [dR_{n,t}]$ is the product of the risk-adjusted excess return on capital and the portfolio share in physical capital $\phi_{k,t}$. The asset pricing condition on the Tobin's Q says that financial intermediaries value wealth more, when they obtain positive and higher risk-adjusted excess returns on capital.

2.4 Households' Portfolio Problem

Let $n_{h,t} \geq 0$ denote the net worth of households. The portfolio investment decisions of households comprise $\{k_{h,t}, m_{h,t}/p_t = 0; \iota_t\}$. Households take no positions on reserves because deposits weakly dominate reserves in terms of return: The opportunity cost of holding reserves is always non-negative, $i_t \geq 0$.

The consumption and portfolio investment decisions of households, together with the transfers that they receive, dictate the evolution of the worth $n_{h,t}$ accordingly

$$dn_{h,t} = dR_{h,t}q_tk_{h,t} + (i_t - \pi_t)(n_{h,t} - q_tk_{h,t})dt - c_tdt + Transfers_t \quad (8)$$

The quantity $n_{h,t} - q_tk_{h,t}$ is the amount of deposits that households hold. The process $Transfers_t$ denotes the transfers that households receive from financial intermediaries and from the Monetary Authority. The net transfers that households receive from financial intermediaries amount to $(\tilde{\rho}N_{f,t} - \kappa q_tK_t)dt$. In this notation, upper case variables denote aggregate variables. We specify the transfers that households receive from the Monetary Authority in subsection 2.5. For convenience, we automatically deduct from the latter the lump-sum taxes that households pay to finance the interest rate payments on government bonds.

The objective of households is to maximize the present discounted value of their utility flows

$$E_t \int_t^\infty e^{-\rho(s-t)} \frac{c_s^{1-\gamma}}{1-\gamma} ds \quad (9)$$

The parameter γ is the risk aversion coefficient. The SDF of households is $\Lambda_t \equiv e^{-\rho t} c_t^{-\gamma}$.

Solving the Portfolio Problem The portfolio problem of households is to maximize (9) subject to $n_{h,t} \geq 0$ and (8).

Proposition 2 *The optimality conditions in the portfolio problem of households are three:*

1. *The internal investment condition*

$$I'(\iota_t) = 1/q_t$$

2. *The asset pricing condition on deposits*

$$(i_t - \pi_t)dt = -E_t[d\Lambda_t/\Lambda_t] \equiv \rho dt + \gamma E_t[dc_t/c_t] - \frac{1}{2}\gamma(\gamma+1)Var_t[dc_t/c_t]$$

3. The asset pricing condition on capital

$$E_t [dR_{h,t}] - (i_t - \pi_t) dt + Cov_t [d\Lambda_t/\Lambda_t, dR_{h,t}] \leq 0$$

with equality if $\bar{k}_{h,t} > 0$.

Proof.²⁵ ■

The internal investment condition is the same for both households and financial intermediaries. The reason is that both agents face the same static investment optimization problem. The internal investment rate ι_t is then independent of who manages the capital stock.

The asset pricing conditions are similar to the optimality conditions in the C-CAPM. The asset pricing condition on deposits implies that households are indifferent on the margin between consumption and deposits. The asset pricing condition on capital describes the preference relationship of households between physical capital and deposits. The term in the LHS is the risk-adjusted excess return on capital over deposits that households earn. If the risk-adjusted excess return is null, households are indifferent on the margin between capital and deposits. Otherwise, households strictly prefer on the margin deposits to capital, and $\bar{k}_{h,t} = 0$.²⁶

2.5 Monetary Policy and Seigniorage Revenues

2.5.1 Monetary Policy

Monetary policy has three independent policy instruments. We characterize policy instruments in terms of rates of return. Monetary policy comprises a process for the nominal interest rate i_t ; a process for the discount window rate $r_{w,t}$; and a process for the interest rate on excess reserves $r_{e,t}$. Policy instruments generate seigniorage revenues and/or expenditures.

²⁵See the Appendix A.

²⁶We disregard the possibility of the opposite inequality, i.e. " $>$ ", because households are not subject to portfolio constraints. If the opposite inequality were to hold, households would take unbounded positions of physical capital, funded with unbounded negative position on deposits, and, in equilibrium, markets would not clear (see subsection 2.6).

2.5.2 Seigniorage Revenues

Open Market Operations The Monetary Authority conducts open market operations to implement a process for the nominal interest rate. Open market operations generate seigniorage revenues because government bonds dominate reserves in terms of return. We assume that the Monetary Authority transfers to the household all seigniorage revenues from open market operations on the spot.

To conduct open market operations, the Monetary Authority issues reserves, and whence borrows at the real rate of return of $-\pi_t dt$, to invest in government bonds at a real rate of return of $(i_t - \pi_t) dt$. Gross seigniorage revenues from open market operations therefore amount to $i_t M_t / p_t dt$.²⁷ The corresponding net seigniorage revenues amount $\pi_t M_t / p_t dt$. The reason is that the Monetary Authority deducts from the gross seigniorage revenues the lump-sum taxes that households pay to finance the interest rate payments $(i_t - \pi_t) G_t dt$ on government bonds.

Discount Window and Excess Reserves The Monetary Authority trades reserves with financial intermediaries to sustain the rates $r_{w,t}$ and $r_{e,t}$. Lending reserves in the discount window generates seigniorage revenues. Paying interest rates on excess reserves generates seigniorage expenditures. We assume that the Monetary Authority transfers to the household on the spot all net seigniorage revenues from trading reserves with financial intermediaries. Net seigniorage revenues from trading reserves amount to

$$[(r_{w,t} - r_{e,t}) \mathbf{1}\{E_F[(\omega - \bar{\omega}_t) \mathbf{1}_{\omega > \bar{\omega}_t}] > \bar{\omega}_t\} + r_{e,t}] * [E_F[(\omega - \bar{\omega}_t) \mathbf{1}_{\omega > \bar{\omega}_t}] - \bar{\omega}_t]$$

per unit of aggregate deposits. If the financial intermediary system is illiquid, i.e. $E_F[(\omega - \bar{\omega}_t) \mathbf{1}_{\omega > \bar{\omega}_t}] > \bar{\omega}_t$, financial intermediaries on aggregate lack enough reserves holdings to meet all of their liquidity needs. Financial intermediaries then have to borrow the remanent reserves $E_F[(\omega - \bar{\omega}_t) \mathbf{1}_{\omega > \bar{\omega}_t}] - \bar{\omega}_t$ in discount window, and net seigniorage revenues therefore amount to $r_{w,t} * [E_F[(\omega - \bar{\omega}_t) \mathbf{1}_{\omega > \bar{\omega}_t}] - \bar{\omega}_t]$ per unit of aggregate deposits. If the financial intermediary system is liquid, i.e. $E_F[(\omega - \bar{\omega}_t) \mathbf{1}_{\omega > \bar{\omega}_t}] < \bar{\omega}_t$, the opposite happens: Financial intermediaries on aggregate have enough reserves holdings to meet all of their liquidity needs, and whence park the remanent reserves $\bar{\omega}_t - E_F[(\omega - \bar{\omega}_t) \mathbf{1}_{\omega > \bar{\omega}_t}]$ in the balance sheet of the Monetary Authority. In this second case, net seigniorage expenditures amount to $r_{e,t} * [\bar{\omega}_t - E_F[(\omega - \bar{\omega}_t) \mathbf{1}_{\omega > \bar{\omega}_t}]]$ per unit of aggregate deposits.

²⁷The aggregate stock of real reserves M_t/p_t equals the position that the Monetary Authority takes on government bonds G_t . The reason is that all seigniorage revenues are paid on the spot.

2.6 Competitive Equilibrium

We restriction attention to a competitive equilibrium in which inflation is locally risk-free and in which endogenous variables do not jump.

Let $\eta_t \equiv N_{f,t}/q_t K_t$ denote the wealth share of financial intermediaries. Let $\varphi_t \equiv (M_t/p_t)/q_t K_t$ denote the wealth share of reserves. The total wealth in the economy is $q_t K_t$ because physical capital is the only real asset.

