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ABSTRACT

A Job Ladder Model with Stochastic Employment Opportunities*

We set up a model with on-the-job search in which firms infrequently post vacancies for which workers occasionally apply. The model nests the standard job ladder and stock-flow models as special cases while remaining analytically tractable and easy to estimate from standard panel data sets. Structurally estimating the model, the parameters are significantly different from the stock-flow or the job ladder model. Further, the estimated parameters governing workers search behavior are found to be consistent with recent survey evidence documented in Faberman et al. (2016). The search behavior implied by the standard job ladder model significantly understates the search option associated with employment (and thus underestimates the replacement ratio). Finally, the standard model is unable to generate the decline in the job finding rate and starting wage with duration of unemployment, both of which are present in our data.

JEL Classification: J31, J64

Keywords: on-the-job search, wage dispersion, wage posting, stock-flow

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1 Introduction

Due to their effectiveness in replicating labor turnover and wage dynamics, search models are extensively used to evaluate labor market policies.[1] Embedded in the standard job ladder model is *a single* friction that prevents the reallocation of workers into more productive jobs. In this paper, we set up a model in which, for a given worker, suitable vacancies are posted infrequently by firms and applications to these are made intermittently by workers. As special cases, our model nests the standard job ladder model and a version of the stock-flow model. In the estimation, we reject the restrictions implied by the standard job ladder model and we find that imposing these restrictions significantly underestimates the search option associated with employment. Furthermore, the model generates a declining job finding rate with the duration of unemployment and substantial wage losses following displacement, which increase in the duration of unemployment.

In the model, only some job openings are suitable for a given worker.[2] This set of job opportunities is treated as a latent variable that follows a stochastic process. Firms create suitable job openings at some Poisson rate similar to McCall (1970). At some Poisson rate, a firm will stop looking for workers. On the other hand, similar to Stigler (1962), workers infrequently send out multiple applications. The (potentially) differential rates by employment status is supposed to capture the differential search behavior; see, for example, Blau and Robins (1990) and Faberman et al. (2016). The worker subsequently accepts the best offer, if it is better than her current job. Firms differ in productivity and workers have an individual skill component. We close the model by assuming

[1] Recent evaluations include enforcement policy on informal firms (Meghir et al., 2015), public sector wage and employment policy (Bradley et al., 2017) and tax policy (Sleet and Yazici, 2017).

[2] There is a number of papers emphasizing differences, (e.g., skill, location) between available vacancies and the unemployed, creating thin markets, and thereby resulting in the simultaneous coexistence of unemployed workers and vacancies; see, for example, Lucas and Prescott (1974), Coles and Smith (1998), Shimer (2007), Alvarez and Shimer (2011), and Carrillo-Tudela and Visschers (2013). In our model, the stock of prospects can be interpreted as the local conditions for the worker and a number of the unemployed have no prospects and are therefore "mismatch" unemployed. Furthermore, the mismatch and stock-flow family of models generate similar employment dynamics as our model and are also able to generate the declining job finding rate with the duration of unemployment.

that firms post wage schedules in worker productivity, prior to meeting the worker.[3] In deciding on the optimal wage, a firm trades off the higher chance of hiring a worker and the longer expected duration of the match against the higher wage cost. The resulting model nests the workhorse empirical labor models of Burdett and Mortensen (1998), and models of stock-flow matching, pioneered by Coles and Smith (1998), as special cases. Like Burdett and Mortensen (1998), the model has an analytical closed form solution and is well identified and empirically tractable, allowing us to estimate all of the parameters using a panel data set on wage and employment dynamics.

We estimate the model using a two-step procedure for different skill groups, assuming that the labor market is segmented by the workers' level of education. In the first step, the parameters governing workers' search behavior are identified by the flows between labor markets states and the duration dependence of the transition rate from unemployment to employment. These moments are calculated from the Current Population Survey (CPS). In the second step, the parameters governing worker and firm productivity distributions are identified from the distribution of average wages across workers as well as the overall distribution of wages. In the second step, we use the Survey of Income and Program Participation (SIPP). The estimated model matches the transition rates from employment to unemployment and out of the labor force and the declining job finding rate with the duration of unemployment. The decline in the job finding rate with the duration of unemployment is due to the fact that those newly unemployed do, on average, have better prospects than the long-term unemployed. In the job ladder model, all unemployed are the same, i.e., unemployment is a single state, which implies that the model is unable to match the falling job finding rate with the duration of unemployment.[4] In the context of our model, the decline of the

[3]The main results of the paper are unchanged if the worker and the firm instead bargain over the wage after the match has been formed. The implications of alternative assumptions on the nature of wage setting are discussed in Section 2.6.

[4]Belot et al. (2016) run an experiment whereby the job seekers are exposed to greater number of vacancies. The treated had an increased number of interviews and particularly so for the long-term unemployed. This result is broadly consistent with our estimated model which suggests that an important driver of unemployment is a lack of labor market opportunities rather than the frequency at which unemployed workers apply. A labor market opportunity, in the context of our model, is defined as the point of application, a job that a worker is: (i) aware of; and (ii) suitable for. We thus interpret a policy increasing the set of jobs that workers are aware of as increasing the frequency at which labor market opportunities arise.

job finding rate is informative of the importance of worker's employment prospects. In the CPS data, the chance that an unemployed worker has a job in a month's time halves over the first three months of an unemployment spell.

The estimated parameters imply that fewer posted vacancies are suitable for unemployed workers. On the other hand, unemployed workers send out applications more often: twice a month compared to less than twice a year for their employed counterparts. Faberman et al. (2016) document that the unemployed send out a much larger number of applications but the number of contacts are similar. (We explore the relation between the empirical observation of Faberman et al. (2016) and our estimated search process in greater detail in section 3.8.) In our model, unemployment is both due to the infrequent applications made by workers as well as the workers lack of suitable jobs. If all workers were to have some prospects, i.e., the limit as their arrival rate goes to infinity, the unemployment rate would fall by a bit more than a half. In contrast, in the benchmark model of Burdett and Mortensen (1998), unemployment disappears as the arrival rate of job contacts increase. The relative importance of applications versus availability of prospects differs starkly by employment state. This suggests that a one friction representation of the labor market might be particularly poor. In particular, compared to the standard job ladder model, the estimated baseline model implies that the search option is much greater for employed workers (thus, the model generates much more frictional wage dispersion for a given replacement ratio). Second, the model fits the hazard rate out of unemployment and earnings losses associated with job displacement that increase with the duration of unemployment, without relying on human capital depreciation.

In the canonical job ladder model, the arrival rate of job offers differs by employment status. Taking a job is thus associated with a change in the search options. The monetary value of the search option depends on the differences, by employment status, in arrival rates and wage offer distributions. Hornstein et al. (2011) suggest that a particularly suitable metric for assessing the option value of search is the ratio of the mean to the minimum wage, hereafter the mean-min ratio (Mm). The probability that an unemployed worker is employed in a month's time is about

40% when calculated in the CPS. In the same data set, the probability that an employed worker changes employers is about 3%. In an estimated job ladder model, these numbers (together with the separation rate) imply that the unemployed get jobs much more frequently. The difference between lowest wage and the flow benefit must at least offset this loss in search option. Hornstein et al. (2011) find that the monetary value of the search option is very large which implies that the standard job ladder model generates very little frictional wage dispersion, with a flow benefit in unemployment consistent with the macro labor literature.

In order to generate a Mm ratio of close to two, our estimation of the Burdett and Mortensen (1998) model requires a negative flow benefit associated with unemployment. On the other hand, our estimated baseline model with replacement ratios in the order of 25-50% matches the same frictional wage distribution. The search option in Burdett and Mortensen (1998) is the difference in the job offer arrival rates multiplied by the expected increase in worker value. This is not the case in our model since employed workers will, on average, have more offers to choose from. This enters the worker value function through two channels. First, it is as if the employed workers sample wages from a distribution that stochastically dominates that of their unemployed counterparts. This is consistent with recent evidence from Faberman et al. (2016). Second, consistent with the data, after losing her job, a worker will, on average, find a job more quickly than the long-term unemployed. Interpreting the data through the lens of the Burdett and Mortensen (1998) model thus overestimates the foregone search option and hence underestimates the flow benefit in unemployment. This occurs as the model, only relying on the different transition rates, misses the better wage offers received in employment and the better position the worker is in if she subsequently gets fired. The replacement ratio is also consequential for the ability of the model to generate cyclical fluctuations in the unemployment rate.[5]

[5] Shimer (2005) shows that observed labor productivity is not variable enough compared to the empirically observed variations seen in market tightness (level of vacancies divided by unemployment) in the standard calibration of the Mortensen and Pissarides (1994) model. A free entry condition pins down the level of market tightness and any cyclical fluctuations come through variation in profit. The variation in labor productivity over the business cycle is typically found to be small. In order for this to translate into large changes in profit, it has to be that the level of profit is also small (Ljungqvist and Sargent, 2015). If the flow benefit of unemployment is high, then the profit will be low and the model is able to generate a sufficient amplification to productivity shocks (Hagedorn and Manovskii,

Empirically, workers displaced in mass layoffs experience a substantial and persistent earnings loss (Jacobson et al., 1993; Davis and Wachter, 2011). In addition, the average starting wages and the job finding rate decrease with the duration of unemployment. In a search model without OJS or stochastic match quality, there is no wage loss following displacement as the average outstanding wage is equal to the average starting wage. In a job ladder model, on the other hand, workers gradually select into better paying jobs. The average employed worker will thus receive a higher wage than the average worker coming from unemployment. Thus, a displaced worker will, on average, experience a wage loss, but these losses do not increase with the duration of unemployment. General human capital depreciation in unemployment generates wages falling with the duration of unemployment and thereby additional losses. For this reason, researchers studying earnings losses using job ladder models have incorporated falling general human capital in unemployment as a key driving mechanism.[6] However, falling general human capital would entail falling reservation wages with the duration of unemployment which is inconsistent with recent evidence from Krueger and Mueller (2016). In addition, since all unemployed search in the same market, the standard assumption of log linear production (and benefits) generates a constant job finding rate with the duration of unemployment. If instead the matching set were to decrease, the continuously falling human capital would imply the same for the job finding rate which is inconsistent with the empirical observation that the job finding rate falls quickly in the first three months but is broadly constant thereafter. Our model, in addition to featuring a positive selection into better jobs, also features an additional state variable - employment prospects. When the model is estimated, we find that the newly unemployed do, on average, have more prospects than the long-term unemployed. This implies that the job finding rate falls with the duration of unemployment as, via dynamic

2008). Elsby and Michaels (2013) consider a model in which firms exhibit decreasing returns to scale. Marginal productivity is then less than average productivity and therefore movements in marginal productivity can be greater than movements at the aggregate level. In our paper, we find that the flow value of unemployment is indeed high compared to the marginal job.

[6] See, for example, Krolikowski (2017), Jarosch (2015), and Burdett et al. (2017). The papers incorporate stochastic human capital into a standard job ladder model, although with different wage setting mechanisms. Depreciation of human capital in unemployment is required to explain the size and, in particular, the persistence of earnings losses following job displacement.

selection, workers with more prospects exit and those without prospects remain. Similarly, via this mechanism, the average starting wage also falls with the duration of unemployment. Our model thus jointly fits: a falling job finding rate with the duration of unemployment and a large wage loss following displacement that is increasing in the duration of unemployment.

Similar to our paper, there is a number of search papers with multiple meetings and applications. However, the key friction in these models differs from our own. A large number of papers have modeled a thick market where workers make multiple applications and are able to direct their search (Albrecht et al., 2006; Kircher and Galenianos, 2009; Kircher, 2009; Wolthoff, 2017).[7] Closer to our paper are papers where workers are unable to direct their search but meet multiple firms (Elliott, 2014; Wolthoff, 2014; Gautier and Holzner, 2017). In all these models, either the number of applications a worker can make is exogenous, or each application carries an additional cost. Workers are ex ante homogeneous in their market conditions and ex post heterogeneous in their position in a network. Our model takes a complementary approach where instead workers are ex-ante heterogeneous in the thickness of their individual markets, and the number of potential opportunities follows a stochastic process.

Following Hornstein et al. (2011), a number of recent studies have examined the ability of search models to generate sufficient frictional wage dispersion. Either there has to be a counterweighting effect that offsets the foregone search option, or it must be that the search option is not measured correctly. For example, if human capital depreciates quickly in unemployment, then that can motivate workers to take a low paid job (Ortego-Marti, 2016). Such an explanation would entail reservation wages falling quickly with the duration of unemployment which contradicts recent survey evidence (Krueger and Mueller, 2016). Within a sequential auctions framework, like in Postel-Vinay and Robin (2002) and Cahuc et al. (2006), the bargaining position of the worker increases when the worker takes a new job. These models can then generate more wage dispersion via this foot in the door effect (Papp, 2013). A foot in the door effect is also present in Carrillo-Tudela (2009) where there is no search option in unemployment. Faberman et al. (2016) is the closest to this

[7] In these models, the worker faces two problems: a portfolio problem in deciding which jobs to apply for; as well as the optimum number of applications to send out.

paper. They consider a job ladder model with exogenously different wage offer distributions for employed and unemployed workers. When a worker is employed, the lower arrival rate of job offers is partly offset by a better offer distribution. In our model, the offer distribution is a time varying object and the average distribution faced by the employed stochastically dominates the distribution facing the unemployed. Our paper can thus be seen as a micro foundation for the two different offer distributions documented in Faberman et al. (2016).

Outline. The rest of this paper is structured as follows. In section 2, we set up the model and provide the analytical solution. In section 3, we present the estimation of the model and the quantitative results. Section 4 concludes the paper.

2 Model

2.1 The Environment

Time is continuous and the labor market is populated by risk-neutral workers and firms. Workers leave the labor force at a Poisson rate μ and are replaced by a doppelgänger in unemployment. Workers are ex-ante heterogeneous in their productivity x, distributed with the cumulative distribution $\Gamma_x(\cdot)$ and ex-post vary in their employment state $s \in \{u, e\}$, their employment opportunities j, and if employed, their wage w. Firms are infinitely lived and are heterogeneous in their productivity y. The cumulative distribution of productivity amongst firms is given by $\Gamma_y(\cdot)$. Both worker and firm productivity distributions are primitives of the model. The total output of a match is the product of worker and firm types, xy. In unemployment, workers earn a flow income proportional to their productivity type, bx. Jobs become unprofitable at an exogenous rate δ which results in the worker entering the pool of unemployed. Finally, we do not allow workers to quit, other than to move to a new job.

The Frictions. The labor market is characterized by search frictions. Individual workers differ in their labor market opportunities which is a latent variable. These opportunities evolve stochastically. A worker amasses job opportunities according to a Poisson process as firms post vacancies. This Poisson rate, denoted by λ_s, differs by the employment state s. A firm stops hiring workers, i.e., the worker loses the job prospect, at a rate υ. The parameter υ captures, in a reduced form way, a variety of mechanisms: the job becomes unprofitable; the job is taken by another worker; the vacancy expires.

Unlike the sequential search literature, pioneered by McCall (1970), these opportunities are not continuously sampled. Instead, there is a stock of outstanding vacancies, which we refer to as the worker's employment prospects, or simply her stock. In the stock-flow model of Coles and Smith (1998), a worker can always match with the stock, whereas we assume that this opportunity arises at a Poisson rate γ_s. If a worker can always match with the stock, the model has an "instantaneous" property: after being fired, the worker will either immediately match or otherwise wait for the inflow of new vacancies. A finite value for the application rate γ_s implies that workers match with the stock intermittently. Some workers will have no prospects and will wait for the flow to increase the stock of prospects, which is analogous to a worker waiting to match with the flow in a standard discrete time stock-flow matching model. When the worker matches with the stock of vacancies, she chooses the most appropriate option. We assume that all rejected vacancies will no longer consider the worker. We argue that this captures a realistic feature of the labor market and is similar to the assumption that workers cannot return to their previous employer. In Appendix A.1.1, we describe the flow equations for the distribution of job opportunities, $j \in \mathbb{N}_+$.

Each employment prospect arrives with a wage that is set optimally by profit maximizing firms. These firms can post wages conditional on worker type x. In this environment, a firm's optimal strategy will be to post piece rate contracts in worker type, as in Barlevy (2008). Workers draw piece rate wage offers from a cumulative distribution function $F(w)$ – an endogenous object of the model.

Relation to existing models. Notice that a number of commonly used search models are nested in our framework. First, in the absence of dynamic market thickness, i.e., as $\gamma_s \to \infty$ $\forall s \in \{e, u\}$, the model converges to the standard job ladder model of Burdett and Mortensen (1998). As workers continuously apply, the number of prospects j is either one or zero and hence, there are no dynamics in the thickness of markets. Second, with no on-the-job search, $\gamma_e = 0$, and with applications made continuously by the unemployed, $\gamma_u \to \infty$, the model nests a version of the stock-flow matching function in continuous time; see Coles and Smith (1998). In this case, after separating from a firm, the worker will immediately match with the stock of available prospects. If the stock is non-empty, the worker will directly transition to a new job, whereas if the stock is empty, the worker has to wait for the inflow of new vacancies. Lastly, if $\gamma_e = 0$, and the unemployed infrequently apply for jobs, $\gamma_u \in (0, \infty)$, then the model shares the feature of stock-flow matching but with search frictions. This case corresponds to a wage setting environment similar to a dynamic version of the Burdett and Judd (1983) model. We estimate the baseline model as well as the model without dynamics of market thickness (NDT), and without on-the-job search (NOJS).

