Interest Rate Uncertainty as a Policy Tool*

Fabio Ghironi†  Galip Kemal Ozhan‡
University of Washington, University of St. Andrews
CEPR, EABCN, and NBER

May 1, 2018

Preliminary
(click here for the latest version)

Abstract

This paper studies the implications of using the volatility of domestic interest rate as a policy instrument in an open economy. We develop a two-region macroeconomic model in which short-term capital flows are carefully distinguished from Foreign Direct Investment (FDI). We focus on the effectiveness of using domestic interest rate volatility as a policy tool to affect the composition of capital inflows between short-term securities and FDI, and we identify the trade-offs that are faced in navigating external balance and price stability. We find that three channels of uncertainty transmission arise in affecting the composition of external account. First, a precautionary savings channel discourages short-term debt and contributes to an outflow. Second, a precautionary pricing channel contributes to an upward-bias in price setting and diminishes the return from FDI, causing net FDI outflows. Finally, introducing time-to-build to FDI dampens net FDI outflows due to irreversibility implied by time-to-build (growth options channel). Currency of export invoicing and risk aversion of outside agents are important determinants in generating these results. Furthermore, we conclude that under every scenario, an increase in policy uncertainty is inflationary. Our analysis is both normative and positive.

JEL codes: E32, F21, F32, F38, G15.

Keywords: International Financial Policy, Stochastic Volatility, Short-term and Long-term Capital Movements, Unconventional Monetary Policy.

*First version: January 8, 2018. We would like to thank Brent Bundick, Refet Gürkaynak, Hande Küçük, Gülçin Özkan, and Ricardo Reis and the seminar participants at Bilkent University and the University of Helsinki for fruitful discussions. All remaining errors are ours.
†Department of Economics, University of Washington, Savery Hall, Box 353330, Seattle, WA 98195, U.S.A. or ghiro@uw.edu URL: http://faculty.washington.edu/ghiro
‡School of Economics and Finance, University of St. Andrews, Castlecliffe, The Scores, St. Andrews KY16 9AR, U.K. or gkozhan@st-andrews.ac.uk URL: http://sites.google.com/site/gozhan
1 INTRODUCTION

This paper studies the implications of using interest rate uncertainty as a policy tool in an open economy to control the capital account, and to attract Foreign Direct Investment (FDI) while discouraging short-term inflows.

Starting with the colonial pattern of foreign investment in 19th century, emerging and developing nations have been subject to ebbs and flows of capital, affecting their economic management. However, the recent episode distinguishes itself with a surge in the size and volatility of flows with the development of financial markets and with the exceptionally expansionary policy in advanced economies after the Global Financial Crisis (GFC). Figure 1 exhibits the change in the portfolio flows to Emerging Markets Economies (EMEs) between 2006-2014. The recent surge in the size and volatility of inflows can cause dislocations —mainly, financial stability concerns and inflationary pressures (Obstfeld, 2015). Hence, the conduct of monetary policy in EMEs has become a very modern contrivance and intended to prevent the adverse effects of feast-famine order of capital flows. The central bankers who are working under multiple mandates are forced to be even more “innovative” when facing similar challenges.

The recent unorthodox policy experiment of the Central Bank of the Republic of Turkey (CBRT) provides an example for the innovative policy response to the changing nature of capital flows, while aiming to achieve its multiple mandates of contributing to financial strength and maintaining price stability. In response to intense capital inflows, the interest rate corridor widened from below to discourage carry trade and channel inflows towards long-term foreign direct investments (FDI), and

---

1See Nurkse (1954) for a comparison of 19th century vs early 20th century capital flows.
2Among others, see Ahmed and Zlate (2013) and Rajan (2013).
3The change in the size and volatility of capital flows recently raised eyebrows on the applicability of independent monetary policy (given flexible exchange rates) in recipient countries. On the one hand, Rey (2013) argues that independent monetary policy is not possible without capital controls, and on the other, Woodford (2010) implies financial globalization does not affect the ability of domestic monetary policy to control inflation. Besides, Taylor (2015) claims that the volatility of capital flows would be ex-ante lower, if there were no deviations from rules-based monetary policy, but Obstfeld (2015) provides a middle ground, and highlights that monetary authorities have multiple goals in reality, and the surge in the volatility of capital flows affect the trade-off that authorities face between macro objectives and other targets, instead of making monetary policy ineffective.
4See IMF (2013) for a summary on the policy responses of several EMEs to capital inflows.
5Calvo et al. (1996) also argue that the countries that had been the most successful in managing capital flows during the Tequila crisis introduced a set of policy options instead of relying on a single instrument.
6The Turkish Central Bank Law, which was amended in 2001, provides the Bank with the instrumental independence to contribute to financial stability, in addition to its primary mandate of achieving price stability.
7Interest rate corridor refers to the window between overnight lending and borrowing rates. The main policy rate of CBRT, one-week repo rate, fluctuates within this band and a widening of the corridor implies an increase in the uncertainty for the future path of main policy rate.
in response to powerful capital outflows, the interest rate corridor was narrowed by raising overnight borrowing rates, with an aim of preventing excessive outflows (see Başçı, 2012). In Figure 2, the period between November 2010 and October 2011 coincides with the horizon of the above policy. However, to the best of our knowledge, there is no structural model that studies such unorthodox attempt.

This paper fills this gap by providing a laboratory for thinking about the effectiveness of the above policy on distinguishing short-term capital flows from long-term investment flows, and identifies the trade-offs that are faced in navigating external balance and price stability. We build a New Keynesian two-region macroeconomic model (EME and the Rest of the World (RoW)) with incomplete international financial markets and deviations from PPP. The model is augmented with interest rate uncertainty shocks, which the EME central bank uses to discourage short-term capital flows and attract FDI, while aiming inflation and output stabilization.

The model stands on the New Keynesian Open Economy framework such as in Bergin (2006), Kollmann (2001), and Obstfeld and Rogoff (1995). We differentiate from this early literature mainly by introducing FDI. We define FDI as the investment in physical capital that will be used in overseas production activity. Introducing FDI in this fashion allows us to decompose the current account into bond flows and FDI flows easily. In our extensions, we impose a time-to-build capital requirement for agents investing in other region’s productive capital à la Kydland and Prescott (1982). By doing so, the model further captures the irreversibility of FDI. The joint analysis of interest rate uncertainty and the different types of international asset transactions yields new insights on the transmission and propagation of interest rate uncertainty both domestically and internationally.

Three key channels of uncertainty transmission operates in affecting the external account in

---

8 Figure 3 exhibits an increase in FDI inflows during the application of interest rate corridor policy, however it is uncertain that whether the increase is due to a mean reversion of inflows after the GFC, or due to the success of the policy.

9 The changes in the size of the interest rate corridor implies a change in the variance of domestic interest rate.

10 Our goal is to capture foreign direct investment dynamics by introducing this feature. Introducing FDI in terms of investing in capital for another country’s use and receiving capital gains from this transaction is also in line with the IMF definition of FDI. IMF definition is as follows: “The term describes a category of international investment made by a resident entity in one economy (direct investor) with the objective of establishing a lasting interest in an enterprise resident in an economy other than that of the investor (direct investment enterprise). ... Direct investment involves both the initial transaction between the two entities and all subsequent capital transactions between them and among affiliated enterprises, both incorporated and unincorporated.” Link: https://www.imf.org/external/np/sta/di/glossary.pdf
our model. First, a precautionary savings channel arises in affecting short-term capital flows. In response to increased risk in the EME, agents shift away from EME debt, and smooth consumption using RoW securities. This contributes to a fall in the short-term financement of the EME current account deficit. Second, when the EME is subject to pricing frictions in production sector, an increase in the uncertainty yields an upward pricing bias for EME firms, leading to a precautionary pricing channel. Firms adjust their prices higher than they would otherwise do, and this behavior contributes to a fall in output and rental rates of physical capital obtained from domestic and international markets. Lower profits for RoW investors in the EME productive capital induces a decline in the net FDI flows into the EME. Third, when FDI is irreversible through a time-to-build condition, sunk costs are partially offset by future profits from investing in the EME (from the perspective of RoW agents), and increased uncertainty in the interest rate contributes to a dampening in the net outflow of FDI from the EME. This gives rise to a growth option channel.

We further show that these results are dependent on the nature of price rigidity (whether prices are set in producer’s currency or in local currency) and the degree of risk aversion of rest of the world agents. Under all of the cases, we also find using uncertainty as a policy tool is inflationary.

Our contribution is three-fold. First, we contribute to the literature that study the effects of uncertainty shocks on economic activity. This paper differentiates from this literature, first, by asking the implications of using uncertainty as a policy tool, instead of a taken-as-given phenomenon. In addition, our paper is also the first that studies the effects of uncertainty on different types of capital flows. There is also very limited work on studying uncertainty in an open economy framework. Our setting is the first that studies implications of uncertainty in an international macro model that exhibit incomplete international financial markets, deviations from PPP, price rigidities and investment dynamics. Moreover, this paper provides a laboratory to study FDI dynamics.

Second, we contribute to the literature that study capital flows to the EMEs. We differentiate

---


13 The literature is still expanding with the recent contributions of Ahmed and Zlate (2013), Aoki et al. (2015),
from those that study an economy integrated into international financial markets by introducing FDI and treating it as an investment for receiving capital gains from the overseas production, rather than international trade of equity. Moreover, none of the papers in this literature study the effects of uncertainty on economic activity.\textsuperscript{14}

This paper, finally, contributes to the literature that study the interdependence between macro-prudential and monetary policy.\textsuperscript{15} In addition to the highlighted features above, this paper sheds light on the effectiveness of using interest rate volatility as a new macroprudential tool that influences financial variables through international markets, and evaluates its success depending on the trade-off between the monetary targets and external balance.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 discusses calibration and model dynamics when several features in the model are turned on and off. Section 4 introduces additional results when new ingredients are included in the baseline economy. Section 5 concludes.