Definition A competitive equilibrium is a set of stochastic processes adapted to the filtration generated by Z : the price of capital $\{q_t\}$; the nominal price $\{p_t\}$; the inflation rate $\{\pi_t\}$; consumption $\{C_t\}$; the capital position of households $\{K_{h,t}\}$; the portfolio share in physical capital $\{\phi_{k,t}\}$; the portfolio share in reserves $\{\phi_{m,t}\}$; the liquidity ratio $\{\bar{\omega}_t\}$; the wealth share of financial intermediaries $\{\eta_t\}$; the Tobin's Q of financial intermediaries $\{v_t\}$; the internal investment rate $\{\iota_t\}$; the aggregate capital stock $\{K_t\}$; the aggregate stock of nominal reserves $\{M_t\}$; the wealth share of reserves $\{\varphi_t\}$; the discount window rate $\{r_{w,t}\}$; the interest rate on excess reserves $\{r_{e,t}\}$; the market rate $\{r_{m,t}\}$ and the policy schedule $\{\tau_{1,t}, \tau_{2,t}(\omega)\}$; such that:

1. Optimality conditions

- (a) $\{\phi_{k,t}, \phi_{m,t}, \bar{\omega}_t; \iota_t\}$ solve the problem of financial intermediaries
- (b) $\{C_t, K_{h,t}; \iota_t\}$ solve the problem of households
- (c) $\{r_{m,t}\}$ solves the Nash Bargaining problem

2. Market clearing conditions

- (a) The market for the consumption good clears

$$C_t + \iota_t K_t = [a_f * \phi_{k,t} \eta_t + a_h * (1 - \phi_{k,t} \eta_t)] K_t$$

- (b) The market for capital holdings clears

$$K_{h,t}/K_t = 1 - \phi_{k,t} \eta_t$$

- (c) The market for real reserve holdings clears

$$\phi_{m,t} \eta_t = \varphi_t$$

(d) The OTC market for reserves clears

3. Conditions for law of motions

The nominal price p_t evolves according to (2) and the aggregate stock of physical capital K_t evolves according to (1).

4. Condition for locally risk-free inflation rate

The aggregate stock of nominal reserves M_t evolves accordingly

$$\frac{dM_t}{M_t} = \pi_t dt + \frac{d(\phi_{m,t}\eta_t)}{\phi_{m,t}\eta_t} + \frac{d(q_t K_t)}{q_t K_t} + \frac{d(\phi_{m,t}\eta_t)}{\phi_{m,t}\eta_t} \frac{d(q_t K_t)}{q_t K_t}$$

5. No-jump condition

The policy schedule $\{\tau_{1,t}, \tau_{2,t}(\omega)\}$ is

$$\tau_{1,t} \equiv E_F [-R_t(\omega; \bar{\omega}_t) * (\phi_{k,t} + \phi_{m,t} - 1)] \theta$$

$$\tau_{2,t}(\omega) \equiv -R_t(\omega; \bar{\omega}_t) * (\phi_{k,t} + \phi_{m,t} - 1)$$

The market clearing condition for the consumption good guarantees that consumption and investment equal output. The process $\phi_{k,t}\eta_t$ in the RHS is the share of the aggregate capital stock that financial intermediaries manage. The market clearing conditions for asset holdings guarantee that all asset markets clear: The market for deposits automatically clears due to Walras Law. The OTC market clearing condition for reserves determines the effective borrowing rate $r_{b,t}$ and the effective lending rate $r_{l,t}$ in equilibrium. Notice that in equilibrium $\tilde{\omega}_t = \bar{\omega}_t$.

The law of motion for M_t describes the evolution of nominal reserves that implements the locally risk-free inflation rate $dp_t/p_t = \pi_t dt$. The law of motion for M_t follows from applying Ito's Lemma to both sides of the market clearing condition for real reserves holdings. The RHS describes the evolution of the aggregate demand of nominal reserves. The LHS has the same interpretation but for the aggregate supply of nominal reserves.

The transfers $\tau_{2,t}(\omega)$ in condition (5) perfectly insure financial intermediaries against withdrawal risks on deposits. The tax rate $\tau_{1,t}$ in condition (5) guarantees that the policy

schedule $\{\tau_{1,t}, \tau_{2,t}(\omega)\}$ is self-financing. We assume that $\{\tau_{1,t}, \tau_{2,t}(\omega)\}$ can be financed by borrowing and lending at an exogenous real interest rate of $\check{\rho}$, with $\check{\rho} \rightarrow 0$. In the limit with $\check{\rho} \rightarrow 0$, the tax rate $\tau_{1,t}$ amounts to the expected value of the transfers $\tau_{2,t}(\omega)$.

Markov Competitive Equilibrium We conjecture that a Markov equilibrium exists. We conjecture furthermore that the state variables of the Markov equilibrium are $\{\eta, K\}$. The level of the aggregate capital stock K_t is irrelevant because the equilibrium outcome is scale invariant with respect to K_t .²⁸ The dynamics of K_t is nonetheless relevant because the evolution of the aggregate capital stock influences the growth rate of the economy (see section 4.1). Both the level and the dynamics of the wealth share of financial intermediaries η_t are relevant (see section 4.1).

The Markov equilibrium adds a consistency condition to the conditions of the competitive equilibrium. Additionally, the Markov equilibrium requires η_t to evolve in accord with the conditions of the competitive equilibrium.

3 Parameter Values and Functional Forms

Table 1 below describes the parameter values and the functional forms that we use in our numerical analysis. The time frequency is annual.

²⁸Namely, the endogenous variables in the equilibrium are either linear on, or independent of, K_t .

Parameter Values and Functional Forms

Parameter Values / Functional Forms		Value	Target
Panel A: Technology			
a_f	Productivity of fin. intermediaries	2%	Ratio of output to capital stock
a_h	Productivity of households	1.3%	Average Sharpe ratio
σ	Fundamental risk	3.5%	Volatility of utilization-adj. TFP
$I(\iota)$	Internal investment technology	$Sqrt(\iota)$	Literature
δ	Depreciation rate of physical capital	1%	Literature
Panel B: Financial Intermediation			
λ	Fraction of divertable assets	2	Average leverage multiple
$\tilde{\rho}$	Arrival rate of retirement shock	10%	Average survival frequency
κ	Initial capital endowment	1.5%	Average wealth-to-capital ratio
Panel C: Settlement			
θ	Arrival rate of withdrawal shocks	5	Average liquidity ratio
$F(\omega)$	Distribution of withdrawal shocks	<i>Logistic</i>	Literature
ξ	Nash bargaining power	0.5	Corridor system
Panel D: Preferences			
ρ	Time discount rate	2%	Literature
γ	Risk aversion coefficient	1	Literature

Panel A: Technology

The functional form for the internal investment technology $I(\iota)$ is similar to Brunnermeier and Sannikov (2014). Specifically, $I(\iota)$ is

$$I(\iota) \equiv \frac{1}{B} \left(\sqrt{A^2 + 2B\iota} - A \right)$$

Notice that $I(\iota)$ has quadratic adjustment costs.

We choose A , B and a_f to match aggregate quantities and prices in the frictionless economy. The frictionless economy is such that there are no financing constraints nor withdrawal risks on deposits (i.e. $1/\lambda, 1/\theta \rightarrow 0$).²⁹ In the frictionless economy, the productivity coefficient a_f equals the ratio of aggregate output to aggregate capital Y_t/K_t (financial

²⁹We assume that in the frictionless economy there is no inflation either. Namely, $\pi_t = 0$.

intermediaries always manage all of the aggregate capital stock). We set a value of a_f of 2% which is standard. In the frictionless economy, additionally, the internal investment rate attains its efficient value ι_E , with

$$\iota_E \equiv \frac{q_E^2 - A^2}{2B}$$

The price of capital, and the real interest rate, attain their efficient value q_E , and r_E , with³⁰

$$q_E \equiv B * \left[\sqrt{\rho^2 + 2a_f/B + (A/B)^2} - \rho \right]$$

$$r_E \equiv \rho + \left(\frac{q_E - A}{B} - \delta \right) \gamma - \frac{1}{2} \gamma (\gamma + 1) \sigma^2$$

We set A and B to jointly target a ratio of investment to output ι_E/a_f of 20% and a real interest rate r_E of 4.05%. A rate $r_E = 4.05\%$ yields an annual real interest rate of 2%.

The depreciation rate of physical capital δ is set equal to 1% which is standard. The coefficient that measures fundamental risk, i.e. σ , is set equal to 3.5%. A value $\sigma = 3.5\%$ matches the unconditional standard deviation of the Utilization-Adjusted Series on Total Factor Productivity (see [Basu, Fernald and Kimball 2006](#); [Basu, Fernald, Fisher and Kimball 2006](#); and [Fernald 2014](#)). Using such TFP series is consistent with interpreting dZ_t as shocks to the effective units of physical capital.