2.2 Worker problem

An individual's utility is given by the present expected discounted value of her future income stream. This will depend on her employment status and opportunity stock and, if employed, her wage. The number of vacancies and the wage of a given offer will only become clear to a worker when she matches with the stock. The value function for an unemployed worker of type x with j offers in hand, is given by (1). $F \in [0, 1]$ is the rank of wages from the job offer sampling distribution.[8]

The value function for unemployed and employed workers is given by

$$U(x, h_t) = \sum_{j=0}^{\infty} p(j; h_t)\tilde{U}(x, j) \quad \text{and} \quad W(x, w, h_t) = \sum_{j=0}^{\infty} p(j; h_t)\tilde{W}(x, w, j),$$

where $p(j; h_t)$ denotes the probability that the latent variable is equal to j, given the employment history h_t of the worker. We specify the model in this way to be agnostic on the exact information set of the agents. It is useful to illustrate the behavior of the model using the value functions conditional on the latent variable j. For the unemployed, the value $\tilde{U}(x, j)$, is defined by

$$\mu\tilde{U}(x, j) = bx + \gamma_u \int_0^1 (\tilde{W}(x, w(x, F), 0) - \tilde{U}(x, j))dF^j + \lambda_u(\tilde{U}(x, j+1) - \tilde{U}(x, j)) + \upsilon j(\tilde{U}(x, j-1) - \tilde{U}(x, j)). \tag{1}$$

[8] Notice that this corresponds perfectly with a firm's rank in the productivity distribution. This is because, as will be seen, firms pay all workers they employ the same piece rate wage and this is a monotonically increasing function in a firm's productivity type.

The value function of an unemployed worker, discounted by the rate at which she leaves the market, is the sum of the flow value of unemployment bx. The search option of the worker is given by the rate at which she accesses the market, γ_u, multiplied by the expected returns to matching with the stock. $\tilde{W}(\cdot)$ is the value of employment and defined in (2). Notice that when an unemployed worker takes up a job offer, she begins her employment spell with no opportunities. This is because the worker has rejected all other offers and we assume there is no recall of offers previously turned down. While in unemployment, the number of employment opportunities of a worker follows a stochastic process with suitable jobs arriving at a rate λ_u, and losing a given opportunity at a rate υ.

Given the underlying latent parameter, the value function associated with employment can be written as the sum of the flow wage, the option value of the j opportunities in the worker's stock, the option value of the stochastic process that governs the evolution of j and the option value of becoming unemployed which occurs with probability δ

$$\begin{aligned}\mu\tilde{W}\left(x,w(x,F),j\right) &= w(x,F)+\gamma_e\int_F^1(\tilde{W}(x,w(x,\tilde{F}),0)-\tilde{W}(x,w(x,F),0))d\tilde{F}^j\\ &+\gamma_e(\tilde{W}(x,w(x,F),0)-\tilde{W}(x,w(x,F),j))+\lambda_e(\tilde{W}(x,w(x,F),j+1)-\tilde{W}(x,w(x,F),j))\\ &+\upsilon j(\tilde{W}(x,w(x,F),j-1)-\tilde{W}(x,w(x,F),j))+\delta(\tilde{U}(j)-\tilde{W}(x,w(x,F),j)). \qquad (2)\end{aligned}$$

As in a standard sequential search model, a worker's decision is whether to accept or reject a given offer.[9] Once matching with the stock, a worker has potentially more than one offer to contend with. Since the wage lasts forever and all jobs are otherwise homogeneous, a worker will always prefer the highest wage job available to them, be it in the stock of opportunities or the job they are currently employed in. An unemployed worker accepts a wage if it yields a higher present value than continuing in unemployment. Since firms post wages optimally, assuming at least a negligible cost in wage posting, no firm would post a wage less than this value and therefore, we are solving

[9]One could imagine a more sophisticated set of strategies depending on what the worker is aware of (e.g., the number of vacancies in the stock, her job tenure or the wages of individual job opportunities). In such an environment, employed workers would under some conditions optimally quit to unemployment. However, this is beyond the scope of this paper.

for the infimum of the wage support, $\phi(x) = w(x, 0)$. This is found by solving the equality

$$\tilde{U}(x,0) = \tilde{W}(x, \phi(x), 0). \tag{3}$$

Solving the reservation wage is slightly more difficult than usual due to the evolution of the additional state variable - the number of employment opportunities. Appendix A.2 explains how one can compute the value functions and derives an expression for $\phi(x)$.

2.3 Steady-state distribution of match quality

In order to solve for the distribution of wages and outstanding matches, we proceed in two steps. First, we define the probability generating function Σ_s for each employment state s as

$$\Sigma_s(F) = \sum_{j=0}^{\infty} p_s(j) F^j,$$

where $p_s(j)$ is a probability mass function that gives the probability that a worker in state s has exactly j employment opportunities. The function $\Sigma_s(F)$ evaluates the probability that when a random worker in state s matches with the stock, she has no vacancy above the rank F. The function $\Sigma_s(F)$ has the steady-state solution

$$\Sigma_e(F) = \frac{1}{1-F}\int_F^1 \exp\left[-\lambda_e/\delta\left(\tilde{F}-F\right)\right]\left(\frac{1-\tilde{F}}{1-F}\right)^{\frac{\gamma_e+\mu}{\delta}} \frac{\gamma_e+\mu+\delta}{\delta} d\tilde{F},$$

$$\Sigma_u(0) = \frac{\int_0^1 \exp\left[-\lambda_u/\delta \tilde{F}\right]\left(1-\tilde{F}\right)^{\frac{\gamma_u-\delta}{\delta}}\left[\frac{\gamma_u \Sigma_e(\tilde{F})}{\delta}\right] d\tilde{F}}{1-\int_0^1 \exp\left[-\lambda_u/\delta \tilde{F}\right]\left(1-\tilde{F}\right)^{\frac{\gamma_u-\delta}{\delta}}\left[\frac{\gamma_u}{\delta}\left(1-\Sigma_e(\tilde{F})\right)\right] d\tilde{F}},$$

$$\Sigma_u(F) = \frac{1}{1-F}\int_F^1 \exp\left[-\lambda_u/\delta(\tilde{F}-F)\right]\left(\frac{1-\tilde{F}}{1-F}\right)^{\frac{\gamma_u-\delta}{\delta}}\left[\frac{\gamma_u \Sigma_u(0)}{\delta} + \frac{(\delta+\mu)(1-u)/u\Sigma_e(\tilde{F})}{\delta}\right] d\tilde{F}.$$

The derivation of this function is in Appendix A.1.2. The rate of inflow into unemployment from employment is given by $\delta + \mu$. Similarly, the rate of outflow from unemployment is given by $\gamma_u (1 - p_u(0))$. In steady-state, the inflow is equal to the outflow which, using the definition of $\Sigma_u(F)$, gives an expression for the unemployment rate

$$u = \frac{(\delta+\mu)}{(\delta+\mu+\gamma_u(1-\Sigma_u(0)))}. \tag{4}$$

The total unemployment rate contains both the friction at which workers qualify for jobs and the frequency at which they apply. In a hypothetical case in which, $\lambda_u \to \infty$, all workers have some employment prospects, that is $p_u(0) = 0$, and then the unemployment rate will purely be due to workers not sending out enough applications and given by

$$\tilde{u} = \frac{(\delta + \mu)}{(\delta + \mu + \gamma_u)}. \tag{5}$$

Comparing the *true* unemployment rate with the one in which all workers have employment opportunities reveals the relative importance of the two frictions for the unemployment rate. Using the function $\Sigma_s(F)$, we can further solve for the distribution of outstanding matches $G(F)$. Note that the inflow of matches below F is $\gamma_u \sum_{j=1}^{\infty} F^j p_u(j)$, i.e., the probability that an unemployed worker matches with an offer less than F. Similarly, the outflow of matches below F is the exogenous separation $\delta + \mu$ plus the endogenous quit of $\gamma_e \left(1 - \sum_{j=0}^{\infty} F^j p_e(j)\right)$. In steady-state, the inflow must equal the out flow which gives

$$(1-u)G(F)\left(\delta + \mu + \gamma_e\left(1 - \sum_{j=0}^{\infty} F^j p_e(j)\right)\right) = u\gamma_u\left(\gamma_u \sum_{j=1}^{\infty} F^j p_u(j)\right). \tag{6}$$

Using the definition of Σ_s, we get

$$G(F) = \frac{u\gamma_u\left(\Sigma_u(F) - \Sigma_u(0)\right)}{(1-u)\left(\delta + \mu + \gamma_e\left(1 - \Sigma_e(F)\right)\right)}. \tag{7}$$

The associated density function and its derivative are given in Appendix A.1.3.

2.4 Firm problem

The firm commits to a wage schedule in worker productivity at the time of vacancy creation (section 2.6 discusses the assumptions on wage setting). The firm then sets the wage to optimally trade off the increased retention and hiring with the increased cost associated with a higher wage. The expected profits per vacancy for a firm with match quality rank F posting a wage w are made up by three terms: the probability that a worker is hired; the expected duration; and the markup. Combining these gives the expression for the expected profits at the time of vacancy creation

$$\Pi^e(x, w, F) = \Pr(\text{hire}|x, w)\text{E}(\text{duration}|x, w)(y(F) - w). \tag{8}$$

Hiring. Search is random, a firm posting a vacancy can either meet an employed or unemployed worker. For the worker to accept the offer, the wage has to be higher than any other offer the worker holds and, if employed, her current wage. Absent of market thickness dynamics, the Burdett and Mortensen (1998) model, workers match instantaneously which means that the offer is always the best amongst the new job offers. The wage is acceptable if it is above the current wage or the reservation wage for the unemployed. In contrast, without OJS, all agents are unmatched but potentially receive more than one offer as in Burdett and Judd (1983). Either of these mechanisms will generate equilibrium wage dispersion. Our model combines both aspects as there is both search on the job and workers match with the stock. Define m as the probability that a vacancy meets a worker. The probability that a worker is hired can then be calculated using

$$\begin{aligned}\Pr(\text{hire}|w \geq \phi) &= m \Pr(\text{meet an unemployed worker}) \Pr(\text{unemployed worker accepts}|w) \\ &\quad + m \Pr(\text{meet an employed worker}) \Pr(\text{employed worker accepts}|w).\end{aligned}$$

The probability that a vacancy meets an unemployed worker, conditional on a meeting, is the flow rate of meetings with unemployed workers divided by the total flow rate of all meetings. The flow rate of meetings for unemployed workers comprises the product of three terms: the rate at which the worker engages in active search; the stock of unemployed; and the expected number of opportunities. The expected number of job opportunities is given by $\sum_{j=1}^{\infty} j p_s(j)$.

The probability that the worker accepts the offer, conditional on meeting with the vacancy, can be broken up into two parts. The probability that the offer is better than her current offer (1 for the unemployed and $G(w)$ for the employed) times the probability that the offer is the highest among all offers the worker has received. The probability that the offer F is the highest offer among all offers for the worker in state s is the probability that the vacancy meets a worker with j offers $\frac{j p_s(j)}{\sum_{j=1}^{\infty} j p_s(j)}$, multiplied by the probability that the offer is higher than the $j-1$ alternative offers (F^{j-1}). This gives

$$\frac{\sum_{j=1}^{\infty} j p_s(j) F^{j-1}}{\sum_{j=1}^{\infty} j p_s(j)}.$$

Combining the expressions above and using the definition of Σ_s we get

$$\begin{aligned}
\Pr(\text{hire}|w \geq \phi) &= m\frac{\gamma_u u \sum_{j=1}^{\infty} jp_u(j)}{\gamma_u u \sum_{j=1}^{\infty} jp_u(j) + \gamma_e(1-u)\sum_{j=1}^{\infty} jp_e(j)} \frac{\sum_{j=1}^{\infty} jpu(j)F^{j-1}}{\sum_{j=1}^{\infty} jp_u(j)} \\
&\quad + m\frac{\gamma_e(1-u)\sum_{j=1}^{\infty} jp_e(j)}{\gamma_u u \sum_{j=1}^{\infty} jp_u(j) + \gamma_e(1-u)\sum_{j=1}^{\infty} jp_e(j)}(1-u)G(F)\frac{\sum_{j=1}^{\infty} jp_e(j)F^{j-1}}{\sum_{j=1}^{\infty} jp_e(j)} \\
&= m\frac{(\gamma_u u \Sigma_u'(F) + \gamma_e(1-u)G(F)\Sigma_e'(F))}{(\gamma_u u \Sigma_u'(1) + \gamma_e(1-u)\Sigma_e'(1))}.
\end{aligned}$$

Duration of a job. Unlike in Burdett and Mortensen (1998), the duration of a job is not exponentially distributed. Instead, the quit rate is a time varying object. At the beginning of a job, the worker has matched with the stock and is therefore unlikely to leave right away; as time progresses, the expected number of offers and hence their quit rate increase. It turns out that even though the leaving rate is not constant, the average leaving rate is a sufficient statistic for the expected duration at the time of hiring due to Little's law. We can calculate the expected duration at the time of hiring. The average rate at which a worker working in a firm of productivity rank F leaves the job is given by $\delta + \mu + \gamma_e(1 - \Sigma_e(F))$. The average duration in a job F is therefore just $1/(\delta+\mu+\gamma_e(1-\Sigma_e(F)))$.

2.5 Equilibrium

An equilibrium in this economy is characterized by the function $\{\tilde{W}(x, w, j), \tilde{U}(x, j), w(F), \Pi(x, w, F)\}$ such that the firm profits are given by (8) and the present value for workers by (26) in the Appendix. The wage function $w(F)$ solves the firm problem, such that (8) is maximized, and the worker is indifferent between the lowest wage and unemployment, both absent of opportunities $\tilde{W}(x, \phi(x), 0) = \tilde{U}(x, 0)$ with $\phi(x) = xw(0)$.

2.6 Discussion of assumptions

Within this subsection, we review two modeling assumptions that we believe warrant further discussion. They are the wage setting mechanism and the recall of previously turned down job offers.

Wage setting. We follow Burdett and Mortensen (1998) and Burdett and Judd (1983) and opt for an environment in which firms commit to wages ex ante of meeting the worker. This is common in the literature and thus proves convenient in comparing our model to the existing literature. In addition, since wages are set prior to meeting the worker, the information structure h_t does not affect wages. It is useful to discuss how our extension of the job ladder model interacts with these particular assumptions on wage setting.

When wages are set prior to meeting the worker, the firm has an incentive to set a higher wage in order to increase hiring and retain the worker for longer. In contrast, when wages are decided ex-post, the firm has a lower incentive to agree on a higher wage since it does not increase hiring. Thus, negotiating of wages ex-post reduces competition; this effect is particularly strong in our model. The reason for this is that hiring entails a Burdett and Judd (1983) feature whereby the firm must offer a higher wage than all the other offers the worker contemplates (which makes the hiring motive particularly strong). In the estimation, as the same wage distribution is matched, this shows up via a more fat-tailed distribution (with ex-post wage setting). Similarly, if workers have some bargaining power, the productivity can be lower in order to match the same wage moments. The main results of the paper, in terms of the differences in, e.g., replacement rates are unchanged as it operates through the worker value functions and not the specificities of wage setting. In particular, since in the estimation to come, we match the same wage distributions, there is no change in the worker's value function (except for the replacement ratio which might be lower as the worker is not necessarily indifferent between unemployment and employment). If wages are infrequently renegotiated, the problem is more difficult. When agents can only commit to a wage for some period of time, the incentive for a firm to set a higher wage is then less, as retention increases less with the wage (Gottfries, 2017). In addition, with short contracts, the exact information structure h_t will affect the wage as it affects the worker's outside option and also the retention motive. Thus, we consider an ex-ante wage setting appealing as we do not have to take a stance on the exact information structure while staying close to the existing literature.

There are some additional mechanisms at play when firms are able to make counteroffers as in Postel-Vinay and Robin (2002) and Cahuc et al. (2006). With this wage setting, the wage is raised as counteroffers arrive which means that the worker initially accepts lower wages via this foot-in-the-door effect. In our model, this becomes exacerbated as the expected number of counteroffers received at the same time is higher if the worker takes a job. With the posting of wages, the exact timing of the offers is not important whereas with counteroffers it is. In particular, the matching set will increase. For example, without dynamic thickness and when firms have all the bargaining power as in Postel-Vinay and Robin (2002), the lowest productive firm that is able to hire workers has a productivity equal to the unemployment benefits. However, in our model, firms with lower productivity would also be able to hire workers as they increase the chance that the worker subsequently has multiple offers to contemplate. However, this mechanism is very sensitive to the assumption that the worker has access to the offers at exactly the same time.

No recall of rejected offers. In the model, after an offer has been turned down, be it for an alternative job prospect or staying with a current employer, a worker cannot subsequently return to that offer. This is analogous to the standard assumption in the job ladder models that previous jobs cannot be recalled.[10] An alternative modeling choice would be to assume that workers can hold on to rejected offers and retain them. This would not complicate the exposition to any considerable extent; instead an unemployed worker's reservation wage would be a function of the number of her prospects, j. However, quantitatively, the modeling assumption should be fairly innocuous as vacancies and therefore prospects have a fairly short shelf life. We calibrate the rate of expiry υ later based on the duration of a vacancy. We find that prospects last approximately one month. This is an order of magnitude larger than the rate at which employed workers, for example, switch jobs and would likely therefore have little effect on our results.