\section{The Model}

The world is composed of two regions, Home and the Rest of the World (RoW)\textsuperscript{16}. The total measure of the world economy is normalized to unity, with Home and RoW having measures of $n$ and $1-n$, respectively. The model shares several basic features of those in the literature that are characterized by microeconomic foundations in combination with nominal rigidities. International financial markets are incomplete as only non-contingent assets are internationally traded. An additional important feature is that, in addition to international trade of short-term securities, RoW agents can invest in productive capital that will be used as an input in Home’s production activity. RoW variables are denoted with an asterisk.

Households consume a basket of final goods which is an Armington aggregator of Home and Banerjee et al. (2015), Devereux and Sutherland (2009), Ghironi et al. (2015), Gourinchas et al. (2016).
\textsuperscript{14}An exception is Fernández-Villaverde et al. (2011) but they only show how changes in the volatility of the world real interest rate affect other variables in a small economy RBC model, abstracting from different types of capital flows and nominal rigidities.
\textsuperscript{15}In a closed economy setting, among others, see Angeloni and Faia (2013) and Kannan et al. (2009). In an open economy setting, see Aoki et al. (2015) and Unsal (2013).
\textsuperscript{16}RoW can be treated as the cluster of countries which engage in international transactions with the Home economy. Alternatively, it can be thought as the main trading partner and the origin of major FDI received by the Home economy after adjusting for the respective country sizes.
RoW goods. Domestic intermediate goods are produced by monopolistically competitive firms which combine labor with real capital from domestic and foreign agents. The baseline setup assumes that the law of one price holds, firms engage in producer currency pricing, and the source of PPP deviations is home bias in preferences. Figure 4 exhibits the model architecture.

In what follows, we focus on Home economy, and otherwise indicated, RoW is symmetric.

2.1 Households

The economy is populated by atomistic households. Each household is a monopolistic supplier of a specific labor input. The representative household, indexed by $h$, maximizes the expected inter-temporal utility from consumption, $C_t(h)$, net of disutility from supplying labor to intermediate good producers, $L_t(h)$:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t(h), L_t(h))$$

where $U(C_t(h), L_t(h)) = \frac{C_t(h)^{1-\rho}}{1-\rho} - \chi \frac{L_t(h)^{1+\rho}}{1+\rho}$ and $\beta \in (0, 1)$ is the discount factor.

Households also accumulate physical capital in Home and RoW consumption units, that are used in the respective region’s production of intermediate goods, $K$ and $K^*$. They rent these two types of capital to intermediate firms at Home and RoW. The rental rate they receive from Home and RoW producers are also in Home and RoW consumption units, respectively. Investments in the respective physical capital stock, $I$ and $I^*$, require use of same composite of goods as in the final goods, $Y$ and $Y^*$. Law of motion of capital is as standard in the literature for both types of capital:

$$K_{t+1}(h) = (1 - \delta)K_t(h) + I_t(h),$$

$$K^*_{t+1}(h) = (1 - \delta)K^*_t(h) + I^*_t(h),$$

where $\delta$ denotes the depreciation rate of capital.

Households supply differentiated labor input, which gives them some pricing power in setting their own wage. The composite labor index is in Dixit-Stiglitz form:

$$L_t(h) = \int_0^1 L_t(h)^{\frac{\epsilon_{W-1}}{\epsilon_W}} dh$$

where $\epsilon_W > 0$ is the elasticity of substitution between the differentiated labor inputs. The aggregate

\footnote{Similarly, RoW agents invest in physical capital that will be used in RoW, $K^*$, and in physical capital that will be rented to Home, $K^*$. As discussed in more detail later on, we interpret the transactions related with the latter as FDI into the EME.}
nominal wage index is \( W_t \equiv \left[ \int_0^1 W_t(h)^{1-\epsilon_W} \, dh \right]^{\frac{1}{1-\epsilon_W}} \), where \( W_t(h) \) is the nominal wage received by household \( h \). The optimization problem for the competitive labor packer yields the labor demand equation:

\[
L_t(h) = \left( \frac{W_t(h)}{W_t} \right)^{-\epsilon_W} L_t. \tag{4}
\]

The nominal wage, \( W_t(h) \), is set by households subject to (4) when maximizing utility. There is quadratic cost of adjustment for the nominal wage rate between period \( t-1 \) and \( t \), à la Rotemberg (1982):

\[
\frac{\kappa_W}{2} \left( \frac{W_t(h)}{W_{t-1}(h)} - 1 \right)^2 W_t(h) L_t(h),
\]

where \( \kappa_W \geq 0 \) determines the size of adjustment cost (if \( \kappa_W = 0 \), then wages are flexible). The size of this cost is proportional in labor income.

Households can hold one-period non-contingent nominal bonds supplied by domestic and RoW agents, \( B \) and \( B^* \). Nominal exchange rate is denoted by \( S \). International asset markets are incomplete as only risk-free bonds are traded across countries. Home bonds are issued by Home households and are denominated in Home currency, whereas RoW bonds are issued by RoW households and denominated in foreign currency. There are convex adjustment costs that ensure that zero international bond holding is the unique steady state, and hence the economy goes back to their initial position after temporary shocks. These costs are rebated back to households in equilibrium in lump-sum fashion.

The period budget constraint of the household can be written as:

\[
P_tC_t(h) + B_{t+1}(h) + S_tB_{s,t+1}(h) + \frac{\eta}{2} P_t \left( \frac{B_{s,t+1}(h)}{P_t} \right)^2 + \frac{\eta}{2} S_t P_t^* \left( \frac{B_{s,t+1}(h)}{P_t^*} \right)^2 + P_t I_t(h) + S_t P_t^* I_{s,t}(h) + R_{t+1} B_t(h) + S_t R_{s,t}^* B_{s,t}(h) + P_t r_K K_t(h) + S_t P_t^* r_K^* K_{s,t}(h) + W_t(h) L_t(h) - \frac{\kappa_W}{2} \left( \frac{W_t(h)}{W_{t-1}(h)} - 1 \right)^2 W_t(h) L_t(h) + d_t(h) + T_t(h), \tag{5}
\]

where \( \frac{\eta}{2} \xi_t(B_{s,t+1})^2 \) is the cost of adjusting holdings of internationally traded securities, \( T_t(h) \) is a fee rebate, taken as given by the household. \( d_t(h) \) represents the profits obtained by household \( h \) from engagements in production. \( R_{t+1} \) and \( R_{t+1}^* \) are gross nominal interest rates on Home and RoW bond holdings between \( t \) and \( t+1 \). Finally, \( d_t(h) \) is the profits from producers, and \( r_K \) and \( r_K^* \) are real rental rates for the capital produced by Home households and used in Home and RoW
production functions.

The household maximizes (1) subject to (2), (3), (4), and (5). The Euler equations for bond holdings are as follows:

\[ 1 + \eta b_{t+1} = R_{t+1} \mathbb{E}_t \left[ \frac{\beta_{t,t+1}}{\Pi_{t+1}} \right], \quad (6) \]

\[ 1 + \eta b_{st,t+1} = R^*_{t+1} \mathbb{E}_t \left[ \frac{\beta_{st,t+1} \text{rer}_{t+1}}{\Pi^*_{t+1}} \right], \quad (7) \]

where \( \beta_{t,t+s} = \frac{U_{C,t+s}}{U_{C,t}} \) is the discount factor with \( U_{C,t} \) denoting marginal utility from consumption in period \( t \). \( \Pi_t \) and \( \Pi^*_t \) represent the gross inflation between \( t - 1 \) and \( t \) in Home and RoW. \( b_{t+1} = \frac{B_{t+1}(h)}{P_t} \) and \( b_{st,t+1} = \frac{B_{st,t+1}(h)}{P^*_t} \) are real holdings of Home and RoW bonds, and \( \text{rer} \) is the consumption-based real exchange rate (units of Home consumption per units of RoW).\(^{18}\) We omit the transversality conditions for bond holdings. With \( \eta > 0 \), no-arbitrage condition\(^{19}\) implies:

\[ \frac{R_{t+1}}{R^*_t} = \frac{(1 + \eta b_{t+1}) \mathbb{E}_t[\frac{\beta_{t,t+1} \text{rer}_{t+1}}{\Pi_{t+1}}]}{(1 + \eta b_{st,t+1}) \mathbb{E}_t[\frac{\beta_{t,t+1}}{\Pi^*_{t+1}}]} . \]

The Euler equations for the accumulation of capital used in Home and RoW production of intermediate inputs are:

\[ 1 = \mathbb{E}_t \left[ \beta_{t,t+1} \left( r_{K,t+1} + 1 - \delta \right) \right], \quad (8) \]

\[ 1 = \mathbb{E}_t \left[ \beta_{t,t+1} \frac{\text{rer}_{t+1}}{\text{rer}_t} \left( r_{K,st,t+1} + 1 - \delta \right) \right], \quad (9) \]

with real prices of each type of capital being

\[ q_t = 1, \quad (10) \]

\[ q_{st} = \text{rer}_t. \quad (11) \]

\(^{18}\)Real exchange rate is defined as \( \text{rer} = \frac{S^*}{P} \). A decrease in \( \text{rer} \) implies appreciation, whereas an increase indicates depreciation of the real exchange rate.