We look at the original economy with financing constraints and withdrawal risks to assign numerical values to the remaining parameters in the model. In the original economy, the spread between a_f and a_h affects that between $dR_{f,t}$ and $dR_{h,t}$, and, whence, the fluctuations on the return on capital in equilibrium. We set a_h equal to 1.3% to target an average Sharpe ratio of 30%. [Van der Gote \(2017\)](#) follows a similar methodology.

Panel B: Financial Intermediation

We set the fraction of divertable assets λ equal to 2 to target an average leverage multiple of 3.5. The arrival rate of the retirement shock of financiers, i.e. $\tilde{\rho}$, targets an average survival frequency of financial intermediaries of 10 years. The initial capital endowment κ targets the average wealth-to-capital ratio in the financial intermediary sector. Using Fed fund data, we estimate an average wealth-to-capital ratio of 20%. This estimate is consistent with [Hirakata, Sudo and Ueda \(2013\)](#). The value of κ is 1.5%.

³⁰The efficient price of capital q_E solves the asset pricing condition on capital for financial intermediaries. See the Appendix B for further details.

Panel C: Settlement

The cumulative probability distribution function of withdrawal shocks $F(\omega)$ is similar to Bianchi and Bigio (2014). Using individual US commercial banks Call Reports, Bianchi and Bigio (2014) construct an empirical distribution of withdrawal shocks, and obtain that the corresponding empirical distribution fits a logistic distribution $F(\omega; \mu, s)$ with location $\mu = -0.0029$ and scale $s = 0.022$. We set the location μ equal to 0 to guarantee that $\int \omega dF(\omega; \mu, s) = 0$. Additionally, we accommodate the support of the logistic distribution to the open interval $(-\infty, 1)$.

We set the arrival rate θ of the Poisson process J_t to target an average liquidity ratio of 40%. The value of θ is 5.

We set a Nash Bargaining coefficient ξ of 0.5. A value $\xi = 0.5$ is consistent with a corridor system of reserves: An even bargaining power between lending and borrowing orders of reserves implies a market rate of $r_{m,t} = (r_{w,t} + r_{e,t})/2$.

Panel D: Preferences

We set a time discount rate ρ of 2% which is standard. The risk aversion coefficient γ is equal to 1 to obtain log-preferences for consumption.

4 Liquidity Management and Leverage Decisions

To examine the interactions between liquidity management and leverage decisions we take monetary policy as given. We consider a monetary policy that is similar to the pre-Global Financial Crisis of 2008 era. Specifically, monetary policy follows a strict-inflation-targeting rule; charges a positive rate in the discount window; and pays no interest rate on excess reserves. We set a target of 2% for annual inflation and a discount window rate of 5%. We study first the economy in which financial intermediaries arbitrary choose to hold no reserves, i.e. $\bar{\omega}_t = \phi_{m,t} = 0$. We interpret this first economy in which there is no liquidity as a benchmark of comparison for the more interesting economy in which liquidity management decisions are endogenous.

4.1 Benchmark Economy without Liquidity

If $\bar{\omega}_t = \phi_{m,t} = 0$, the equilibrium outcome has two well-demarcated regions (Figure 1). The underlying difference between these two regions concerns the size of the borrowing capacity of financial intermediaries. In a first region, financial intermediaries have a low borrowing

capacity relative to the total wealth in the economy. Specifically, financial intermediaries, on aggregate, lack enough borrowing capacity to absorb all of the aggregate capital stock, i.e. $\lambda v_t \eta_t < 1$. In the second region, the opposite happens: Financial intermediaries have a high borrowing capacity relative to the total wealth in the economy, and, on aggregate, they have a enough borrowing capacity to absorb all of the aggregate capital stock, i.e. $\lambda v_t \eta_t \geq 1$.

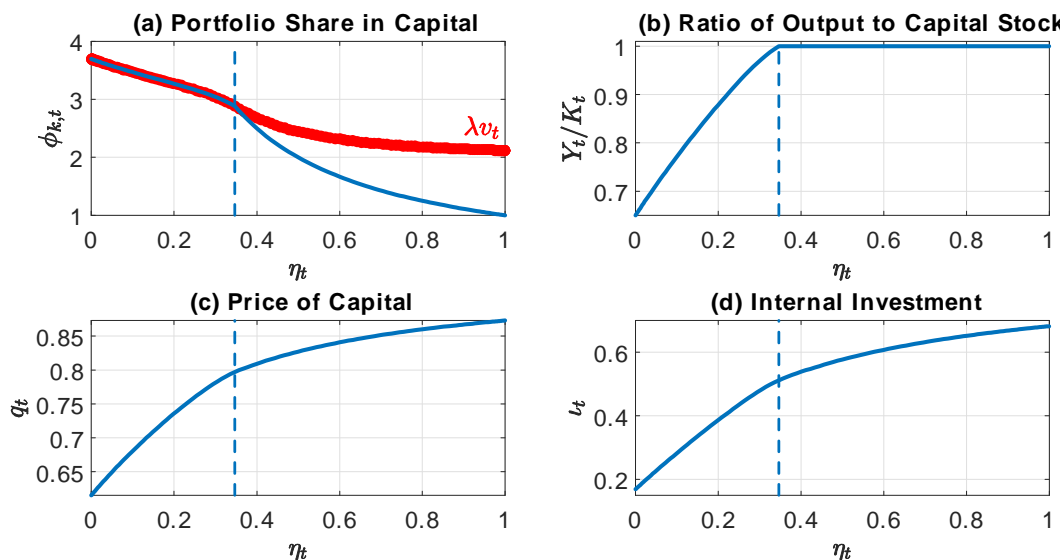


FIGURE 1: In Figures 1b, 1c, and 1d the dependent variable is deflated by its corresponding value in the frictionless economy.

When $\lambda v_t \eta_t < 1$, in equilibrium, both financial intermediaries and households hold physical capital (otherwise, the market for capital would not clear). Households are the marginal investors on capital because they are not subject to financing constraints.

The price of capital reflects the low valuation that households have for physical capital relative to financial intermediaries (figure 1c, LHS of dotted line). The price of capital attains a value considerably below its efficient value of q_E , because otherwise households would earn low net dividend yields $(a_h - \iota_t)/q_t$ as well as negative risk-adjusted excess returns. Financial intermediaries borrow until hitting their financing constraint (figure 1a, LHS of dotted line). Financial intermediaries take the largest possible levered position on capital, $\phi_{k,t} = \lambda v_t$, because they earn net high dividend yields $(a_f - \iota_t)/q_t$ as well as high

and positive risk-adjusted excess returns. The ratio $Y_t/K_t \equiv \phi_{k,t}\eta_t a_f + (1 - \phi_{k,t}\eta_t) a_h$ attains a value below its efficient level of a_f (figure 1b, LHS of dotted line) because the allocation of physical capital is inefficient. The internal investment rate ι_t also attains a value below its efficient level of ι_E but because $q_t \ll q_E$.

When $\lambda v_t \eta_t \geq 1$, in equilibrium, financial intermediaries manage all of the aggregate capital stock, and households take no positions on capital. Financial intermediaries borrow until clearing the market for capital and whence $\phi_{k,t} = 1/\eta_t \leq \lambda v_t$ (figure 1a, RHS of dotted line). Financial intermediaries become the marginal investors on capital because financing constraints are slack (financial intermediaries cannot borrow until hitting their financing constraint because otherwise the market for capital would not clear).

The price of capital reflects the high valuation that financial intermediaries have for physical capital relative to households (figure 1c, RHS of dotted line). Households refrain themselves from holding capital because they earn low net dividend yields as well as negative risk-adjusted excess returns. The ratio Y_t/K_t attains its efficient level of a_f (figure 1b, RHS of dotted line). The internal investment rate takes a value closer to its efficient value of ι_E (figure 1d, RHS of dotted line).

The dynamics of the equilibrium outcome differ between the two regions as well. To study dynamic behaviors we decouple time series into a trend variable and a cyclical variable. The cyclical variable measures the fluctuations of the time series around their corresponding trend variable.

Dynamics in general are governed by the law of motions of the state variables K_t and η_t . The law of motion of the aggregate capital stock K_t dictates the evolution of the trend variables. The reason is that the equilibrium outcome is scale invariant with respect to K_t . Because all endogenous variables that exhibit a positive trend have indeed a common trend (i.e. all of them are linear on K_t), the law of motion of K_t also dictates the evolution of the trend rate of economic growth. The law of motion of the wealth share of financial intermediaries η_t dictates the evolution of the cyclical variables. Endogenous variables in general exhibit different cyclical behaviors because their corresponding cyclical variables have a different mapping with respect to η_t .