2.7 Identification

Identification of worker and firm productivities is by standard arguments non-parametrically identified. For the transition parameters, the rate of application γ_u and γ_e determines the job finding rate in unemployment and the job-to-job transition rate, respectively. The values of λ_u and λ_e instead determine how the job finding rate changes with duration in unemployment and employ-

[10]Recall of previous jobs has recently been explored by Fujita and Moscarini (2013) and Carillo-Tudela and Smith (2016) in a Mortensen and Pissarides (1994) and a sequential auctions model of the labor market, respectively.

ment, respectively. In addition, appendix A.3 provides a proof that the transition parameters of our model are identified. Although the exact moments discussed in the Appendix A.3 are not practically implemented in the estimation, we use similar moments for the purpose of estimation. The aim of our estimation is to minimize a criterion defined as a distance between simulated and empirically observable moments. While our proof does not guarantee that the estimated parameters to be presented constitute a global minimum of the criterion, it does imply that the parameters are identifiable. However, since our model is computationally inexpensive we make a global search over the parameter space to ensure that the estimates correspond to a global minimum.

3 Estimation

Our estimation will focus on estimating the model presented in the previous sections. In addition, we estimate the special cases of no dynamics of market thickness (NDT) and without OJS (NOJS).

3.1 Data

The data used in estimation are taken from the Current Population Survey (CPS) and the Survey of Income and Program Participation (SIPP). Moments relating to labor mobility are taken from the CPS, due to its larger cross-sectional component. Wage moments are taken from the SIPP, as for these, we rely on panel data. The sample is stratified according to observable skill level into three distinct strata, consistent across data source. They are: the college educated; those whose highest academic achievement is a high school diploma; and those who have not completed their high school education. We restrict the attention to male workers aged between 25 and 45 since both models rely on steady-state assumptions and to best mitigate issues associated with early retirement. In order to give the alternative specifications ample chance of matching the level of frictional wage dispersion, we trim the bottom ten per cent of the wage distribution. Moreover, we restrict the attention to the relatively short and stable period between the years 1996 and 1999, inclusive. As will be seen, key parameters will be identified from labor mobility by duration and we do not want cohort effects to play any role. In a supplementary appendix (S.2), we plot the separation and job

Table 1: Descriptive Statistics

	All	Low-skill	Medium-skill	High-skill
proportions:				
CPS	100%	12%	52%	37%
SIPP	100%	10%	51%	39%
mean earnings ($/hour)				
CPS	15.8	9.8	13.9	20.2
SIPP	16.5	10.2	13.9	21.4
mean weekly hours worked:				
CPS	43.1	41.4	42.5	44.2
SIPP	43.7	41.7	43.1	45.0

Note: Data come from the CPS and SIPP, moments are based on male workers aged between 25 and 45 between 1996 and 1999, inclusive.

finding rates by age and highlight our estimation window in shaded gray. Separation rates exhibit a clear downward trend and the pattern of the finding rate is less clear. Since our model assumes a constant separation rate, we choose a window where this seems to be a fair approximation of the data. Specific details regarding sample selection are provided in the supplementary appendix.

Identification will rely on employment dynamics and the cross-sectional wage distribution. Table 6 reports moments on hourly earnings and the number of hours worked per week for each stratum. These are computed by dividing the self-reported weekly earnings by self-reported hours worker per week. Since in the estimation, wage data are taken from the SIPP and employment dynamics from the CPS, we want to show that the data in both look quantitatively similar. The two data sets are broadly consistent. The SIPP implies a greater number of hours worked for more pay. There are large systematic differences in hourly earnings across skills. These differences are the motivation for stratification. Comparing hourly earnings across strata seems sensible as there is little cross strata variation in hours, with all subgroups working, on average, between 40 and 45 hours per week. Finally, it is worth bearing in mind that the medium-skilled, those with a high school diploma but without a college degree, account for about half of the labor force.

Table 2 presents employment dynamics, estimated from a three-state Markov process. The rows represent a worker's employment status at period t and in the columns in $t+1$, all changes are conditional on a change in employers. All moments presented in Table 2 will be exactly identified in the estimation to come. An inspection of these matrices reveals the large flow from inactivity to employment and differences across strata. Particularly pronounced differences relate to the duration of jobs, with a worker being less likely to switch from employment to inactivity, unemployment or to another employer if she is of a higher skill.

Table 2: Transition Matrices

All

	Inact.	Unemp.	Emp.
Inact.	—	—	0.098
Unemp.	—	—	0.312
Emp.	0.010	0.012	0.024

Low-skill

	Inact.	Unemp.	Emp.
Inact.	—	—	0.078
Unemp.	—	—	0.297
Emp.	0.019	0.026	0.032

Medium-skill

	Inact.	Unemp.	Emp.
Inact.	—	—	0.096
Unemp.	—	—	0.316
Emp.	0.010	0.014	0.025

High-skill

	Inact.	Unemp.	Emp.
Inact.	—	—	0.131
Unemp.	—	—	0.319
Emp.	0.006	0.006	0.022

Note: The transition rates are monthly. The rows do not add up to one. The emp-emp entries represent the fractions of individuals *changing jobs* while remaining employed. **Source:** Data are taken from the CPS and relate to 25-45 year old males between 1996 and 1999, inclusive.

3.2 Parameterization

The set of parameters to be estimated is given by the vector θ

$$\theta = \left(\mu, \delta, \upsilon, \lambda_u, \lambda_e, \gamma_u, \gamma_e, b, \Gamma_x\left(x\right), \Gamma_y\left(y\right)\right)'. \tag{9}$$

Notice that (9) contains the entire distributions of $\Gamma_x\left(y\right)$ and $\Gamma_y\left(y\right)$. We make further parametric assumptions on the primitive initial distribution of worker and firm types. We assume that they follow a transformed log-normal and beta distribution, respectively. The specific distributions were chosen as we found that they performed better than alternative parameterizations in fitting the data. With a slight abuse of notation, we define a worker's rank in the distribution as F_x and recall that a firm's rank is F. Φ denotes a standard normal distribution and B the beta distribution. We include location and shape parameters μ_x and σ_x. A firm draws productivity from a generalized log-beta distribution. α_y and β_y are the underlying distributional parameters and, for additional flexibility, we include a shape parameter σ_y.

$$x(F_x) = \exp(\mu_x) + \exp\left[\sigma_x \Phi^{-1}(F_x)\right]$$
$$y(F) = \exp\left[\sigma_y \mathrm{B}^{-1}(F; \alpha_y, \beta_y)\right]$$

The rate at which workers lose employment opportunities, υ, is the only parameter not directly estimated and calibrated to match the mean duration of a vacancy. Vacancy duration is estimated using the "The Conference Board Help Wanted Online Data Series" (HWOL). Details of exactly how this parameter is calibrated are provided in Appendix A.4. It should be noted that these data do not cover our estimation window nor can we look at the vacancy duration by skill requirement of the job opening. The implied value of the mean duration ($1/\upsilon$) is approximately one month.

After these assumptions, equation (9) can be reduced to the following vector of scalars. The focus of the rest of this section is the estimation of the vector θ

$$\theta = (\mu, \delta, \lambda_u, \lambda_e, \gamma_u, \gamma_e, b, \mu_x, \sigma_x, \mu_y, \sigma_y, \alpha_y, \beta_y)'.$$

3.3 Estimation Protocol

The model is estimated by indirect inference in two steps. In a first step, employment transitions are matched, based on CPS data. The second step matches auxiliary wage moments computed from the SIPP to uncover the underlying primitive productivity distributions of workers and firms and the value of home production. To estimate the models of no OJS search and no dynamic market thickness, we use an identical first step. In order to match the same degree of frictional wage dispersion, we compute the distribution predicted by our baseline model and target this directly.

To do this, we use an even more flexible beta distribution to guarantee a satisfactory fit.[11] Thus, for the two alternative specifications, we do not estimate the distribution of worker types.

Step one. The first step matches aggregate job to job and employment to unemployment transition rates. The rate at which workers leave the labor market and finally the job finding rate of the unemployed are computed by duration, matching the monthly probability at a weekly frequency for 52 weeks. We thus match 55 moments, which we weight by the precision at which they are estimated in the data. This step is matched varying $\theta^t = (\mu, \delta, \lambda_u, \lambda_e, \gamma_u, \gamma_e)$ and can be done independently of all other parameters. Formally, θ^t is the solution to the following, where $m^t(\theta_t)$ and m^t are the 55 targeted moments, from the model and data, respectively and $\hat{V}$ is the diagonal of the variance-covariance matrix of m^t. Note that for transition calculations based on fewer than 20 observations we replace the diagonal with zeros.

$$\hat{\theta}^t := \arg\max_{\theta^t \in \Theta^t} \left(m^t(\theta^t) - m^t\right)' \hat{V}^{-1} \left(m^t(\theta^t) - m^t\right).$$

Step two. This step estimates the value of home production and worker and firm productivity parameters, $\theta^p := (b, \mu_x, \sigma_x, \alpha_y, \beta_y, \sigma_y)$. We simulate data generated from our model as in the SIPP. That is, we simulate a monthly panel with the same number of individuals, over the same time frame, with the same rate of attrition. Since we only rely on the seam of the SIPP, where wages are not based on recall, we treat the simulated data in the same way. In order to distinguish between the relative contribution of the worker and firm productivities, we match each (1^{st} to 99^{th}) percentile of the mean wage of a worker over our time horizon. Further, we match the same percentiles of the overall wage distribution including the infimum of the support. Given all other elements in θ, the value of home production pins down the lowest wage. This leaves a total of 199 empirical moments to fit, which we denote by the vector m^p. To review this, m^p consists of the 100 quantiles of the wage support $\{w_q\}_{q=0,\dots 99}$ and the 99 quantiles of the mean worker wage $\{\overline{w}_q\}_{q=1,\dots 99}$.

Unlike step one, we do not have any analytical expressions for our moments, but instead rely

[11] We include an additional location parameter such that $y(\Gamma_y) = \xi_y + \zeta_y \exp\left[\sigma_y \mathrm{B}^{-1}(\Gamma_y; \alpha_y, \beta_y)\right]$. Notice that for $\xi_y = 0$ and $\zeta_y = 1$ it becomes identical to the distribution of firm types in the baseline.

on Monte Carlo simulations. We simulate the model M times and take the mean of each model predicted moment condition, given by $M^{-1}\sum_{i=1}^{M} m_i^p(\theta^p)$. Further, since the empirical distribution of wages contains many mass points and there is simulation error, a bootstrapped weighting matrix does not seem appropriate. Instead, we implement a two-step GMM estimation in which the first step estimates the asymptotically efficient weighting matrix, $W(\theta)^{-1}$. In the first step, take an initial guess at $W(\theta)^{-1}$ as the identity matrix and estimate the model and set $M = 1$. We then simulate $m_i^p(\theta^p)$ numerous times and compute a variance-covariance matrix as our estimate of $W(\theta)$. The second step estimates θ^p as the solution to

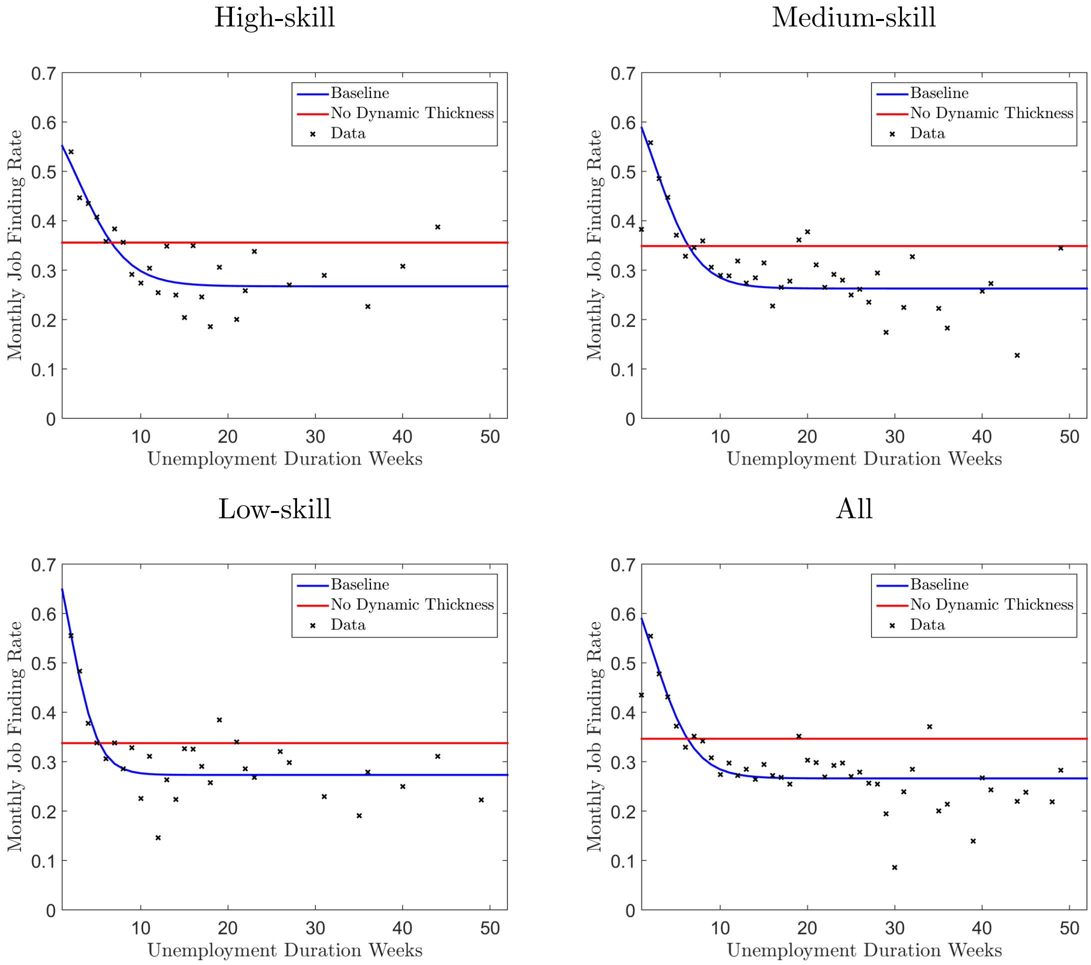
$$\hat{\theta^p} := \arg\max_{\theta^p \in \Theta^p} \left(\frac{\sum_{i=1}^{M} m_i^p(\theta^p;\theta^t)}{M} - m^p \right)' \hat{W}(\theta)^{-1} \left(\frac{\sum_{i=1}^{M} m_i^p(\theta^p;\theta^t)}{M} - m^p \right).$$

Practicalities. In the second step of the estimation, we re-simulate our model twenty times, $M = 20$. In order to isolate differences across specifications, we first estimate our model. Then, in the two special cases, we fix the distribution of worker productivity to be identical to our model. In this estimation of the two special cases (no market thickness and no OJS), we include an additional two parameters to be estimated (ξ_y, ζ_y) to ensure a satisfactory fit. In all three specifications the first step is identical, but for the two nested cases, the second step matches the distribution of wages simulated from our baseline model. That is, target 99 percentiles of the $G(w)$ distribution predicted by the baseline model as above, using the identity matrix as a weighting matrix. Finally, the data are re-sampled and the estimation repeated on each re-sample. The bootstrap procedure is implemented in order to make an inference on our parameters and results. For computational expediency, bootstrapped standard errors of the second step are performed assuming perfect precision in step one. This saves us from resimulating large data sets in every re-estimation, which would become quite cumbersome. Since the two alternative specifications are estimated using frictional wages generated by our baseline model, it did not seem informative to compute standard errors for the productivity parameters.

3.4 Fit

Step one. Figure 1 shows the probability that an unemployed worker moves to employment, by the duration of her unemployment spell. The horizontal red line represents that predicted with dynamic thickness (NDT). The declining blue line is our baseline model and the black crosses are the targeted estimates from the data. We omit the special case of no OJS as it is indistinguishable from the baseline. All models do almost exactly match the aggregate transition rates.