\(^{19}\)As discussed in calibration, we will set \( \eta \) to a very small value that will minimally affect the model dynamics, and its sole implication will be on ensuring that zero international bond holding is the unique non-stochastic steady state of the model. However, our experiments with volatility shocks and the solution method of the model will imply that there will be deviations from the uncovered interest rate parity based on a time-varying risk component.
Equations (9) and (11) imply that Home households’ investment in capital that will go into RoW production is not only dependent on the rental rate but also on the fluctuations in real exchange rate. The benefit of an additional unit of new capital that will be used by foreign production is the present discounted stream of extra profits (marginal products). Equation (11) says that the cost is equal to the real exchange rate, and hence, for an additional unit of capital, \( K_{*,t+1} \), investment will be adjusted by the movements in the real exchange rate such that future profits from renting the capital abroad will not be affected.

The first-order-condition with respect to \( W_t(h) \) shows that real wage, \( w_t \), is a time-varying markup over the marginal rate of substitution between labor and consumption:

\[
w_t = \mu^W_t \left( \frac{\lambda L^\varphi_t}{C_t^{\rho}} \right),
\]

where \( \mu^W_t \) is the time-varying wage markup:

\[
\mu^W_t \equiv \frac{\epsilon_W}{(\epsilon_W - 1) \left( 1 - \frac{\kappa^W}{2} (\Pi^W_t - 1)^2 \right) + \kappa^W (\Pi^W_t (\Pi^W_t - 1) - \mathbb{E}_t \left( \frac{\beta_{t+1}}{\Pi_{t+1}} (\Pi^W_{t+1} - 1) (\Pi^W_{t+1})^2 L_{t+1} \right) \right)},
\]

with \( \Pi^W_t ≡ \frac{w_t}{w_{t-1}} \Pi_t \) being the gross nominal wage inflation. The response of output will be less than it would if wages were flexible, when markups move in response to shocks.

### 2.2 Firms

Final goods in the economy, \( Y_t \), are produced by aggregating a variety of differentiated intermediate Home goods indexed by \( i \in [0, 1] \) along with a variety differentiated intermediate RoW goods indexed by \( j \in [0, 1] \). The aggregation technology for producing final goods is

\[
Y_t = \left( a^\frac{1}{\sigma} Y_{H,t}^{\frac{\sigma-1}{\sigma}} + (1 - a)^{\frac{1}{\sigma}} Y_{F,t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},
\]

where \( Y_{H,t} = \left( \int_0^1 Y_{H,t}(i)^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}} \) represents an aggregate of Home goods sold domestically, and \( Y_{F,t} = \left( \int_0^1 Y_{F,t}(j)^{\frac{\sigma-1}{\sigma}} \, dj \right)^{\frac{\sigma}{\sigma-1}} \) is an aggregate of imported RoW goods. Home bias is denoted by \( a \). The producer of final good is competitive and demand is allocated between Home and RoW goods.
according to:

\[ Y_{H,t} = a \left( \frac{P_{H,t}}{P_t} \right)^{-\omega} Y_t, \]  

\[ Y_{F,t} = (1 - a) \left( \frac{P_{F,t}}{P_t} \right)^{-\omega} Y_t, \]  

where \( P_{H,t} \) and \( P_{F,t} \) are nominal prices of aggregate Home goods sold domestically, and aggregate imported goods from RoW. \( P_t \) is aggregate price index as

\[ P_t = \left( a P_{H,t}^{1-\omega} + (1 - a) P_{F,t}^{1-\omega} \right) ^{\frac{1}{1-\omega}}. \]  

Each differentiated intermediate Home good is produced by using capital rented from Home households, \( K_t(i) \), capital rented from RoW households, \( K^*_t(i) \), and homogenous labor input rented from Home households, \( L_t(i) \):

\[ Y_{H,t}(i) + \left( \frac{1 - n}{n} \right) Y^*_H(i) = K_t(i)^{\alpha_1} K^*_t(i)^{\alpha_2} L_t(i)^{1-\alpha_1-\alpha_2}, \]

where \( Y^*_H \) is the exported intermediate Home good, and \( \alpha_1, \alpha_2 \) and \( \alpha_1 + \alpha_2 \in (0, 1). \)

The producer of each differentiated intermediate Home good is monopolistically competitive and faces demand curves for its product sold domestically, \( Y_{H,t}(i) = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon} Y_{H,t} \), and for its product sold in RoW, \( Y_{H,t}(i) = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon} Y_{H,t} \), where \( P_{H,t}(i) \) is the nominal price of domestically sold Home good \( i \), and \( P_{H,t}(i) \) is the domestic currency price of the exported good \( i \) with the price in the foreign market being \( P_{H,t}(i) = \frac{P_{H,t}(i)}{S_t} \). Finally, \( P_{H,t} = \left( \int_0^1 P_{H,t}(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}} \) is the nominal price of domestically sold Home good bundle and \( P_{H,t}^* = \left( \int_0^1 P_{H,t}(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}} \) is the nominal foreign currency price of the export bundle.

Let \( r_{K,t} \) be the rental price of capital rented from Home households, and \( r^*_{K,t} \) be the rental price of capital rented from RoW households. The real marginal cost of producing intermediate Home good is

\[ mc_t = \frac{w_t^{1-\alpha_1-\alpha_2} r^*_{K,t}^{\alpha_1}}{(1 - \alpha_1 - \alpha_2)^{1-\alpha_1-\alpha_2} \alpha_1^\alpha_2}. \]

The monopolistic producer \( i \) chooses a rule \( (P_{H,t}(i), P_{H,t}(i), Y_{H,t}(i), Y_{H,t}(i)) \) to maximize the

---

20 Three input Cobb-Douglas production function implies RoW capital and domestically produced capital are neither substitutes nor complements, and hence can be treated as the general case for the complementarity of FDI.
expected discounted profit:

\[
\mathbb{E}_t \left[ \sum_{s=t}^{\infty} \beta_{t,t+s} \left( 1 - \frac{\epsilon}{2} \left( \frac{P_{H,t+s}(i)}{P_{H,t+s-1}(i)} - 1 \right)^2 \right) \frac{P_{H,t+s}(i)}{P_{C,t+s}} \left( Y_{H,t+s}(i) \right) \right] - m_{t} \left( Y_{H,t+s}(i) + \left( \frac{1-n}{n} \right) Y_{H,t+s}^{\star}(i) \right)
\]

where the quadratic terms are the adjustment costs of respective prices. From the first-order-conditions with respect to \( P_{H,t+s}(i) \) and \( P_{H,t+s}^{\star}(i) \) evaluated under symmetric equilibrium, we obtain the real price of Home output for domestic sales (i.e. \( r_{PH} \equiv \frac{P_{H}}{P_{C}} \)) as a time-varying markup, \( \mu_{H,t} \) over the marginal cost:

\[
r_{PH,t} = \mu_{H,t} m_{t} \tag{17}
\]

and the real price of Home output for export sales (in units of RoW consumption) as a time-varying markup, \( \mu_{H,t}^{\star} \) over the marginal cost

\[
r_{PH,t}^{\star} = \frac{\mu_{H,t}^{\star} m_{t}}{r_{PH,t}^{rer}_{t}} \tag{18}
\]

where

\[
\mu_{H,t} \equiv \frac{\epsilon}{(\epsilon - 1) \left( 1 - \frac{\kappa}{2} (\Pi_{H,t} - 1)^2 \right) + \kappa \left( \Pi_{H,t} (\Pi_{H,t} - 1) - \mathbb{E}_t \left[ \frac{\beta_{t,t+1}}{\Pi_{t+1}} (\Pi_{H,t+1} - 1) (\Pi_{H,t+1} - 1)^{2} \frac{Y_{H,t+1}}{Y_{H,t}} \right] \right)},
\]

\[
\mu_{H,t}^{\star} \equiv \frac{\epsilon}{(\epsilon - 1) \left( 1 - \frac{\kappa^{\star}}{2} (\Pi_{H,t}^{\star} - 1)^2 \right) + \kappa^{\star} \left( \Pi_{H,t}^{\star} (\Pi_{H,t}^{\star} - 1) - \mathbb{E}_t \left[ \frac{\beta_{t,t+1}}{\Pi_{t+1}} (\Pi_{H,t+1}^{\star} - 1) (\Pi_{H,t+1}^{\star} - 1)^{2} \frac{Y_{H,t+1}^{\star}}{Y_{H,t}} \right] \right)},
\]

with \( \Pi_{H,t} \equiv \frac{r_{PH,t}}{r_{PH,t-1}} \Pi_{t} \) and \( \Pi_{H,t}^{\star} \equiv \frac{r_{PH,t}^{\star}}{r_{PH,t-1}^{rer}} \Pi_{t}^{rer} \).

Given the cost of adjusting prices in domestic and export markets, firms has to move their markups to smooth price changes over time.

### 2.3 Equilibrium

Under symmetric equilibrium, we also learn

\[
Y_{H,t} + \left( \frac{1-n}{n} \right) Y_{H,t}^{\star} = K_t^{\alpha_1} K_t^{\alpha_2} L_t^{1-\alpha_1-\alpha_2}, \tag{19}
\]
where \( K_t = \int_0^1 K_t(i) \, di \), \( K^*_t = \int_0^1 K^*_t(i) \, di \), and \( L_t = \int_0^1 L_t(i) \, di \). The cost minimization implies

\[
\alpha_1 w_t L_t = (1 - \alpha_1 - \alpha_2) r_{K,t} K_t,
\]

\[
\alpha_2 r_{K,t} K_t = \alpha_1 r^*_{K,t} K^*_t.
\]

Hence, the trade-off between domestic capital, RoW capital, and labor inputs depends on the relative cost of each.