The trend rate of economic growth evolves accordingly

$$dK_t/K_t = [I(\iota_t) - \delta] dt + \sigma dZ_t$$

The trend rate evolves stochastically. The expected trend rate is endogenous and depends on the position of η_t . Specifically, when the wealth share of financial intermediaries η_t is low, the expected trend rate is low as well, because the price of capital is depressed, and the internal investment rate ι_t is low. The opposite happens when the wealth share of financial intermediaries η_t is high.

The wealth share of financial intermediaries η_t evolves according to

$$d\eta_t/\eta_t = \mu_{\eta,t}dt + \sigma_{\eta,t}dZ_t$$

with

$$\begin{aligned} \mu_{\eta,t}dt &= \frac{a_f - \iota_t}{q_t}dt + (\phi_{k,t} - 1) [E_t [dR_{f,t}] + E_F [R_t(\omega; \bar{\omega}_t)] \theta dt - (i_t - \pi) dt - Var_t [dR_{f,t}]] \\ &\quad - \phi_{m,t} [i_t - E_F [R_t(\omega; \bar{\omega}_t)] \theta] dt + \left(\frac{\kappa}{\eta_t} - \tilde{\rho} \right) dt \end{aligned}$$

$$\sigma_{\eta,t}dt = (\phi_{k,t} - 1) (\sigma_{q,t} + \sigma) dt$$

The wealth share of financial intermediaries also evolves stochastically. The drift process $\mu_{\eta,t}$ measures the expected growth rate of η_t . The first three terms in $\mu_{\eta,t}$ add up to the risk-adjusted excess return on the investment portfolio of financial intermediaries over the total wealth in the economy. The first term accounts for the excess return on internal financing, namely, on capital positions funded with net worth. The second term accounts for the risk-adjusted excess return on external financing, i.e. on capital positions funded with deposits. The third term is the effective return from holding reserves.³¹ The second and third terms include the policy schedule $\{\tau_{1,t}, \tau_{2,t}(\omega)\}$ that financial intermediaries face in equilibrium (see subsection 2.6). The expectation $E_F [R_t(\omega; \bar{\omega}_t)] \theta dt$ in the second term deducts the costs of withdrawal risks that follow from the deposits positions that finance the levered capital positions $\phi_{k,t} - 1$. The same expectation $E_F [R_t(\omega; \bar{\omega}_t)] \theta dt$, but now in the third term, incorporates the benefits of holding reserves into the effective return of reserves. The last term in $\mu_{\eta,t}$ is the net transfers that financial intermediaries payout to households (see subsection 2.3).

The diffusion process $\sigma_{\eta,t}$ measures the volatility of the wealth share of financial intermediaries. The process $\sigma_{\eta,t}$ is the product between the levered capital positions $\phi_{k,t} - 1$

³¹In this particular economy without liquidity, the third term is null, as $\phi_{m,t} = 0$.

and the aggregate risk $\sigma_{q,t} + \sigma$. Aggregate risk amounts to the volatility of the growth rate of the total wealth in the economy, i.e. $Var_t [d(q_t K_t) / q_t K_t] = (\sigma_{q,t} + \sigma)^2 dt$.

Figures 2a and 2b illustrate the dynamic behavior of η_t by isolating the behaviors of $\mu_{\eta,t}\eta_t$ and of $\sigma_{\eta,t}\eta_t$.

The bottom line from Figure 2a is that the wealth share of financial intermediaries η_t is mean-reverting. The mean at which η_t reverts is the stochastic steady state, i.e. the state at which $\mu_{\eta,t} = 0$, (the red dotted line in Figure 2a).³² The bottom line from Figure 2b is that fluctuations on η_t gets amplified through endogenous fluctuations on the price of capital q_t . The red line in Figure 2b plots the process $\sigma_{\eta,t}\eta_t$ assuming that the price of capital is constant, i.e. $\sigma_{q,t} = 0$. The spread between the blue and the red lines is $(\phi_{k,t} - 1) \sigma_{q,t}$.³³

The processes $\mu_{\eta,t}\eta_t$ and $\sigma_{\eta,t}\eta_t$ together shape the invariant distribution of the wealth share of financial intermediaries (see the Appendix). The invariant distribution of η_t shows that the regions $\{\lambda v_t \eta_t < 1\}$ and $\{\lambda v_t \eta_t \geq 1\}$ occur frequently often in equilibrium (Figure 2c). Additionally, the invariant distribution of η_t , together with the processes $\mu_{\eta,t}\eta_t$ and $\sigma_{\eta,t}\eta_t$, show that the economy recurrently transitions from episodes in which financial intermediaries are undercapitalized, and $\{\lambda v_t \eta_t < 1\}$, to episodes in which financial intermediaries are well-capitalized, and $\{\lambda v_t \eta_t \geq 1\}$, and vice versa.

The invariant distribution of Y_t/K_t results from dynamic behavior of the wealth share of financial intermediaries (Figure 2d). The ratio Y_t/K_t is pro-cyclical as well as mean-reverting. The ratio Y_t/K_t is pro-cyclical because Y_t/K_t is positively related to η_t (Figure 1d).

³²The process η_t mean-reverts because the first three terms in $\mu_{\eta,t}$ are inversely related to the marginal valuation for capital of the marginal investor (Van der Ghote 2017).

³³The endogenous fluctuations on the price of capital result from the combination of financing constraints and deposit contracts (Van der Ghote 2017). These fluctuations are larger when financing constraints bind, and financial intermediaries have large aggregate effects, because the typical pecuniary externality of economies with incomplete financial markets and occasionally binding financing constraints is present in our model economy as well (Lorenzoni 2008, Jeanne and Korinek 2010, Bianchi 2011, Bianchi and Mendoza 2012, Van der Ghote 2017).

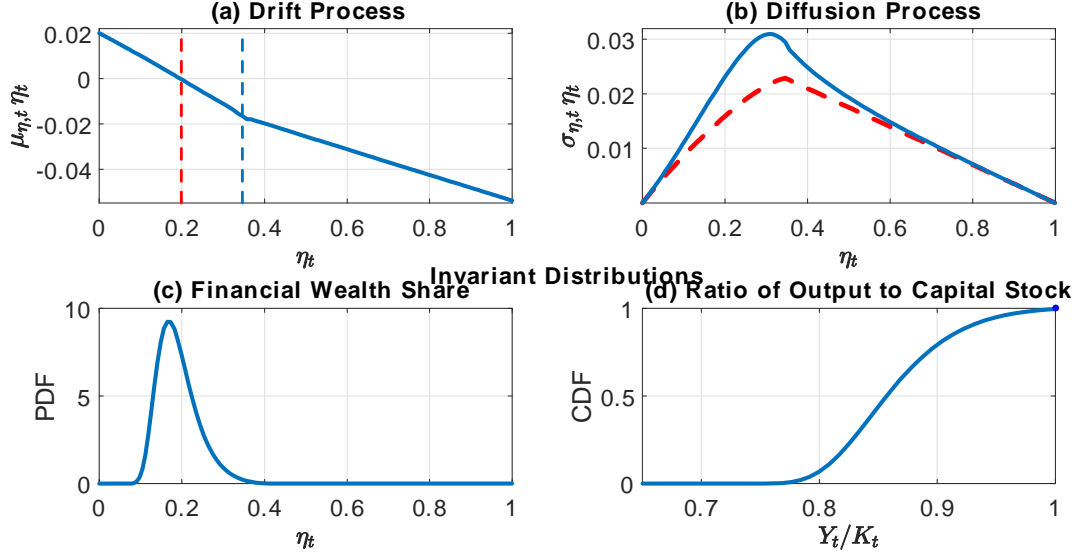


FIGURE 2

4.2 Economy with Liquidity

Regions in the equilibrium outcome remain the same independently of liquidity management decisions. Specifically, in equilibrium, independently of the processes for $\phi_{m,t}$ and $\bar{\omega}_t$, there are always two well-demarcated regions (i.e. the regions $\{\lambda v_t \eta_t < 1\}$ and $\{\lambda v_t \eta_t \geq 1\}$) that share the same features described for the benchmark economy without liquidity (see subsection 4.1). The behavior of the equilibrium outcome nonetheless in general depends on $\bar{\omega}_t$ and $\phi_{m,t}$.