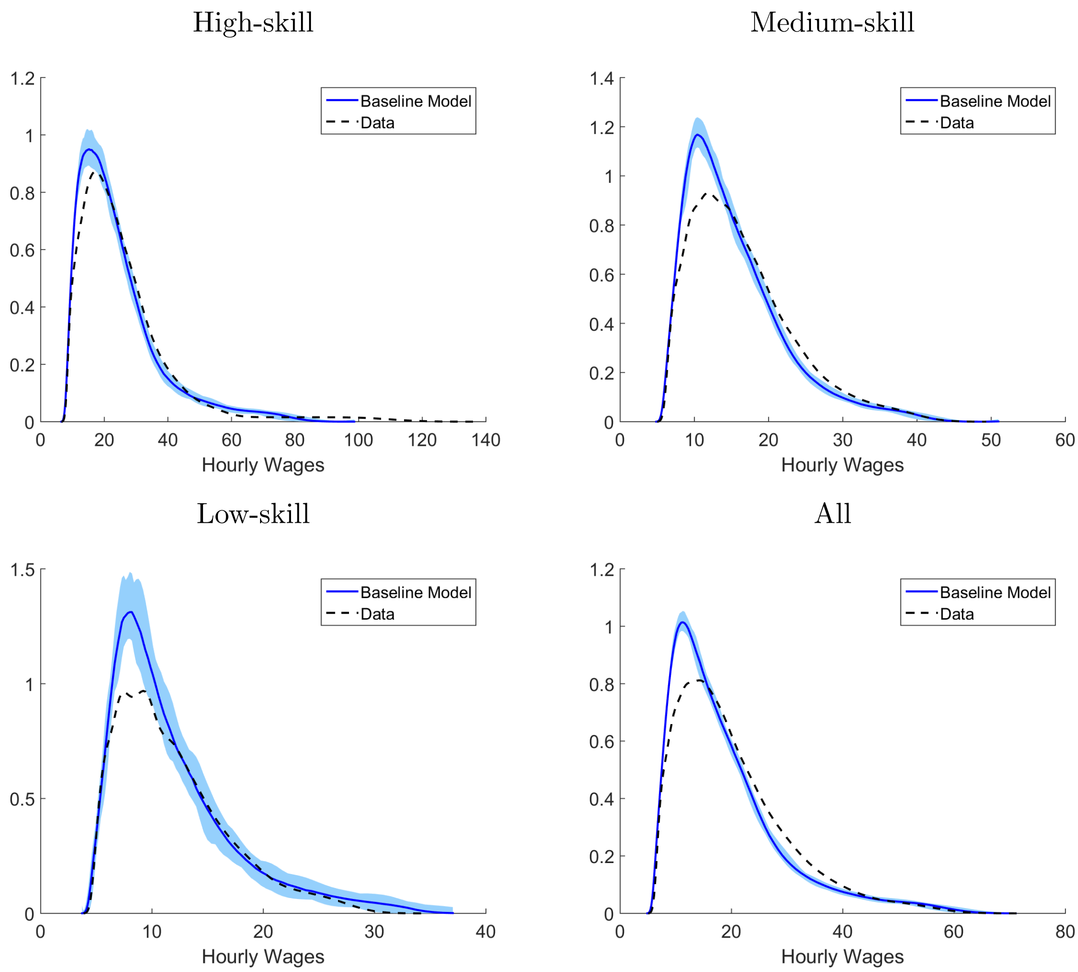
Figure 1: Job finding rate by duration of unemployment



Note: The empirical job finding rate, black 'x' is computed for each week, conditional on there being at least 20 individuals with an appropriate length of observed unemployment duration.

Step two. Figure 2 shows the fit of the overall wage distribution for the baseline model.

Figure 2: Fit of the Wage Distributions



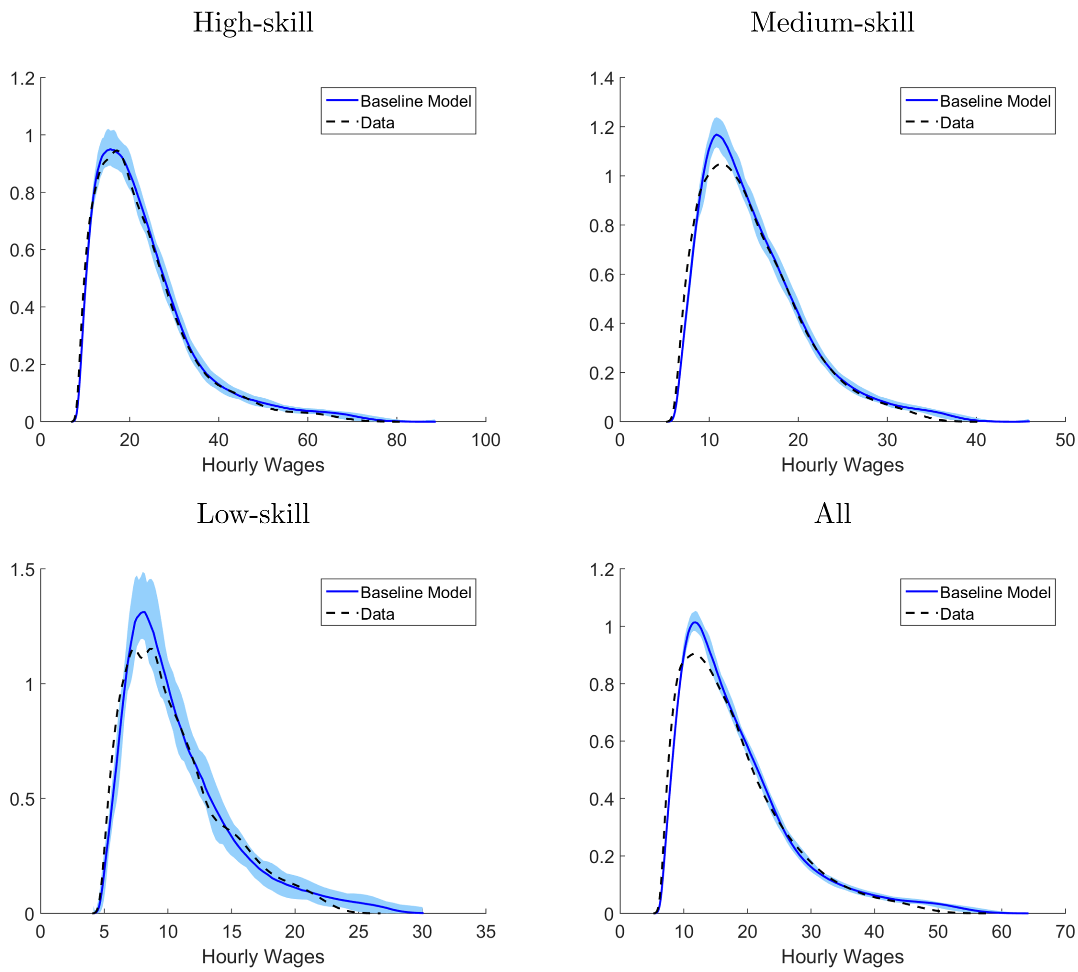
Note: Distributions are kernel density plots of the simulated and empirical data. The shaded blue areas represent 99% confidence intervals based on a repeated resimulation of the model.

Figure 3 displays the fit of the distribution of mean worker wages. Both distributions are skewed to the right for all worker types. In a supplementary appendix (S.3), the fit of the frictional wage dispersion for the two alternative specifications of the model (without market thickness and OJS, respectively) are presented.

3.5 Results

Running the multi-step estimation procedure as described yields the parameter estimates presented in Table 3. Bootstrapped standard errors are given in parentheses and all parameters are statis-

Figure 3: Fit of the Distribution of Mean Wages



Note: Distributions are kernel density plots of the simulated and empirical data. The shaded blue areas represent 99% confidence intervals based on a repeated resimulation of the model.

tically significant to any conventional significance level. The rest of this section will discuss each cell of the table in turn and then some implications. Finally, we discuss the importance of the mechanisms of the model for the measured search option associated with taking a job.

Transitional Parameters. Inspecting the transitional parameters, the upper cell of the table, it is immediately apparent that each model has a very different interpretation of the functioning of the labor market. First, across all skill groups, in the model without market thickness (NDT) $\lambda_u > \lambda_e$ which implies that workers are exposed to a greater number of job offers in unemployment than in employment. The higher rate at which the unemployed find new jobs must be rationalized by a lower

Table 3: Parameter Estimates

	All			High-skill			Medium-skill			Low-skill		
	NDT	NOJS	Baseline	NDT	NOJS	Baseline	NDT	NOJS	Baseline	NDT	NOJS	Baseline
μ	0.01 (1.346e−4)	0.01 (1.302e−4)	0.01 (1.305e−4)	0.006 (1.663e−4)	0.006 (1.573e−4)	0.006 (1.687e−4)	0.011 (1.91e−4)	0.011 (1.757e−4)	0.011 (1.968e−4)	0.02 (6.274e−4)	0.02 (6.112e−4)	0.02 (5.925e−4)
δ	0.015 (1.904e−4)	0.02 (6.689e−4)	0.02 (7.026e−4)	0.007 (2.031e−4)	0.009 (5.367e−4)	0.009 (7.963e−4)	0.017 (2.659e−4)	0.022 (9.699e−4)	0.022 (9.45e−4)	0.033 (8.952e−4)	0.055 (0.007)	0.055 (0.007)
λ_u	0.434 (0.005)	0.54 (0.02)	0.537 (0.02)	0.445 (0.012)	0.613 (0.056)	0.607 (0.054)	0.439 (0.007)	0.544 (0.026)	0.541 (0.025)	0.431 (0.011)	0.474 (0.028)	0.473 (0.028)
λ_e	0.114 (0.002)	2.119 (0.128)	2.397 (0.162)	0.16 (0.007)	2.43 (0.586)	2.832 (0.675)	0.11 (0.002)	2.273 (0.23)	2.557 (0.237)	0.114 (0.004)	1.968 (0.164)	2.253 (0.181)
γ_u	—	1.445 (0.14)	1.463 (0.155)	—	1.116 (0.243)	1.137 (0.328)	—	1.38 (0.181)	1.398 (0.179)	—	2.39 (0.527)	2.408 (0.55)
γ_e	—	—	0.095 (0.002)	—	—	0.122 (0.008)	—	—	0.091 (0.003)	—	—	0.104 (0.005)
$b/\mathbb{E}[w]$	−78.1% (−−−)	−341.7% (−−−)	19.9% (0.762)	−43% (−−−)	−677.1% (−−−)	44% (0.898)	−197.8% (−−−)	−230.7% (−−−)	27.1% (1.64)	−321.9% (−−−)	−50% (−−−)	41% (1.891)
μ_x	—	—	1.768 (0.01)	—	—	2.056 (0.015)	—	—	1.727 (0.021)	—	—	1.442 (0.024)
σ_x	—	—	1.253 (0.022)	—	—	1.405 (0.042)	—	—	1.079 (0.03)	—	—	1.014 (0.047)
α_y	1.402 (−−−)	0.309 (−−−)	0.679 (0.018)	0.638 (−−−)	0.547 (−−−)	0.371 (0.023)	1.737 (−−−)	1.289 (−−−)	0.784 (0.048)	1.558 (−−−)	0.925 (−−−)	0.811 (0.048)
β_y	4.362 (−−−)	0.288 (−−−)	3.993 (0.083)	3.685 (−−−)	0.477 (−−−)	1.866e4 (6.408e2)	5.148 (−−−)	0.56 (−−−)	3.918 (0.201)	3.853 (−−−)	0.315 (−−−)	1.35 (0.077)
σ_y	5.713 (−−−)	2.258 (−−−)	2.447 (0.037)	4.749 (−−−)	2.563 (−−−)	9.443e3 (2.719e2)	5.769 (−−−)	2.625 (−−−)	2.084 (0.057)	5.076 (−−−)	2.302 (−−−)	1.119 (0.035)
ξ_y	0.879 (−−−)	1.452 (−−−)	—	0.742 (−−−)	1.255 (−−−)	—	0.897 (−−−)	0.76 (−−−)	—	0.977 (−−−)	0.76 (−−−)	—
ζ_y	0.16 (−−−)	0.412 (−−−)	—	0.254 (−−−)	0.351 (−−−)	—	0.151 (−−−)	0.268 (−−−)	—	0.171 (−−−)	0.256 (−−−)	—

contact rate in NDT. In our model, employment prospects arrive at a similar rate in both states. Instead, the higher finding rate among the unemployed is due to a more frequent active search, $\gamma_u > \gamma_e$. This large disparity between γ_u and γ_e is what allows the model to replicate the declining hazard rate with unemployment duration. This also implies that the employed and the newly unemployed do, on average, have better employment prospects than the long-term unemployed and thus have a higher job finding rate. Finally, across all skill groups, the job destruction rate δ is higher in the model with stochastic market thickness. In that version, the newly unemployed find jobs more quickly than they would in the NDT framework and, consequently, more workers lose their jobs and find employment within a month.

Unemployment decomposition. In Burdett and Mortensen (1998), there is only one source of unemployment due to the infrequent arrival of job offers. However, in our model, there are two sources of labor market frictions. Not only must a worker apply for jobs, she must also have positive employment prospects. Equation (4) denotes the true unemployment rate u and equation

(5) an unemployment rate in which the only impediment to finding work is the frequency in which one applies to jobs, $\tilde{u}$. A comparison of the two rates reveals the relative quantitative importance of the two sources of frictions.

The exit rate from unemployment occurs after a γ_u shock and on top of this, the worker must also have at least one potential job, which occurs with probability $(1 - \Sigma_0(0))$. The frictional rate is governed by only the primitive γ_u which prevents a worker from matching with her current opportunities. The relative importance of the two frictions, by skill group, is reported in Table 4. The quantitative importance of a lack of opportunity is apparent, with this mechanism being responsible for approximately half of the unemployment rate.

Table 4: Unemployment Decomposition

Unemployment Rate	All	High-skill	Medium-skill	Low-skill
$\tilde{u}$	2%	1.3%	2.3%	3.0%
u	4.5%	2.6%	4.9%	8.3%

Note: This table computes and compares u and $\tilde{u}$ as defined in equations (4) and (5).

To further understand the relative importance of the frictions on unemployment, we compute the elasticity of unemployment with respect to the four frictional parameters.[12] Consistent with Table 4, the parameters governing the frequency at which opportunities arrive, the λ's, have a greater (absolute) combined elasticity than the frequency at which workers apply for jobs, the γ's. The single most consequential parameter in determining unemployment is γ_u. The effectiveness of reducing unemployment through an increase in the frequency of applications increases with the skill of the worker. However, encouraging the employed to apply more frequently will have the opposite effect, since by applying for jobs when in employment a worker exhausts the opportunities that she otherwise could have relied on after a layoff in the future. Finally, it is interesting to note that the arrival rate of offers to the employed have a similar impact on the overall unemployment rate as offers to the unemployed.

[12] For λ_u for example, this is computed as ${\partial \log(u)}/{\partial \log(\lambda_u)}$. The derivative is approximated using two-sided differences.

Table 5: Elasticity of Unemployment

Parameter	All	High-skill	Medium-skill	Low-skill
γ_u	-0.874	-0.964	-0.898	-0.691
γ_e	0.069	0.074	0.066	0.089
λ_u	-0.608	-0.564	-0.589	-0.644
λ_e	-0.519	-0.428	-0.505	-0.647

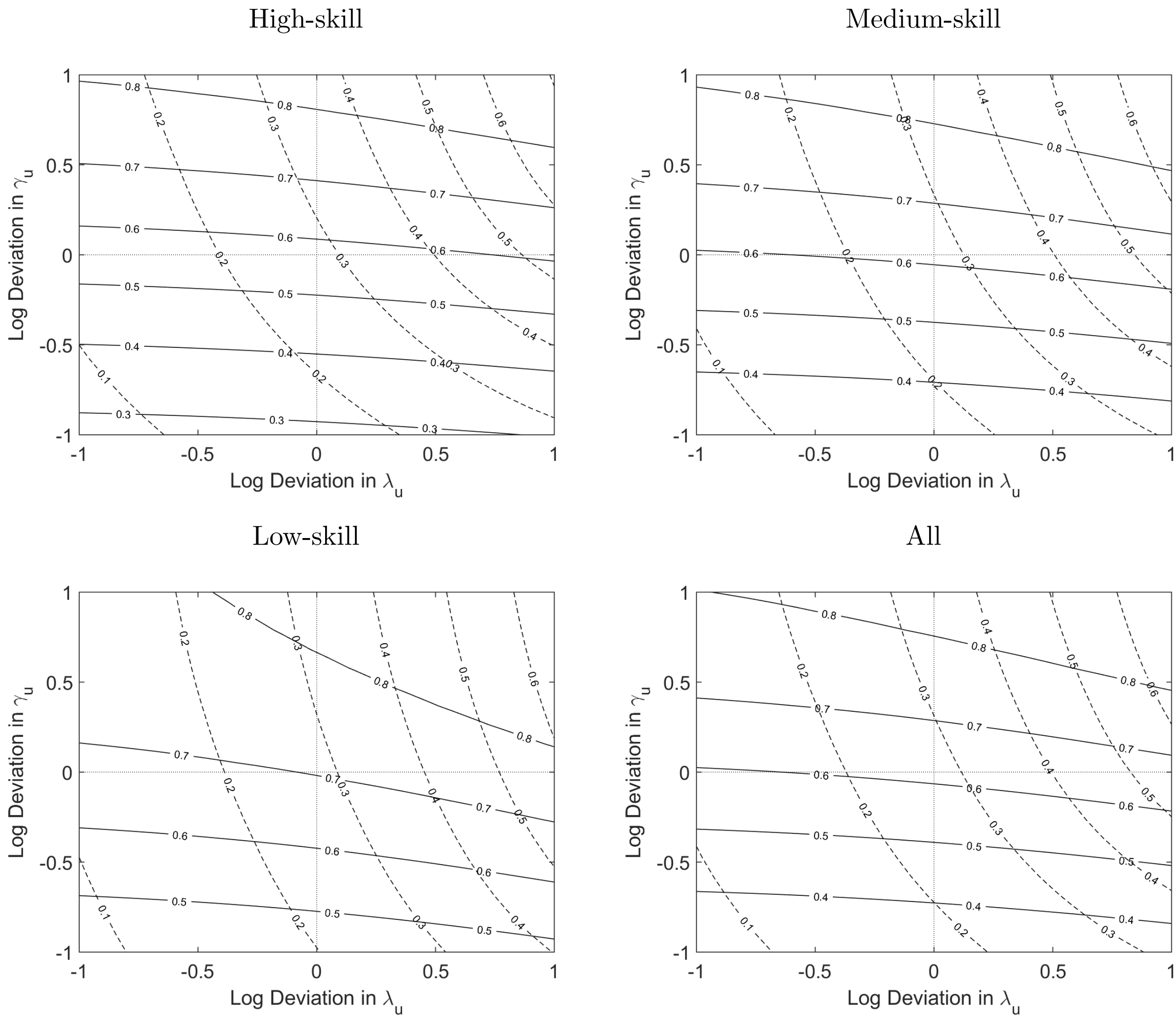
Note: This presents elasticities of unemployment with respect to transition parameters of the model. In the case of λ_u, the elasticity is $\partial \log(u)/\partial \log(\lambda_u)$. In practice, this is approximated by two-sided differences, where the elasticity of λ_u, for example, is given by $\frac{\lambda_u(\log(u(\theta^{\sim}))-\log(u(\theta_{\sim})))}{2\lambda_u \epsilon}$ where $\theta^{\sim}$ is identical to $\hat{\theta}$ but $\lambda_u^{\sim} := \lambda_u(1+\epsilon)$, where ϵ is arbitrarily small and for $\theta_{\sim}$ $\lambda_{u\sim} := \lambda_u(1-\epsilon)$.

