Safety traps, liquidity and information-sensitive assets

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Abstract

We investigate the implications of safe assets scarcity in a framework in which the safety of an asset is an equilibrium outcome. The intrinsic characteristics and supply of the assets determine their liquidity properties and degree of safeness. The equilibrium can be inefficient although assets are plentiful and information-insensitive. Only a sufficiently broad expansion of a particular class of safe information-insensitive assets can achieve the first-best allocation, while a marginal increase in their supply can be ineffective. We conclude that microfounding assets safety is fundamental to understand the effects and policy implications of safe assets scarcity.
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1 Introduction

The economic literature has recently begun to explore the role of safe assets in the economy and the resulting implications for monetary policy and financial stability. Safe assets, and in particular their scarcity, may be an essential factor to understand the secular decline of real interest rates or why global macroeconomic imbalances build up.[1] However, what makes an asset safe? What does exactly mean for safe assets to be scarce?

In most of this literature, securities are safe when they have a non-stochastic payoff. However, many assets that we usually consider safe do not fall under this definition. For example, government bonds can be considered default-free in nominal terms but are still subject to inflation risk. For this reason, we adopt a different approach: safety should be an equilibrium outcome.

In this paper we explicitly consider the role of information in the determination of the degree of *safeness* of the assets. Following Gorton (2017), an asset is safe as long as there is no incentive to produce private information about its quality.[2] Indeed, information can generate volatility in the value of an asset, making it not suitable for facilitating transactions. This implies that opacity may be preferred to transparency.[3]

Regarding scarcity, instead, the *shortage* of safe assets matters if they have a special role compared to other assets. Intuitively, we mean that some transactions cannot be realized if agents do not have a sufficient amount of safe assets. Only if their supply is sufficiently large the economy can reach the first-best equilibrium, regardless of the availability of other assets. Otherwise, it is stuck in a *safety trap*, as in Caballero and Farhi (2018).[4]

[1] Caballero, Farhi and Gourinchas (2017) discuss these issues and provide empirical evidence of a shortage of safe assets. Gorton (2017) reviews the empirical literature and the implications for financial stability.

[2] For alternative definitions or microfoundations of the assets safety see Caballero, Farhi and Gourinchas (2017) and He, Krishnamurthy and Milbradt (2018).

[3] See for example: Hirshleifer (1971); Andolfatto (2010); Gorton and Ordoñez (2013, 2014); Andolfatto, Berentsen and Waller (2014); Dang et al. (2017).

[4] A different notion of safety trap has been introduced by Benhima and Massenot (2013), that refers to situations in which risk aversion and habit consumption can lead to an inefficient over-accumulation of assets with the non-stochastic payoff. In this paper we always refer to the definition of Caballero and Farhi (2018).

Our first contribution is to show that a safety trap can arise naturally in a general framework which microfounds when and why an asset is safe and why people demand safe assets. When safety is an equilibrium outcome there is no need to resort to nominal rigidities or extreme risk-aversion as in Caballero and Farhi (2018). More generally, we show that being explicit about the determinants of assets safety is fundamental to understand the impact and the policy implications of safe assets scarcity.

We consider a general equilibrium environment à la Lagos and Wright (2005), in which limited commitment and the absence of a record-keeping technology make unsecured credit unfeasible. Assets are essential because they allow the realization of profitable bilateral exchanges that, otherwise, would not be feasible. As in Gorton and Ordoñez (2013, 2014), the critical friction is that assets can be *information-sensitive*. It means that agents choose to produce costly private information about the assets payoffs before the latter become public knowledge. Information acquisition introduces uncertainty about the outcome of the transactions, which will depend on the information that has been produced. Assets are safe, instead, when they are *information-insensitive*, in the sense that there is no endogenous production of information.

Information-insensitive assets can have different abilities in facilitating transactions. Suppose there is a divisible asset with a stochastic payoff, called $\mathcal{A}$. The option to produce costly private information can generate a haircut on the asset's value or an endogenous upper bound for the amount of assets that can be transacted.[5] Agents could be constrained in the use of the asset and, differently from Gorton and Ordoñez (2013, 2014), the first best could not be attained, regardless of how abundant the asset is and although in equilibrium the asset is information-insensitive. Assets are definitively *safer* when agents can use them in the desired amount and trade them at face value without the threat of information acquisition. It is the scarcity of these *safer* assets – called $\mathcal{B}$ – that keeps the output of the economy below its optimal level. Only increasing their supply

[5]Intuitively, the private value of information is increasing in the amount of asset transacted, while the cost of information acquisition is fixed. Both the haircut and the endogenous upper bound allow to keep the profit from information acquisition lower than its cost.

can improve welfare, regardless of the supply of the other assets.

This result is equivalent to the safety trap described in Caballero and Farhi (2018), although both the rational and the policy implications are different. For example, suppose both assets $\mathcal{A}$ and $\mathcal{B}$ coexist. For a small initial provision of the asset $\mathcal{B}$, a costless marginal increase in its supply can affect only asset prices, and surprisingly there could be no benefits concerning welfare. Agents do not change their level of consumption but only the mix of assets used in their transactions. They use more of the asset $\mathcal{B}$ and less of the asset $\mathcal{A}$. Only when the initial supply of the asset $\mathcal{B}$ is sufficiently large, a marginal increase in its provision is welfare improving and leads to an expansion of trade and production in the economy. Therefore, differently from the previous literature the benefits of increasing the supply of the safest assets depend on their initial amount and the magnitude of the expansion, even under the extreme assumption that changing their supply is costless. We conclude that microfounding assets safety is fundamental to understand the policy implications of safe assets scarcity.

Related literature. The effects of a shortage of safe assets have been extensively analyzed by Caballero in a series of contributions since 2006 (for a review see Caballero, Farhi and Gourinchas, 2017). Caballero and Farhi (2018) introduced the notion of safety trap in a Keynesian model in which the demand for safe assets determines the natural interest rate.[6] Differently from them, our results do not depend on risk aversion but information frictions. Moreover, here prices are flexible, while their results rely on nominal rigidities.

This paper endogenizes assets liquidity by allowing for the possibility to produce private information about the quality of the assets, as in Dang, Gorton and Hölmstrom (2015*a*,*b*). While their focus is the determination of the optimal security that agents use to trade, here we are interested in the implications of the coexistence of different assets as a medium of exchange. The most related works are Gorton and Ordoñez (2013, 2014). Gorton and Ordoñez (2013) look to the

[6]The scarcity of safe assets pushes the natural (safe) interest rate down. When the economy hits the zero lower bound the real rate cannot clear the market for safe assets. Since nominal prices cannot adjust, this excess demand can only be absorbed if output goes down.

coexistence of assets with a stochastic and non-stochastic payoff, respectively, showing that the latter is redundant when the first is information-insensitive. In our case, instead, the threat of information acquisition can make the asset with a non-stochastic payoff essential also when the other securities are information-insensitive and abundant. Differently from Gorton and Ordoñez (2013), we make different assumptions about preferences and assets are divisible.

Andolfatto, Berentsen and Waller (2014) consider a framework similar to this paper, but they are concerned about the desirability for the social planner to disclose information about the quality of the assets. Also Andolfatto (2010) and Andolfatto and Martin (2013) show that information may make an asset not suitable as a mean of payment. In their case the disadvantage is that the asset can have a low valuation in a situation in which there is a need for liquidity, while here informational frictions affect the ability of the assets to be used as medium of exchange.

Like the two previous contributions, this paper is related to the New Monetarist literature (Lagos, Rocheteau and Wright, 2017; Nosal and Rocheteau, 2011). We endogenize assets liquidity in the Lagos and Wright (2005) framework, but we abstract from fiat money and other nominal assets.[7] Also in Lester, Postlewaite and Wright (2011, 2012) agents can invest in a costly technology to recognize the quality of assets. Differently from here, in their case this choice must be made before meeting a counterpart and receiving an offer. Li, Rocheteau and Weill (2012) extend their model to the case in which agents can produce at a positive cost counterfeited assets, and this generates an endogenous upper bound on the amount of assets that can be transferred in bilateral matches, similarly to our model. Rocheteau (2011) studies a signaling game in which some agents have superior information about the quality of an asset and make a take-it-or-leave-it offer to their counterparts. Asymmetric information makes assets partially illiquid, preventing to attain the first-best allocation. Here, a similar result arises from just the threat of asymmetric information.

[7]The role of real assets in facilitating transactions has been already studied by Geromichalos, Licari and Suarez-Lledo (2007) and Lagos (2010, 2011), although in their models the liquidity properties of the assets are taken as given.

In section 2 we present the structure of the model and we show when agents produce information. In section 3 we discuss the equilibria of the model and the implications of safe assets scarcity. Finally, in section 4 we draw the main conclusions, and we illustrate the direction for future research.

2 The model

Time is discrete, starts at t = 0 and continues forever. Similarly to Lagos and Wright (2005), each period is divided in two sub-periods. In the first sub-period trades occur in a decentralized market (DM), while in the last sub-period trades take place in a Walrasian centralized market (CM). There are two perishable consumption goods, one in each sub-period. There is a continuum of infinitely-lived agents divided into two types, both with measure 1. We call them *buyers* and *sellers*, and they differ regarding when they produce and consume.

In each period the utility of a buyer is $u(q_t) - h_t$, where q is the consumption of the DM good and h is the disutility of work during the second sub-period. The utility function $u(\cdot)$ is twice continuously differentiable, with $u(0) = 0$, $u'(0) = \infty$, $u'(\infty) = 0$, $u'(\cdot) > 0$ and $u''(\cdot) < 0$. The utility of a seller is $-q_t + c_t$, where the first term is the disutility to produce q_t units of goods in the DM and c_t is the linear utility from consuming in the CM. All agents discount future utility at a rate $\beta \in (0, 1)$.[8]

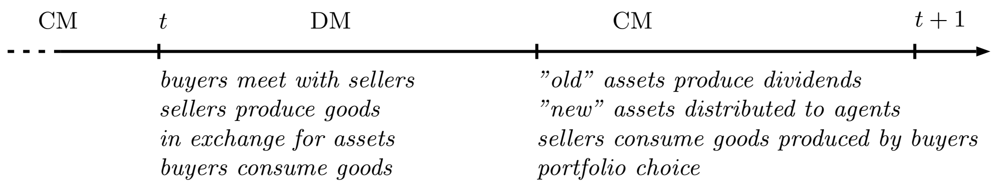

Figure 1: Timeline

During the day each buyer meets randomly with a seller and consumes the good produced by the seller, while during the night buyers produce and sellers

[8]Thanks to these assumptions we can make clear that the specialness of safe assets does not rely exclusively on risk aversion. See Gu, Mattesini and Wright (2016) for a discussion about potential generalizations.

consume. In this economy welfare is maximized by the following first-best allocation. In the DM buyers consume an amount q^* of goods produced by sellers, where q^* satisfies $u'(q^*) = 1$. In the CM $h = c = q^*$.

Unsecured debt would support the first-best allocation, but it is ruled out by the absence of a record-keeping technology and the impossibility for agents to make binding commitments (Lagos, Rocheteau and Wright, 2017). Assets can facilitate trade, meaning that they can allow buyers to reach a level of consumption otherwise unfeasible.

There are two one-period-lived divisible real assets, in positive net supply $A, B > 0$. Both assets pay dividends in terms of units of the CM good once this market opens. Then, there is a supply of A, B new units of both assets, that agents can buy at a price ρ^a and ρ^b (also in terms of the CM good), respectively.[9] Let us call the first asset $\mathcal{A}$ and the second one $\mathcal{B}$. The two securities differ because of their payoffs. Each unit of the asset $\mathcal{A}$ generates a stochastic dividend: with probability π_l its payoff is $\delta_l > 0$, while with probability $\pi_h = 1 - \pi_l$ is $\delta_h > \delta_l$. We define the expected dividend as δ, and we assume there is no serial correlation in the realization of the returns. The asset $\mathcal{B}$ has a non-stochastic payoff, that we normalize to 1. The crucial assumption is that the actual realization of the dividend of the asset $\mathcal{A}$ becomes public knowledge only at the beginning of the CM.[10]

2.1 Markets

We start from the analysis of the CM. We will define the quantities of the assets that an agent owns at the beginning of each sub-period using lowercase letters: a for the asset $\mathcal{A}$, b for the asset $\mathcal{B}$. Notice that we ignore time subscripts because we restrict the attention to stationary equilibria.