Figure 3b shows how to determine the liquidity ratio $\bar{\omega}_t$ in equilibrium. Figure 3b plots the responses of $i_t dt$ and $E_F [R_{m,t}(\omega; \bar{\omega}_t)] \theta dt$ to a one-shot deviation of the liquidity ratio $\bar{\omega}_t$ from the process that it follows in equilibrium. Figure 3b takes the state η_t as given. Figure 3b shows that a one-shot deviation generates a small effect on the opportunity cost of holding reserves, $i_t dt$, but a large effect on the marginal benefits of holding reserves, $E_F [R_{m,t}(\omega; \bar{\omega}_t)] \theta dt$. The effect on $i_t dt$ is small because the liquidity ratio does not affect aggregate consumption directly but only indirectly through its effect on the evolution of net worth $n_{f,t}$. The effect on $E_F [R_{m,t}(\omega; \bar{\omega}_t)] \theta dt$ is large because $R_{m,t}(\omega; \bar{\omega}_t)$ depends directly on the liquidity ratio (see subsection 2.3).

The response functions in Figure 3b pin down the value of the liquidity ratio in equi-

librium. In equilibrium, if the liquidity ratio is positive, the intersection between $i_t dt$ and $E_F [R_{m,t}(\omega; \bar{\omega}_t)] \theta dt$ determines the value of $\bar{\omega}_t$. Otherwise, if the liquidity ratio is null, $i_t dt$ has to be weakly greater than $E_F [R_{m,t}(\omega; \bar{\omega}_t)] \theta dt$ for any given $\bar{\omega}_t \in [0, 1]$. The slope of $E_F [R_{m,t}(\omega; \bar{\omega}_t)]$ in Figure 3b determines the sign of the co-movement between $\bar{\omega}_t$ and i_t . The slope of $E_F [R_{m,t}(\omega; \bar{\omega}_t)]$ is negative because the effective lending rate $r_{l,t}$ and the effective borrowing rate $r_{b,t}$ are inversely related to the liquidity ratio $\bar{\omega}_t$ (see subsection 2.2 and Figure 3a).³⁴ The slope of $E_F [R_{m,t}(\omega; \bar{\omega}_t)]$ implies that the liquidity ratio and the nominal interest rate negatively co-move in equilibrium. Intuitively, in equilibrium, financial intermediaries reduce their leverage ratios, and whence holds less reserves, when the opportunity costs of holding reserves is high.

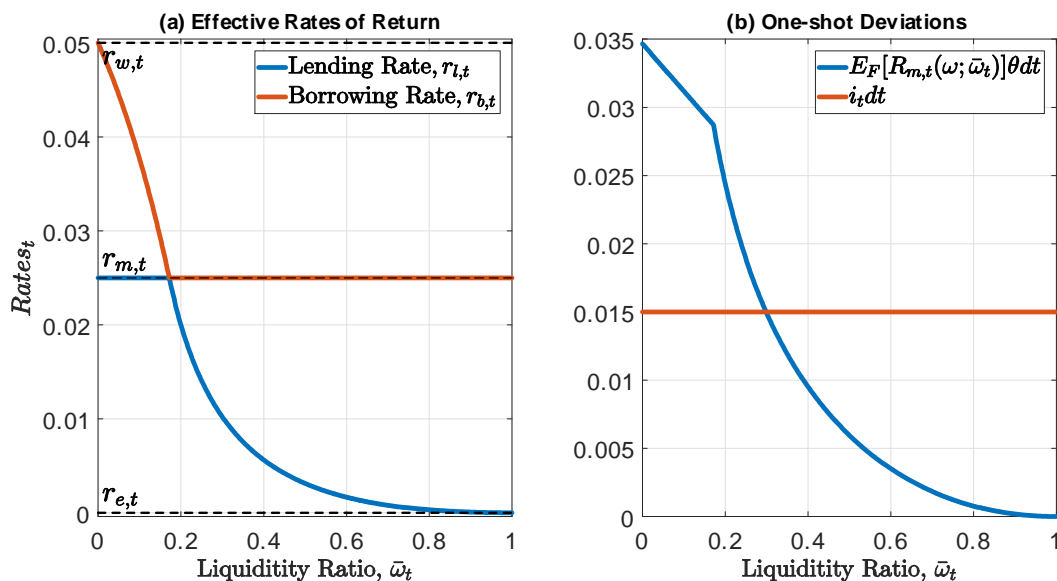


FIGURE 3

Figure 4 plots the equilibrium outcome in the economy in which liquidity management decisions are endogenous.

Figure 4b plots the liquidity ratio $\bar{\omega}_t$ as a function of the state η_t . Figure 4b shows that liquidity ratio is counter-cyclical. The liquidity ratio in Figure 4b is counter-cyclical because the real interest rate, and whence the opportunity costs of holding reserves, are

³⁴Figure 3a plots the expressions for $r_{l,t}$ and $r_{b,t}$ in subsection 2.2 setting $\bar{\omega}_t = \bar{\omega}_t$.

pro-cyclical.³⁵

Figure 4a plots the portfolio positions in capital $\phi_{k,t}$ as a function of the state η_t . The purpose of Figure 4a is to contrast the behavior of $\phi_{k,t}$ between the economies with, and without, liquidity management decisions. Figure 4a shows that $\phi_{k,t}$ is higher in the economy in which liquidity management decisions are endogenous. Figure 4a shows also that $\phi_{k,t}$ is higher when the state η_t is low, i.e. $\eta_t < \bar{\eta}$, and financing constraints bind, and that the threshold state $\bar{\eta}$ shifts rightward.

The results in Figure 4a follow from the positive effects that optimal liquidity ratios have on the borrowing capacity λv_t . Optimal liquidity ratios provide a better hedge against withdrawal risk on deposits. Optimal liquidity ratios therefore reduce the costs of leverage $(\phi_{k,t} - 1) E_F [R_t(\omega; \bar{\omega}_t)] \theta dt$, that result from the withdrawal risks on the deposits positions that finance the levered capital positions $\phi_{k,t} - 1$. Lower costs of leverage boosts the profitability in financial intermediation as well as the Tobin's Q of financial intermediaries v_t . Agency problems in financial markets then relax and the borrowing capacity of financial intermediaries expands.

Figures 4c and 4d plot invariant distributions. Figure 4c shows that the economy with endogenous liquidity management decisions spends more time around states η_t in which financial intermediaries are well-capitalized. Figure 4d show that it also spends more time around states in which the ratio Y_t/K_t is higher.

The results in Figures 4c and 4d also follow from the positive effects that optimal liquidity ratios have on the dynamics of net worth $n_{f,t}$. Optimal liquidity ratios boost the profitability in financial intermediation and whence improve the risk-adjusted excess return on the investment portfolio of financial intermediaries over the total wealth in the economy.

³⁵The cyclical behavior of the real interest rate depends on the cyclical behaviors of Y_t/K_t and of u_t . Two counteracting forces shape the cyclical behavior of the real interest rate. On the one hand, a pro-cyclical ratio Y_t/K_t pushes for a counter-cyclical real interest rate. On the other hand, a pro-cyclical investment rate u_t , together with a pro-cyclical expected trend rate $I(\iota_t) - \delta$, pushes for a pro-cyclical real interest rate. The second effect in general dominates.

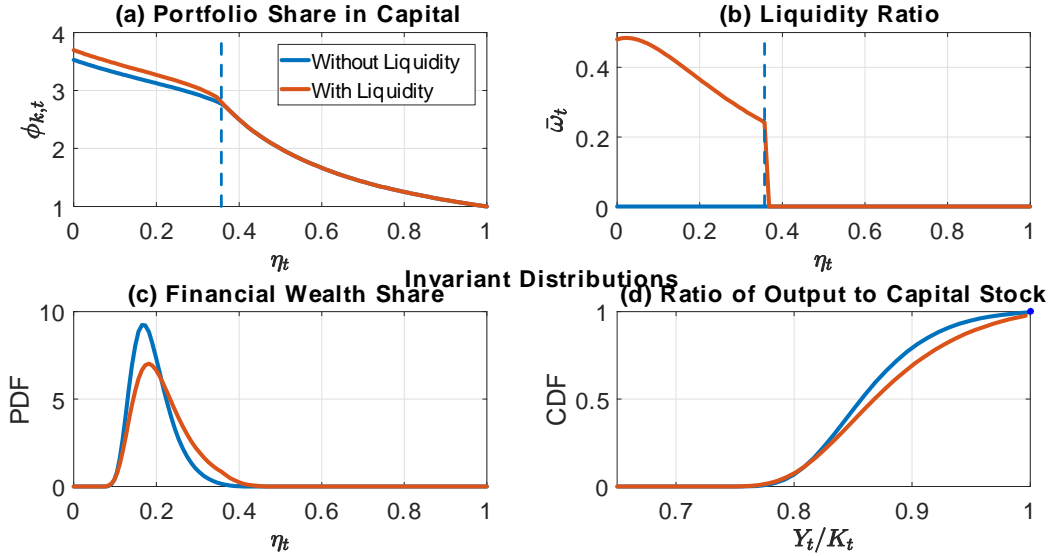


FIGURE 4

5 Policy Experiments

We conduct two simple policy experiments to examine the real effects of monetary policy. In the first experiment, the target of inflation falls from 2% to 1%. In the second experiment, the width of the corridor (i.e. the spread between the discount window rate and the interest rate on excess reserves) falls by 0.5% leaving the market rate constant. The second experiment explores the real effects of the policy that is traditionally known as narrowing the corridor.