Figure 4 shows the impact of the two most consequential parameters γ_u and λ_u on the monthly job finding rate for the newly unemployed and those unemployed for three months. The figure is produced by simulating the model on a grid, varying the parameter values of γ_u and λ_u. The short and long-term unemployed exhibit large differences in the sensitivity of the job finding rate to the parameters. For the newly unemployed, the contour lines are significantly shallower than the 45-degree line implying that γ_u is more important for exiting unemployment. As discussed, the long-term unemployed have, on average, fewer opportunities than their short-term counterparts. Consequently, it is the lack of opportunities which arrive at rate λ_u that prevents them finding work. The contour lines are steeper for the long-term unemployed, reflecting the higher relative importance of λ_u. The lesser importance of λ_u for the short term unemployed comes as many already have a number of employment prospects. These patterns are consistent across the worker skill distribution implying the cause of not finding work is primarily determined by the duration of your unemployment spell, rather than your level of human capital.

Productivity Parameters. The final cell of Table 3 presents the parameters from the underlying distributions of worker and firm productivity. On their own, they are not easy to interpret so, instead, we discuss the degree of wage competition (i.e., the amount of competition in the different models).

The percentage increase in firm type, $\ell'(F)/\ell(F)$, is just $\ell'(F)/\ell(F) = G''(F)/G'(F) = h(F) + r(F)$

Figure 4: Impact of γ_u and λ_u on the Monthly Job Finding Rate of the Unemployed



Note: For a given job finding rate, solid lines connect the values of γ_u and λ_u where that rate occurs for the newly unemployed. Dashed lines correspond to the job finding rate of those unemployed for exactly three months. These contour plots are computed by resimulating the model, varying the size of the parameters γ_u and λ_u over a fine grid. The numbers on the plot represent the monthly job finding rate. The plot is centered around the parameter values from our baseline estimation.

and we can rewrite the first-order condition as

$$\Pi_w(w, F) \quad = \quad \frac{\ell'(F)}{\ell(F)}(y(F) - w(F)) - w'(F) = 0. \tag{10}$$

We refer to $G''(F)/G'(F)$ as the degree of competition. If $G''(F)/G'(F)$ is low, then the firm size is unresponsive to the wage and there is little reason to increase the pay. The degree of competition, using our formula, can be written as

$$\frac{\ell'(F)}{\ell(F)} = \frac{\left[\Sigma_u''(F) + \frac{\gamma_e(\Sigma_u(F)-\Sigma_u(0))}{(\delta+\mu+\gamma_e(1-\Sigma_e(F)))}\Sigma_e''(F)\right]}{\Sigma_u'(F) + \frac{\gamma_e(\Sigma_u(F)-\Sigma_u(0))}{(\delta+\mu+\gamma_e(1-\Sigma_e(F)))}\Sigma_e'(F)} + 2\gamma_e \frac{\Sigma_e'(F)}{(\delta+\mu+\gamma_e(1-\Sigma_e(F)))},$$

whereas the competition term for the Burdett and Mortensen (1998) model is

$$\frac{\ell'(F)}{\ell(F)} = 2\lambda_e \frac{1}{(\delta+\mu+\lambda_e(1-F))}.$$

Note that $\Sigma_s(F)$ is a convex function. When the firm considers the hiring margin in the Burdett and Mortensen (1998) model, it need only consider the probability that the worker is working at a lower paying firm. In our setup, the firm also needs to consider the probability that the worker has a better offer in hand. The competition in our model therefore increases more as we move to the tail of the distribution.

The Burdett and Mortensen (1998) model fails to generate as much wage competition in the upper tail of the distribution. Our model includes a further competition term via a Burdett and Judd (1983) mechanism. In order to show the difference, we plot expression 10 to calculate the competition for different firm types for Burdett and Mortensen (1998) and our model. Figure A.5, in the appendix, reveals, that for all skill groups, high productive firms exhibit a stronger wage competition in our model as compared to Burdett and Mortensen (1998). The intuition for this is that since some workers have many offers, it is relatively more likely that the highest offer is in the upper part of the distribution. The competition at the lower type firm is, on the other hand, similar in our model and that in the Burdett and Mortensen (1998) model. The introduction of thin markets into the Burdett and Mortensen (1998) model thus shifts competition from the lower to the upper part of the distribution, thereby increasing the dispersion of wages for a given primitive firm productivity distribution.

Wage Posting Motivative. In Burdett and Mortensen (1998) and Bontemps et al. (2000), like in our model, the firm trades off the hiring and retention of the worker against a higher wage, equation (8). The first-order condition for the logarithm of expected profits gives a differential equation for the optimal wage. Defining $h(F) = {}^{\partial \log \Pr(\text{hire})}/_{\partial F}$ and $r(F) = {}^{\partial \log E(\text{duration})}/_{\partial F}$ and

$w^m(F) = \int_0^F m(F)(y(F) - w(F))dF$ where, $m \in \{h, r\}$, we can decompose the wage $w(F)$ into three terms, the wage increase from the retention motive ($w^r(F)$) and the hiring motive ($w^h(F)$) and the wage that satisfies the participation constraint for the worker $w(0)$,

$$w(F) = w^r(F) + w^h(F) + w(0).$$

In the Burdett and Mortensen (1998) model, the motive to pay for retention and hiring is

$$r(F) = h(F) = \frac{\lambda_e}{\delta + \mu + \lambda_e(1 - F)}.$$

In our model, the incentive is

$$\begin{aligned} h(F) &= \frac{(\gamma_u u \Sigma_u''(F) + \gamma_e(1-u)G'(F)\Sigma_e'(F) + \gamma_e(1-u)G(F)\Sigma_e''(F))}{(\gamma_u u \Sigma_u'(F) + \gamma_e(1-u)G(F)\Sigma_e'(F))}, \\ r(F) &= \frac{\gamma_e \Sigma_e'(F)}{(\delta + \mu + \gamma_e(1 - \Sigma_e(F)))}. \end{aligned}$$

In Figure A.6, we show the fraction of the wage that is paid due to the incentive to retention workers. The results suggest that the hiring motive is quantitatively more important, but relatively less so higher up in the upper support of the wage distribution.

3.6 Frictional wage dispersion

As has been discussed, in order to generate the level of frictional wage dispersion observed in the data as measured by the mean-min ratio, the typical search model requires an implausibly low or even negative flow benefit associated with unemployment. Our results, see Table 3, show that the implied replacement ratio, the ratio of the flow benefit to the mean wage in the economy needed to justify the observed wage distribution, is much higher under the baseline model. Across all skill groups, only the baseline model predicts a positive replacement ratio and depending on the skill, the other two specifications require enormous costs associated with unemployment. To put our numbers in some context, estimates from a field experiment put the replacement ratio at 58% (see Mas and Pallais (2017)).

The replacement ratio in the Burdett and Mortensen (1998) model can be decomposed into the min-mean ratio and the search option,

$$\underbrace{\frac{b}{\mathrm{E}[w]}}_{\text{Rep. ratio}} = \underbrace{\frac{w(0)}{\mathrm{E}[w]}}_{Mm} + \underbrace{\int_0^1 \frac{w'(\tilde{F})}{\mathrm{E}[w]} \frac{(\lambda_e - \lambda_u)(1-\tilde{F})}{\delta + \mu + \lambda_e(1-\tilde{F})} d\tilde{F}.}_{\text{Search}} \tag{11}$$

The flow value in our model is instead

$$\underbrace{\frac{b}{\mathrm{E}[w]}}_{\text{Rep. ratio}} = \underbrace{\frac{w(0)}{\mathrm{E}[w]}}_{Mm} + \underbrace{\int_0^1 \frac{w'(\tilde{F})}{\mathrm{E}[w]} \frac{\gamma_e(+1-\Sigma_e(\tilde{F})) - \gamma_u(1-\Sigma_{uu}(\tilde{F}))}{\delta + \mu + \gamma_e(1-\Sigma_e(\tilde{F}))} d\tilde{F}}_{\text{Search}}$$
$$\underbrace{+\delta\left(\frac{\gamma_u \int_0^1 \frac{w'(\tilde{F})}{\mathrm{E}[w]} \frac{\Sigma_{uu}(\tilde{F})-\Sigma_u(\tilde{F})}{\delta+\mu+\gamma_e(1-\Sigma_e(\tilde{F}))} d(\tilde{F})}{(\mu + \gamma_u(1-\Sigma_u(0)))}\right)}_{\text{Insurance}}. \tag{12}$$

The terms are intuitive. There are differences in the flow value and in the search option, captured by how often search occurs, the sampling distribution of wages and finally because, in our model, workers who separate from a job are in a different position as compared to the average unemployed. The second term, $\gamma_i(1-\Sigma_i(\tilde{F}))$, differs because the unemployed and employed do, on average, sample from different distributions, $\Sigma_i(\tilde{F})$, and at different rates, γ_i. Faberman et al. (2016) provide evidence that the employed on average sample from a better distribution. Similarly, the third term, $\Sigma_{uu}(\tilde{F}) - \Sigma_u(\tilde{F})$, captures the effect that workers moving from employment to unemployment do, on average, have a higher number of prospects than the average unemployed which generates the declining job finding rate with the duration of unemployment observed in the data. These effects are missed in the standard job ladder model. Table 6 provides a thorough decomposition of the replacement ratios for the different specifications and skill groups. A consistent finding across all skill groups is that only the baseline specification can accommodate the degree of frictional wage dispersion with a positive replacement ratio. An inspection of Table 6 reveals that while the insurance option helps, it is the reduction in the search option which is more important quantitatively. While still negative, the better average prospects that the employed are exposed to significantly reduce the value of waiting in unemployment and, consequently, unemployed workers for the same value of b are prepared to accept much lower wages.

Table 6: Decomposition of the Flow Value of Unemployment

	Replacement Ratio	Min-mean Ratio	Search Option	Insurance Option	Replacement Ratio	Min-mean Ratio	Search Option	Insurance Option
	All				**High-skill**			
NDT	-78.1%	49.7%	-127.8%	0%	-43.0%	50.6%	-93.6%	0%
NOJS	-341.7%	47.6%	-412.6%	23.3%	-677.1%	48.6%	-745.3%	19.7%
Baseline	19.9%	50.7%	-36.4%	5.5%	44.0%	50.7%	-9.4%	2.3%
	Medium-skill				**Low-skill**			
NDT	-197.8%	53.2%	-251.0%	0%	-321.9%	57.9%	-379.8%	0%
NOJS	-230.7%	55.7%	-307.2%	20.7%	-50.0%	60.7%	-131.7%	21.0%
Baseline	27.1%	54.2%	-32.9%	5.8%	41.0%	59.7%	-28.3%	9.6%

Note: This table provides results from decomposing the replacement ratio into its three constituent parts derived in equations (11) and (12).

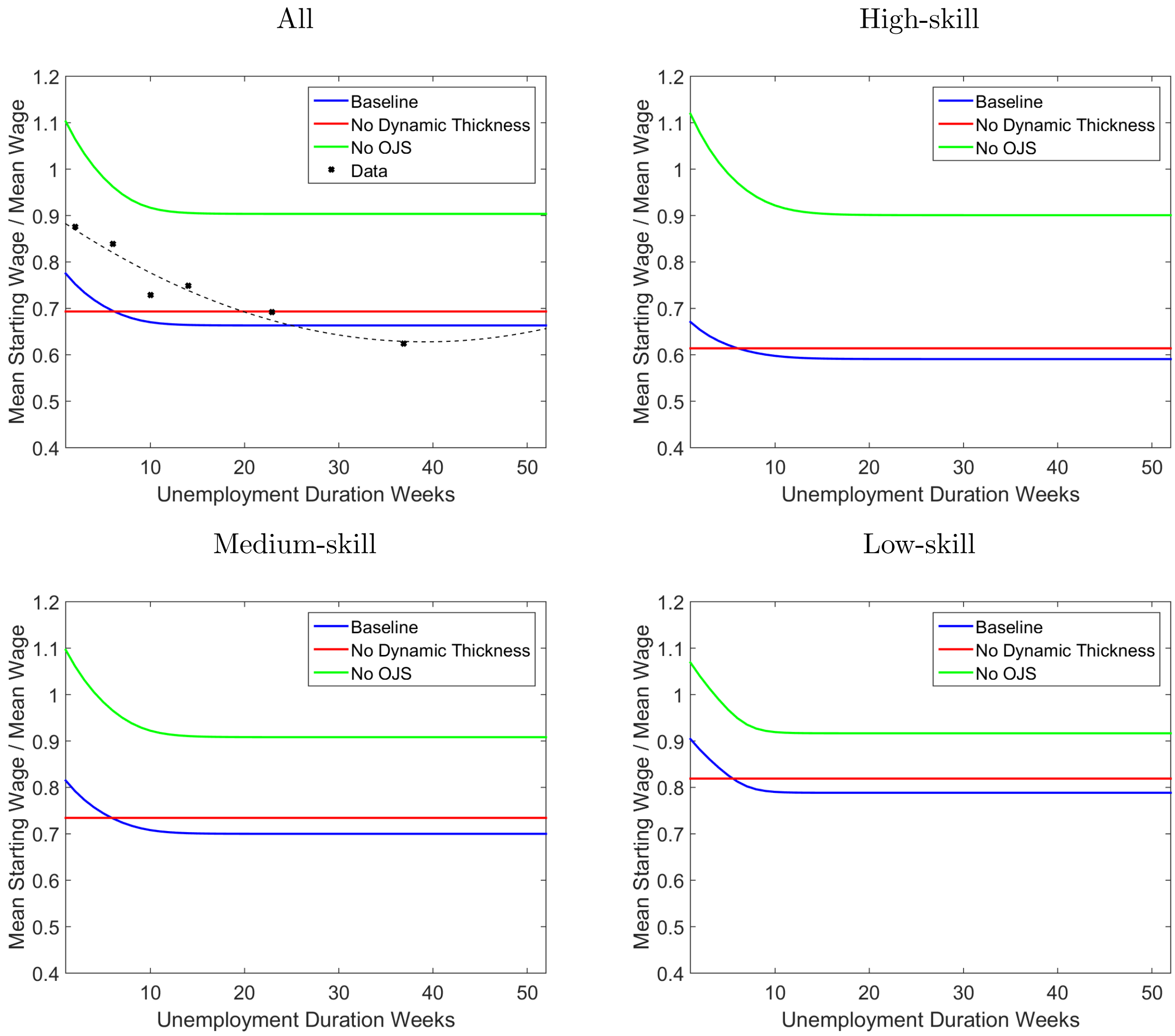
3.7 Earnings loss

Our baseline model and the two alternative specifications provide very different predictions regarding the average wage an unemployed worker receives in employment as a function of the duration of her unemployment spell. As is displayed in Figure 5, with no dynamic market thickness, an unemployed worker samples from the same distribution of wages independent of the duration of the unemployment spell to date. However, because of selection into better jobs, the job ladder, this wage is lower than the mean wage amongst employed workers. Without OJS, there is no selection into better jobs for the employed. Thus, the average wage taken by an unemployed worker equals the average wage amongst the employed workers. However, because of dynamic selection, a worker with a longer duration of unemployment will, on average, have fewer prospects and thus samples from a distribution with a lower mean wage. This results in a decline in the average starting wage within the first couple of months. Our baseline model has both these features and thus generates both an average earnings loss, via selection in employment, and increasing losses with the duration of unemployment via dynamic selection.

While a more thorough empirical analysis is required, we plot the starting wages relative to the mean wage for the SIPP sample, following individuals reporting consecutive spells of unemployment.

There appears to be a consistent story between model and data, one of an initial fall from the job ladder followed by a further, more gradual decline, with the duration of unemployment.