[9] We assume that buyers receive all the endowment of the new assets. Given the linearity of preferences this is without loss of generality.

[10] It would be easy to introduce fiat money or long term assets in this framework, but this is not essential for the purposes of this paper.

Centralized Market. The value function of a buyer in the CM is:

$$\begin{aligned} W^b(a,b) = \max_{h,a',b'} & \quad -h + \beta V^b(a',b') \\ \text{s.t.} & \quad \rho^a a' + \rho^b b' = h + \delta_j a + b + \rho^a A + \rho^b B \end{aligned}$$

where $j \in \{l,h\}$. The buyer chooses the amount of work to supply and the portfolio of assets to bring in the next period, (a',b'). He takes into account the continuation utility in the DM, V^b, the initial wealth, $\delta_j a + b$, and the endowment of new assets. Notice that δ_j becomes public knowledge at the beginning of this sub-period.

Substituting the budget constraint into the objective function, we get

$$W^b(a,b) = \delta_j a + b + \rho^a A + \rho^b B + \max_{a',b' \geq 0} \left[-\rho^a a' - \rho^b b' + \beta V^b(a',b') \right] \tag{1}$$

Since the value function is linear in the initial wealth and the stochastic dividends are i.i.d., the choice of asset holdings is independent of the state variables and the realization of δ_j.

The problem of the seller is derived equivalently, and his value function W^s is linear in the initial wealth. As we will discuss in section 3, in equilibrium sellers never bring assets in the next period, then we do not report their problem.

Decentralized market. In the DM each buyer is randomly matched with a seller and can use claims on his assets holdings as a medium of exchange.[11] Buyers and sellers know only the probability distributions of the payoff of asset $\mathcal{A}$. For the moment, agents do not receive any informative private signal. Buyers make take-it-or-leave-it offers denoted by $\mathbf{x} \equiv (q, d^a, d^b)$, where d^a and d^b are the quantities of the two assets that a buyer transfers to the seller in exchange for q units of the good. Given the information set and the linearity of the CM value functions, the surplus of the buyer is $S^b(\mathbf{x}) \equiv u(q) - \delta d^a - d^b$, while for the seller is $S^s(\mathbf{x}) \equiv -q + \delta d^a + d^b$.[12]

[11] Lagos (2011) and Venkateswaran and Wright (2014) show that in a large class of models it is irrelevant if an asset is used as a medium of exchange or as a collateral. We will discuss this assumption at the end of the section.

[12] It is also useful to introduce the ex-post utility of buyers and sellers, defined as S^b_j and S^s_j,

Because of symmetric ignorance about the future realization of δ_j, asset $\mathcal{A}$ is valued at its fair value δ, that is predictable. A buyer entering in the DM with this asset incurs no risk related to the consumption of DM goods, as in the case of a risk-free security.

Proposition 1 *If $\delta a + b \leq q^*$, then $q = \delta a + b$, $d^a = a$ and $d^b = b$. Otherwise, $q = q^*$, $\delta d^a + d^b = q^*$, while d^a and d^b are undetermined.*

If the buyer owns a sufficient amount of assets he can afford to consume the first-best quantity of goods. Otherwise, he deploys all his holdings of both assets consuming a quantity $\delta a + b$ of goods. In both cases the buyers extracts all the gains from trade. However, the main implication of Proposition 1 is that the two assets are equivalent, in the sense that they support the same allocations. If $\delta a \geq q^*$, asset $\mathcal{A}$ supports the efficient allocation and asset $\mathcal{B}$ is redundant.

2.2 Information acquisition

We assume that sellers can produce private information about the payoff of asset $\mathcal{A}$ by incurring a positive disutility cost θ. Sellers can learn the exact realization of δ_j after they have received an offer from a buyer, and before to decide to accept or reject.[13] Buyers do not have access to this technology and cannot observe if sellers acquired information.[14] The game proceeds as follows:

Stage 1. The buyer makes a take-it-or-leave-it offer to the seller.

Stage 2. The seller decides to produce information or not.

Stage 3. The seller decides to accept or reject the offer. If there is a rejection, the buyer cannot make a new offer.

Since the uninformed party moves first, this game has stagewise perfect information, so we look for pure strategy subgame perfect equilibria. Stage 2 is

with $j \in \{l, h\}$. Ex-post utility differs from ex-ante utility only because δ is replaced by δ_j.

[13] For example, this asset may be an asset-backed security and the seller can hire a financial expert that has the ability to assess the underlying assets.

[14] Based on Rocheteau (2011) we do not expect buyers to have a gain from information acquisition because they cannot take advantage of that. Sellers would understand that buyers are informed. Then, buyers should play a signaling game and sellers would end up extracting rents (see Rocheteau, 2011).

the crucial step: the problem of the seller is to compare the expected gain from acquiring the information with the cost θ. At stage 1, the buyer makes an offer that do or do not provide to the seller the incentive to produce information. The buyer compares the continuation utility of the strategy with no information acquisition, V^N, with the utility derived from an offer that gives the seller the incentives to acquire information, V^I. Let us define the indicator function $\tau(a, b) \equiv \mathbb{1}_{V^N(a,b) \geq V^I(a,b)}$. The value function of a buyer in the DM is defined as:

$$V^b(a, b) = \tau(a, b)V^N(a, b) + [1 - \tau(a, b)]V^I(a, b) \tag{2}$$

Before to move to the derivation of V^N and V^I, we briefly discuss the choice of the trading arrangement. We assume that assets are used as a medium of exchange because it can be shown that the incentives to produce private information would be the same with collateralized debt. Moreover, this assumption allows us to isolate the implications of endogenous private information in a microfounded model with minimal and, in particular, well-accepted assumptions about the environment (limited commitment and no record-keeping). Collateralized debt, instead, requires the specification a richer environment whose details can have strong implications for the final results.[15]

2.2.1 Strategy with no information acquisition

We first consider the strategy in which the buyer avoids the production of private information.

Suppose that the buyer makes an offer $\mathbf{x}$. If the seller does not produce information his expected utility is $S^s(\mathbf{x})$. An informed seller, instead, accepts to trade only if observes the payoff δ_h – in any equilibrium $q \geq d^b + \delta_l d^a$ – and his expected utility is $\pi_h S_h^s(\mathbf{x})$. As a consequence, the seller will not produce information if the expected profit from acquiring information, $-\pi_l S_l^s(\mathbf{x})$, is smaller than the cost of information acquisition, θ. That is $\pi_l \left(q - \delta_l d^a - d^b\right) \leq \theta$.

[15]We claim that our results can be retrieved with collateralized debt, but not in any model in which assets are used as collateral. We address this discussion in a companion paper.

Therefore, the problem of the buyer can be defined as follows:

$$V^N(a,b) = \max_{q,d^a \le a, d^b \le b} \; S^b\left(q, d^a, d^b\right) + \mathbb{E}W^b(a,b)$$

$$\text{s.t.} \quad S^s\left(q, d^a, d^b\right) \ge 0 \tag{3}$$

$$-\pi_l S_l^s\left(q, d^a, d^b\right) \le \theta \tag{4}$$

The buyer maximizes his surplus, keeping into account the following constraints. First, $d^a \le a$ and $d^b \le b$. Second, the seller gets a non-negative surplus (equation 3), and he has no incentive to produce private information (equation 4). A main implication of (4) is that sellers are more inclined to produce information as d^a increases, because the cost of information per unit of asset decreases. In particular, there exists a threshold value $\bar{a} \equiv \theta\left[\pi_l(\delta - \delta_l)\right]^{-1}$ such that if $d^a < \bar{a}$ the incentive constraint (4) is always slack. This threshold depends positively on the cost of information acquisition, θ, negatively on a proxy of the dispersion of the stochastic dividend, $\delta - \delta_l$, and the probability of realization of the bad state, π_l.

We define $\tilde{q}$ as the solution to $u'(\tilde{q}) = \delta/\delta_l$, $\tilde{a}(b) \equiv \max\left\{\left(\tilde{q} - b - \theta/\pi_l\right)/\delta_l, \bar{a}\right\}$, $\tilde{b} \equiv \max\left\{\tilde{q} - \delta\bar{a}, 0\right\}$ and $\bar{b}(a) \equiv q^* - \delta \min\left\{a, \bar{a}\right\}$. The following proposition summarizes the solution of the problem.

Proposition 2 *If $a \le \bar{a}$ or $b \ge \tilde{b}$, then $q = \min\left\{q^*, \delta d^a + b\right\}$, with $d^a = \min\{a, \bar{a}\}$; when $q = q^*$ the amount of d^b and d^a are undetermined (but $d^a \le \bar{a}$). If $a > \bar{a}$ and $b \in [0, \tilde{b})$, whenever $\tilde{b} > 0$, then $q = \min\left\{\tilde{q}, \delta_l a + b + \theta/\pi_l\right\}$, with $d^a = \min\left\{a, \tilde{a}(b)\right\}$.*

We start considering the case in which $b = 0$ (Figure 2a). If $a < \bar{a}$ the participation constraint of the seller (3) is binding, while (4) is slack. The buyer consumes an amount of goods $q = \min\left\{q^*, \delta a\right\}$ and extracts all the surplus from this transaction. If $\bar{a}$ is sufficiently large, $\bar{a} \ge q^*/\delta$, this is the only solution for all a and Proposition 1 applies. The most interesting case is when $\bar{a} < q^*/\delta$ and $a \ge \bar{a}$. Now (4) is binding, and the problem has two possible solutions.

First, if $\tilde{q} > \delta\bar{a}$ the constraint (3) is slack and buyers and sellers split the

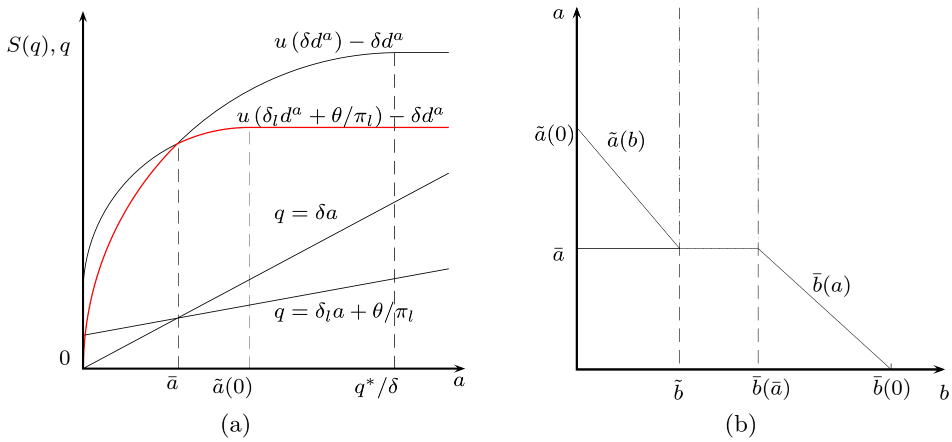

Figure 2: Optimal offer with no information acquisition.

gains from trade. Buyers give away an amount δd^a of initial wealth in the CM in exchange for $\delta_l d^a + \theta/\pi_l < \delta d^a$ units of consumption goods in the DM. The buyer must provide to the seller an informational rent to avoid the production of private information. Since this rent is increasing in d^a, a trade-off arises, and consumption of DM goods is at most $\tilde{q} < q^*$. At $\tilde{q}$ the marginal utility from using the asset to buy goods in the DM is equal to the marginal utility from using the asset in the CM, $u'(\tilde{q})\,\delta_l = \delta$.