5.1 Real Effects of Lower Inflation Targets

A reduction in the target of inflation reduces the opportunity cost of holding reserves. The reason is that a lower inflation rate boosts the real rate of return of reserves.

Financial intermediaries respond to a reduction in the target of inflation by increasing their liquidity ratio (Figure 5b). Financial intermediaries increase their liquidity ratio relatively more when they are undercapitalized, i.e. when $\lambda v_t \eta_t < 1$, and financing constraints bind, because in that region financial intermediaries take relatively larger levered positions

on physical capital, and whence are relatively more exposed to withdrawal risks on deposits (Figure 5a).

The reduction in the target of inflation, combined with the subsequent liquidity responses of financial intermediaries, expand the borrowing capacity λv_t . The reason is that a lower opportunity cost of holding reserves, along with a lower exposure to withdrawal risks, boost the profitability of financial intermediation. The Tobin's Q of financial intermediaries v_t therefore increases and agency problems in financial markets relax. The main consequence of expanding the borrowing capacity λv_t is that financial intermediaries take larger levered positions on physical capital when financing constraints bind (Figure 5a).

The reduction in the target of inflation affects also the dynamic behavior of the equilibrium outcome. Specifically, when the target of inflation falls, the economy spends more time in states in which financial intermediaries are well-capitalized and the ratio Y_t/K_t is high (Figures 5c and 5d).

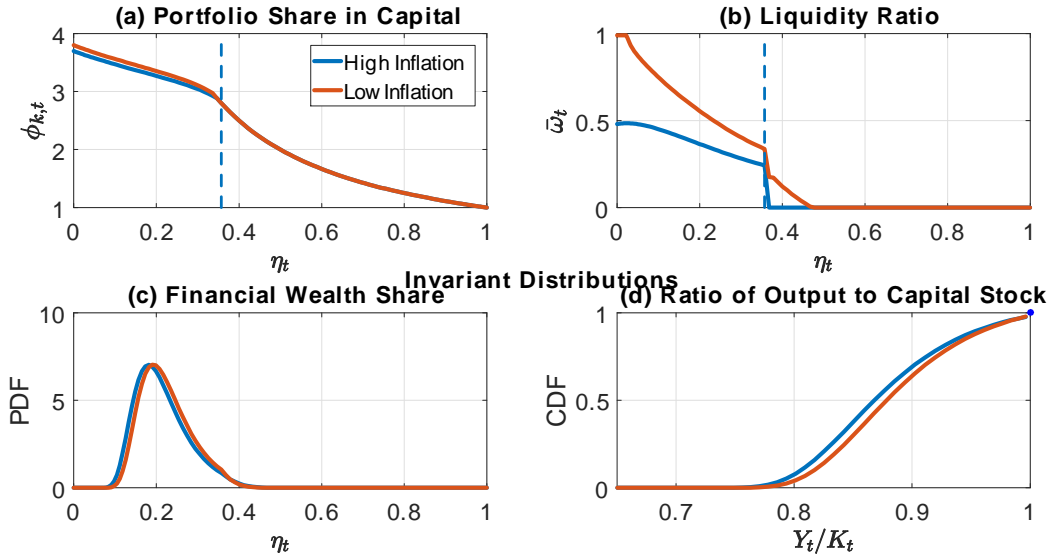


FIGURE 5

5.2 Real Effects of Narrower Corridors

A narrower corridor improves the terms of trade of trading reserves with the Monetary Authority. The reason is that a narrower corridor reduces the discount window rate $r_{w,t}$

as well as increases the interest rate on excess reserves $r_{e,t}$, to keep the market rate $r_{m,t}$ constant.

The liquidity response of financial intermediaries to a narrower corridor depends on how liquid the financial intermediary system initially is (Figure 6). If the financial intermediary system is liquid, meaning the economy spends most of its time in the region in which $\bar{\omega}_t > E_F[(\omega - \bar{\omega}_t) \mathbf{1}_{\omega > \bar{\omega}_t}]$, financial intermediaries increase their liquidity ratio $\bar{\omega}_t$ as well as their portfolio share in reserves $\phi_{m,t}$ (Figure 6a). Otherwise, financial intermediaries reduce their liquidity ratio $\bar{\omega}_t$ and $\phi_{m,t}$ (Figure 6b).

If the financial intermediary system is liquid, financial intermediaries almost never borrow reserves in the discount window. The reason is that the financial intermediary system almost always has enough reserves to meet all of its aggregate liquidity needs (see subsection 2.6). Narrowing the corridor therefore improves the terms of trade of lenders of reserves while keeping constant those of borrowers of reserves. The natural response of financial intermediaries is to increase their liquidity ratio and their liquidity positions.

If the financial intermediary system is illiquid, (i.e. the economy spends most of its time in the region in which $\bar{\omega}_t < E_F[(\omega - \bar{\omega}_t) \mathbf{1}_{\omega > \bar{\omega}_t}]$) the opposite happens. In this second case, financial intermediaries almost never park excess reserves in the balance sheet of the Monetary Authority because the financial intermediary system almost never has enough reserves to meet all of its aggregate liquidity needs. Narrowing the corridor then improves the terms of trade of borrowers of reserves while keeping constant those of lenders of reserves. The natural response of financial intermediaries is to reduce their liquidity ratio and their liquidity positions.

How liquid the financial intermediary system initially is in equilibrium is endogenous and depends on monetary policy. For a given width of corridor, the ratio of the target of inflation π to the market rate $r_{m,t}$ determines the liquidity status of the financial intermediary system in equilibrium. If π is low relative to $r_{m,t}$, the opportunity cost of holding reserves is low relative to its benefits. As a consequence, liquidity ratios are high, and the financial intermediary system is highly liquid. If π is high relative to $r_{m,t}$, the opposite results happens.

For any given initial liquidity status of the financial intermediary, narrowing the corridor reinforces such status (Figure 6 again). Specifically, if initially the financial intermediary system is liquid, a narrower the corridor system boosts liquidity ratios $\bar{\omega}_t$ and $\phi_{m,t}$ as well as the share of time that the economy spends in the liquid region of $\{\bar{\omega}_t > E_F[(\omega - \bar{\omega}_t) \mathbf{1}_{\omega > \bar{\omega}_t}]\}$. If initially the financial intermediary system is illiquid, the op-

posite happens: A narrower the corridor system reduces liquidity ratios $\bar{\omega}_t$ and $\phi_{m,t}$ as well as the share of time that the economy spends in the illiquid region of $\{\bar{\omega}_t < E_F[(\omega - \bar{\omega}_t) \mathbf{1}_{\omega > \bar{\omega}_t}]\}$.