Market clearing requires that final production equals consumption and investment received from Home and RoW agents, net of cost of adjusting nominal prices:

\[
Y_t = C_t + I_t + I_t^r + \frac{k^W}{2} (\Pi^W - 1)^2 w_t L_t + \frac{k}{2} (\Pi_{H,t} - 1)^2 r_{P,H,t} Y_{H,t} + \left(1 - \frac{n}{n}\right)^2 Y^*_{H,t} + \left(1 - \frac{n}{n}\right)^2 r_{P,H,t} Y^*_{H,t}.
\]

Finally, bonds are in zero net supply, which implies \( b_{t+1} + b^*_{t+1} = 0 \) and \( b^*_{s,t+1} + b_{s,t+1} = 0 \) at all periods. The lump sum transfer is \( T_t = \frac{n}{2} \left[ P_t \left( \frac{B_{t+1}(h)}{P_t} \right)^2 + S_t P^*_t \left( \frac{B^*_{s,t+1}(h)}{P^*_t} \right)^2 \right] \). We show in Appendix A that Home net foreign assets are determined by:

\[
\begin{align*}
&b_{t+1} + rer_t b_{s,t+1} + \left(1 - \frac{n}{n}\right) rer_t K^*_{s,t+1} - K^*_t \\
&= \frac{R_t}{n} b_t + \frac{R^*_t}{n} rer_t b_{s,t} + \left(1 - \frac{n}{n}\right) rer_t (r_{K,s,t} + 1 - \delta) K_{s,t} - \left(1 - \frac{n}{n}\right) r^{*}_{K,t} + TB_t
\end{align*}
\]

where trade balance is given as follows: \( TB_t \equiv \left(1 - \frac{n}{n}\right) \mu^*_{H,t} mc_t Y^*_{H,t} - rer_t \mu_{F,t} mc^*_{F,t} Y_{F,t} \).

The above equation differs from those in standard open-economy models by the terms that indicate the stock of physical capital received from RoW net of physical capital installed into RoW, and by the terms that indicate the respective rental gains from this transaction. The change in net foreign assets between \( t \) and \( t + 1 \) is determined by the current account, \( CA_t \):

\[
\begin{align*}
&\left( b_{t+1} - b_t \right) + rer_t (b_{s,t+1} - b_{s,t}) + \left(1 - \frac{n}{n}\right) rer_t (K^*_{s,t+1} - K_{s,t}) - \left(1 - \frac{n}{n}\right) (K^*_{t+1} - K^*_t) \equiv CA_t
\end{align*}
\]

As shown under brackets, the current account is decomposed into short-term security flows component and FDI flows component.
2.4 Monetary Policy

To close the model, we must describe the monetary authority. The central bank sets the nominal interest rate according to a Taylor rule that reacts to inflation and output which displays stochastic volatility:

\[ \frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left( \frac{\Pi_t}{\Pi} \right)^{(1-\rho_R)\rho_{\Pi}} \left( \frac{Y_t}{Y} \right)^{(1-\rho_R)\rho_Y} e^{u_t}, \]  

(24)

where \( \rho_R \) is a smoothing parameter to capture gradual movements in interest rates, and the parameters \( \rho_{\Pi} \) and \( \rho_Y \) are responsiveness of the nominal interest rate to deviations of inflation and output from their steady state values. The monetary policy shock, \( u_t \), follows an AR(1) process:

\[ u_t = \rho^u u_{t-1} + \sigma^R \varepsilon_t, \]  

(25)

where \( \varepsilon_t \) is a normally distributed shock with zero mean and unit variance. Moreover, the standard deviation, \( \sigma^R_t \), follows an AR(1) process:

\[ \sigma^R_t = (1 - \rho^{\sigma^R}) \sigma^R + \rho^{\sigma^R} \sigma^R_{t-1} + \varepsilon_t^\sigma, \]  

(26)

where \( \varepsilon_t^\sigma \) is a normally distributed shock with zero mean and unit variance. Parameter \( \sigma^R \) controls the mean volatility of the exogenous component in the Taylor rule. An increase in the volatility of the exogenous process increases uncertainty about the future path of monetary policy.

2.5 Summary

Table 1 summarizes the key equilibrium conditions of the model. The equations ((2), (3), (6), (7), (8), (9), (12), (13), (14), (15), (16), (17), (18), (19), (20), (21), (22), (24)) and their RoW counterparts, together with the net foreign asset condition in equation (23) determine 37 endogenous variables of interest: \( (Y_t, C_t, I_t, I_{st}, K_t, K_{st,t}, L_t, Y_{H,t}, Y_{H,t}^*, mc_t, r_{pH,t}, r_{pH,t}^*, w_t, r_{K,t}, r_{K,s,t}, b_t, R_t, \Pi_t) \) and their foreign counterparts, and \( rer_t \). Auxiliary variables and exogenous processes are described above.

12
3 Model Calibration and Simulations

In this section, we illustrate the model dynamics and highlight the role of using interest rate uncertainty in adjusting external capital account.

3.1 Calibration

We set the discount factor, $\beta$, to match 2% real rate, to 0.9804 as in the literature that focus on emerging markets. Relative risk aversion, $\rho$, relative weight of labor in the utility function, $\chi$, and Frisch elasticity, $\varphi$, are set to their conventional values in the literature as well. We set them to 2, 1, and 0.25, respectively. As regards to the convex adjustment costs of bond holdings to the households, $\eta$, we use 0.0025 as in Ghironi and Melitz (2005). This value implies that the cost of adjusting has negligible impact on model dynamics, other than pinning down the non-stochastic steady state and ensuring mean reversion when shocks are transitory. Following Christiano et al. (2005), we assume nominal wages are sticky for 3 periods on average, and set $\kappa^W$ to 116 to replicate this behavior. Moreover, we set the elasticity of substitution of differentiated labor inputs, $\varepsilon^W$, to 11, which implies a wage markup of 10% under flexible prices.

For the parameters that are importance for the firm side, we set the home bias in final production to 0.65 as in Unsal (2013), and the shares for domestic and foreign capital in intermediate good productions, $\alpha_1$ and $\alpha_2$, to 0.30 and 0.15 as in Aoki et al. (2015). Rotemberg price adjustment parameters for domestic and exported goods, $\kappa$ and $\kappa^*$, are set to 116, to replicate 3-period stickiness as in Barattieri et al. (2018). We again follow them on setting the elasticity of substitution of differentiated labor inputs, $\varepsilon$, to 11, and replicate a 10% price markup in both sectors when prices are flexible.

Finally, our choices for the parameters in the Taylor rule is also in line with the previous literature, and set the smoothing coefficient, $\rho_R$, the steady state response to inflation, $\rho_\Pi$, and the steady state response to output, $\rho_Y$, to 0.7, 1.5, and 0.5/4, respectively.

Table 2 summarizes the parametrization of the model.
3.2 Solution Method

We solve the model using third-order perturbation techniques. A first-order approximation will be certainty equivalent and will neglect higher order effects. A model solution under a second-order approximation will not be able to study the direct effects of a volatility change, as the model solution includes cross products of exogenous volatility and level variables. Hence, to single out the individual effects of volatility shocks, a third-order approximation of the model is needed (Fernández-Villaverde et al. (2011)).

As highlighted by Kim et al. (2008), solutions using higher-order perturbation techniques tend to yield explosive time-paths due to accumulation of terms of increasing order. To overcome this problem, Andreasen et al. (2013) uses pruning all higher order terms, and we integrate their method in our simulations.

Moreover, higher-order approximation solutions move the ergodic distribution of the model endogenous variables away from their non-stochastic steady state values (Fernández-Villaverde et al. (2011)). Therefore, calculating impulse responses from non-stochastic steady state is not informative. To overcome this difficulty, we follow the previous literature and calculate impulse responses as deviations from the stochastic steady states of the endogenous variables. In defining the stochastic steady state, we follow Born and Pfeifer (2014b) and Fernández-Villaverde et al. (2011), and characterize it as the fixed point of the third-order approximated policy functions in the absence of shocks. This is the point in which agents choose to remain with taking future uncertainty into account. Hence, this method allows us to study the effects of an increase in the uncertainty of the future path of interest rate without imposing any changes in the realized volatility of the interest rate per se.

3.3 Results

We, first, study how our model reacts to a 1% interest rate level shock, before studying the implications of increasing interest rate uncertainty on capital flows. Then, we focus on our main experiment, and look at how the model economy reacts to an increase in interest rate uncertainty. We, later, concentrate on the changes in the composition of capital account after an increase in interest rate uncertainty, and to identify the transmission channels of volatility shocks, we present the model
dynamics when several key features in our model are turned on and off.

### 3.3.1 Interest Rate Shocks versus Interest Rate Uncertainty Shocks

Figure 5 exhibits impulse responses after a 1% increase in $u_t$. To make comparison with the standard monetary open-economy models in the literature, we solve the model using a first-order approximation, and calculate impulse responses as deviations from non-stochastic steady state values.

The model reacts to a one-time exogenous increase in the interest rate reasonably: the effects to the economy are contractionary with national absorption. Most of the collapse in output is due to fall in investment for domestic physical capital. There is downward pressure on prices, and markups fall accordingly. The fall in household demand on goods is followed by a fall in labor supply. Firms lower their demand on both types of capital. Moreover, from uncovered interest parity condition, real exchange rate appreciates and investment in Home economy becomes more expensive. Hence, RoW agents’ investment in capital for Home production falls, and this leads to an outflow in the FDI component of the current account. Finally, the fall in consumption diminishes the supply of Home bonds and indicates an outflow in the short-term component of the current account. Most of the correction in the capital account is due to changes in the FDI component as investment dynamics are more volatile.