Second, if $\tilde{q} \leq \delta\bar{a}$ also (3) is binding. Buyers choose $d^a = \bar{a}$ and $q = \delta\bar{a}$, also when $a > \bar{a}$. They keep all the gains from trade, but they cannot get a greater consumption.

We now consider the general case, in which $b > 0$ and $\delta\bar{a} < \tilde{q} < q^*$ (Figure 2b). When the holding of asset $\mathcal{A}$ is sufficiently small, $a \leq \bar{a}$, buyers extract all the surplus, and consumption is $q = \delta a + b$. Otherwise, when $a > \bar{a}$ we can establish a pecking order in the use of the two assets. Buyers first deploy all their holdings of the asset $\mathcal{B}$, then use the asset $\mathcal{A}$. If $b \geq \tilde{b}$ buyers keep all the gains from trade, but the threat of asymmetric information gives rise to an endogenous upper bound: $d^a = \bar{a} < a$. Consumption is $\delta\bar{a} + b$. When $b < \tilde{b}$, instead, (3) is slack and the consumption of buyers is $q = \min\{\tilde{q}, \delta_l a + b + \theta/\pi_l\}$. Notice that asset $\mathcal{B}$ is always valued at face value. Therefore, buyers deploy their holdings of this asset to minimize the informational rent of the sellers, that is increasing in d^a.

2.2.2 Strategy with information acquisition

Since avoiding information acquisition may be penalizing in terms of forgone consumption, a buyer can prefer to let the seller produce information.

In this equilibrium of the game the seller discovers the payoff of asset $\mathcal{A}$. Therefore, it makes sense to consider a strategy in which the buyer proposes a menu of offers with the following properties. First, the seller has the incentive to produce private information. Second, the seller chooses one of the menu offers. Third, the seller reveals his private information about asset $\mathcal{A}$ through its choice. Since asset $\mathcal{A}$ has two possible payoffs, δ_l (low state) and δ_h (high state), we consider a menu including two offers. We denote this menu by $\{\mathbf{x}_l, \mathbf{x}_h\}$, where $\mathbf{x}_j = \left\{q_j, d_j^a, d_j^b\right\}$ and $j \in \{l, h\}$.

The menu is designed in order to maximize the expected continuation utility of the buyer, $\pi_l S_l^b(\mathbf{x}_l) + \pi_h S_h^b(\mathbf{x}_h) + \mathbb{E}W^b(a,b)$, subject to a set of participation and incentive constraints; the related value function is denoted by V^I.[16] The seller produces information if his expected utility is nonnegative, $\pi_l S_l^s(\mathbf{x}_l) + \pi_h S_h^s(\mathbf{x}_h) \geq \theta$. Besides, the production of information is not observable. Therefore, producing information must be not dominated by a trivial strategy in which the seller saves the cost of information acquisition and always choose one of the two offers, $\mathbf{x}_l$ or $\mathbf{x}_h$, without being informed. This turns out to be a generalization of the standard truth-telling conditions in asymmetric information problems, and is represented by the following constraints: $S_h^s(\mathbf{x}_l) \leq S_h^s(\mathbf{x}_h) - \theta/\pi_h$ and $S_l^s(\mathbf{x}_h) \leq S_l^s(\mathbf{x}_l) - \theta/\pi_l$. These constraints also guarantee that a seller observing the payoff δ_l (δ_h) chooses the offer $\mathbf{x}_l$ ($\mathbf{x}_h$). Finally, the seller must always get a non-negative surplus, namely $S_j^s(\mathbf{x}_j) \geq 0$ for $j \in \{l, h\}$.[17]

Let us define $\hat{q}_l$ as the unique solution to $[u'(\hat{q}_l) - 1]\,\pi_l \delta_l = \delta - \delta_l$. The following proposition shows how the menu is composed:

[16] The full statement of the problem is reported in Appendix A.

[17] A further possibility is that the buyer does not propose a menu but a single offer $\mathbf{x}'$ that violates (4). In this case the seller would accept only if the payoff of asset $\mathcal{A}$ is δ_h. This possibility is encompassed in our formulation, e.g. $\mathbf{x}_h = \mathbf{x}'$ and $\mathbf{x}_l = \{0, 0, 0\}$.

Proposition 3 *Suppose $b \in [0, q^*)$. The menu $\{\mathbf{x}_l, \mathbf{x}_h\}$ is implementable if and only if $a \geq \bar{a}$. In the optimal menu $d_l^b = b$, $q_l = \delta_l d_l^a + b$ and*

$$d_h^a - d_l^a \geq \bar{a} \tag{5}$$

$$(\delta_h - \delta_l)d_l^a + \frac{\theta}{\pi_h} = -q_h + \delta_h d_h^a + d_h^b \tag{6}$$

For each b there exists a threshold $\hat{a}(b)$ (weakly decreasing in b) such that if $a \geq \hat{a}(b)$ we have: $q_h = q^*$, $d_l^a = \max\{0, (\hat{q}_l - b)/\delta_l\}$ and (6) pins down $\delta_h d_h^a + d_h^b$. If $a < \hat{a}(b)$ we have: $d_h^b = b$, $d_h^a = a$ and (q_h, d_l^a) are jointly determined by (5), (6) and

$$u'(\delta_l d^a + b) - 1 = \frac{\pi_h}{\pi_l}\frac{\delta_h - \delta_l}{\delta_l} u'(q_h) + \zeta \tag{7}$$

where ζ is the lagrangian multiplier associated to (5).

This strategy is implementable if and only if $a \geq \bar{a}$ because the menu of offers must satisfy (5). Intuitively, the seller has the incentive to produce information and reveal the actual payoff of the asset $\mathcal{A}$ only if the terms of trade are sufficiently different between $\mathbf{x}_l$ and $\mathbf{x}_h$.[18]

In the low state the seller extracts no rents, as $q_l = \delta_l d_l^a + b$. Instead, in the high state the seller gets a surplus equal to $\theta/\pi_h + (\delta_h - \delta_l)d_l^a$. The first component is the compensation for the cost of information acquisition. The last component provides the incentive to truthfully reveal if δ_h realized. If the seller does not acquire information and always chooses $\mathbf{x}_l$, he gets a null payoff with probability π_l and a strictly positive surplus with probability π_h. For the seller the expected payoff of this strategy, which is increasing in d_l^a, should not be greater than the payoff of the strategy in which he incurs the cost to produce information. Then, the buyer must give an informational rent to the seller (increasing in d_l^a) in the high state and a null surplus in the low state. Since this is costly, d_l^a is endogenously bounded by (5) and when $d_l^a > 0$ the maximal amount of consumption in the low state is $\hat{q}_l < \tilde{q}$.

[18]It should be clear from the previous section that this is not an issue. When $a < \bar{a}$ the constraint associated with the threat of information acquisition is not binding. Therefore, the seller will never let the seller produce private information.

Asset $\mathcal{B}$ relaxes the incentive constraints of the seller and allows the buyer to extract a larger share of the gains from trade. Then, the buyer deploys all his holdings b. By increasing d_l^b, the buyer can reduce d_l^a and the informational rent in the high state. When $b \geq \hat{q}_l$ the buyer must compensate the seller only for the cost to produce information, and increasing q_l does not affect the incentive constraints.

The optimal quantity of good demanded by the buyer in the high state can be equal to the first-best q^*, provided that the buyer hold at least $\hat{a}(b)$ units of the asset $\mathcal{A}$, given a holding b of the other security. The informational rent that the buyer provides to the seller in the high state is not correlated with d_h^a but depends on the sunk cost θ/π_l and d_l^a.[19]

3 Portfolio choice and equilibrium

Once we have derived V^N and V^I, we can characterize V^b according to (2). An obvious result is that buyers avoid asymmetric information when a is sufficiently small or b is relatively large.

Lemma 1 *If $a \leq \bar{a}$ or $b \geq \bar{b}(\bar{a})$, then $V^b(a, b) = V^N(a, b)$.*

The first result is trivial, because when $a \leq \bar{a}$ the buyer can extract the maximum surplus from trading with the seller without facing consumption risk. The second conclusion derives from the strict concavity of $u(\cdot)$ and $\theta > 0$.

The trade-off between the two strategies is the following. With no information acquisition the buyer does not face consumption risk. He is constrained in the use of asset $\mathcal{A}$ and consumption can be low. With information acquisition there is no such a constraint. Eventually the buyer can consume the optimal level of goods, q^*, but he faces consumption risk. In general, V^N is greater or smaller than V^I depending on the characteristics of the assets, the cost to acquire information and the initial asset holdings (see for example Figure 4 and

[19] For a sufficiently large a constraint (5) is slack, otherwise also in this case the buyer would internalize the negative effect of increasing d_h^a via the informational rent $(\delta_h - \delta_l)d_l^a$.

footnote 23). The buyer may prefer information acquisition when his holding of asset $\mathcal{A}$ is large, θ is small and δ_h/δ_l is high.

Once V^b has been defined, we have to discuss the portfolio choice in the CM. As shown in (1), the buyer has to choose the optimal amount of assets to bring in the next DM, given V^b and asset prices. Notice that we restrict our attention only to the problem of the buyers because sellers have no need to bring assets in the DM.[20] Finally, the clearing of the market for assets requires $A = A^d(\rho^a)$ and $B = B^d(\rho^b)$, where A and B are the fixed supply of the two assets, respectively, and $A^d(\rho^a)$ and $B^d(\rho^b)$ are the relative aggregate demand correspondences:

$$A^d(\rho^a) = \int_{[0,1]} a'(i)di, \qquad B^d(\rho^b) = \int_{[0,1]} b'(i)di \tag{8}$$

where $a'(i)$ and $b'(i)$ are the quantities chosen by the i^{th} buyer. At this point we can state the definition of equilibrium.

Definition 1 (Equilibrium) *A stationary equilibrium is a list of value functions* $\{W^b, V^b, V^N, V^I\}$*, a list of portfolio* $\{a'(i), b'(i)\}$*, a vector of prices* (ρ^a, ρ^b)*, a decision rule in the DM* $\tau(a', b')$ *and a list* $\{\mathbf{x}, \mathbf{x}_l, \mathbf{x}_h\}$ *such that:* $\{a'(i), b'(i)\}$ *solves* (1) *for each buyer i given* V^b*;* $\tau(a', b')$ *returns the strategy chosen in the DM given* V^N *and* V^I*;* V^b *is determined according to* (2)*;* $\mathbf{x}$ *maximizes* V^N *and* $\{\mathbf{x}_l, \mathbf{x}_h\}$ *maximizes* V^I*;* (ρ^a, ρ^b) *are such that* $A^d(\rho^b) = A$ *and* $B^d(\rho^b) = B$*.*

The prices of the two assets are greater or equal to their expected discounted dividend in the next CM, that is $\rho^a \geq \beta\delta$ and $\rho^b \geq \beta$. The term of the right-hand side is the fundamental value of each asset and reflects the role of the asset as a store of value. The price of an asset departs from its fundamental value when it facilitates trade in the DM, namely when an additional unit of this asset allows a buyer to get a greater utility when trading in the DM. In this case, following Lagos, Rocheteau and Wright (2017), we will say that the asset bears a liquidity premium. Hereafter, we find more useful to focus directly on a measure of this liquidity premium: $R^a = \rho^a/(\beta\delta) - 1$ and $R^b = \rho^b/\beta - 1$.

[20]Since sellers care only about the payoff of the assets in the next CM, they do not demand assets if $\rho^a > \beta\delta$ and $\rho^b > \beta$, while they are indifferent if $\rho^a = \beta\delta$ and $\rho^b = \beta$. Then, we can assume that they do not demand any asset.