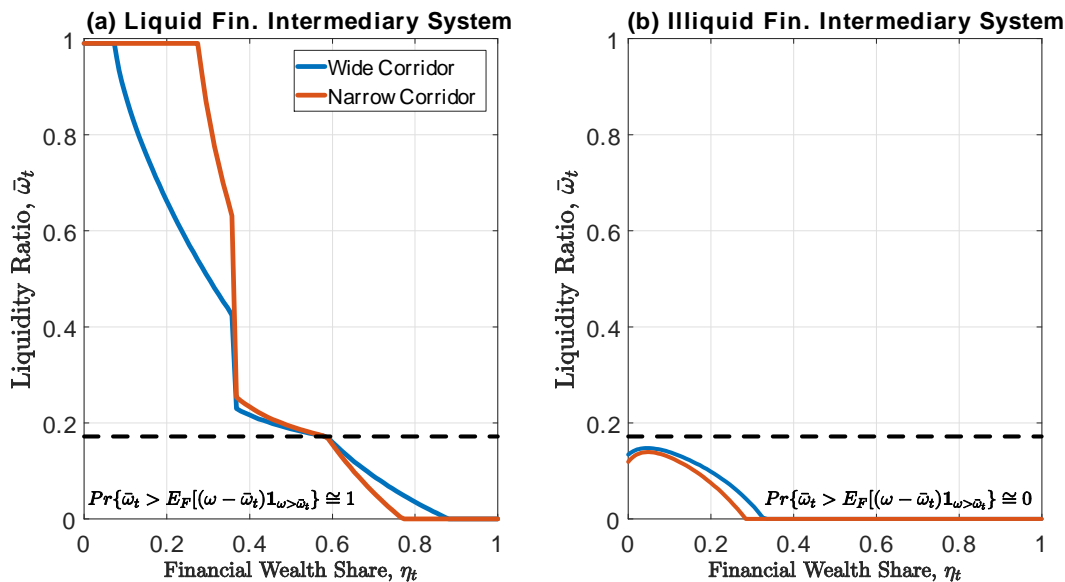


FIGURE 6

The responses of the other endogenous variables to a narrower corridor are independent of the initial liquidity status of the financial intermediary system. The other variables respond mainly to the fact that a narrower corridor improves the terms of trade of trading with the Monetary Authority.

The portfolio share of physical capital $\phi_{k,t}$ always increase, because better terms of trade of borrowing reserves against, and of lending excess reserves to, the Monetary Authority (along with the subsequent liquidity response of financial intermediaries) reduce the costs of leverage associated with withdrawal risks on deposits. The portfolio share $\phi_{k,t}$ increase relatively more when financial intermediaries are undercapitalized, and financing constraints bind, because in that region financial intermediaries are relatively more exposed to withdrawal risks. The economy spends more time in regions in which financial intermediaries are well-capitalized and the ratio Y_t/K_t is high, because financial intermediation because more profitable when leverage costs fall.

6 Conclusion

The recent Global Financial Crisis of 2008 has underscored the importance of the financial system in the transmission mechanism of monetary policy. This paper proposes a framework in which complementarities in the provisions of settlement and of financial intermediary services matter for assessing the real effects of monetary policy. A key insight that follows is that interest-rate policies such as policies on the nominal interest rate, on the discount window rate, and on interest rate on excess reserves, affect the interplay between liquidity management and leverage decisions at the financial intermediary level and whence the course of the real economy.

The analysis conducted in this paper can be extended in two notable directions. A first direction consists in marrying this paper with Van der Ghote (2017). The resulting marriage would yield a new framework in which the interplay between liquidity management and leverage decisions interacts with sluggish price adjustments. The second direction consists in allowing endogenous variables to jump along with the Poisson process J_t . This second extension would relate to the bank runs framework of Gertler and Kiyotaki (2015).

References

- [1] Afonso, Gara and Ricardo Lagos, 2012, Trade Dynamics in the Market for Federal Funds, *Econometrica* 83, No. 1, January 2015, 263–313
- [2] Bagehot, Walter, 1873, Lombard Street: A Description of the Money Market, *Wiley*
- [3] Bernanke, Ben and Alan Blinder, 1988, Credit, Money, and Aggregate Demand, *American Economic Review* 78, 435–439
- [4] Bianchi, Javier and Saki Bigio, 2014, Banks, Liquidity Management and Monetary Policy, *Working paper*
- [5] Brunnermeier, Markus, and Lasse Heje Pedersen, 2009, Market liquidity and funding liquidity, *Review of Financial studies* 22, 2201–2238
- [6] Brunnermeier, Markus, and Yuliy Sannikov, 2014, A macroeconomic model with a financial sector, *American Economic Review* 104, 379–421
- [7] Brunnermeier, Markus, and Yuliy Sannikov, 2016, The I theory of money, *Working paper*
- [8] Cipolla, Carlo, 1956, Money, Prices and Civilization in the Mediterranean World, Fifth to Seventeenth Century, *New York: Gordian Press*
- [9] Di Tella, Sebastian, 2014, Uncertainty Shocks and Balance Sheet Recessions, *Working Paper*
- [10] Drechsler, Itamar, Alexi Savov, and Philipp Schnabl, 2015a, A Model of Monetary Policy and Risk Premia, *Working paper*
- [11] Drechsler, Itamar, Alexi Savov, and Philipp Schnabl, 2015b, The deposits channel of monetary policy, *Working paper*
- [12] Gertler, Mark and Peter Karadi, 2011, A Model of Unconventional Monetary Policy, *Journal of Monetary Economics* 58, 17–34
- [13] Gertler, Mark, and Nobuhiro Kiyotaki, 2010, Financial intermediation and credit policy in business cycle analysis, in Benjamin M. Friedman, and Michael Woodford, eds., *Handbook of Monetary Economics, volume 3 of Handbook of Monetary Economics*, chapter 11, 547–599 (Elsevier)

- [14] Gertler, Mark, and Nobuhiro Kiyotaki, 2015, Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy, *American Economic Review* 105, 2011-43
- [15] Gurley, John and Edward Shaw, 1964, *Money in a Theory of Finance*, *Brookings Institution*
- [16] Hanson, Samuel, Andrei Shleifer, Jeremy C Stein, and Robert W Vishny, 2015, Banks as Patient Fixed-Income Investors, *Journal of Financial Economics*, Forthcoming
- [17] Holmström, Bengt, and Jean Tirole, 1998, Private and public supply of liquidity, *Journal of Political Economy* 106, 1–40
- [18] Kahn, Charles, and William Roberds, 2009, Why pay? An introduction to payments economics, *Journal of Financial Intermediation*, 18, Issue 1, January 2009, Pages 1–23
- [19] Piazzesi, Monika, and Martin Schneider, 2016, Payments, Credit and Asset Prices, *Working paper*
- [20] Roberds, William, 2008, Payment systems, *The New Palgrave Dictionary of Economics, Second Edition*. Eds. Steven N. Durlauf and Lawrence E. Blume, *Palgrave Macmillan*
- [21] Stein, Jeremy C, 2012, Monetary policy as financial stability regulation, *The Quarterly Journal of Economics* 127, 57–95
- [22] Van der Ghote, Alejandro, 2017, Coordinating Monetary and Financial Regulatory Policies, *Working paper*

Appendixes

Let $B_{j,t}$ with $j = 1, 2$ denote Ito processes with drift process $\mu_{B_j,t}$ and diffusion process $\sigma_{B_j,t}$. In the Appendixes, we use the expressions $E_t [dB_{j,t}/B_{j,t}]$ and $\mu_{B_j,t}dt$ interchangeably. We also use interchangeably the expressions $Cov_t [dB_{1,t}/B_{1,t}, B_{2,t}/B_{2,t}]$ and $\sigma_{B_{1,t}}\sigma_{B_{2,t}}dt$.

Appendix A

In the Appendix A, we derive the optimality conditions. Firstly, we derive the optimality conditions of the portfolio problem of financial intermediaries. Secondly, we derive the optimality conditions of the portfolio problem of households.

Financial Intermediaries

HJB Let G_t denote the gain process associated with the franchise value of financial intermediaries V_t . The gain process G_t satisfies

$$G_t \equiv E_t \int_0^\infty \tilde{\rho} e^{-\tilde{\rho}s} \Lambda_s n_{f,s} ds = \int_0^t \tilde{\rho} e^{-\tilde{\rho}s} \Lambda_s n_{f,s} ds + e^{-\tilde{\rho}t} \Lambda_t v_t n_{f,t}$$

The equality at the RHS follows from the conjecture that $V_t = v_t n_{f,t}$ and from the definition of V_t . The drift process of G_t is null because G_t is the conditional expectation of a random variable. From applying Ito's Lemma to the RHS, and then equalizing the resulting drift process to zero, we obtain the following Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned} \tilde{\rho} v_t &= \max_{\phi_{m,t}, \phi_{k,t}, \iota_t \geq 0} \left\{ \tilde{\rho} + [\tilde{\mu}_{n,t} + \mu_{v,t} + \mu_{\Lambda,t} + \sigma_{v,t} \tilde{\sigma}_{n,t} + \sigma_{\Lambda,t} \tilde{\sigma}_{n,t} + \sigma_{\Lambda,t} \sigma_{v,t}] v_t \right\} \\ s.t. : \phi_{k,t} &\leq \lambda v_t \end{aligned}$$

where the drift and diffusion processes of net worth are

$$\begin{aligned} \tilde{\mu}_{n,t} &= \left[\frac{a_f - \iota_t}{q_t} + I(\iota_t) - \delta + u_{q,t} + \sigma_{q,t} \sigma \right] \phi_{k,t} - \pi_t \phi_{m,t} - (i_t - \pi_t) (\phi_{k,t} + \phi_{m,t} - 1) - \tau_{1,t} \\ &\quad + E_F [R_t(\omega, \bar{\omega}) * (\phi_{k,t} + \phi_{m,t} - 1) + \tau_{2,t}(\omega)] \theta \end{aligned}$$

$$\tilde{\sigma}_{n,t} = (\sigma_{q,t} + \sigma) \phi_{k,t}$$

respectively. To derive the second line in $\tilde{\mu}_{n,t}$, we have used the conjecture that v_t follows an Ito process (notice that $v_t^+ = v_t$). The Tobin's Q of financial intermediaries v_t is the solution to the HJB above.