Now, we turn to our main experiment and focus on the dynamics after a 10 standard deviation increase in the exogenous component of the interest rate while there is no change in the level of the process. Figure 6 show the impulse responses. Our instant observations are that Home agents decrease their consumption and investment for domestic capital, which contribute to a fall in overall output. Households start to invest in physical capital that will be used in RoW production, where risk is lower. Because Home FDI going into RoW has a small share in Home economy, it does not compensate for the fall in output although there is negative comovement with the investment for Home physical capital. Increase in interest uncertainty makes the Home economy risky and agents decrease their consumption due to precautionary motives. Future uncertainty on marginal costs force firms to set higher prices, leading to amplification in price markups in both domestic and export markets. Markups rise because, for one reason, with consumption (demand) falling, prices do not fully accommodate the lower demand under rigidities. Increase in markups combined with
low rental payments to RoW for their capital in Home production and the appreciation in real exchange rate contribute to a decrease in FDI inflows to Home, leading to an increase in the FDI component of the current account. Home agents’ lower consumption further decreases supply of Home bonds, leading to an increase in the short-term component of the current account. Due to combined effect of rising markups and shift in investment, FDI component of the current account moves more significantly.

To further understand why real exchange rate appreciates, let’s rewrite equation (9) in the following form:

\[
1 = \mathbb{E}_t [\beta_{t,t+1}] \mathbb{E}_t \left[ \frac{rer_{t+1}}{rer_t} (r_{K,*t+1} + 1 - \delta) \right] + Cov \left( \beta_{t,t+1}, \frac{rer_{t+1}}{rer_t} (r_{K,*t+1} + 1 - \delta) \right).
\]

After an increase in the interest rate uncertainty, the covariance term in the above expression turns out to be negative, and hence, one can write:

\[
1 < \mathbb{E}_t [\beta_{t,t+1}] \mathbb{E}_t \left[ \frac{rer_{t+1}}{rer_t} (r_{K,*t+1} + 1 - \delta) \right].
\]

Furthermore, a fall in consumption implies a positive change in the stochastic discount factor, leaving the return from investing in RoW production positive. However, due to fall in marginal costs, our model implies a fall in the rental rate of capital that is produced for RoW production (in RoW consumption units). From the capital Euler equation above, the latter implies an appreciation in the real exchange rate. The intuition is simple: Given the increase in interest rate uncertainty, and the increase in the risk of Home economy, agents want to shift away their resources. However, in doing so, they demand higher return premium to continue investing in physical capital that will be used by RoW. With rental rates in RoW consumption units falling, only option is an appreciation in real exchange rate.

3.3.2 Interest Rate Uncertainty in Controlling the Current Account

In this subsection, we highlight the role of price rigidities in the model in transmission of interest rate uncertainty, and turn on/off the adjustment costs for wage and price settings to show their individual effects on model dynamics.
Precautionary Savings Channel and the Role of the Real Exchange Rate

Figure 7 represents dynamics after an increase in interest rate uncertainty as in the above experiment, but in the model version with no Rotemberg adjustment costs for the goods produced for domestic and export markets (i.e. $\kappa = \kappa^* = 0$). In the absence of adjustment costs in production, there are no time-varying inefficient wedges in the link between relative prices and marginal costs, and hence, prices move in a flexible manner. Flexible prices remove any distortion in FDI dynamics that is a result of rising markups in the production sector. In the absence of pricing frictions in production, inflation goes up due to rise in wage markup.

After an increase in the uncertainty in Home economy, consumption falls because of precautionary motives. Labor supply still falls, but in a less pronounced manner vis-a-vis to the baseline model. This is, again, because of an upward movement in the markup over marginal substitution that contributes to inflation. Output exhibits an instant positive jump due to shift of resources to investment in domestic capital. Uncertainty in Home economy makes Home debt riskier and RoW agents are deleveraging their holdings of Home debt. Hence, the short-term component of the current account increases.

The RoW counterpart of equation (9) sheds light on RoW investment in physical capital that is used in Home production:

$$1 = \mathbb{E}_t \left[ \beta^*_{r+1} \frac{r_{t+1}}{r_{t+1}} (r^*_{K,t+1} + 1 - \delta) \right].$$

Due to comovement in consumption between Home and RoW, RoW agents’ stochastic discount factor increases. When Home households start to invest in more for domestic capital, marginal product of capital falls for Home agents’ holdings of capital used in Home production. Factor pricing equation implies a similar for for the RoW agent’s physical capital used in Home production. The Euler equation above indicates that real exchange rate needs to be adjusted to generate future profits from renting capital to Home. The latter implies that expected change in real exchange rate has to be depreciation (from Home’s perspective). Depreciation in Home real exchange rate further makes investment to Home more profitable, and Home starts to attract FDI.

This scenario is in line with the policymaker’s goal of attracting FDI and discouraging short-term inflows in Home economy.
**Upward Pricing Bias Channel**

Oi (1961), Hartman (1972), and Abel (1983) show that if profits are convex in demand or costs, then uncertainty increases expected profits. More recently, Fernández-Villaverde et al. (2015) presented this effect in a closed economy where firms engage in precautionary pricing and move their prices upwards when production is subject to adjustment costs. A similar mechanism is at work in our model for FDI dynamics. To single out the effects of pricing frictions in the production sector, in Figure 8, we show model dynamics after removing wage adjustment frictions in the baseline economy.

Because Home firms’ profits are convex in prices, and asymmetric in response to price changes, under interest rate uncertainty, firms set a higher price than they would otherwise do. For example, if a factory expects to halve the profits in response to a price decrease, and only lose one-quarter of profits in response to a price increase, given induced uncertainty by the central bank, firms prefer to increase their prices. As can be seen in Figure 8, asymmetric profits contributes to a rise in markups in domestic and export markets, and therefore to a hike in inflation. Contraction in output further contributes to a fall in demand on capital, and Home investment falls for physical capital that is rented to Home firms. Instead, Home households shift their resources to rent capital to RoW agents, but this is a relatively small share of Home economy and does not help output from falling down. Due to the relationship between factor prices, renting less capital from Home households imply renting less capital from RoW households, and therefore a fall in the received FDI from RoW. This is reflected as a hike in the FDI component of the current account. Again, due to more volatile nature of investment, changes in the FDI composition of current account is more severe than in the short-term component of the current account.

This case suggests that if the economy is subject to pricing frictions in the production sector, using interest rate uncertainty as a policy tool does not act in line with the policymaker’s objective of attracting FDI and discouraging short-term inflows, simultaneously. Rather, net FDI entering into the economy diminishes.
4 Additional Results

In this section, we discuss how our results react to introducing new relevant features to the baseline model. First, to capture the long-run nature of FDI, we introduce time-to-build capital for investment in physical capital that will be used in overseas production. Second, due to central role of the real exchange rate in our results, we also study dynamics when firms engage in local currency pricing. Third, we look at the implications of using Epstein-Zin-Weil preferences, and deviate from time-separable utility. Finally, the years in which CBRT applied interest rate uncertainty policy coincides with near zero interest rates in advanced economies, and to capture the role of the zero-lower-bound (ZLB) in advanced economies in affecting outgoing FDI, we impose a zero-lower-bound constraint for RoW.

4.1 Time-to-build FDI

Given the long-run nature of FDI, we also study dynamics under the condition that it requires multiple periods for Home and RoW agents to build the physical capital that will be used in the overseas production process. In doing so, equation (3) will be replaced by the following conditions:

\[
K_{*,t+1}(h) = (1 - \delta)K_{*,t}(h) + I_{*,1,t}(h),
\]

\[
I_{*,j-1,t+1}(h) = I_{*,j,t}(h); \quad j = 2, \ldots, J.
\]

\[
I_{*,t}(h) = \sum_{j=1}^{J} \frac{1}{J} I_{*,j,t}(h)
\]

where \( \frac{1}{J} \) determines the fixed fraction of total investment expenditures allocated to projects that are \( j \) periods away from completion. \( I_{*,j,t}(h) \) is the project that is initiated in period \( t \) and \( j \) periods away from completion.

The conditions in (27) imply that, each period, households initiate projects that will be completed in \( J \) periods, and complete partially finished projects that are initiated in previous periods. Home households’ optimization problem subject to above constraints lead to following Euler equation for Home capital that will be used in RoW and the respective pricing equation for the outgoing FDI:

\[
q_{*,t+J-1} = \mathbb{E}_{t+J-1} \left[ \beta_{t+J-1,t+J} \left( r_{per_{t+J}} K_{*,t+J} + q_{*,t+J} (1 - \delta) \right) \right],
\]
\[ \mathbb{E}_t [\beta_{t,t+J-1}q_{*,t+J-1}] = \frac{1}{J} \left( r_{rt_1} + \mathbb{E}_t [\beta_{t,t+1}r_{rt+1}] + \ldots + \mathbb{E}_t [\beta_{t,t+J-1}r_{rt+J-1}] \right), \] (29)

Equations (28) and (29) show that the period \( t + J - 1 \) return of investment for the capital that will be completely built in period \( t + J \) depends on the rental rate (in foreign consumption units) and the expected fluctuations in real exchange rate. Equation (29) links the discounted marginal cost of each project initiated in subsequent periods to the expected discounted one-period beforehand price of investment.

Furthermore, now the current account can be written as:

\[
(b_{t+1} - b_t) + r_{rt} (b_{*,t+1} - b_{*,t}) + \frac{1}{J} \left( \left( \frac{1 - n}{n} \right) r_{rt} (K_{*,t+J} - K_{*,t}) - (K^*_{t+J} - K^*_t) \right) \equiv CA_t
\]

Appendix B shows the modifications in net foreign asset condition when FDI is subject to time-to-build.