3.1 Equilibrium without information acquisition

First, we consider the case in which $V^b = V^N$ for all $a, b \in \Re_+$.[21] Both assets are information-insensitive and safe, because in equilibrium there is never uncertainty about their valuations in the DM. Buyers know ex-ante the quantity of consumption goods they can get.

The portfolio optimization problem in (1) is a concave program. All buyers enter in the DM with the same portfolio, and we can restrict attention to symmetric equilibria.

Both assets are information-insensitive but they may not be equivalent. Indeed, the threat of information acquisition may dampen the liquidity of asset $\mathcal{A}$. An asset can be freely used as a medium of exchange only if producing private information is not possible or not convenient – because in equilibrium the profit from information acquisition is always strictly smaller than its cost.

Proposition 4 *Suppose $A, B > 0$, $\tilde{q} > \delta\bar{a}$ and $V^b = V^N$ for all $a, b \in \Re_+$. If $A \neq \bar{a}$ there always exists a unique symmetric equilibrium. For any A, $R^a = R^b = 0$ and $q = q^*$ if and only if $B \geq \bar{b}(A)$. Suppose $B < \bar{b}(A)$. If $A < \bar{a}$ then $R^b = R^a > 0$. If $A > \bar{a}$ then $R^b > R^a \geq 0$, with $R^a = 0$ if $A \geq \tilde{a}(B)$.*

According to Proposition 4, the first-best allocation can be attained provided there be a sufficient amount of securities, in particular of the asset $\mathcal{B}$. In this equilibrium (A, B) must be such that $B + \delta \min\{A, \bar{a}\} \geq q^*$. Since assets are abundant, buyers consume the first-best level of DM goods. Additional units of both assets are valued only for their role as a store of value and $R^b = R^a = 0$.

When the total amount of assets is scarce and $A < \bar{a}$, buyers cannot afford to consume the optimal quantity of goods. In this situation a marginal increase in the supply of any of the two assets allows the buyer to expand his consumption and surplus in the DM. Then, a buyer is willing to pay for both assets a price greater than the fundamental value, and asset prices incorporate a liquidity premium: $R^b = R^a = u'(B + \delta A) - 1$.

[21] In the Appendix C we derive sufficient conditions for δ_h/δ_l and θ that guarantee this result.

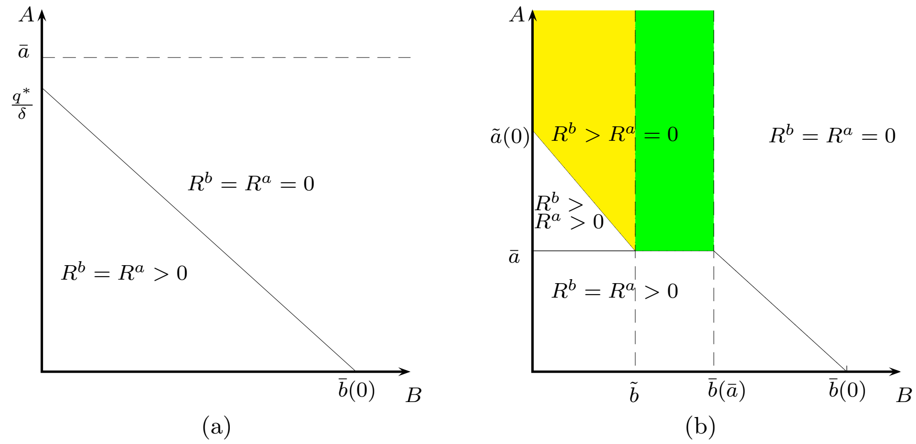

Figure 3: Equilibrium with information-insensitive assets

If $\bar{a} > q^*/\delta$, this would be the full characterization of all the possible equilibria (Figure 3a). The constraint (4) would be always slack, and the two assets would be equivalent, because they could support the same set of allocations. If $A > q^*/\delta$ the first best would be attained, independently of the level of B. However, under the hypothesis of Proposition 4 this is not the case.

When $B < \tilde{b}$ and $\bar{a} < A < \tilde{a}(B)$ buyers consume an amount of goods $q < \tilde{q}$. Then, an additional unit of both types of assets would allow an expansion of consumption. The prices of both assets are greater than their fundamental value, but $R^b > R^a$ because asset $\mathcal{A}$ has a lower degree of liquidity:

$$R^b = u\left(\delta_l A + B + \theta/\pi_l\right) - 1 > u\left(\delta_l A + B + \theta/\pi_l\right)\frac{\delta_l}{\delta} - 1 = R^a$$

When $A \geq \tilde{a}(B)$ or $B \geq \tilde{b}$, instead, there exists an endogenous upper bound on the amount of the asset $\mathcal{A}$ that buyers may want to transfer to sellers (yellow and green area in Figure 3b). An additional unit of this asset does not allow the buyer to increase his utility in the DM, then $R^a = 0$. The price of the other security, instead, includes a positive liquidity premium. In this equilibrium $q < q^*$, and only a sufficiently large increase in B can bring the economy to the first best. The two assets are not equivalent, then they have different degrees of safeness. The asset $\mathcal{A}$ is safe, but only because agents discipline themselves in its use. The other asset, at the opposite, can be freely used.

Proposition 5 *Suppose $B < \bar{b}(A)$ and $A > \tilde{a}(B)$. A marginal increase in B is welfare improving if and only if $B \geq \tilde{b}$.*

Depending on the initial amount of the asset $\mathcal{B}$, a marginal increase in its supply does not imply that the aggregate consumption (total surplus) in the DM rises. In particular, when B is small (yellow area in Figure 3b) total welfare does not change. From Proposition 2, in this region the total surplus into the DM is $u(\tilde{q}) - \tilde{q}$. Following a marginal increase in the holdings of asset $\mathcal{B}$, the buyer keeps the amount of consumption unchanged to $\tilde{q}$, but replaces asset $\mathcal{A}$ with asset $\mathcal{B}$, increasing its share of the gains from trade. This redistribution happens up to $\tilde{b}$, when the buyer takes all the surplus. Only from $\tilde{b}$ onward a buyer with a larger holding of the asset $\mathcal{B}$ does expand his consumption to increase his surplus from the trade (green area in Figure 3b). In a symmetric equilibrium this behavior holds in aggregate. Then, a marginal increase of B does not necessarily cause an increase of the aggregate welfare, but it may only involve a redistribution of surplus from sellers to buyers. We have positive effects only if the asset $\mathcal{B}$ is already relatively abundant.[22]

It should be noted that the price of the asset $\mathcal{B}$ is always affected by a variation in B, provided that $B < \bar{b}(A)$. In equilibrium, the price of the asset is related to the surplus of the buyer. Then, as long as $q < q^*$ an expansion of B leads to a reduction of ρ^b, although q may not change.

3.2 Equilibrium with information acquisition

We now consider the case in which for some combination of A and B in equilibrium buyers let sellers produce private information. We restrict the discussion to the identification and the analysis of safety traps equilibria.

The identification of which asset is safe is trivial. When the asset $\mathcal{A}$ is information-sensitive, buyers face uncertainty about its valuation and then about consumption. The information-insensitive asset is special because it always guar-

[22] In Appendix B we show that this *irrelevance* with respect to consumption can be retrieved in a more general setting in which sellers have a strictly convex cost of production in the DM and a strictly concave utility function in the CM.

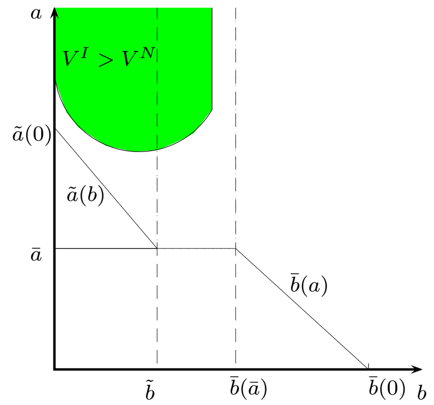

Figure 4: Optimal strategy in the DM with information acquisition

antees the same level of consumption. When $b < \tilde{q}_l$ and $a \geq \hat{a}(b)$ the buyer can consume q^* in the high state and $\hat{q}_l$ in the low state; his expected payoff is:

$$\pi_l \left[u(\hat{q}_l) - \hat{q}_l\right] + \pi_h \left[u(q^*) - q^* - \frac{\theta}{\pi_h} - (\delta_h - \delta_l)\, d_l^a\right] \tag{9}$$

Therefore, an additional unit of the asset $\mathcal{A}$ will be valued only for its expected dividend in the CM. At the opposite, an additional unit of the safe asset allows the buyer to reduce the informational rent in the high state (when $b < \tilde{q}_l$) or to increase the consumption in the low state (when $b \geq \tilde{q}_l$). Then, buyers are willing to pay for the safe security a price greater than its fundamental value.

The main question is how a change in the supply of asset $\mathcal{B}$ can affect the equilibrium. The following proposition reports the key result of this section.

Proposition 6 *Suppose* $A \geq \max\{\tilde{a}(0), \hat{a}(0)\}$. *There exists a* $B' < \bar{b}(\bar{a})$ *such that for* $B > B'$ *the equilibrium is symmetric and* $V^N(A,B) \geq V^I(A,B)$.

Lemma 1 implies the trivial result that for $B \geq \bar{b}(\bar{a})$ in equilibrium all assets are information-insensitive and the first best is reached. This is because the asset $\mathcal{B}$ is so abundant that (4) is always slack and buyers reach the optimal level of consumption. Proposition 6, instead, implies that in a safety trap all assets become information-insensitive provided that the provision of the asset $\mathcal{B}$ becomes sufficiently large, although lower than $\bar{b}(\bar{a})$ (see Figure 4).[23] The

[23]An output qualitative similar to Figure 4 can be produced by assuming $u(q) = \frac{q^{1-\eta}-1}{1-\eta}$,

economy would still be in a safety trap, but now both assets are safe and buyers make the same portfolio choice. Therefore, the safety of an asset can depend not only on its intrinsic characteristics but also on the provision of other types of safe assets. Intuitively, avoiding information acquisition may not be a preferred strategy when the buyer has a large holding of asset $\mathcal{A}$. To avoid information acquisition the buyer should limit the use of asset $\mathcal{A}$. At the opposite, with information acquisition the buyer can potentially (with probability π_h) deploy his holding of the asset $\mathcal{A}$ enjoying a large consumption. A greater holding of the asset $\mathcal{B}$ reduces the relative benefits of information acquisition.[24]

There are two final remarks. First, marginally increasing the supply of the asset $\mathcal{B}$ may have no effects on welfare. Let us suppose that $B < \hat{q}_l$ and we are are in a symmetric equilibrium in which in all DM meetings sellers acquire information. Then, buyers take advantage of a marginal increase in the supply of the safe security by reducing the informational rent $(\delta_h - \delta_l)d_l^a$, while both q_l and q_h do not change.

Second, since V^b is not necessarily convex a symmetric equilibria may fail to exist. In this case we should look for asymmetric equilibria, in which a different fractions of buyers choose different portfolio of assets and, therefore, different strategies in the next DM.

4 Conclusions

We discussed the role of safe assets in a model in which their status is endogenously determined. Following Gorton (2017), we introduced endogenous private information in a standard model in which unsecured credit is unfeasible, and agents can use different assets to trade. We showed that assets with different degrees of safeness can coexist. Securities for which the threat of private information is never relevant are a preferred medium of exchange. When they are scarce, agents choose a suboptimal level of consumption, regardless of their

with $\eta = 0.5$, $\theta = 0.01$, $\pi_l = 0.5$, $\delta_l = 0.75$ and $\delta_h = 1.25$. If we set $\delta_l = 0.8$ and $\delta_h = 1.2$, instead, we would get $V^b = V^N$ for all possible values of a and b.

[24]When $b \geq \tilde{q}_l$ a greater holding of the asset $\mathcal{B}$ is beneficial only in the low state. It allows an increase of q_l, while q_h is capped at q^*.

holdings of other assets.