Figure 5: Wage Loss after Job Loss



Note: The data are taken from the SIPP, presented for the 'all' group, where we have sufficient observations to plot credibly. The points represent a comparison of the mean wage in the economy and the mean wage following reporting being unemployed, plotted at the mean duration of their bin. Note that this is not quite consistent with the model, as workers could have had intermediate employment/unemployment spells between observation dates. The dashed black line is a fitted quadratic function of the data.

3.8 Search process

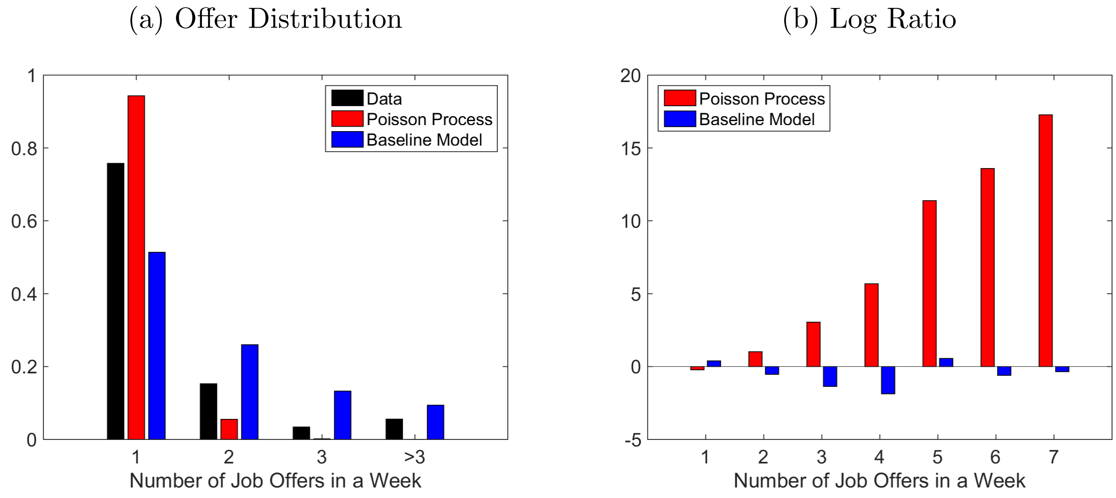
To demonstrate that our model replicates workers' search behavior in a realistic manner, we compare the underlying theoretical mechanism with direct evidence on workers' search behavior. For this exercise, we rely on two data sources not used in our estimation. They constitute a supplement to the Survey of Consumer Expectations (SCE) provided by the New York Fed from 2013 and 2014 and is a subset of that used in Faberman et al. (2016). The survey is a repeated cross-section, nationally representative and has approximately 1,200 individuals per year. In addition, we use the Survey of Unemployed Workers in New Jersey. The data and their construction are detailed in Krueger and Mueller (2011). 6,025 unemployed workers in the New Jersey area are surveyed at a weekly frequency for up to 24 weeks. A feature of the data is that it asks workers about the job offers they receive (not necessarily take) and their contemporaneous reservation wage.[13] Results from both data sets are weighted by the weights provided and described in Faberman et al. (2016) and Krueger and Mueller (2016), respectively.

First, we exploit data from the Survey of Unemployed Workers in New Jersey. In Figure 6, we present the number of offers received by unemployed workers in a month and compare this to what is predicted by our model and a memoryless Poisson process. The memoryless Poisson process is computed, given the proportion of people in the data with no offers. Our baseline model is a representation of the distribution $p_u(j)$, the solution of the flow equations (15) and (16). Since the data cover a different time period and only focus on New Jersey, there is no reason to assume that the model will fit the data well. However, what is clear from panel (b) is that a memoryless Poisson process cannot generate the number of people with large numbers of offers that is observed in the data – a feature that our baseline model replicates.

Turning to the supplement of the Survey of Consumer Expectations (SCE), it is worth noting that the statistics presented here are merely to demonstrate that the underlying search process reported in the survey is quite different from what is assumed in a standard search model and bears some resemblance to the mechanism in our model. Any further inference is difficult to make

[13]The data for this analysis are available for public download at http://opr.princeton.edu/archive/njui/.

Figure 6: Number of Job Offers



The data come from the survey of unemployed job seekers in New Jersey. We restrict our attention to male workers between the age of 25 and 45. Panel (a) shows the distribution of reported job offers received in a week, conditional on receiving at least one offer: as observed empirically (weighted by sampling weights); implied by a Poisson process and as implied by the baseline model. Panel (b) is the log ratio of the implied distributions with the data.

since although representative, the sample has a fairly small cross-section, meaning that inference about the unemployed is based on 61 (26) individuals (males). Table 7 shows by employment status, over a four-week period, the mean number of applications, the proportion of those making at least one application and the mean number of contacts received. We present these from the model and the data, and we further distinguish between unemployed and long-term unemployed in our model. The unemployed do, on average, send out more applications and more frequently engage in active search as compared to their employed counterparts. A fact that our model successfully reconciles. The number of applications sent out in the data is an order of magnitude larger as a few people are observed sending out hundreds of applications, something not present in our model. By trimming the distributions, the unemployed remain more active while the levels in the data begin to look somewhat more similar to the model. The number of contacts received in a month, on the other hand, is similar across the two groups. However, since we have relatively few unemployed in the sample, it is hard to establish this clearly. What is certain is that, as implied by a standard job ladder model, the unemployed do not receive an order of magnitude larger contacts than their employed counterparts. Finally, we have included the model's predictions for the long-term unemployed to further inform the reader. However, with so few unemployed in the data, the same moments in the data are uninformative.

Table 7: Mean Job Prospects by Employment Status

	# of applications		prop. who apply		# of contacts	
	Model	Data	Model	Data	Model	Data
Unemployed	0.52	7.23	64%	88%	0.54	0.77
Employed	0.18	1.31	8%	23%	2.4	0.74
L.T. Unemployed	0.08	—	31%	—	0.54	—

The data are taken from the Survey of Consumer Expectations and the attention is restricted to male workers. Applications are calculated based on the question "*How many potential employers, if any, did you apply to for employment within the LAST 4 WEEKS? Please include all applications made in person, online, or through other direct methods. Do not include inquiries that did not lead to a job application.*". Similarly, the number of contacts are computed based on the question "*In the LAST 4 WEEKS, how many potential employers contacted you about a job opening? Please include all contacts, even those that were not solicited by you.*". All moments are computed based on appropriate sampling weights. Long-term unemployed is defined as having reached the ergodic distribution of prospects, in practice this occurs in under three months.

Finally, we aim to assess the importance of an assumption made in estimation which dictates the distribution of employment opportunities across workers. The only parameter calibrated outside of estimation is the frequency in which job opportunities/vacancies expire, υ. Clearly, all estimates are conditional on the specific value of this calibration. Appendix A.7 shows that our estimate of the average duration of a vacancy, by which we calibrate υ, sits within the range of estimates taken from the literature. Further, we show that calibrating υ across this broad range of values and re-estimating the model does not change our results in a quantitatively meaningful way.

4 Conclusion

This paper sets up a model which extends the standard job ladder model to incorporate thin markets. The model is solved analytically and estimated on U.S. survey data. The estimated model delivers declining job finding rates by the duration of unemployment as observed in the data.

Further, the flow value associated with unemployment required to match the wage distribution does not need to be large and negative. Our estimates of the replacement ratio, in the order of a quarter to a half of the workers' average wage, are consistent with the numbers used in the macro labor literature. On the other hand, the estimation of the Burdett and Mortensen (1998) model and a model without on-the-job search requires large and negative replacement ratios. Additionally, to generate a wage penalty associated with the duration of an unemployment spell, the standard job ladder model requires decreasing general human capital in unemployment. Our model generates this via the stochastic process for employment prospects. This has implications for the persistence in earnings losses following job displacement. Whether this mechanism can generate a sufficient persistence in earnings remains an open question and could prove fruitful for future research.

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A Appendix

A.1 Distribution of match quality

A.1.1 Steady-state distribution of vacancy stock

We denote the number of vacancies in the stock by $j \in \mathbb{N}_+$. The probability that a worker in employment state s has j vacancies in the stock is denoted by $p_s(j)$. The inflow of employed workers with $j \geq 1$ offers comes from two sources: (i) those with $j-1$ offers who receive an additional offer and (ii) those with $j+1$ offers who lose an offer. The inflow for employed workers with no offers $j = 0$ is from two sources: employed workers with one offer which they lose and workers who just matched with the stock, independent of employment state

$$\begin{aligned} \text{inflow} &= \lambda_e p_e(j-1) + \upsilon(j+1)p_e(j+1) \quad \forall j \geq 1, \\ \text{inflow} &= \upsilon p_e(1) + \gamma_e + \gamma_u(1-p_u(0))\frac{u}{1-u} \quad j = 0. \end{aligned}$$

Similarly, the outflow can be due to separation from a job, losing or losing an offer in hand or because the worker was matched with the stock. The outflow is then given below

$$\text{outflow} = (\lambda_e + \gamma_e + \mu + \delta + \upsilon j)p_e(j).$$

The steady-state distributions are given by equalizing the outflow and inflow of a given number of job offers j. The number of outstanding offers is then

$$(\lambda_e + \gamma_e + \mu + \delta + \upsilon j)p_e(j) = \lambda_e p_e(j-1) + \upsilon(j+1)p_e(j+1) \quad \forall j \geq 1, \tag{13}$$

$$(\lambda_e + \gamma_e + \mu + \delta)p_e(0) = \upsilon p_e(1) + \gamma_e + \gamma_u(1-p_u(0))\frac{u}{1-u} \quad j = 0. \tag{14}$$

The inflow of unemployed workers with the stock of $j \geq 1$ vacancies can either be because a worker had a stock of $j-1$ vacancies and accrues one more, or because a worker with a stock $j+1$ loses one or an employed worker with that number of opportunities is hit by a job destruction shock. The inflow for $j = 0$ is from unemployed workers who lose an offer and employed workers with no offers that are hit by a destruction shock

$$\begin{aligned} \text{inflow} &= \lambda_u p_u(j-1) + \upsilon(j+1)p_u(j+1) + (\delta+\mu)\frac{u}{1-u}p_e(j) \quad \forall j \geq 1, \\ \text{inflow} &= \upsilon p_u(1) + (\delta+\mu)\frac{u}{1-u}p_e(0). \end{aligned}$$

For the unemployed, the outflow can be due to workers taking job offers when they match with the stock at a rate γ_u. In addition, they also acquire new offers at a rate λ_u and lose offers at rate δ

$$\begin{aligned}
\text{outflow} &= (\lambda_u + \gamma_u + \upsilon j)p_u(j) \quad \forall j \geq 1, \\
\text{outflow} &= \lambda_u p_u(0) \quad j = 0.
\end{aligned}$$

The steady-state distribution solves the equations

$$(\lambda_u + \gamma_u + \upsilon j)p_u(j) = \lambda_u p_e(j-1) + \upsilon(j+1)p_u(j+1) + (\delta + \mu)\frac{u}{1-u}p_e(j) \quad \forall j \geq 1, \tag{15}$$

$$\lambda_u p_u(0) = \upsilon p_u(1) + (\delta + \mu)\frac{u}{1-u}p_e(0) \quad j = 0. \tag{16}$$

A.1.2 Derivations of Σ

Employed Σ_e. Define the probability generating function (pgf) of the stationary distribution as

$$\Sigma_e(F) = \sum_{j=0}^{\infty} F^j p_e(j). \tag{17}$$

Summing equations (13) and (14) over j and using the definition of $\Sigma_e(F)$ gives

$$0 = -(\lambda_e(1-F) + \gamma_e + \mu + \delta)\Sigma_e(F) + \upsilon(1-F)\Sigma_e'(F) + \gamma_e + \mu + \delta.$$

Solving the differential equation gives

$$\Sigma_e(F) = \frac{1}{1-F}\int_F^1 \exp\left[-\lambda_e/\upsilon\left(\tilde{F} - F\right)\right]\left(\frac{1-\tilde{F}}{1-F}\right)^{\frac{\gamma_e+\mu+\delta}{\upsilon}-1}\frac{\gamma_e + \mu + \delta}{\upsilon}d\tilde{F}.$$

The limits are

$$\Sigma_e(1) = 1, \tag{18}$$

$$\frac{\partial \Sigma_e(F)}{\partial F}|_{F=1} = \frac{\frac{\lambda_e}{\upsilon}}{1 + \frac{\gamma_e+\mu+\delta}{\upsilon}} = \frac{\lambda_e}{(\gamma_e + \mu + \delta + \upsilon)}, \tag{19}$$

$$\frac{\partial^2 \Sigma_e(F)}{\partial F^2}|_{F=1} = \frac{2\lambda_e^2}{(\gamma_e + \mu + \delta + \upsilon)(\gamma_e + \mu + \delta + 2\upsilon)}. \tag{20}$$

Unemployed Σ_u. Define the pgf for the average unemployed as

$$\Sigma_u(F,t) = \sum_{j=0}^{\infty} F^j p_u(j,t). \tag{21}$$

Summing equations (15) and (16) over j and using the definition of $\Sigma_u(F)$ gives

$$0 = -(\lambda_u(1-F)+\gamma_u)\Sigma_u(F) + \upsilon(1-F)\Sigma'_u(F) + (\delta+\mu)(1-u)/u\Sigma_e(F) + \gamma_u\Sigma_u(0).$$

Solving the differential equation using $\Sigma_u(1) = 1$ gives

$$\Sigma_u(0) = \frac{\int_0^1 \exp\left[-\lambda_u/\upsilon\tilde{F}\right]\left(1-\tilde{F}\right)^{\frac{\gamma_u}{\upsilon}-1}\left[\frac{\gamma_u\Sigma_e(\tilde{F})}{\upsilon}\right]d\tilde{F}}{1-\int_0^1 \exp\left[-\lambda_u/\upsilon\tilde{F}\right]\left(1-\tilde{F}\right)^{\frac{\gamma_u}{\upsilon}-1}\left[\frac{\gamma_u}{\upsilon}\left(1-\Sigma_e(\tilde{F})\right)\right]d\tilde{F}},$$

$$\Sigma_u(F) = \frac{1}{1-F}\int_F^1 \exp\left[-\lambda_u/\upsilon(\tilde{F}-F)\right]\left(\frac{1-\tilde{F}}{1-F}\right)^{\frac{\gamma_u}{\upsilon}-1}\left[\frac{\gamma_u\Sigma_u(0)}{\upsilon} + \frac{(\delta+\mu)(1-u)/u\Sigma_e(\tilde{F})}{\upsilon}\right].$$

Unemployed Σ_{uu}. Lastly, we derive the distribution of wages that a worker expects who starts in unemployment with no prospects. The flow equations are then given by

$$0 = -(\lambda_u+\gamma_u+\upsilon j)p_{uu}(j) + \lambda_u p_{uu}(j-1) + \upsilon(j+1)p_e(j+1)\forall j \geq 1,$$

$$0 = -(\lambda_u+\gamma_u)p_{uu}(0) + \upsilon p_{uu}(1) + \gamma_u.$$

Rewriting in terms of the probability generating function gives

$$0 = -(\lambda_u(1-F)+\gamma_u)\Sigma_{uu}(F) + \upsilon(1-F)\Sigma'_{uu}(F) + \gamma_u.$$

Again, solving the differential equation gives

$$\Sigma_{uu}(F) = \frac{1}{1-F}\int_F^1 \exp\left[-\lambda_u/\upsilon\left(\tilde{F}-F\right)\right]\left(\frac{1-\tilde{F}}{1-F}\right)^{\frac{\gamma_u}{\upsilon}-1}\frac{\gamma_u}{\upsilon}d\tilde{F}.$$

A.1.3 Distribution of outstanding matches G

The first and second derivative of $G(\cdot)$ are given by

$$u = \frac{(\delta+\mu)}{(\delta+\mu+\gamma_u(1-\Sigma_u(0)))}, \tag{22}$$

$$G(F) = \frac{(\delta+\mu)\left(\Sigma_u(F)-\Sigma_u(0)\right)}{\left(1-\Sigma_u(0)\right)\left(\delta+\mu+\gamma_e(1-\Sigma_e(F))\right)}, \tag{23}$$

$$G'(F) = \frac{(\delta+\mu)\Sigma'_u(F)}{\left(1-\Sigma_u(0)\right)\left(\delta+\mu+\gamma_e(1-\Sigma_e(F))\right)} + \frac{\gamma_e(\delta+\mu)\left(\Sigma_u(F)-\Sigma_u(0)\right)\Sigma'_e(F)}{\left(1-\Sigma_u(0)\right)\left(\delta+\mu+\gamma_e(1-\Sigma_e(F))\right)^2}, \tag{24}$$

$$\begin{aligned} G''(F) &= \frac{(\delta+\mu)\Sigma''_u(F)}{\left(1-\Sigma_u(0)\right)\left(\delta+\mu+\gamma_e(1-\Sigma_e(F))\right)} + \frac{\gamma_e(\delta+\mu)\left(\Sigma_u(F)-\Sigma_u(0)\right)\Sigma''_e(F)}{\left(1-\Sigma_u(0)\right)\left(\delta+\mu+\gamma_e(1-\Sigma_e(F))\right)^2} \\ &+ 2\gamma_e\Sigma'_e(F)\frac{(\delta+\mu)\Sigma'_u(F)}{\left(1-\Sigma_u(0)\right)\left(\delta+\mu+\gamma_e(1-\Sigma_e(F))\right)^2} \\ &+ 2\frac{\gamma_e^2(\delta+\mu)\left(\Sigma_u(F)-\Sigma_u(0)\right)\Sigma'_e(F)^2}{\left(1-\Sigma_u(0)\right)\left(\delta+\mu+\gamma_e(1-\Sigma_e(F))\right)^3}. \end{aligned} \tag{25}$$