Figure 9 shows the impulse responses to an increase in interest rate uncertainty when FDI is subject to 4-period time-to-build. Irreversibility of FDI contributes to a dampening in the net outflow of FDI from EME. EME households shift their investments toward the physical capital that will be used in RoW production, where risk is lower. However, because FDI takes 4-periods to be completed, they cannot generate immediate rental gains from shifting their investments, and instead, borrow instantaneously from international financial markets to prevent a further fall in their consumption. On the other hand, because EME debt is now riskier, RoW investors run away from EME debt and the bond outflows are observed two-periods after the rise in uncertainty in EME interest rate. Markups rise and contribute to an increase in inflation as before.

A comparison of current account dynamics under different periods of time-to-build is given in Figure 13. The figure shows that as the periods of time-to-build increase from two periods to four periods, net outflow of FDI becomes milder. However, as the sought rental gain from shifting investments is not received immediately, the instantaneous short-term debt is being amplified as it takes more time to build FDI.

Dampening in net FDI outflows when FDI is subject to time-to-build is related with the “growth options” argument that Bar-Ilan and Strange (1996) highlighted: uncertainty can encourage investment if there are delays in completing projects. Intuition is that if the costs are sunk, and the
expected profits are not constrained from above, firms still have incentives to invest in response to rising uncertainty. The policymaker’s objective of attracting long-run and discouraging short-run flows is still not achieved under this version of the model. Moreover, irreversibility of FDI increases short-run inflows to the EME immediately after using interest rate uncertainty as a policy tool. The policy is, again, inflationary as in previous versions.

4.2 Currency of Trade Invoicing

We assumed that domestic prices are always set in the currency of the consumer and the foreign price is set in the producer currency, in our baseline setting. Since the nature of price rigidity is central for the real effects of exchange rate fluctuations, in this subsection, we investigate the consequences when foreign price is set in the local currency, referred to as local currency pricing (LCP)\textsuperscript{21}.

Under LCP, the cost of adjustment in changing prices for the export market is also dependent on the fluctuations in the nominal exchange rate in addition to price differentials. Therefore, the movements in the nominal exchange rates affect the relative price of the firm in domestic and foreign markets, violating the law of one price. Appendix C provides the details about firms’ problem under LCP.

Figure 10 shows impulse response functions after an increase in interest rate uncertainty in a setting where firms engage in LCP. The immediate observation is that LCP reverses the behavior of net FDI flows in response to interest rate uncertainty. The increase in domestic inflation does not pass-through to export prices, making exports cheaper under LCP. Relative cheapness of exports contribute to an increase in export demand and a pressure on domestic firms to increase production. The latter generates a demand on capital, both from Home and RoW agents, contributing to the increase in net FDI inflows to the EME. FDI component of the current account gives a deficit as an instant response to an increasing uncertainty in the future path of domestic interest rate. This outcome is in line with the policymaker’s desire to attract long-run capital while discouraging short-run capital inflows.

Figure 14 shows the difference between the current account behavior under PCP versus LCP.\textsuperscript{21}

\textsuperscript{21}We focus on LCP and leave the analysis with dominant currency pricing (DCP) recently highlighted by Gopinath (2016) for future work.
Short-run debt of the EME is again riskier with an increase in uncertainty and it is discouraged. The outflow is similar under both price settings. If FDI is subject to time-to-build, the pass-through effect gets smaller over time due to long-run nature of FDI. Figure 15 shows that the instant net inflow of FDI gets smaller as FDI is subject to longer periods to build. While the net-inflow is getting smaller, export revenues are diminishing. With the required rental gains are not delivered instantaneously, an immediate rise in the short-term current account deficit takes place, as in the case under PCP. When FDI is subject to four periods to build, it is observed from Figure 16 that short term component of the current account experiences a larger initial deficit.

### 4.3 Epstein-Zin-Weil Preferences

Computing the equilibrium with recursive preferences are being used to break the link between risk aversion and inverse of the elasticity of intertemporal substitution (EIS). Because the source of fluctuations is an increase in uncertainty of the central bank policy rate, it is informative to identify the trade-offs between agents’ incentives on smoothing across states versus smoothing across time, in response to changes in interest rate risk. Therefore, in this subsection, we extend our analysis by assuming different degrees of risk aversion for the RoW agents and study their interaction with the EME.

We follow the literature and generalize equation (1) to an Epstein-Zin-Weil (Epstein and Zin (1989) and Weil (1989)) specification:

\[
V_t \equiv (1 - \beta)U(C_t(h), L_t(h)) - \beta \left[ \mathbb{E}_t (-V_{t+1})^{1-\alpha} \right]^{1/(1-\alpha)}, \quad (30)
\]

where \(\alpha \in \mathbb{R}\).\(^{22}\) When \(\alpha = 0\), (30) reduces to the standard expected utility in (1). With \(U \leq 0\) everywhere, lower values of \(\alpha\) correspond to greater degrees of risk aversion.

Now, the discount factor of RoW agents, \(\beta_{t,t+1}^w\), becomes

\[
\beta_{t,t+1}^w \equiv \frac{\beta U_{C,t+1}^w}{U_{C,t}^w} \left( \frac{-V_{t+1}^w}{\mathbb{E}_t [-V_{t+1}^{1-\alpha^*}]} \right)^{-\alpha^*}. \quad (31)
\]

\(^{22}\)Because our utility kernel employs an elastic intertemporal substitution (i.e. \(\rho = 2\)), \(U \leq 0\) everywhere and recursion is formulated as in (30). As highlighted by Rudebusch and Swanson (2012), when \(U \geq 0\) everywhere, it is natural to reformulate the recursion as \(V_t \equiv (1 - \beta)U(C_t(h), L_t(h)) + \beta \left[ \mathbb{E}_t (V_{t+1})^{1-\alpha} \right]^{1/(1-\alpha)}\).
With Epstein-Zin-Weil preferences, the discount factor now has an additional term reflecting the early resolution of uncertainty. With \( \alpha^* < 0 \), unfavorable changes in utility imply a higher discount factor for RoW agents.

Using (31) and its Home counterpart we can express the no-arbitrage condition as:

\[
\frac{R_t^{*+1}}{R_t^{*+1}} = \frac{(1 + \eta b_{t+1}^*)E_t\left[ \frac{\beta U_{C,t}^{*+1}}{U_{C,t}^{*+1}} \left( \frac{-V^{*+1}_{t+1}}{E_t[-V^{*+1}_{t+1}]} \right)^{\frac{1}{1-\alpha^*}} \right]}{(1 + \eta b_{t+1}^*)E_t\left[ \frac{\beta U_{C,t}^{*+1}}{U_{C,t}^{*+1}} \left( \frac{-V^{*+1}_{t+1}}{E_t[-V^{*+1}_{t+1}]} \right)^{\frac{1}{1-\alpha^*}} \right]} .
\]

Deviations from the uncovered interest parity is now additionally based on the discounted future utility stream and its time-varying risk component. Employing a third-order perturbation solution technique implies the covariance between the stochastic discount factor and the change in nominal exchange rates will be important to understand whether EME bonds are a better hedge with respect to RoW bonds, from the perspective of RoW agents.

Figure 17 compares the impulse responses to rising uncertainty in EME interest rate when RoW agents are becoming more risk averse. To highlight the role of RoW risk aversion on model dynamics, \( w.l.o.g. \), we compare simulations with \( \alpha^* \) being equal to 0, -48, and -148. Our calibration implies an approximate risk aversion of 25, and 80 for the values of \( \alpha^* \) we set.\(^{23}\)

As we increase RoW risk aversion, it is observed from the figure that there is a decrease in the investment made by RoW for the capital that is rented to Home economy, leading to an initial increase in the FDI component of the EME current account. The intuition is similar as in the baseline version of the model: Precautionary pricing of Home firms lead to higher markups, contributing to a decrease in profits. However, markups increase milder than in baseline model because a change in EME production prices have a more dominant effect on RoW investment behavior as RoW agents are more risk averse now. The change in prices is transformed into lower rental rates for the capital that is rented from RoW. The rental rate decreases quicker for higher degrees of RoW risk-aversion, reflecting the desire for the early resolution of uncertainty. In return, agents in RoW cut their investment in a more pronounced manner vis-a-vis to the behavior in the baseline model.

International linkages with EME imply a decrease in RoW consumption and increase in RoW

\(^{23}\)Swanson (2018) calculates risk aversion in models with flexible labor margin and we follow his calculations in setting risk aversion.
savings. Lower rental gains of RoW agents from investing in EME capital is transformed into a noticable increase in the real exchange rate appreciation (from EME perspective). Through the uncovered interest parity condition, a more pronounced increase in real exchange rate combined with higher RoW discount factor induces a lower short-term return from holding RoW bonds vis-a-vis to holding EME bonds. Although the source of risk is in EME interest rate, RoW agents increase their holdings of EME bonds, causing an inflow to EME. Hence, the behavior of the bond component of the current account is reversed with more risk-averse RoW agents.

The jump in markups is lower and they are transformed into a lower hike in EME inflation. The increase in EME policy uncertainty leads to short-term inflows together with long-term outflows and an increase in inflation.

4.4 ZERO-LOWER-BOUND IN THE REST OF THE WORLD

TBW

5 CONCLUSION

This paper presented a two-region macroeconomic model that is characterized by microeconomic foundations in combination with nominal rigidities, in which the effectiveness of using interest rate uncertainty to control the capital account, and to attract FDI while discouraging short-term inflows are assessed. Whether policymaker is successful in controlling the current account by using interest rate uncertainty a a policy tool depends on the type of price frictions, the long-run nature of FDI coming into the EME, and the currency of export invoicing.