The benefits of an increase in the provision of the safest assets would depend on their initial supply and the magnitude of the expansion. There are positive welfare effects only when they are already sufficiently abundant, or when the magnitude of the increase is significant. Otherwise welfare effects are null. This result is significant because so far the main concerns have been about the feasibility or the implicit costs associated with an increase in the supply of safest assets. Consider, for example, the case of government bonds. A vast expansion of their supply can jeopardize their status of safe assets (Caballero and Farhi, 2018), and its benefits must be traded off with the distortions introduced by the taxes that are levied to support the new debt (Gorton and Ordoñez, 2013). Here, instead, it could be the case that there are no benefits at all, although we consider the extreme case in which there are no costs associated with this policy.

Our results have potential implications also for monetary policy. Caballero and Farhi (2018) argue that the effectiveness of some unconventional monetary policies, such as large-scale open market operations, may be dampened by the adverse effect of a reduction in the supply of safe assets. However, according to our results this should also depends on their initial supply. The analysis of the implications of this type of unconventional monetary policies is left for future research.

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A Proofs

Proof of Proposition 1. The optimization problem of the buyer is the following:

$$\begin{aligned} \max_{q,d^a,d^b} \quad & u(q) - \delta d^a - d^b \\ \text{s.t.} \quad & -q + \delta d^a + d^b \geq 0 \qquad (10) \\ & 0 \leq d^a \leq a, \quad 0 \leq d^b \leq b \end{aligned}$$

The Lagrangian can be written as follows:

$$\mathcal{L} = u(q) - \delta d^a - d^b + \lambda_1(-q + \delta d^a + d^b) + \lambda_2 d^a + \lambda_3(a - d^a) + \lambda_4 d^b + \lambda_5(b - d^b)$$

and the first order conditions are

$$[q] : u'(q) - \lambda_1 = 0 \qquad (11)$$

$$[d^a] : -\delta + \lambda_1 \delta + \lambda_2 - \lambda_3 = 0 \qquad (12)$$

$$[d^b] : -1 + \lambda_1 + \lambda_4 - \lambda_5 = 0 \qquad (13)$$

From (11) we have $\lambda_1 > 0$ and (10) is binding, otherwise $q = \infty$. We can rewrite (12) and (13) as $\lambda_1 = 1 + (\lambda_3 - \lambda_2)/\delta$ and $\lambda_1 = 1 + \lambda_5 - \lambda_4$, where it is clear that $\lambda_3 > 0$ implies $\lambda_5 > 0$. $\lambda_1 \geq 1$, otherwise $q > q^*$ and $\lambda_2, \lambda_4 > 0$. But $\lambda_2, \lambda_4 > 0$ imply $q = 0$, then $\lambda_1 \geq 1$. If $\lambda_3 = 0$, then $\lambda_5 = 0$, $\lambda_1 = 1$ and $q = q^*$ from (11). To have this solution we need $\delta a + b > q^*$. Notice that d^a and d^b are undetermined but (10) pins down $\delta d^a + d^b = q^*$. If If $\lambda_3 > 0$, then $\lambda_5 > 0$, $\lambda_1 > 1$ and (10) pins down $q = \delta a + b$, while $d^a = a$ and $d^b = b$. ■

Proof of Proposition 2. The optimization problem of the buyer is the following:

$$\begin{aligned} \max_{q,d^a,d^b} \quad & u(q) - \delta d^a - d^b \\ \text{s.t.} \quad & -q + \delta d^a + d^b \geq 0 \qquad (14) \\ & \pi_l(q - \delta_l d^a - d^b) \leq \theta \qquad (15) \\ & 0 \leq d^a \leq a, \quad 0 \leq d^b \leq b \end{aligned}$$

The Lagrangian can be written as follows:

$$\mathcal{L} = u(q) - \delta d^a - d^b + \lambda_1(-q + \delta d^a + d^b) + \lambda_2 \left(-q + \delta_l d^a + d^b + \frac{\theta}{\pi_l}\right) +$$

$$\lambda_3 d^a + \lambda_4(a - d^a) + \lambda_5 d^b + \lambda_6(b - d^b)$$

and the first order conditions are

$$[q] : u'(q) - \lambda_1 - \lambda_2 = 0 \tag{16}$$

$$[d^a] : -\delta + \lambda_1\delta + \lambda_2\delta_l + \lambda_3 - \lambda_4 = 0 \tag{17}$$

$$[d^b] : -1 + \lambda_1 + \lambda_2 + \lambda_5 - \lambda_6 = 0 \tag{18}$$

From (16) at least one constraint among (14) and (15) has to be binding, otherwise $\lambda_1 = \lambda_2 = 0$ and $q = \infty$.

Case I: $\lambda_1 > 0,\ \lambda_2 = 0$**.** We can rewrite (17) and (18) as $\lambda_1 = 1 + (\lambda_4 - \lambda_3)/\delta$ and $\lambda_1 = 1 + \lambda_6 - \lambda_5$, where it is clear that $\lambda_4 > 0$ implies $\lambda_6 > 0$. $\lambda_1 \geq 1$ by using the same argument of the proof of Proposition 1. The optimal quantity of good that the buyer want to consume is q^*, the solution to $u'(q) = 1$. If $\delta a + b > q^*$ then $q = q^*$ but d^a and d^b are undetermined. Otherwise, $d^a = a$, $d^b = b$ and $q = \delta a + b$. Since this solution exists if (15) is slack, we have to verify that this is true. Substituting (14) in (15) and rearranging we get $\pi_l(\delta - \delta_l)d^a \leq \theta$. Therefore, this solution exists as long as $a \leq \bar{a} \equiv \theta\left[\pi_l(\delta - \delta_l)\right]^{-1}$ or $b \geq \bar{b}(a) \equiv q^* - \delta \min\{a, \bar{a}\}$. In the latter case $d^a \leq \bar{a}$.

Case II : $\lambda_1 = 0,\ \lambda_2 > 0$**.** If $0 < d^b < b$, then $\lambda_5 = \lambda_6 = 0$ and from (18) $\lambda_2 = 1$. Since $\delta/\delta_l > 1$, from (17) necessarily $\lambda_3 > 0$, that implies $d^a = 0$. But if $d^a = 0$, then (15) cannot be binding. Then $d^b = b$. Having $d^b = b$ and $d^a > 0$, substituting (17) in (16) we get $u'(q) = \delta/\delta_l + \lambda_4$ and $q \leq \tilde{q}$, where the latter is the solution to $u'(q) = \delta/\delta_l$. The existence of this solution requires $a > \bar{a}$ and $b < \tilde{b} \equiv \tilde{q} - \delta\bar{a}$, otherwise (14) is binding. Notice that for $b < \tilde{b}$ and $a = \bar{a}$ we have $u\left(\delta\bar{a} + b\right) - \delta\bar{a} - b = u\left(\delta_l\bar{a} + b + \theta/\pi_l\right) - \delta\bar{a} - b$.

Case III : $\lambda_1 > 0,\ \lambda_2 > 0$**.** Both (14) and (15) are binding. Substituting (14) in (15) we pin down $d^a = \bar{a}$ and the whole problem can be rewritten as

$$\max_{d^b \leq b} u(\delta\bar{a} + d^b) - \delta\bar{a} - d^b$$

When $\delta\bar{a} + b \leq q^*$ we have $d^b = b$ and $q = \delta\bar{a} + b$. The existence of this solution requires $\tilde{b} \leq b < \bar{b}(a)$ and $a > \bar{a}$. ■

Proof of Proposition 3. Let $b \in [0, q^*)$ and define $S_i^s(\mathbf{x}_j) = -q_j + \delta_i d_j^a + d_j^b$, with $i, j \in \{l, h\}$. The problem is the following

$$\max_{q_l, q_h, d_l^a, d_l^b, d_h^a, d_h^b} \quad \pi_l\left[u\left(q_l\right) - \delta_l d_l^a - d_l^b\right] + \pi_h\left[u\left(q_h\right) - \delta_h d_h^a - d_h^b\right]$$

$$
\begin{aligned}
\text{s.t.} \quad & \pi_l S_l^s(\mathbf{x}_l) + \pi_h S_h^s(\mathbf{x}_h) \geq \theta && (19)\\
& S_l^s(\mathbf{x}_l) \geq 0 && (20)\\
& S_h^s(\mathbf{x}_h) \geq 0 && (21)\\
& \pi_l S_l^s(\mathbf{x}_l) + \pi_h S_h^s(\mathbf{x}_l) \leq \pi_l S_l^s(\mathbf{x}_l) + \pi_h S_h^s(\mathbf{x}_h) - \theta && (22)\\
& \pi_l S_l^s(\mathbf{x}_h) + \pi_h S_h^s(\mathbf{x}_h) \leq \pi_l S_l^s(\mathbf{x}_l) + \pi_h S_h^s(\mathbf{x}_h) - \theta && (23)\\
& S_l^s(\mathbf{x}_l) \geq S_l^s(\mathbf{x}_h) && (24)\\
& S_h^s(\mathbf{x}_h) \geq S_h^s(\mathbf{x}_l) && (25)\\
& q_l, q_h \geq 0 \quad 0 \leq d_l^a \leq a \quad 0 \leq d_l^b \leq b \quad 0 \leq d_h^a \leq a \quad 0 \leq d_h^b \leq b
\end{aligned}
$$

Equations (22) and (23) imply (24) and (25), respectively. Therefore the latter can be ignored. Moreover, given (20) equation (22) implies (21). Consider now equation (20). Since the buyer maximizes his surplus, for any given d_l^a and d_l^b it is possible to increase q_l up to (20) be binding and (22) and (23) are still satisfied. To see this, let us rewrite the two constraints as

$$
\begin{aligned}
S_l^s(\mathbf{x}_l) + (\delta_h - \delta_l) d_l^a &\leq S_h^s(\mathbf{x}_h) - \frac{\theta}{\pi_h} \\
S_h^s(\mathbf{x}_h) - (\delta_h - \delta_l) d_h^a &\leq S_l^s(\mathbf{x}_l) - \frac{\theta}{\pi_l}
\end{aligned}
$$