A.2 Value Functions

The value function can be calculated using the (expected) average flow value and the (expected) average duration using the formula

$$\text{(Avg. Duration)}W(w(F),0) \quad = \quad \text{Avg. Flow benefit.}$$

The average duration in a job with quality F is $\delta+\mu+\gamma_e(1-\Sigma_e(F))$. The average flow benefits are given by the wage $w(F)$ plus the search option $\gamma_e \int_F^1 W(w(\tilde{F}),0)d\Sigma_e(\tilde{F})$ and the separation value δU_{eu}. Defining $W_F(w(F),j) = W_w(w(F),j)w'(F)$. The value function at the time of hiring is then given by[14]

$$W(w(F),0) \quad = \quad \frac{w(F)+\gamma_e \int_F^1 W(w(\tilde{F}),0)d\Sigma_e(\tilde{F})+\delta U_{eu}}{\delta+\mu+\gamma_e(1-\Sigma_e(F))}, \tag{26}$$

$$W_F(w(F),0) \quad = \quad \frac{w'(F)}{\delta+\mu+\gamma_e(1-\Sigma_e(F))}, \tag{27}$$

$$W(w(F),0)-W(0,0) \quad = \quad \int_u^F \frac{w'(\tilde{F})}{\delta+\mu+\gamma_e(1-\Sigma_e(\tilde{F}))}d\tilde{F}. \tag{28}$$

[14]Note that differentiating (2) with respect to F gives

$$(\gamma_u(1-F^j)+\mu+\lambda_e+\delta+j\delta)W_F(w(F),j) = w'(F)+\lambda_e W_F(F,j+1)+\delta j W_F(F,j-1)+\gamma_u(1-F^j)W_F(F,0).$$

One can write the above expression in the form below, where the row and column of the matrix correspond to the number of job offers, starting with zero

$$W_F = \begin{bmatrix} (\rho+\lambda_e+\delta+\mu) & -\lambda_e & 0 & 0 & \dots \\ -\delta-\gamma_u(1-F) & (\gamma_u(1-F)+\mu+\lambda_e+\delta+\upsilon) & -\lambda_e & 0 & \dots \\ -\gamma_u(1-F^2) & -2\upsilon & (\gamma_u(1-F^2)+\mu+\lambda_e+\delta+2\upsilon) & -\lambda_e & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}^{-1} w'(F).$$

Inverting the matrix and using the first element, we get the following

$$W_F(w(F),0) \quad = \quad \frac{w'(F)}{\delta+\mu+\gamma_e(1-\Sigma_e(F))},$$

Similarly, for the unemployed, we have

$$\begin{aligned}
U(0) &= \frac{b+\gamma_u \int_0^1 W(w(\tilde{F}),0)d\Sigma_{uu}(\tilde{F})}{\mu+\gamma_u(1-\Sigma_{uu}(0))},\\
U_{ue} &= \frac{b+\gamma_u \int_0^1 W(w(\tilde{F}),0)d\Sigma_{u}(\tilde{F})}{\mu+\gamma_u(1-\Sigma_{u}(0))},\\
U_{ue}-U(0) &= \frac{b+\gamma_u \int_0^1 W(w(\tilde{F}),0)d\Sigma_{u}(\tilde{F})}{\mu+\gamma_u(1-\Sigma_{u}(0))} - \frac{b+\gamma_u \int_0^1 W(w(\tilde{F}),0)d\Sigma_{uu}(\tilde{F})}{\mu+\gamma_u(1-\Sigma_{uu}(0))}.
\end{aligned}$$

Evaluating the value function for an employed worker at the worst match ($F = 0$) and using $W(w(0), 0) = U(0)$ gives

$$\begin{aligned}
b &= w(0)+\gamma_e\int_0^1 \frac{w'(\tilde{F})(1-\Sigma_e(\tilde{F}))}{\delta+\mu+\gamma_e(1-\Sigma_e(\tilde{F}))}d\tilde{F} - \gamma_u\int_0^1 \frac{w'(\tilde{F})(1-\Sigma_{uu}(\tilde{F}))}{\delta+\mu+\gamma_e(1-\Sigma_e(\tilde{F}))}d\tilde{F}\\
&+\delta\left(\frac{\gamma_u\int_0^1 \frac{w'(\tilde{F})}{\delta+\mu+\gamma_e(1-\Sigma_e(\tilde{F}))}\left(\Sigma_{uu}(\tilde{F})-\Sigma_u(\tilde{F})\right)d(\tilde{F})}{(\mu+\gamma_u(1-\Sigma_u(0)))}\right).
\end{aligned}$$

A.3 Proof of Identification

The lowest wage worker, $w(0) = \phi$, is identified from the data. The transition rates μ and δ can be estimated using the rate at which the employed workers leave employment for unemployment and to be out of the labor force, respectively. υ is calibrated outside the estimation. From the data, we can estimate the job finding rate of someone employed in the lowest job $F = 0$ as a function of tenure. We denote the quit rate in the lowest match quality by $\gamma_e(1 - \Sigma_{e0}(0, t))$. The differential equation for $\gamma_e(1 - \Sigma_{e0}(F, t))$ is

$$\frac{\partial\gamma_e(1-\Sigma_{e0}(F,t))}{\partial t} = -\gamma_e\left(\upsilon(1-F)\frac{\partial\Sigma_{e0}(F,t)}{\partial F} - \lambda_e(1-F)\Sigma_{e0}(F,t)+\gamma_e\Sigma_{e0}(0,t)(1-\Sigma_{e0}(F,t))\right). \tag{29}$$

Replacing $\gamma_e(1 - \Sigma_{e0}(F, t))$ by $H_0(t, F)$ and $\gamma_e\frac{\partial\Sigma_{e0}(F,t)}{\partial F}$ by $-\frac{\partial H_0(t,F)}{\partial F}$, $\Sigma_{e0}(F, t) = 1 - \frac{H_0(t,F)}{\gamma_e}$ gives

$$\frac{\partial H(t,F)}{\partial t} = \frac{\partial H(t,F)}{\partial F}\upsilon(1-F) - (\gamma_e - H(t,0))\,H(t,F) - \lambda_e(1-F)\,(\gamma_e - H(t,F)). \tag{30}$$

Note that $H(t, F)$ is identified in the date from the rate at which the worker takes a job that pays $w(F)$. Evaluating this expression at at $t = 0$, we have $H(0, F) = \gamma_e(1 - \Sigma_e(F, 0)) = 0$ and

$\frac{\partial H(0,F)}{\partial F} = 0$ which implies

$$\lambda_e = \frac{-\frac{\partial H(t,F)}{\partial t}|_{t=0}}{\gamma_e(1-F)}. \quad (31)$$

Using this in the "original" expression, we get

$$\frac{\partial H(t,F)}{\partial t} = \frac{\partial H(t,F)}{\partial F}\upsilon(1-F) - (\gamma_e - H(t,0))\,H(t,F) + \frac{1}{\gamma_e}\frac{\partial H(t,F)}{\partial t}|_{t=0}\,(\gamma_e - H(t,F))\,. \quad (32)$$

This gives the quadratic equation above in γ_e with the solution

$$\gamma_e = \frac{-\left[-\frac{\partial H(t,F)}{\partial t} + \frac{\partial H(t,F)}{\partial F}\upsilon(1-F) + H(t,0)H(t,F) + \frac{\partial H(t,F)}{\partial t}|_{t=0}\right]}{-2H(t,F)}$$
$$\frac{\pm\sqrt{\left[-\frac{\partial H(t,F)}{\partial t} + \frac{\partial H(t,F)}{\partial F}\upsilon(1-F) + H(t,0)H(t,F) + \frac{\partial H(t,F)}{\partial t}|_{t=0}\right]^2 - 4H(t,F)\frac{\partial H(t,F)}{\partial t}|_{t=0}H(t,F)}}{-2H(t,F)}.$$

Noting that $H(t,F) \geq 0$ and $\frac{\partial H(t,F)}{\partial t}|_{t=0} < 0$ implies that there is a unique positive solution. The equation therefore solves for γ_e. Using the previous equation, we get λ_e. Having identified λ_e, γ_e, it is straightforward to identify γ_u. Note that we can calculate $\Sigma_e(F)$ and the instantaneous job finding rate following separation is

$$\gamma_u(1 - \Sigma_e(F)), \quad (33)$$

which gives γ_u. We can then identify λ_u by the job finding rate of the long-term unemployed.

A.4 Vacancy Duration

The data are taken from the "The Conference Board Help Wanted Online Data Series" (HWOL). The HWOL aims at an exhaustive coverage of all job vacancies advertised online. Data are thus collected from over 16,000 online job boards. The data contain two time series, starting in May 2005 and updated contemporaneously. The first is 'new ads', that is, the number of unduplicated ads that did not appear in the previous reference period. An ad is only counted as 'new' in the first reference point in which it appears. The second variable is 'total ads'. This is the total number of unduplicated ads appearing in the reference period. This is the sum of 'new ads' and reposted ads from previous periods. Finally, it is worth noting that a reference period is centered on the first of the months. For example, 'total ads' for October is the sum of all posted ads from September 14$^{\text{th}}$ until October 13$^{\text{th}}$.

Expiry Rate of a Vacancy. We use these data to infer the rate at which vacancies expire. A steady-state approximation implies that the inflow of new vacancies in month t (n_t) is equal to the total amount of vacancies expiring, the product of the stock (υ_t) and the expiry rate (σ_t).

$$n_t \approx \sigma_t \upsilon_t$$

Unfortunately, we do not observe a snapshot of the stock of vacancies. Instead, we observe the total vacancies that have accumulated over that reference period, which we call V_t. Since the stock of vacancies is constant over a reference period, given our steady-state assumption, we can approximate υ_t as

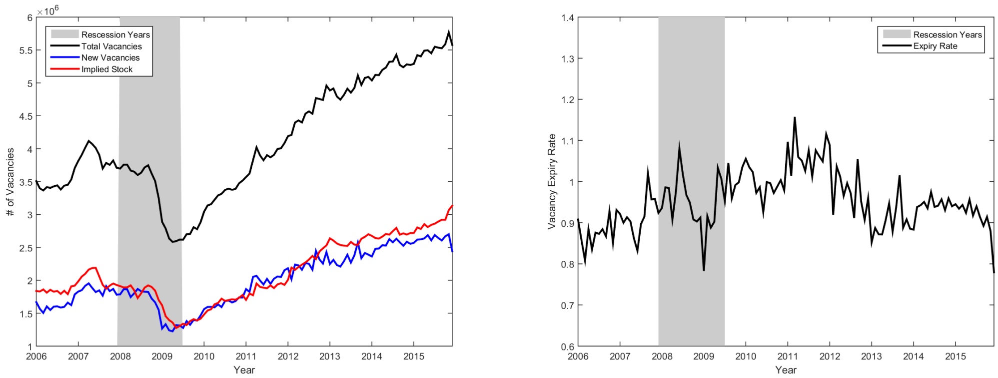
$$\upsilon_t \approx V_t - n_t.$$

Combining the above gives a straightforward approximation of the monthly rate at which vacancies expire for a reference period t.

$$\sigma_t \approx \frac{n_t}{V_t - n_t}$$

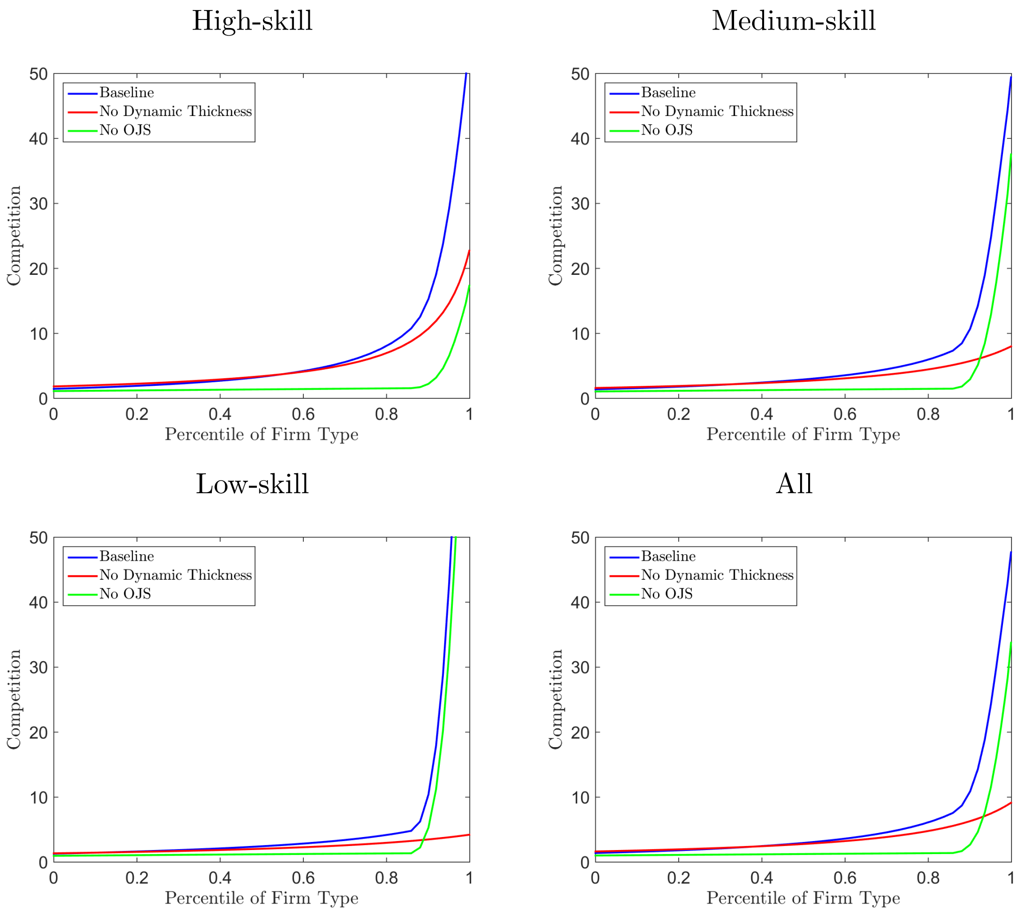
We restrict the attention to the decade January 2006 to December 2016. Changing the time horizon does little to change the mean monthly expiration date which is computed as 0.95, implying that vacancies last a little longer than a month. The series are presented in Figure A.4. The first panel shows the raw series of total and new vacancies as well as the implied number of vacancies in the stock at that point in time. The second panel shows the implied expiry rate of vacancies over the period.

Figure A.4: Vacancy Series



A.5 Wage Competition

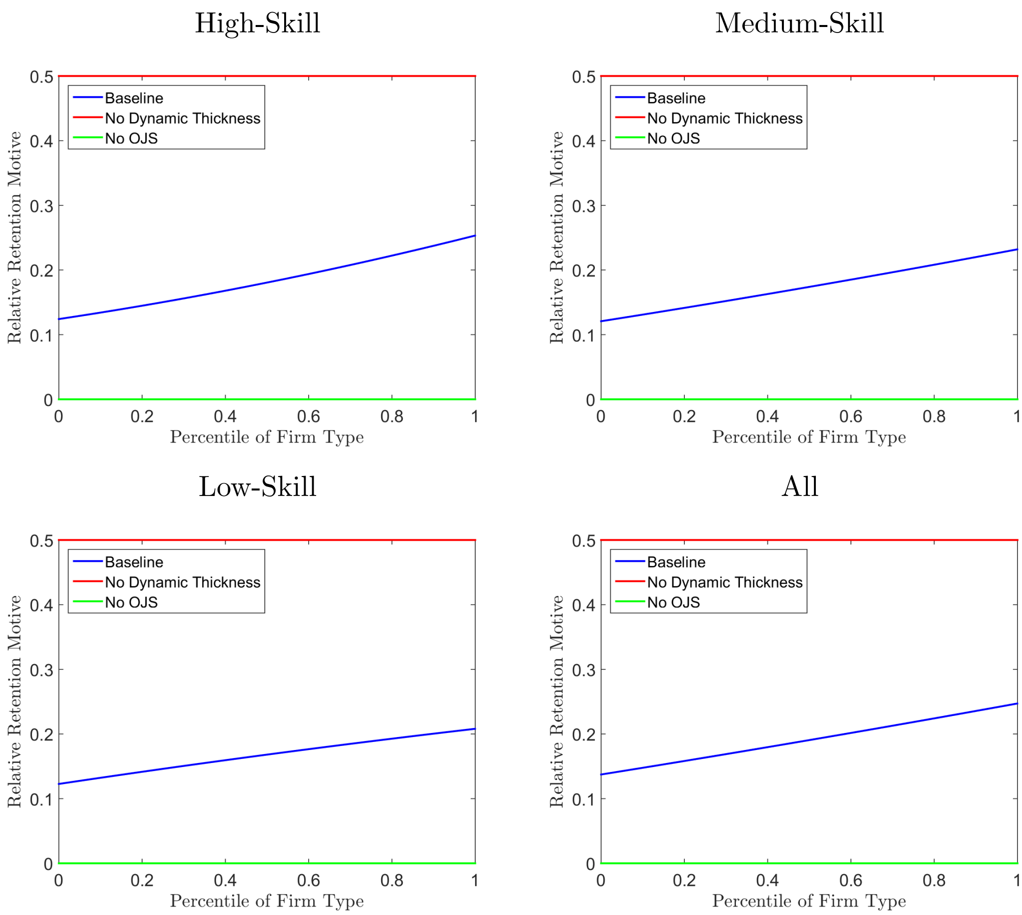
A.5: Wage Competition



Note: This figure plots the degree of competition in each model, as defined by ${\ell'(F)}/{\ell(F)}$.