In response to increased uncertainty in Home economy (EME), agents shift away from EME debt when smoothing consumption as a result of the precautionary saving motives in the model. This effect contributes to an increase in the short-term component of the current account. However, when production is subject to price rigidities, firms adjust their prices to higher levels than they would otherwise do due to asymmetry in their profit function. Increase in markups together with a fall in consumption imply lower rental rate paid to RoW households for their investment in EME capital, and contributes to a fall in FDI received by the EME. Precautionary pricing motive yields an increase in the FDI component of the current account, against the policymaker’s will. Whether
firms engage in PCP or LCP in trade invoicing changes the reaction of net FDI flows in response to interest rate uncertainty. There is a comovement of FDI flows with bond flows if the increase in domestic inflation pass-through to export prices. The direction of the bond component of the current is also dependent on the degree of risk aversion of RoW agents. More risk averse RoW agents tweak the uncovered interest parity condition, making savings through EME bonds more profitable. Finally, introducing time-to-build in FDI dampens the fluctuations in FDI flows. Our main takeaway from our analysis is that uncertainty can be used to adjust the external account depending on the pricing frictions that apply to the EME, but at the expense of higher inflation and lower output.

The results are relevant because the Central Bank of the Republic of Turkey (CBRT) conducted a similar policy in the near past with an objective of adjusting the composition of external account. This is the first paper which shows the transmission of interest rate uncertainty in an open economy framework that distinguishes short-term capital flows from FDI flows. Hence, our results contribute to both positive and normative aspects of such experiment.

An extension of the model that will capture an additional Emerging Market Economy feature will be to introduce an imperfect banking sector in the EME. Under this setting, increased uncertainty in the policy rate can aggravate financial frictions in the banking sector, contributing to a new channel in affecting capital flows. We leave this extension for future work.
References


Taylor, J. (2015): “Rethinking the International Monetary System”, Prepared manuscript for presentation at the Cato Institute Monetary Conference on Rethinking Monetary Policy.


### Tables

Table 1: Model Summary (Baseline)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 + \eta b_{t+1} = R_{t+1} E_t \left[ \tilde{b}<em>{t+1} \right] / \tilde{H}</em>{t+1}$</td>
<td>Euler equation, domestic bonds</td>
</tr>
<tr>
<td>$1 + \eta b_{st+1} = R^*<em>{t+1} E_t \left[ \tilde{b}</em>{t+1} \right] / \tilde{H}_{t+1}$</td>
<td>Euler equation, RoW bonds</td>
</tr>
<tr>
<td>$K_{t+1}(h) = (1 - \delta) K_t(h) + I_t(h)$</td>
<td>Law of motion of capital (Home)</td>
</tr>
<tr>
<td>$K_{s,t+1}(h) = (1 - \delta) K_{s,t}(h) + I_{s,t}(h)$</td>
<td>Law of motion of capital (FDI)</td>
</tr>
<tr>
<td>$1 = E_t \left[ \beta_{t+1} (r_{K,t+1} + 1 - \delta) \right]$</td>
<td>Euler equation, Home capital</td>
</tr>
<tr>
<td>$1 = E_t \left[ \beta_{t+1} \left( r_{K,t+1} + 1 - \delta \right) \right]$</td>
<td>Euler equation, FDI</td>
</tr>
<tr>
<td>$w_t = \mu_t W \left( \chi L_t \right) / C_t$</td>
<td>Real wage</td>
</tr>
<tr>
<td>$Y_{H,t} = a \left( \frac{p_{H_t}}{P_t} \right)^{-\omega} Y_t$</td>
<td>Demand functions</td>
</tr>
<tr>
<td>$Y_{F,t} = (1 - a) \left( \frac{p_{F,t}}{P_t} \right)^{-\omega} Y_t$</td>
<td>Price index</td>
</tr>
<tr>
<td>$1 = \left( a \cdot r_{H,t}^{1-\omega} + (1 - a) r_{F,t}^{1-\omega} \right) \frac{1}{1-\omega}$</td>
<td>Marginal cost of intermediate good production</td>
</tr>
<tr>
<td>$r p_{H,t} = \mu_t m c_t$</td>
<td>Relative price of goods sold at Home</td>
</tr>
<tr>
<td>$r p_{H,t}^* = \frac{\mu_t m c_t}{r_{rer}^{1-\omega}}$</td>
<td>Relative price of exports</td>
</tr>
<tr>
<td>$Y_{H,t} + \left( \frac{1-n}{n} \right) Y_{H,t}^* = K_t^{\alpha_1} K_t^{\alpha_2} L_t^{1-\alpha_1-\alpha_2}$</td>
<td>Intermediate good production</td>
</tr>
<tr>
<td>$\alpha_1 w_t L_t = (1 - \alpha_1 - \alpha_2) r_{K,t} K_t$</td>
<td>Factors of production</td>
</tr>
<tr>
<td>$\alpha_2 r_{K,t} K_t = \alpha_3 r_{K,t} K_t^*$</td>
<td>Resource constraint</td>
</tr>
<tr>
<td>$I_t = C_t + I_t + \alpha_1 w_t \left( \Pi_{H}^W - 1 \right)^2 w_t L_t + \frac{\alpha_1}{2} (\Pi_{H,t} - 1)^2 r p_{H,t} Y_{H,t} + \left( \frac{1-n}{n} \right) \frac{\alpha_1}{2} (\Pi_{H,t}^h - 1)^2 r p_{H,t}^h Y_{H,t}$</td>
<td>Resource constraint</td>
</tr>
<tr>
<td>$b_{t+1} + r_{rer} b_{s,t+1} + \left( \frac{1-n}{n} \right) r_{rer} K_{s,t+1} - K_{t+1}^*$</td>
<td>Resource constraint</td>
</tr>
<tr>
<td>$= \frac{R_t}{R^<em>} b_t + \frac{R^</em>}{R_t} r_{rer} b_{s,t} + \left( \frac{1-n}{n} \right) r_{rer} (r_{K,s,t+1} + 1 - \delta) K_{s,t}$</td>
<td>Resource constraint</td>
</tr>
<tr>
<td>$= \left( \frac{R_{s,t+1}}{R_t} \right)^{\rho_R} \left( \frac{R_{s,t}}{R_t} \right) (1-\rho_R) p u \left( Y_{s,t} \right)^{(1-\rho_R) \nu} e_{a_t}$</td>
<td>Resource constraint</td>
</tr>
<tr>
<td>$\frac{R_t}{R^*} = \left( \frac{R_{s,t+1}}{R_t} \right)^{\rho_R} \left( \frac{R_{s,t}}{R_t} \right) (1-\rho_R) p u \left( Y_{s,t} \right)^{(1-\rho_R) \nu} e_{a_t}$</td>
<td>Resource constraint</td>
</tr>
</tbody>
</table>

31
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.9804</td>
</tr>
<tr>
<td>Relative risk aversion</td>
<td>$\rho$</td>
<td>2</td>
</tr>
<tr>
<td>Relative weight of labor in utility</td>
<td>$\chi$</td>
<td>1</td>
</tr>
<tr>
<td>Frisch elasticity</td>
<td>$\varphi$</td>
<td>0.25</td>
</tr>
<tr>
<td>Bond adjustment</td>
<td>$\psi$</td>
<td>0.0025</td>
</tr>
<tr>
<td>Rotemberg wage adjustment</td>
<td>$\kappa^W$</td>
<td>116</td>
</tr>
<tr>
<td>Elasticity of substitution of differentiated labor</td>
<td>$e^W$</td>
<td>11</td>
</tr>
<tr>
<td>Home bias</td>
<td>$a$</td>
<td>0.65</td>
</tr>
<tr>
<td>Share of domestic capital</td>
<td>$\alpha_1$</td>
<td>0.30</td>
</tr>
<tr>
<td>Share of foreign capital</td>
<td>$\alpha_2$</td>
<td>0.15</td>
</tr>
<tr>
<td>Rotemberg domestic price adjustment</td>
<td>$\kappa$</td>
<td>116</td>
</tr>
<tr>
<td>Rotemberg export price adjustment</td>
<td>$\kappa^*$</td>
<td>116</td>
</tr>
<tr>
<td>Elasticity of substitution of differentiated goods</td>
<td>$\epsilon$</td>
<td>11</td>
</tr>
<tr>
<td>Interest rate smoothing coefficient</td>
<td>$\rho_R$</td>
<td>0.7</td>
</tr>
<tr>
<td>Steady state response to inflation</td>
<td>$\rho_H$</td>
<td>1.5</td>
</tr>
<tr>
<td>Steady state response to output</td>
<td>$\rho_Y$</td>
<td>0.5/4</td>
</tr>
</tbody>
</table>
FIGURES

Figure 1: Portfolio Flows to EMEs (13-week moving average, bn USD)

Source: Central Bank of the Republic of Turkey

Figure 2: Interest Rate Corridor and Average Funding Cost, Turkey

Source: Central Bank of the Republic of Turkey
Figure 3: Financing the Current Account, Turkey (12-month cumulative, bn USD)

Source: Central Bank of the Republic of Turkey

Figure 4: Model Architecture
Figure 5: Impulse Responses to an Increase in Interest Rate Level
Figure 6: Impulse Responses to an Increase in Interest Rate Uncertainty (baseline)
Figure 7: Impulse Responses to an Increase in Interest Rate Uncertainty ($\kappa = \kappa^* = 0$)
Figure 8: Impulse Responses to an Increase in Interest Rate Uncertainty ($\omega = 0$)
Figure 9: Impulse Responses to an Increase in Interest Rate Uncertainty (4-period time-to-build)
Figure 10: Impulse Responses to an Increase in Interest Rate Uncertainty (LCP)
Figure 11: Impulse Responses to an Increase in Interest Rate Uncertainty (LCP, 4-period time-to-build FDI)
Figure 12: Comparison of Current Account Dynamics I

Figure 13: Comparison of Current Account Dynamics II
Figure 14: Comparison of Current Account Dynamics III