Then

$$
S_l^s(\mathbf{x}_l) + (\delta_h - \delta_l) d_l^a \leq S_h^s(\mathbf{x}_h) - \frac{\theta}{\pi_h} \leq S_h^s(\mathbf{x}_h) \leq S_l^s(\mathbf{x}_l) + (\delta_h - \delta_l) d_h^a - \frac{\theta}{\pi_l}
$$

that implies $S_l^s(\mathbf{x}_l)$ can be set to 0, as only the difference between d_h^a and d_l^a matters. Then, (20) is binding. Finally, given (20) is binding and $d_l^a \geq 0$, (22) implies (19), that becomes redundant. The problem can be simplified as follows:

$$
\begin{aligned}
\max_{q_h, d_l^a, d_l^b, d_h^a, d_h^b} \quad & \pi_l \left[u\left(\delta_l d_l^a + d_l^b\right) - \delta_l d_l^a - d_l^b \right] + \pi_h \left[u\left(q_h\right) - \delta_h d_h^a - d_h^b \right] \\
\text{s.t.} \quad & \pi_h (\delta_h - \delta_l) d_l^a \leq \pi_h \left(-q_h + \delta_h d_h^a + d_h^b \right) - \theta && (26)\\
& \pi_l (q_h - \delta_l d_h^a - d_h^b) \geq \theta && (27)\\
& q_h \geq 0 \quad 0 \leq d_l^a \leq a \quad 0 \leq d_l^b \leq b \quad 0 \leq d_h^a \leq a \quad 0 \leq d_h^b \leq b
\end{aligned}
$$

where (27) follows from (23). The Lagrangian of this problem can be written as:

$$
\mathcal{L} = \pi_l \left[u\left(\delta_l d_l^a + d_l^b\right) - \delta_l d_l^a - d_l^b \right] + \pi_h \left[u\left(q_h\right) - \delta_h d_h^a - d_h^b \right] +
$$

$$+\lambda_1\pi_h\left[-q_h+\delta_h d_h^a+d_h^b-\frac{\theta}{\pi_h}-(\delta_h-\delta_l)d_l^a\right]+$$
$$+\lambda_2\pi_l\left(q_h-\delta_l d_h^a-d_h^b-\frac{\theta}{\pi_l}\right)+\lambda_3 d_l^a+\lambda_4 d_l^b+\lambda_5 d_h^a+\lambda_6 d_h^b+\lambda_7 q_h+$$
$$+\lambda_8\left(a-d_l^a\right)+\lambda_9\left(b-d_l^b\right)+\lambda_{10}\left(a-d_h^a\right)+\lambda_{11}\left(b-d_h^b\right)$$

and the first order conditions are:

$$[q_h]:\pi_h u'\left(q_h\right)-\lambda_1\pi_h+\lambda_2\pi_l+\lambda_7=0 \tag{28}$$
$$[d_l^a]:\pi_l\left[u'\left(\delta_l d_l^a+d_l^b\right)-1\right]\delta_l-\lambda_1\pi_h(\delta_h-\delta_l)+\lambda_3-\lambda_8=0 \tag{29}$$
$$[d_l^b]:\pi_l\left[u'\left(\delta_l d_l^a+d_l^b\right)-1\right]+\lambda_4-\lambda_9=0 \tag{30}$$
$$[d_h^a]:-\pi_h\delta_h+\lambda_1\pi_h\delta_h-\lambda_2\pi_l\delta_l+\lambda_5-\lambda_{10}=0 \tag{31}$$
$$[d_h^b]:-\pi_h+\lambda_1\pi_h-\lambda_2\pi_l+\lambda_6-\lambda_{11}=0 \tag{32}$$

Firstly, $\lambda_7=0$, otherwise $q_h=0$. Secondly, if $0<q_h<\infty$, then from (28) $\lambda_1>0$ and then (26) is binding. Thirdly, substituting (26) in (27) we find the condition

$$d_h^a-d_l^a\geq\frac{\theta}{\pi_l(\delta-\delta_l)} \tag{33}$$

that implies $a\geq d_h^a\geq\theta[\pi_l(\delta-\delta_l)]^{-1}=\bar{a}$, $d_l^a<a$ and $\lambda_5=\lambda_8=0$. Notice that this strategy is feasible iff $a\geq\bar{a}$ and from now on we assume that this condition is satisfied. Forthly, from (29) and (30) we have $d_l^b<b$ iff $q_l=q^*$, but this is clearly not possible because $b<q^*$ by assumption and from (29) $q_l<q^*$ if $d_l^a>0$; therefore $d_l^b=b$.

Claim: $q_h\leq q^*$. From (28) and (32) $q_h>q^*$ implies $d_h^b=0$, then we can focus on the case in which $b=0$. Suppose a is sufficiently large such that $\lambda_{10}=0$. By rearranging (32) we have $\lambda_2\pi_l=(\lambda_1-1)\,\pi_h\delta_h$. If $\lambda_2=0$, then $\lambda_1=1$. From (28) $q=q^*$, and the claim is correct. From (26) and (29) it is possible to retrieve the values of d_l^a and d_h^a consistent with $\lambda_2=0$. We define them $\hat{d}_l^a$ and $\hat{d}_h^a$, respectively. Consider now the case in which $\lambda_2>0$. If $q>q^*$ we must have $d_h^a>\hat{d}_h^a$, and $d_l^a>\hat{d}_l^a$ because (33) is binding. But $\lambda_2>0$ implies $\lambda_1>1$, and from (29) $d_l^a<\hat{d}_l^a$. Since this is a contradiction the claim is proved. Moreover, when $\lambda_{10}=0$ we have $\lambda_2=0$.

The problem reduces to find a possible unique vector $\left(q_h,d_l^a,d_h^a,d_h^b\right)$ such that the following conditions – derived from (28)-(32) – are satisfied:

$$\pi_h\left[u'\left(q_h\right)-1\right]\delta_h=-\lambda_2\pi_l(\delta_h-\delta_l)+\lambda_{10} \tag{34}$$
$$\pi_h\left[u'\left(q_h\right)-1\right]=-\lambda_6+\lambda_{11} \tag{35}$$

$$\pi_l \left[u'\left(\delta_l d_l^a + b\right) - 1\right] \delta_l = \left(\delta_h - \delta_l\right) \left[\pi_h u'\left(q_h\right) + \pi_l \lambda_2\right] - \lambda_3 \tag{36}$$

$$(\delta_h - \delta_l) d_l^a + q_h + \frac{\theta}{\pi_h} = \delta_h d_h^a + d_h^b \tag{37}$$

together with the relative complementary slackness conditions and (33).

Case I. Suppose $d_h^a < a$. Therefore $\lambda_{10} = 0$ and from (34) we have $q_h = q^*$ and $\lambda_2 = 0$. From (35), $\lambda_6 = \lambda_{11} = 0$. From (36), d_l^a is the solution to

$$u'\left(\delta_l d_l^a + b\right) - 1 = \frac{\pi_h}{\pi_l} \frac{\delta_h - \delta_l}{\delta_l} - \lambda_3$$

Let $\hat{q}_l \equiv \xi\left(\frac{\pi_h}{\pi_l}\frac{\delta_h - \delta_l}{\delta_l} + 1\right)$, where $\xi(\cdot)$ is defined as the inverse function of $u'(\cdot)$. Then $d_l^a = \max\{0, (\hat{q}_l - b)/\delta_l\}$, where $d_l^a = 0$ ($\lambda_3 \geq 0$) when $b \geq \hat{q}_l$. Given d_l^a, from (37) we find $\delta_h d_h^a + d_h^b$. We cannot pin down d_h^a and d_h^b, although (33) must hold. Given $b < q^*$, this can be a solution if and only if $a \geq \hat{a}(b)$, where $\hat{a}(b)$ is a function of b that returns the lowest value of a such that $q_h = q^*$ given b. It can be retrived from (36)-(37) by imposing $q_h = q^*$ and $d_h^b = b$:

$$\hat{a}(b) = \max\left\{d_l^a + \bar{a}, \left[(\delta_h - \delta_l) d_l^a + q^* + \frac{\theta}{\pi_h} - b\right] / \delta_h\right\}$$

Notice that $\hat{a}(b)$ is weakly decreasing in b.

Case II. Suppose that $\lambda_{11} > 0$. Then, from the complementary slackness condition $d_h^b = b$, $\lambda_6 = 0$ and from (35) $q_h < q^*$. Then, from (34) we have $\lambda_{10} > 0$ and $d_h^a = a$. It remains to determine d_l^a and q_h, looking separately to the case in which (33) is slack ($\lambda_2 = 0$) or binding ($\lambda_2 > 0$). If $b \geq \hat{q}_l$, from (36) we have $\lambda_3 > 0$, that implies $d_l^a = 0$. In this case (33) is slack, $\lambda_2 = 0$ and, from (37), $q_h = \delta_h a + b - \theta/\pi_h$. Consider now the case in which $b < \hat{q}_l$. If $\lambda_2 > 0$, $d_l^a = a - \theta[\pi_l(\delta - \delta_l)]^{-1}$ and from (37) we have $q_h = \delta_l a + b + \theta/\pi_l$. If the following condition is satisfied:

$$u'\left(\delta_l a + b - \frac{\theta \delta_l}{\pi_l(\delta - \delta_l)}\right) - 1 > \frac{\pi_h}{\pi_l} \frac{\delta_h - \delta_l}{\delta_l} u'\left(\delta_l a + b + \frac{\theta}{\pi_l}\right)$$

λ_2 is actually greater than 0 and we are done. If this is not true, $\lambda_2 = 0$ and (33) is slack. Then, we can jointly solve for q_h and d_l^a in the positive orthant of the (d_l^a, q_h) space. Considering (37), we have that q_h is monotonically non-increasing in d_l^a, it is equal to $\delta_h a + b - \theta/\pi_h$ for $d_l^a = 0$ and it vanishes at $d_l^a = \frac{\delta_h a + b - \theta/\pi_h}{\delta_h - \delta_l}$. Looking to (36), we have: $d_l^a = 0$ for $0 < q_h \leq \hat{q}_h \equiv \frac{\delta_l}{\delta_h - \delta_l} \frac{\pi_l}{\pi_h} \xi\left(\Gamma(b)\right)$, where $\Gamma(b) \equiv u'(b) - 1$; $d_l^a = (\hat{q}_l - b)/\delta_l$ when $q_h = q^*$; d_l^a is monotonically non-decreasing when $\hat{q}_h < q_h < q^*$. Therefore, given

$q_h < q^*$ there exists a unique vector (d_l^a, q_h) that solve (36) and (37).

This is the solution when $\bar{a} \leq a < \hat{a}(b)$. ■

Proof of Proposition 4. Since we consider stationary equilibria with no information acquisition the objective function of the portfolio optimization problem is the following:

$$\max_{a',b',q,d^a,d^b} -\frac{\rho^a - \beta\delta}{\beta}a' - \frac{\rho^b - \beta}{\beta}b' + S^b\left(q, d^a, d^b\right)$$

subject to (14) and (15), the usual nonnegative constraints for the choice variables and $d^a \leq a'$, $d^b \leq b'$. Since the objective function is concave and the inequality constraints are continuously differentiable convex functions, the first order conditions are sufficient to find a global maximum. Therefore, we can focus on symmetric equilibria. We maintain the same notation of the proof of Proposition 1. The first order conditions with respect to q, d^a and d^b are (16), (17) and (18). The additional focs are:

$$[a'] : \quad \frac{\rho^a - \beta\delta}{\beta} = \lambda_4 + \zeta_1 \tag{38}$$

$$[b'] : \quad \frac{\rho^b - \beta}{\beta} = \lambda_6 + \zeta_2 \tag{39}$$

where ζ_1 and ζ_2 are lagrangian multipliers associated with the nonnegative constraints for a' and b'. Since we assumed $A, B > 0$ and the asset market must clear, in equilibrium $\zeta_1 = \zeta_2 = 0$. Let us define $R^a \equiv \rho^a/\beta - \delta$ and $R^b \equiv \rho^b/\beta - 1$. We consider all the possible cases in which $R^a, R^b \geq 0$ (otherwise there would be an infinite demand for assets).

Case I: $R^b = R^a > 0$. In this case from (38) and (39) we have $\lambda_4, \lambda_6 > 0$, therefore $d^a = a'$ and $d^b = b'$. Substituting (17) and (18) in (38) and (39), we see that $R^b = R^a$ requires $\lambda_2 = 0$, then $a' < \bar{a}$. In this equilibrium q is such that $R^a = R^b = u'(q) - 1$ and $q < q^*$. Since we are considering symmetric equilibria, R^a and R^b are retrieved by substituting the market clearing conditions of the assets market in the first order conditions. Then the existence of this equilibrium requires $A < \bar{a}$ and $B < \bar{b}(A)$.

Case II: $R^b = R^a = 0$. In this case we have $\lambda_4, \lambda_6 = 0$, therefore $d^a < a'$, $d^b < b'$ and from (17) or (18) we have $q = q^*$. The existence of this equilibrium requires $B \geq \bar{b}(A)$.

Case III: $R^b > R^a > 0$. In this equilibrium we have $\lambda_4, \lambda_6 > 0$, therefore $d^a = a'$ and $d^b = b'$. However, $R^b > R^a$, then substituting (17) and (18) in (38) and (39) we need $\lambda_2 > 0$. Given $d^a = a'$, we need $q = \delta_l a' + b' + \theta/\pi_l < \tilde{q}$ (otherwise $d^a < a'$ and $\lambda_4 = 0$).