FOC The first-order condition with respect to the internal investment rate is

$$I'(\iota_t) = 1/q_t$$

The first-order condition with respect to the portfolio share in reserves is

$$-i_t + E_F [R_{m,t}(\omega, \bar{\omega})] \theta \leq 0$$

with equality if $\phi_{m,t} > 0$. The function $R_{m,t}(\omega, \bar{\omega})$ is the partial derivative of the total return $R_t(\omega, \bar{\omega}) * (\phi_{k,t} + \phi_{m,t} - 1)$ with respect to $\phi_{m,t}$.

The first-order condition with respect to the portfolio share in physical capital is

$$\frac{a_f - \iota_t}{q_t} + I(\iota_t) - \delta + u_{q,t} + \sigma_{q,t} \sigma - (i_t - \pi_t) + E_F [R_{k,t}(\omega, \bar{\omega})] \theta + (\sigma_{\Lambda,t} + \sigma_{v,t}) (\sigma_{q,t} + \sigma) \geq 0$$

with equality if $\phi_{k,t} < \lambda v_t$. The function $R_{k,t}(\omega, \bar{\omega})$ is the partial derivative of the total return $R_t(\omega, \bar{\omega}) * (\phi_{k,t} + \phi_{m,t} - 1)$ with respect to $\phi_{k,t}$.

HJB Again The asset pricing condition for the Tobin's Q satisfies

$$\frac{\tilde{\rho}}{v_t} + [FOC_k] \phi_{k,t} + E_F [r_{b,t} \mathbf{1}_{\omega > \bar{\omega}_t} + r_{l,t} \mathbf{1}_{\omega < \bar{\omega}_t}] \bar{\omega}_t \theta - \tau_{1,t} + E_F [\tau_{2,t}(\omega)] \theta + \mu_{v,t} - \tilde{\rho} + \sigma_{\Lambda,t} \sigma_{v,t} = 0$$

Households

HJB Let W_t denote the value of households. We conjecture that W_t satisfies

$$W_t = W(n_{h,t}, J_t)$$

where $W : \mathbb{R}^2 \rightarrow \mathbb{R}$ is a twice continuously differentiable function, and J_t is a sufficient statistic of the aggregate state variables in the problem of households. The process J_t is a scalar. We conjecture that J_t follows an Ito process with drift process $\mu_{J,t}$ and diffusion process $\sigma_{J,t}$.

The value W_t is the solution to the Hamilton-Jacobi-Bellman (HJB) equation

$$\rho W_t = \max_{c_t, k_{h,t}, \iota_t \geq 0} \left\{ \begin{array}{l} \frac{1}{1-\gamma} c_t^{1-\gamma} + \frac{\partial W_t}{\partial n_{h,t}} \mu_{n,t} n_{h,t} + \frac{\partial W_t}{\partial J_t} \mu_{J,t} J_t + \\ \frac{1}{2} \frac{\partial^2 W_t}{(\partial n_{h,t})^2} (\sigma_{n,t} n_{h,t})^2 + \frac{\partial^2 W_t}{\partial J_t \partial n_{h,t}} \sigma_{J,t} J_t \sigma_{n,t} n_{h,t} + \frac{1}{2} \frac{\partial^2 W_t}{(\partial J_t)^2} (\sigma_{J,t} J_t)^2 \end{array} \right\}$$

where the drift and the diffusion processes for net worth are

$$\mu_{n,t} n_{h,t} = \left[\frac{a_h - \iota_t}{q_t} + I(\iota_t) - \delta + u_{q,t} + \sigma_{q,t} \sigma \right] q_t k_{h,t} + (i_t - \pi_t) (n_{h,t} - q_t k_{h,t}) + Transfers_t - c_t$$

$$\sigma_{n,t} n_{h,t} = (\sigma_{q,t} + \sigma) q_t k_{h,t}$$

respectively.

FOC The first-order condition with respect to internal investment rate is

$$I'(\iota_t) = 1/q_t$$

The first-order condition with respect to consumption is

$$c_t^{-\gamma} = \frac{\partial W_t}{\partial n_{h,t}}$$

The first-order condition with respect to physical capital is

$$\begin{aligned} & \left[\frac{a_h - \iota_t}{q_t} + I(\iota_t) - \delta + u_{q,t} + \sigma_{q,t} \sigma - (i_t - \pi_t) \right] \frac{\partial W_t}{\partial n_{h,t}} + (\sigma_{q,t} + \sigma) \frac{\partial^2 W_t}{(\partial n_{h,t})^2} \sigma_{n,t} n_{h,t} + \\ & + (\sigma_{q,t} + \sigma) \frac{\partial^2 W_t}{\partial J_t \partial n_{h,t}} \sigma_{J,t} J_t \leq 0 \end{aligned}$$

with equality if $k_{h,t} > 0$.

To derive the asset pricing conditions on deposits and on the price of capital, we follow the same methodology as in Cox, Ingersoll and Ross (1985). Specifically, first, we replace the first-order conditions in the HJB equation; second, we take the first-order condition with respect to $n_{h,t}$ in the expression that we derived in the first step; and, third, we re-arrange the expression obtained in the second step accordingly.

Appendix B

In Appendix B, we characterize the competitive equilibrium.

Competitive Equilibrium The competitive equilibrium is characterized by the following set of conditions

- The internal investment condition

$$I'(\iota_t) = 1/q_t$$

- The asset pricing conditions on capital and on the Tobin's Q

$$\begin{aligned} & \frac{a_h - \iota_t}{q_t} + I(\iota_t) - \delta + u_{q,t} + \sigma_{q,t}\sigma - (i_t - \pi_t) + \sigma_{\Lambda,t}(\sigma_{q,t} + \sigma) + \\ & + \left[\frac{a_f - a_h}{q_t} + E_F[R_{k,t}(\omega, \bar{\omega})]\theta + \sigma_{v,t}(\sigma_{q,t} + \sigma) \right] \mathbf{1}_{f,t} = 0 \end{aligned}$$

$$\begin{aligned} & \left[\frac{a_f - a_h}{q_t} + E_F[R_{k,t}(\omega, \bar{\omega})]\theta + \sigma_{v,t}(\sigma_{q,t} + \sigma) \right] \phi_{k,t} \mathbf{1}_{f,t} + \\ & \bar{\omega}_t (r_{b,t} - r_{l,t}) E_F[\mathbf{1}_{\omega > \bar{\omega}_t}] \theta + r_{b,t} E_F[(\omega - \bar{\omega}_t) \mathbf{1}_{\omega > \bar{\omega}_t}] \theta + \frac{\tilde{\rho}}{v_t} + \mu_{v,t} - \tilde{\rho} - \sigma_{y,t} \sigma_{v,t} = 0 \end{aligned}$$

- The reserves holdings condition

$$-i_t dt + E_F[R_{m,t}(\omega; \bar{\omega}_t)] \theta dt \leq 0$$

with "=" if $\phi_{m,t} > 0$

- The law of motion of η_t

The leverage multiple is $\phi_{k,t} = \min\{\lambda v_t, 1/\eta_t\}$. The real interest rate is $r_t = \rho + \mu_{c,t} - \sigma_{c,t}^2$ (this follows from the asset pricing condition on deposits). The process $\mathbf{1}_{f,t} \in \{0, 1\}$ indicates whether financial intermediaries are the marginal investors on capital. Namely, $\mathbf{1}_{f,t} = 1$ iff $\phi_{k,t} = 1/\eta_t$.