Figure 15: Comparison of Current Account Dynamics IV
Figure 16: Comparison of Current Account Dynamics V

Bond component of the Current Account

- Baseline (PCP, 4-period time-to-build)
- LCP, 4-period time-to-build

FDI Component of the Current Account

% deviation

- $4 \times 10^{-3}$
Figure 17: Impulse Responses to an Increase in Interest Rate Uncertainty (Recursive Preferences)
Figure 18: Impulse Responses to an Increase in Interest Rate Uncertainty (Recursive Preferences, same degree of risk aversion in EME and RoW)
A: Derivation of Net Foreign Assets

Start with Home households’ budget constraint, equation (5), divide it by $P_t$, and impose $T_t = \frac{n}{2} \left[ P_t \left( \frac{B_{t+1}(h)}{P_t} \right)^2 + S_t P_t^* \left( \frac{B_{t+1}(h)}{P_t} \right)^2 \right]$ and equation (22) to obtain:

$$b_{t+1} + \text{rer}_t b_{s,t+1} + \left( \frac{1-n}{n} \right) \text{rer}_t I_{s,t} = \left( \frac{R_t}{P_t} \right) b_t + \left( \frac{R_t^*}{P_t^*} \right) \text{rer}_t b_{s,t} + w_t L_t + \text{rer}_t r_{K,t} K_t + \left( \frac{1-n}{n} \right) \text{rer}_t r_{K,s,t} K_{s,t} + I_t^* + (\mu_{H,t} - 1) m_{c,t} Y_{H,t} + \left( \frac{1-n}{n} \right) \left( \mu_{H,t}^* - 1 \right) m_{c,t} Y_{H,t}^* - Y_t.$$  

Now, use $w_t L_t + \text{rer}_t r_{K,t} K_t = m_{c,t} \left( Y_{H,t} + \left( \frac{1-n}{n} \right) Y_{H,t}^* \right) - r_{K,t} K_t^*$ to get:

$$b_{t+1} + \text{rer}_t b_{s,t+1} + \left( \frac{1-n}{n} \right) \text{rer}_t I_{s,t} - I_t^* = \left( \frac{R_t}{P_t} \right) b_t + \left( \frac{R_t^*}{P_t^*} \right) \text{rer}_t b_{s,t} + \left( \frac{1-n}{n} \right) \text{rer}_t r_{K,s,t} K_{s,t} - r_{K,t} K_t^* + \left( \frac{1-n}{n} \right) \mu_{H,t}^* m_{c,t} Y_{H,t}^* - \text{rer}_t \mu_{F,t} m_{c,t}^* Y_{F,t}.$$  

Use isomorphic equations for the RoW to obtain:

$$b_{s,t+1}^{*} + \frac{b^*_{t+1}}{rer_t} + \left( \frac{n}{n-1} \right) \frac{I_t^*}{rer_t} - I_{s,t} = \left( \frac{R_t^*}{P_t^*} \right) b_{s,t}^{*} + \left( \frac{R_t}{P_t} \right) b_{s,t} + \left( \frac{n}{n-1} \right) \mu_{F,t}^* Y_{F,t} - \frac{mc_t^* Y_{H,t}^*}{rer_t \mu_{H,t}^*} Y_{H,t}.$$  

Now, multiply equation (32) with $rer_t (1-n)$, subtract it from equation (33) and impose the bond market clearing conditions, $nb_{t+1} + (1-n)b_{s,t+1} = 0$ and $nb_{s,t+1} + (1-n)b_{s,t+1} = 0$:

$$2n(b_{t+1} + \text{rer}_t b_{s,t+1}) + 2((1-n)rer_t I_{s,t} - nI_t^*) = 2n \left( \left( \frac{R_t}{P_t} \right) b_t + \left( \frac{R_t^*}{P_t^*} \right) \text{rer}_t b_{s,t} \right)$$  

$$+ 2(1-n)rer_t r_{K,t} K_{s,t} + 2r_{K,t} K_{s,t}^* + 2(1-n)\mu_{H,t}^* m_{c,t} Y_{H,t}^* - 2\text{rer}_t \mu_{F,t} m_{c,t}^* Y_{F,t}.$$  

Finally, divide the above equation with $2n$, and impose law of motion of capital for $K^*$ and $K_s$ to obtain equation (23):

$$b_{t+1} + \text{rer}_t b_{s,t+1} + \left( \frac{1-n}{n} \right) \text{rer}_t K_{s,t+1} - K_{t+1}^* = \frac{R_t}{P_t} b_t + \frac{R_t^*}{P_t^*} \text{rer}_t b_{s,t} + \left( \frac{1-n}{n} \right) \text{rer}_t (r_{K,t} + 1 - \delta) K_{s,t} - \left( r_{K,t}^* + 1 - \delta \right) K_t^* + TB_t,$$

where $TB_t = \left( \frac{(1-n)}{n} \right) \mu_{H,t}^* m_{c,t} Y_{H,t}^* - \text{rer}_t \mu_{F,t} m_{c,t}^* Y_{F,t}$. 

47
B: Modifications with Time-to-Build

Starting from Equation (34), let’s impose bond market clearing conditions and First combine the conditions in (27) and their RoW counterpart to obtain:

\[ I_{t} = \frac{1}{J} \left[ (K_{*,t+1} - (1 - \delta)K_{*,t} + ... + (K_{*,t+J} - (1 - \delta)K_{*,t+J-1}) \right] \]

\[ I_{t}^{*} = \frac{1}{J} \left[ (K_{*,t+1}^{*} - (1 - \delta)K_{*,t}^{*} + ... + (K_{*,t+J}^{*} - (1 - \delta)K_{*,t+J-1}) \right] \]

Now, starting from Equation (34), let’s impose the bond market clearing conditions and the conditions above to obtain the modified net foreign asset equation:

\[ b_{t+1} + rer_{t}b_{*,t+1} + \left( \frac{1-n}{n} \right) rer_{t} \frac{1}{J} \left[ (K_{*,t+1} + \delta K_{*,t+J-1} + ... + \delta K_{*,t+J-1} - \frac{1}{J} \left( (K_{*,t+J} + \delta K_{*,t+J-1} + ... + \delta K_{*,t+J-1} \right) \right] = \frac{R_t}{n} b_t + \frac{R_t}{n} rer_t b_{*,t} + \left( \frac{1-n}{n} \right) rer_t \left( r_{K_{*,t+1}} + \frac{1}{J} (1 - \delta) K_{*,t} - \left( r_{K_{*,t+1}} + \frac{1}{J} (1 - \delta) K_{*,t} + TB_t, \right. \]

(35)

C: Local Currency Pricing

The export price is set in RoW currency under LCP. The cost of adjusting the export price is given as follows:

\[ \left( \frac{1-n}{n} \right) \kappa^{*} \left( \frac{P_{H,t+s}^{*}(i)}{P_{H,t+s-1}^{*}(i)} - 1 \right)^{2} \frac{S_{t+s}P_{H,t+s}^{*}(i)}{P_{t+s}} Y_{H,t+s}^{*}. \]

The monopolistic producer \( i \) chooses a rule \((P_{H,t}(i), P_{H,t}^{*}(i), Y_{H,t}(i), Y_{H,t}^{*}(i))\) to maximize the expected discounted profit:

\[ E_t \left[ \sum_{s=t}^{\infty} \beta_{t,t+s} \left( \left( 1 - \frac{n}{n} \right) \left( 1 - \frac{\kappa^{*}}{2} \left( \frac{P_{H,t+s}^{*}(i)}{P_{H,t+s-1}^{*}(i)} - 1 \right)^{2} \frac{S_{t+s}P_{H,t+s}^{*}(i)}{P_{t+s}} Y_{H,t+s}^{*}(i) \right) \right) \right]. \]

From the first-order-conditions with respect to \( P_{H,t+s}(i) \) and \( P_{H,t+s}^{*}(i) \) evaluated under symmetric equilibrium, we obtain the real price of Home output for domestic sales (i.e. \( rp_{H} = \frac{P_{H}}{P_{F}} \)) as a time-varying markup, \( \mu_{H,t} \) over the marginal cost:

\[ rp_{H,t} = \mu_{H,t}mc_{t}, \]
and the real price of Home output for export sales (in units of RoW consumption) as a time-varying markup, $\mu^*_H,t$, over the marginal cost

$$rp^*_H,t = \frac{\mu^*_H,t mc_t}{rer_t},$$

where

$$\mu^*_H,t \equiv \epsilon \frac{(\epsilon - 1) \left( 1 - \frac{\kappa}{2} (\Pi^*_H,t - 1)^2 \right) + \kappa \left( \Pi^*_H,t (\Pi_H,t - 1) - \mathbb{E}_t \left[ \frac{\beta_{t+1}}{\Pi_{t+1}} (\Pi^*_H,t+1 - 1)(\Pi_H,t+1)^2 \frac{Y_{t+1}}{Y_{H,t}} \right] \right)}{\left( \epsilon - 1 \right) \left( 1 - \frac{\kappa}{2} (\Pi^*_H,t - 1)^2 \right) + \kappa \left( \Pi^*_H,t (\Pi_H,t - 1) - \mathbb{E}_t \left[ \frac{\beta_{t+1}}{\Pi_{t+1}} (\Pi^*_H,t+1 - 1)(\Pi_H,t+1)^2 \frac{rer_{t+1}}{rer_t} \frac{Y_{t+1}}{Y_{H,t}} \right] \right)}.$$

with $\Pi^*_H,t \equiv \frac{rp^*_H,t}{rp^*_H,t-1} \Pi_t$ and $\Pi^*_H,t \equiv \frac{rp^*_H,t}{rp^*_H,t-1} \Pi_t$. 

49