Therefore, this equilibrium exists if $A > \bar{a}$ and $B < \tilde{b}$ and asset prices are such that:

$$R^b = u\left(\delta_l A + B + \theta/\pi_l\right) - 1$$
$$R^a = u\left(\delta_l A + B + \theta/\pi_l\right)\frac{\delta_l}{\delta} - 1$$

Case IV: $R^b > R^a = 0$. In this case we have $\lambda_4 = 0$ and $\lambda_6 > 0$, therefore $d^a < a'$ and $d^b = b'$. Since $R^b > R^a$, by substituting (18) in (39) we need $\lambda_2 > 0$. Now, $\lambda_6 > 0$ implies $q < q^*$ from (18). There are two possible cases. If $\lambda_1 > 0$, then $\lambda_2 > 0$ implies $d^a = \bar{a}$ and $q = \delta\bar{a} + b'$. Therefore, this equilibrium requires $A \geq \bar{a}$ and $B \in \left[\tilde{b}, \bar{b}(A)\right)$. If $\lambda_1 = 0$, then from (17) we have $q = \tilde{q}$. Then this equilibrium requires $B \in \left[0, \tilde{b}\right)$ and $A \geq \tilde{a}(B)$.
Case V: $R^b < R^a$. This cannot part of an equilibrium because by substituting (17) and (18) in (38) and (39) and using (16) we get

$$R^b = u'(q) - 1$$
$$R^a = u'(q) - 1 - \lambda_2\frac{\delta - \delta_l}{\delta}$$

Then $R^a = R^b - \lambda_2\frac{\delta-\delta_l}{\delta}$, but $R^a > R^b$ implies $\delta < \delta_l$, that is obviously impossible. ■
Proof of Proposition 5. See case IV in the proof of Proposition 4. ■
Proof of Proposition 6. The goal is to construct a symmetric general equilibrium in which $A \geq \max\{\tilde{a}(0), \hat{a}(0)\}$, $B < \bar{b}\left(\bar{a}\right)$, prices are such that all buyers choose the same portfolio (A, B) and $V^N(A,B) \geq V^I(A,B)$.
Notice that for a given (a,b) we have $V^N \geq V^I$ if $S^N \geq S^I$, where we define S^N as the surplus of the buyer in the DM trade when private information is avoided, and S^I as the expected surplus in a trade in which sellers acquire information. Let us define $\breve{a} \equiv \max\{\tilde{a}(0), \hat{a}(0)\}$. Since both $\tilde{a}(b)$ and $\hat{a}(b)$ are decreasing in b, for any given b and $\breve{a}' > \breve{a}$ we have $S^N\left(\breve{a}', b\right) = S^N\left(\breve{a}, b\right)$ and $S^I\left(\breve{a}', b\right) = S^I\left(\breve{a}, b\right)$. Suppose $\tilde{b} > 0$ (but this is not necessary). When $a = \breve{a}$, $\partial S^N/\partial b$ is equal to $(\delta - \delta_l)/\delta_l$ for $b \in (0, \tilde{b}]$ and to $u'\left(b + \delta\bar{a}\right) - 1$ for $b \in \left[\tilde{b}, \bar{b}(\bar{a})\right)$, while $\partial S^I/\partial b = (\delta - \delta_l)/\delta_l$ for $b \in (0, \hat{q}_l]$ and equal to $\pi_l u'\left(b\right) - 1 > 0$ for $b \in \left[\hat{q}_l, \bar{b}(\bar{a})\right)$. Both derivatives are continuous.
At $\bar{b}(\bar{a})$ we have $S^N > S^I$ and $\lim_{b \leftarrow \bar{b}(\bar{a})} \partial S^N\left(\breve{a}, b\right)/\partial b < \lim_{b \leftarrow \bar{b}(\bar{a})} \partial S^I\left(\breve{a}, b\right)/\partial b$. Therefore, by continuity there is always some $\breve{b} < \bar{b}\left(\bar{a}\right)$ such that $S^N\left(\breve{a}, \breve{b}\right) \geq S^I\left(\breve{a}, \breve{b}\right)$ and $\partial S^N\left(\breve{a}, \breve{b}\right)/\partial b < \partial S^I\left(\breve{a}, \breve{b}\right)/\partial b$. Then, a vector of asset prices such that $R^a = 0$ and $0 \geq R^b < \partial S^N\left(\breve{a}, \breve{b}\right)/\partial b$ implies that the portfolio optimization problem of the buyers has a unique local optimum. Then, all buyers choose the same portfolio of assets to

bring in the DM. Therefore, if $A > \breve{a}$ and $B > \check{b}$ there exists a symmetric equilibrium with no information acquisition. ■

B Risk averse sellers

We solve the problem of a buyer that wants to avoid the production of private information when the seller is risk averse. We suppose that the utility of the buyer is $U^b = u(q) - h$, as in the baseline model, while the utility of the seller is $U^s = -c(q) + \nu(c)$. We assume that: $c(0) = 0$, $c'(\cdot) > 0$ and $c''(\cdot) > 0$; $\nu(0) = 0$, $\nu'(\cdot) > 0$, $\nu''(\cdot) < 0$ and $\nu(\cdot)$ satisfies the Inada conditions. The problem is:

$$\max_{q,d^a,d^b} \quad u(q) - \delta d^a - d^b$$

$$\text{s.t.} \quad -c(q) + \pi_l \nu(p_l) + \pi_h \nu(p_h) \geq 0 \tag{40}$$

$$c(q) - \nu(p_l) \leq \theta / \pi_l \tag{41}$$

$$d^b \leq b, \qquad d^a \leq a$$

where $p_l = d^b + \delta_l d^a$ and $p_h = d^b + \delta_h d^a$. Notice that the incentive constraint is derived as in the baseline model. Before to solve this problem, let us suppose that (40) is binding and substitute it in (41). We get:

$$\nu(p_h) - \nu(p_l) \leq \frac{\theta}{\pi_l \pi_h} \tag{42}$$

If we assume that (42) is binding and $d^b = b$, we find the threshold $\bar{a}(b)$. Notice that $\bar{a}(b)$ is increasing in b, because of the concavity of $\nu(\cdot)$. Now we can write down the Lagrangian:

$$L(q, d^a, d^b) = u(q) - \delta d^a - d^b + \lambda_1 \left[-c(q) + \pi_l \nu(p_l) + \pi_h \nu(p_h)\right] +$$
$$\lambda_2 \left[-c(q) + \nu(p_l)\right] + \lambda_3 \left(b - d^b\right) + \lambda_4 \left(a - d^a\right) + \lambda_5 d^b + \lambda_6 d^a$$

The first order conditions are:

$$[q]: \quad u'(q) - (\lambda_1 + \lambda_2) c'(q) = 0 \tag{43}$$

$$[d^a]: \quad -\delta + \lambda_1 \left[\pi_l \nu'(p_l) \delta_l + \pi_h \nu'(p_h) \delta_h\right] + \lambda_2 \nu'(p_l) \delta_l - \lambda_4 + \lambda_6 = 0 \tag{44}$$

$$[d^b]: \quad -1 + \lambda_1 \left[\pi_l \nu'(p_l) + \pi_h \nu'(p_h)\right] + \lambda_2 \nu'(p_l) - \lambda_3 + \lambda_5 = 0 \tag{45}$$

Equation (43) implies that at least one constraint among (40) and (41) is binding. Therefore, we consider separately the three possible cases.

Case I: $\lambda_1 > 0$, $\lambda_2 = 0$. We can rewrite (44) and (45) as follows:

$$\pi_l \nu'(p_l)\frac{\delta_l}{\delta} + \pi_h \nu'(p_h)\frac{\delta_h}{\delta} = \frac{c'(q)}{u'(q)}\left(1 + \frac{\lambda_4 - \lambda_6}{\delta}\right) \tag{46}$$

$$\pi_l \nu'(p_l) + \pi_h \nu'(p_h) = \frac{c'(q)}{u'(q)}(1 + \lambda_3 - \lambda_5) \tag{47}$$

As long as $d^a > 0$ we have $\nu'(p_l) > \nu'(p_h)$. Therefore, the LHS of (46) is lower than the LHS of (47): they are both weighted averages of $\nu'(p_l)$ and $\nu'(p_h)$, with the first putting more weight on $\nu'(p_h)$. Then, $\lambda_4 = 0$ implies $\lambda_3 > 0$ and $d^b = b$, while $\lambda_3 = 0$ implies $\lambda_6 > 0$ and $d^a = 0$.

When $\lambda_3 = 0$, from (47) we have $\nu'\left(d^b\right) = c'(q)/u'(q)$, while from (40) we have $c(q) = \nu\left(d^b\right)$. These two equations must be solved for d^b and q and the solution is unique. The first equation implies that q is decreasing in d^b, $q \uparrow \infty$ when $d^b \downarrow 0$ and $q \uparrow 0$ when $d^b \downarrow \infty$. The second equation implies that q is increasing in d^b, $q \uparrow 0$ when $d^b \downarrow 0$ and $q \uparrow \infty$ when $d^b \downarrow \infty$. We define this solution (b^*, q_b^*).

When $\lambda_4 = 0$, from (46) we have

$$\pi_l \nu'(b + \delta_l d^a)\frac{\delta_l}{\delta} + \pi_h \nu'(b + \delta_h d^a)\frac{\delta_h}{\delta} = \frac{c'(q)}{u'(q)} \tag{48}$$

Using this equation and (40) we have a system of two equations in two unknowns, d^a and q. Also in this case the solution is unique. According to equation (48), q is decreasing in d^a. When $d^a = 0$ and $b > 0$, then $0 < q < \infty$ satisfies $\nu'(b) = \frac{c'(q)}{u'(q)}$. When $d^a \uparrow \infty$, $q \downarrow 0$. According to equation (40), q is increasing in d^a. When $d^a = 0$ and $b > 0$, q satisfies $c(q) = \nu(b)$. When $d^a \uparrow \infty$, $q \uparrow \infty$. Since $b < b^*$, we have $d^a > 0$ and the system of equations has a unique solution. We define this solution as (a^*, q_a), where both a^* and q_a depend on b.

When $\lambda_4 > 0$, $d^b = b$, $d^a = a$ and q is determined by (40).

This solution requires $a \leq \bar{a}(b)$.

Case II: $\lambda_1 = 0$, $\lambda_2 > 0$. We can rewrite (44) and (45) as:

$$\nu'(p_l)\frac{\delta_l}{\delta} = \frac{c'(q)}{u'(q)}\left(1 + \frac{\lambda_4 - \lambda_6}{\delta}\right) \tag{49}$$

$$\nu'(p_l) = \frac{c'(q)}{u'(q)}(1 + \lambda_3 - \lambda_5) \tag{50}$$

$\lambda_4 = 0$ implies $\lambda_3 > 0$ and $d^b = b$. When $\lambda_4 = 0$ the optimal consumption $\tilde{q}$ is lower that q_a in Case I. To see this, notice that from (48) we have $\nu'(p_l)\frac{\delta_l}{\delta} = \frac{c'(q_a)}{u'(q_a)} - \frac{\pi_h}{\pi_l}\nu'(p_h)\frac{\delta_h}{\delta}$; it implies $\nu'(p_l)\frac{\delta_l}{\delta} < \frac{c'(q_a)}{u'(q_a)}$. Suppose now that $\tilde{q} = q_a$. From (49) we have $\nu'(p_l)\frac{\delta_l}{\delta} =$

$\frac{c'(q_a)}{u'(q_a)}$, that implies $d^a < a^*$. But this cannot be possible, because (40) is slack. Then, for a given q_a we would need a larger d^a. Therefore, $\tilde{q} < q^a$.