A.6 Proportion of Wages Driven by the Retention Motive

A.6: Proportion of Wages Driven by the Retention Motive



Note: The relative retention motive is bounded on $[0, 1]$ and defined as $r(F)/(r(F)+h(F))$.

A.7 Robustness to the Calibration of υ

The goal of this section to argue that the specific calibration made of υ is inconsequential to the primary findings of the paper. By computing the average vacancy duration in section A.4, υ is calibrated as 0.95. However there is a fairly broad range of *sensible* calibrations, see Table A.7. To assess the implication of the value of υ we re-estimate the first step of the model varying υ from a half to one and a half, implying vacancy durations between three weeks and two months. The productivity parameters are then re-estimated ensuring the level of frictional wage dispersion remains fixed. This is analogous to the way in which the models with no dynamic thickness or no on the job search are estimated. For brevity this exercise is conducted only on the unstratified sample.

Table A.7: Estimates of Mean Vacancy Duration in the U.S.

Paper	Data Source	Time Period	Mean Duration	Implied υ
Brencic and Norris (2012)	Monster.com	2004-2006	44 days	0.68
Davis and Samaniego de la Parra (2017)	JOLTS	2012-2016	41.9	0.72
Davis and Samaniego de la Parra (2017) + Crane et al. (2016)*	JOLTS + SCE	2012-2016	58.1 days	0.52
Davis et al. (2013)$^{\times}$	DHI-DFH	2001-2018	30.5	1.02
Marinescu and Wolthoff (2017)	CareerBuilder.com	2011	15.7 days	1.91
This paper	HWOL	2005-2018	28.5 days	0.95

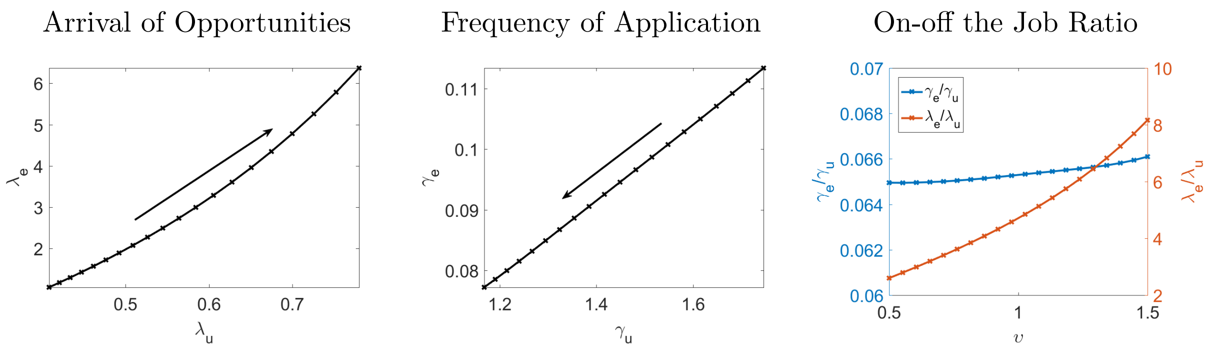
$\star$ The 16.2 days from the addition of Crane et al. (2016) represents the additional lapsed time associated with the 'start lag'. The time taken after a vacancy is filled and when the job commences.

$\times$ This moment is taken from the mean of the "DHI-DFH Mean Vacancy Duration Measure", based on a time series for the U.S. from January 2001 until April 2018. Data are taken from DHI Group, Inc., DHI-DFH Mean Vacancy Duration Measure [DHIDFHMVDM], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/DHIDFHMVDM. Note, this is measured in working days, so to compute calendar days durations are multiplied by 7/5.

Figure A.7a shows how the transitional parameters change varying the parameter υ. As the rate at which vacancies expire increases, in order to match the transitional moments, λ_u and λ_e, the rate opportunities arrive increase. However, since workers now lose their opportunities at a higher rate, differences between the short and long term unemployed reduce. To maintain the same job finding rates by duration of unemployment, the estimates of λ_e increase at a faster rate than λ_u. Changes to γ_u and γ_e, the frequency to which unemployed and employed workers access the market are, by comparison, small. This is because the majority of workers, particularly the employed and recently unemployed, have some opportunities. Further, the ratio of γ_e to γ_u remains fairly stable for any υ whereas implied differences in the arrival rate of opportunities across employment state vary enormously with υ.

Rather than the parameter values, what is perhaps more important is how the implications of the model vary with υ. In particular, in determining the primary cause of unemployment and the ability of the model to replicate wage dispersion. In section 3.5 the unemployment rate is computed

Figure A.7a: Transition Parameters - Varying Upsilon

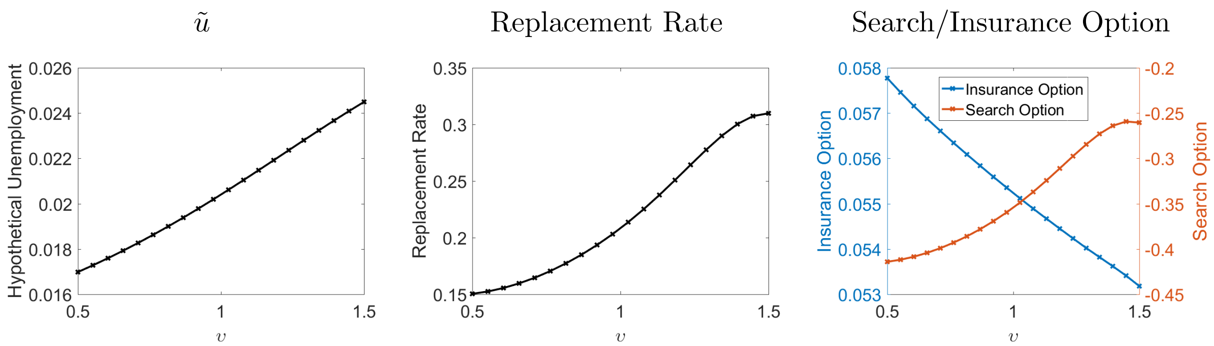


Notes: The connected 'x's represent individual estimates for a given υ. The large arrows in the first two panels show the direction in which υ is increasing, values range from 0.5 to 1.5.

assuming all workers have labor market opportunities. Since as υ increases the estimate of γ_u decreases, so fewer unemployed apply for jobs, thus this hypothetical unemployment rate increases with υ. However, since changes in γ_u are relatively small, so are changes to the unemployment decomposition. Across the entire span of υ both a lack of opportunity and simply not accessing the market are quantitatively important. The latter varies from explaining 38% of total unemployment, when υ is a half, to 56% when υ is one and a half.

Turning to wage dispersion, as is made apparent in the main body of the text, the canonical job ladder model struggles to generate the level of frictional wage dispersion seen in the data without negative replacement rates. Figure A.7b documents how the implied replacement rate changes with υ, fixing the same level of frictional wage dispersion. Over the span of calibrated υ the replacement rate varies from a low of 15% to a high of 30%. Although the implied value of home production clearly depends on the specific value of υ, the model can also generate positive replacement ratios consistent with the macro labor literature for the empirically relevant range of values of υ. Given the level of frictional wage dispersion, the two components that determine the replacement rate are the search option and the insurance option. These two move in opposite directions in response to a change in the expiration of opportunities, again see Figure A.7b. The search option increases, decreasing in absolute terms. Since λ_e is growing relative to λ_u as υ increases, there is an increase in the returns to taking a job for a given wage, and less value in remaining unemployed. The insurance option, the benefits of returning to unemployment in a better position, declines with υ. Since opportunities disappear more quickly, the short and long term unemployed are in similar positions after a shorter lapse of time. Since the search option is an order of magnitude larger than the insurance option, our estimates of the replacement rate increase with larger values of υ.

Figure A.7b: Replacement Rate - Varying Upsilon



S Supplementary Appendix

This Supplementary Appendix contains additional information for the paper by Bradley and Gottfries (2018). All references to sections and equations refer to this main paper.

S.1 Sample Selection

We make a special effort to ensure that the variables circumscribing the samples are consistent across surveys. That is, the following filters are passed through each survey.

(i) The attention is restricted to a sample of only male workers. The sex of a worker is defined in the SIPP by the variable *esex* and *pesex* in the CPS.

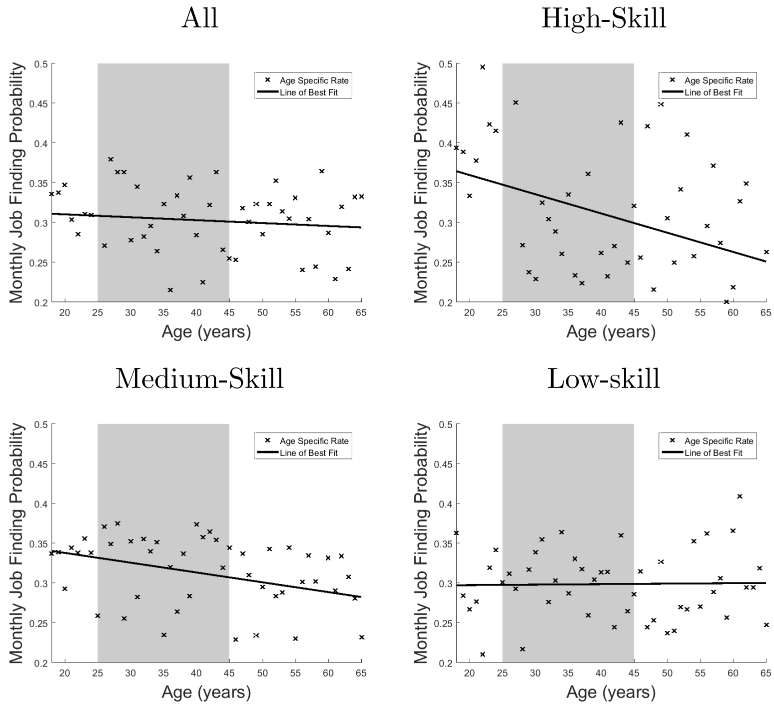
(ii) We use the full-time window in the 1996 SIPP, including early observations based on recall of previous employment. This corresponds to observations from December 1995 until February 2000, inclusive. The identical window is used in the CPS.

(iii) Motivated by differential mobility rates by age, see Appendix S.2, the attention is restricted to only workers between 25 and 45, where age is defined as a respondent's age as of last birthday in the variable *tage* in the SIPP and age as of the end of the survey week in the CPS by the variable *peage*. Note that this will introduce negligible differences across samples when a respondent's birthday occurs in a CPS surveying week.

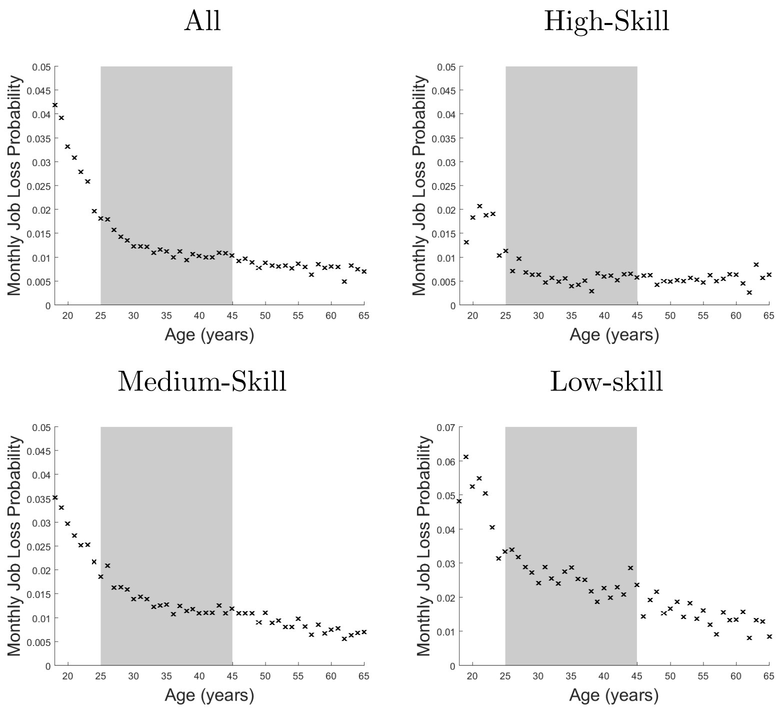
(iv) Skill groups are defined by the variables *eeducate* in the SIPP and *peeduca* in the CPS. The two variables are defined identically with one exception. The CPS variable differentiates between having a '*diploma or certificate from a voc, tech, trade or bus school beyond high school*' and having an '*associate degree in college - occupational/vocational program*' while the SIPP variable agglomerates the two. We treat these two groupings as college educated and include them as high-skill workers. All other groupings are non-controversial.

S.2 Transition Rates by Age

Job Finding Rate



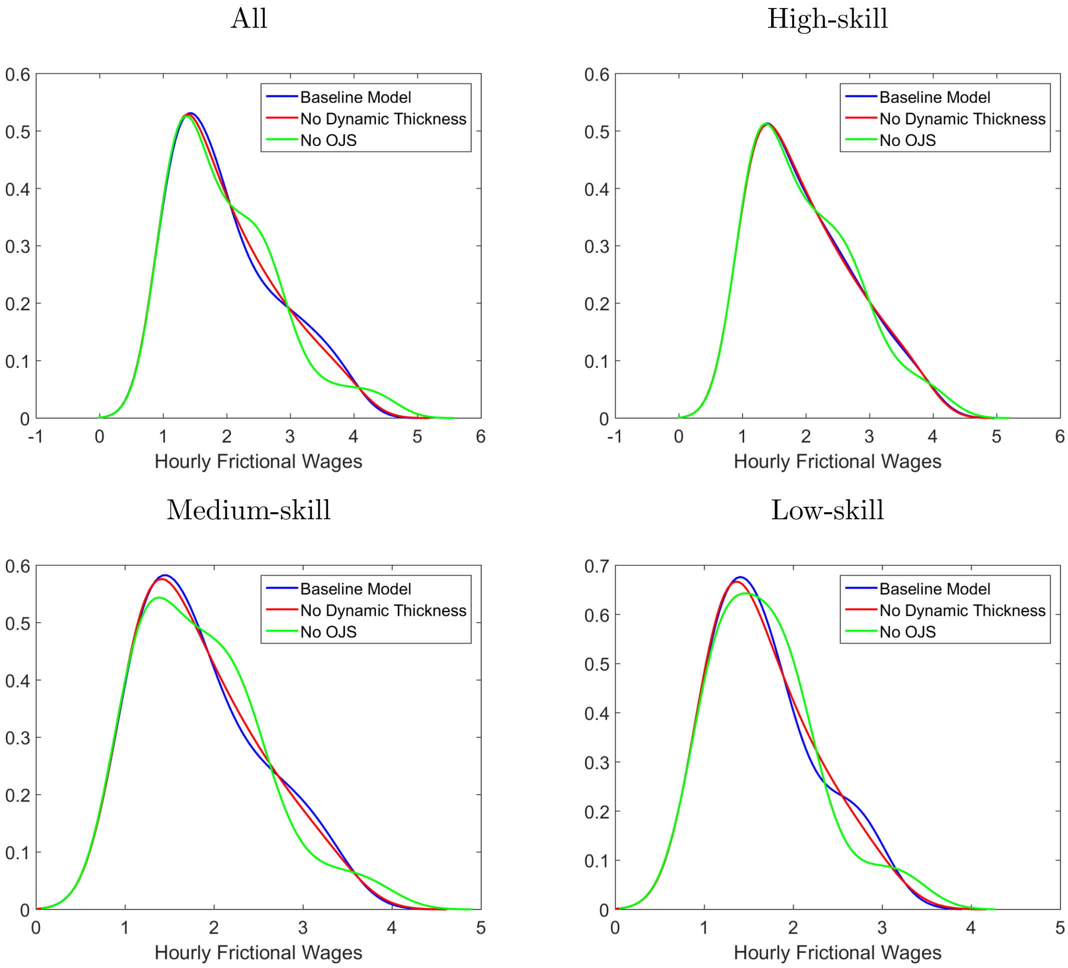
Separation Rate



Note: The ‘x’s represent the appropriate monthly transition probability for a male of that age. The shaded region represents the specific age we will focus on in our analysis. Data come from the CPS, 1996-1999 inclusive.

S.3 Fit of Frictional Wage Distribution

S.3: Fit of Wage Moments



Note: The kernel density plot showing the fit of the two nest models, no dynamic thickness and no on-the-job search to the distributional distribution of frictional wages as predicted by the baseline.

Reference

Bradley, J. and A. Gottfries, "A Job Ladder Model with Stochastic Employment Opportunities", *mimeo* (2018)