When $\lambda_4 = 0$ we use (49) and (41) to pin down $\tilde{q}$ and d^a. By using the same argument for Case I it is possible to show that this system of equations has a unique solution. Moreover, when $\lambda_4 = 0$ we have that $\tilde{q}$ does not change with b. Indeed, suppose that b increases and d^a is unchanged. From (41) we have that q must increase and this implies that $c'(q)/u'(q)$ increases too. But also p_l increases, then the LHS of (49) decreases and we have a contradiction. Therefore, if b increases we need d^a to decrease in order to keep p_l constant.

When $\lambda_4 > 0$, $d^b = b$, $d^a = a$ and q is determined by (41).

The existence of this solution requires $a \geq \bar{a}(b)$ and $\tilde{q} > \bar{q}$, where $\bar{q}$ solve (40) when it is binding and $d^b = b$, $d^a = \bar{a}(b)$.

Case III: $\lambda_1 > 0$, $\lambda_2 > 0$. In this case $d^b = b$, $d^a = \bar{a}(b)$ and q is determined by (40). This solution requires $a > \bar{a}(b)$ and $q \geq \tilde{q}$.

C Sufficient conditions for $V^b = V^N$

In this appendix we derive sufficient conditions for $V^b = V^N$, with $\tilde{q} > \delta\bar{a}$. The definitions of $\tilde{q}$, $\bar{a}$, $\tilde{b}$, $\hat{q}_l$, $\tilde{a}$ and $\hat{a}$ are those in the main text; here we define $\bar{b} \equiv \bar{b}(\bar{a})$. Moreover, we also define $S^I(a,b) \equiv \pi_l S_l^b\left[\mathbf{x}_l^*(a,b)\right] + \pi_h S_h^b\left[\mathbf{x}_h^*(a,b)\right]$ and $S^N(a,b) \equiv S^b\left[\mathbf{x}^*(a,b)\right]$, with $\mathbf{x}_l^*$, $\mathbf{x}_h^*$, $\mathbf{x}^*$ the optimal choice in the DM given (a,b). The result is derived in different steps.

Let us define $\Gamma(b) \equiv \pi_l\left[u(b) - b\right] + \pi_h\left[u(q^*) - q^* - \theta/\pi_h\right]$ and α a constant in the open interval $(1/2, 1)$.

Lemma 2 *There exist a $\kappa \in (0,1)$ and a positive increasing function $\theta^*(\delta_l/\delta)$ such that for $\delta_l/\delta \geq \kappa$ and $\theta = \alpha\theta^*(\delta_l/\delta)$ we have $\tilde{b} > 0$ and $\Gamma(b) \leq \max_a S^N(a,b)$ for all b.*

Proof of Lemma 2. Suppose a is sufficiently big, such that $a > \arg\max_x S^N(x,b)$ holds for all b (it is sufficient that this holds at $b=0$) and $\tilde{b} \in (0, \bar{b})$. Then, we have:

$$S^N(a,b) \equiv \begin{cases} u(\tilde{q}) - \left(\tilde{q} - \frac{\theta}{\pi_l}\right)\frac{\delta}{\delta_l} + \frac{\delta - \delta_l}{\delta_l} b & \text{for } b \in [0, \tilde{b}] \\ u(b + \delta\bar{a}) - b - \delta\bar{a} & \text{for } b \in [\tilde{b}, \bar{b}] \\ u(q^*) - q^* & \text{for } b \geq \bar{b} \end{cases}$$

Let us define $\xi(x) \equiv u'^{-1}(x)$ and $\theta^*(\delta_l/\delta) \equiv \xi(\delta/\delta_l)\,\frac{\pi_l(\delta/\delta_l - 1)}{\delta/\delta_l}$. For a given δ_l/δ, we have that $\theta \leq \theta^*(\delta_l/\delta)$ implies $\tilde{q} \geq \delta\bar{a}$ and $\tilde{b} > 0$. By construction $S^N(a,b)$ is continuous in b for any a, $\delta_l/\delta < 1$ and $\theta \in (0, \theta^*(\delta_l/\delta))$. Note also that for $\delta_l/\delta \uparrow 1$ we have $\tilde{q} \uparrow q^*$ and, provided $\theta = \alpha\theta^*$, $\delta\bar{a} \uparrow \alpha q^*$.

Fix b and $\theta = \alpha\theta^*(\delta_l/\delta)$. If $\delta_l/\delta \uparrow 1$ we have $S^N(a,b) \uparrow u(q^*) - q^* > \Gamma(b)$. As δ_l/δ decreases, $S^N(a,b)$ decreases too, while $\Gamma(b)$ increases (because θ decreases). Then there exist some $\bar{\kappa}(b) > 0$ such that for $\delta_l/\delta \geq \bar{\kappa}(b)$ we have $S^N(a,b) \geq \Gamma(b)$. Then it is sufficient to take $\kappa = \max \bar{\kappa}(b)$. ∎

Lemma 2 is derived with respect to δ/δ_l, but since $\delta = \pi_l\delta_l + \pi_h\delta_h$, the same result holds if we work in terms of δ_h/δ_l. Hereafter, we fix $\alpha \in (1/2, 1)$ and θ^* is a function defined as in Lemma 2.

Lemma 3 *Define $a' = \max\{\tilde{a}(0), \hat{a}(0)\}$. There exists a $\kappa \in (0,1)$ such that if $\delta_l/\delta_h \geq \kappa$ and $\theta = \alpha\theta^*$ then $S^N(a',b) \geq S^I(a',b)$ for all b.*

Proof of Lemma 3. Firstly notice that $\Gamma(b)$ in Lemma 2 is equal to $S^I(a',b)$ for $b \geq \hat{q}_l$. According to Lemma 2 there exists $\kappa \in (0,1)$ such that for $\delta_l/\delta_h \geq \kappa$ and $\theta = \alpha\theta^*$ we have $S^N(a',b) \geq S^I(a',b)$ for $b \geq \hat{q}_l$.

Let us now consider the partial derivatives of S^I and S^N with respect to b for $b \in (0, \bar{b})$:

$$S_b^I = \begin{cases} \pi_h \frac{\delta_h - \delta_l}{\delta_l} = \frac{\delta - \delta_l}{\delta_l} & \text{for } b \in (0, \hat{q}_l) \\ \pi_l \left[u'(b) - 1\right] \leq \frac{\delta - \delta_l}{\delta_l} & \text{for } b \in [\hat{q}_l, \bar{b}) \end{cases}$$

$$S_b^N = \begin{cases} \frac{\delta - \delta_l}{\delta_l} & \text{for } b \in (0, \tilde{b}) \\ \left[u'(\delta \bar{a} + b) - 1\right] \leq \frac{\delta - \delta_l}{\delta_l} & \text{for } b \in [\tilde{b}, \bar{b}) \end{cases}$$

Since $S^N(a', \hat{q}_l) \geq S^I(a', \hat{q}_l)$, if $\hat{q}_l \leq \tilde{b}$ then $S^N(a', b) \geq S^I(a', b)$ also for $b \in [0, \hat{q}_l]$. If $\hat{q}_l > \tilde{b}$, then $S^N(a', b) < S^I(a', b)$ for $b \in [0, \hat{q}_l)$. ■

According to the previous lemma, there exist conditions that guarantee $\max_a S^N(a, b) \geq \max_a S^I(a, b)$ for all b. Given the linearity of W^b, we can fix a $\kappa = \bar{\kappa}$ such that $\max_a V^N(a, b) \geq \max_a V^I(a, b)$ for all b.

Proposition 7 *Suppose $u'''(\cdot) \geq 0$, $\delta_l/\delta_h \geq \max\left\{\bar{\kappa}, \sqrt{\pi_h}\right\}$ and $\theta = \alpha\theta^*$. Then $\tilde{a}(b) < \hat{a}(b)$ and $V^b = V^N$.*

Proof of Proposition 7. We start proving that $V^N(a, 0) > V^I(a, 0)$ for all $a > \bar{a}$. Later, we generalize this result to all $b > 0$.

Let us assume $b = 0$. Firstly, notice that $\lim_{a \to \bar{a}^+} V^N - V^I > 0$, because by strict concavity of $u(\cdot)$ we have $u(\delta_l \bar{a} + \theta/\pi_l) - \delta\bar{a} = u(\delta\bar{a}) - \delta\bar{a} > \pi_l\left[u(\delta_l \bar{a}) - \delta_l \bar{a}\right] + \pi_h\left[u(\delta_h \bar{a}) - \delta_h \bar{a}\right] > \lim_{a \to \bar{a}^+} S^I(a, 0)$.

Now, we show that $\frac{\partial V^N}{\partial a} \leq \frac{\partial V^I}{\partial a}$ for all $a > \bar{a}$. Since $\frac{\partial W^b}{\partial a}$ is the same under both strategies, it is just sufficient to check the marginal payoff in the DM. Then, the relevant partial derivatives are:

$$S_a^I = \begin{cases} \pi_l \left[u'(\delta_l(a - \bar{a})) - 1\right]\delta_l + \pi_h\left[u'(\delta_l a + \theta/\pi_l)\delta_l - \delta_h\right] & \text{for } a \in (\bar{a}, a''] \\ \pi_h\left[u'(q_h) - 1\right]\delta_h & \text{for } a \in [a'', \hat{a}(0)) \\ 0 & \text{for } a \geq \hat{a}(0) \end{cases}$$

$$S_a^N = \begin{cases} u'(\delta_l a + \theta/\pi_l)\delta_l - \delta & \text{for } a \in (\bar{a}, \tilde{a}(0)) \\ 0 & \text{for } a \geq \tilde{a}(0) \end{cases}$$

where a'' is such that for $a \leq a''$ we have $d_h^a - d_l^a = \bar{a}$.

Since $u''' > 0$, if $a'' \geq \tilde{a}(0)$, then $S_a^I \geq S_a^N$. Then $\frac{\partial V^I}{\partial a} \geq \frac{\partial V^N}{\partial a}$ for all $a > \bar{a}$ and $\tilde{a}(0) \leq \hat{a}(0)$. This also implies that if $V^I \geq V^N$ for some $\check{a}$, then $V^I \geq V^N$ for all $a > \check{a}$. But this is not possible, because $\delta_l/\delta_h \geq \bar{\kappa}$ and Lemma 3 implies that

$V^N(\hat{a}(0), b) \geq V^I(\hat{a}(0), b)$.

If $a'' < \tilde{a}(0)$, then $S_a^I > S_a^N$ for $a \in (\bar{a}, a'']$. Since both S_a^I and S_a^N are decreasing, for $a > a''$ we must check that S_a^I has a slope lower than S_a^N in absolute terms; since at a'' we have $S_a^I > S_a^N$, this would guarantee that S_a^I and S_a^N never cross and $\tilde{a}(0) < \hat{a}(0)$. By taking second derivatives, we need $|u''(q_h)\pi_h\delta_h\frac{\partial q_h}{\partial a}| \leq |u''(q_h)\pi_h\delta_h^2| \leq |u''(\delta_l a + \theta/\pi_l)\,\delta_l^2|$, where the first inequality derives from $\frac{\partial q_h}{\partial a} \leq \delta_a^h$. Then

$$|u''(q_h)\pi_h\delta_h^2| \leq |u''(\delta_l a + \theta/\pi_l)\,\delta_l^2| \implies \frac{|u''(q_h)|}{|u''(\delta_l a + \theta/\pi_l)|} \leq \frac{\delta_l^2}{\pi_h\delta_h^2}$$

Since $u'''(\cdot) \geq 0$ the LHS is ≤ 1, while the RHS is ≥ 1 because $\delta_l/\delta_h \geq \sqrt{\pi_h}$. Then $\frac{\partial V^I}{\partial a} \geq \frac{\partial V^N}{\partial a}$ for all $a > \bar{a}$ and $\tilde{a}(0) \leq \hat{a}(0)$. This implies $V^N(a, 0) \geq V^I(a, 0)$ for all a.

For $b > 0$ the proof follows the same logic above as long as $b < \tilde{b}$. For $b \geq \tilde{b}$ the result is immediate from Lemma 3. ■